

# Problem 2:

## Understanding the problem

We have  $n$  heroes and  $m$  monsters. Each hero can kill certain monsters, but only one by default. With a potion, a hero can kill one extra monster, and there are  $k$  potions total. We want the maximum monsters killed.

This is essentially a **bipartite matching** with a global budget of  $k$  heroes that can have capacity  $2$  instead of  $1$ .

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## Initial strategy

At first I thought of:

1. Build a bipartite match between heroes and monsters.
2. Afterwards, use potions to kill leftover monsters adjacent to heroes.

But this greedy idea failed — the optimal choice of potion use sometimes requires reshuffling the original matching.

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## Final design

Model it as a **max flow**:

- $S \rightarrow \text{hero } (1)$  (base capacity).
- $S \rightarrow P(k)$  then  $P \rightarrow \text{hero } (1)$  (potions hub lets at most  $k$  heroes get +1 capacity).
- $\text{hero} \rightarrow \text{monster } (1)$  if the hero can kill that monster.
- $\text{monster} \rightarrow T(1)$  ensures each monster is killed at most once.

Run Dinic's algorithm; the max flow = maximum kills.

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## What went wrong

- **Greedy potion assignment** missed optimal rematching. → Fixed by putting potions directly into the flow network.

- **Indexing errors:** I double-subtracted monster indices and misaligned ranges.  
→ Fixed with a clear mapping `monster = n + (id-1)`.
  - **Duplicate edges:** Initially added `monster→T` multiple times (once per hero). → Moved into a single loop after input.
  - **Graph size confusion:** Off-by-one in total node count. → Fixed by allocating up to `T`.
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## Fixes

- Unified heroes in `[0..n-1]`, monsters in `[n..n+m-1]`, `S=n+m`, `P=S+1`, `T=P+1`.
- Only one `monster→T` edge per monster.
- Optional guard to prevent duplicate hero→monster edges.