

Problem 2:

Understanding the problem

We're given $n < 10^6$ and want to see if, after repeatedly taking square roots, it breaks down into distinct primes. If not, we must multiply by something minimal to make this possible. The difficulty lies in how the prime exponents behave when halved over and over.

Initial strategy

At first I thought:

- Just check if each exponent is itself a power of two.
- If not, mark it as needing an extra factor.

This seemed natural since square roots halve exponents each step.

What went wrong

This check was too strict. For example, 2^{12} fails because 12 isn't a power of two, but in reality you can pad it up to 16 and the process works. By only testing membership in $\{1, 2, 4, 8, \dots\}$, I was rejecting numbers that were actually fixable.

Fixes

I realized the right test is **divisibility**, not equality. Each exponent just needs to be divisible by the largest power of two that fits into it. If not, we multiply by enough to "round it up."

Example:

- 2^8 works: $8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.
- 2^{12} doesn't: halving leaves leftovers, so multiply to reach 16.