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Danish Fiscal Policy under Supply Shocks

Trade-offs and Effects of Interventions

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Danish Fiscal Policy under Supply Shocks

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Abstract

In this study, I explore the dynamics of fiscal policy interventions in the Danish economy, particularly during a global supply-side shock. By building upon the BabyMAKRO model developed by Jeppe Druedahl (Druedahl 2023a), I introduce key modifications to better replicate the economy's response to such shocks. These modifications include adjusting model parameters to emphasize the critical role of foreign supply goods in production, increasing the public debt to GDP ratio, and incorporating delayed government reactions.

The analysis reveals significant trade-offs between contractionary and expansionary fiscal measures, taking into account their intertemporal effects. The results include a trade-off between stabilization and faster convergence to steady state. Additionally, I find that for risk-averse agents, expansionary fiscal policies – characterized simply by tax cuts – can enhance total accumulated utility across short and medium-term horzions.

However, the model's limitations must be acknowledged, particularly its inability to fully encapsulate the government's extensive range of actions and the real-world complexities involved. Despite these constraints, the study provides valuable insights into the nuanced impacts of fiscal policy choices under supply-side constraints.

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1 Introduction

During the corona virus crisis, millions of Danes received 1,000 DKK into their bank account, with the government hoping this would stimulate the economy. (Finance 2020). An instance of expansionary fiscal policy amidst a supply shock. Such policy choices are particularly significant in the context of small, open economies like Denmark, which are bound by the constraints of a fixed exchange rate and are more vulnerable to external shocks. Keynesian theory posits that supply shocks present a dilemma, leading to a trade-off between higher inflation and increased unemployment (Romer 2011), and The Danish government simply chose the latter.

However, this perspective may not encapsulate the entirety of the issue. The intricate dynamics of supply shocks and the corresponding mitigation strategies, especially when modeled in a comprehensive framework, suggest a richer field of study. This thesis is driven by questions surrounding the shaping of fiscal policy to counteract the adverse effects of supply shocks and the deeper trade-offs involved in such interventions. This thesis employs the BabyMAKRO model as the model framework. The BabyMAKRO model, designed by Jeppe Druedahl (Druedahl 2023a), is a simplified counterpart to the more complex MAKRO model used by the Danish Ministry of Finance. While simplified, BabyMAKRO retains sufficient complexity to offer meaningful insights into macroeconomic phenomena. It provides a robust platform for undergraduate research, enabling me to simulate complex economic scenarios in a controlled environment.

One key modification to the BabyMAKRO model lies in the incorporation of fiscal policy. The original model does not account for any fiscal policy mechanisms besides the passive form of benefits to the unemployed and retired. To make government actions more realistic and possible, I introduce lagging to the government responsiveness in terms of taxes and government spending to rectify this. Notably, I also modify the model to allow for exogenous shocks to the otherwise endogenous tax rate parameter, allowing the government in the model to take much larger deficits (and surpluses).
In summary, this thesis aims to make a significant contribution to the field of effectiveness in fiscal policy, specifically in the context of small open economies with fixed exchange rates. By leveraging a modified BabyMAKRO model, I uncover the intricate dynamics and trade-offs involved in using fiscal policy to combat external price shocks. The findings offer insights for policymakers and economists interested in the complex interplay between fiscal policy and economic stability.

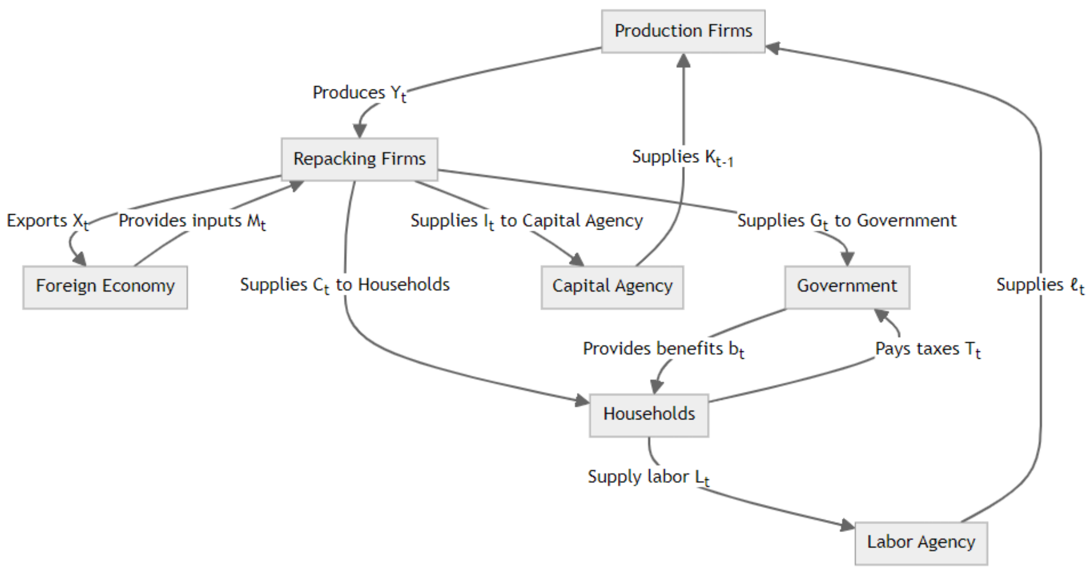
2 Introduction to the Model

BabyMAKRO is a dynamic deterministic general equilibrium (DDGE) model specifically designed for analyzing small open economies under a fixed exchange rate regime, with a particular focus on Denmark. It represents a simplified version of the more comprehensive MAKRO model developed by the Danish Ministry of Finance (Bonde et al. 2023). BabyMAKRO shares several structural similarities with its predecessor, including the incorporation of an overlapping generations (OLG) framework and heterogeneous agents. The model's architecture is modular, with its code divided into discrete segments referred to as ”blocks”, paralleling the ”Modules” structure in the MAKRO model. Prior to a detailed exploration of these blocks, an introductory overview of the BabyMAKRO model and chosen parameters is provided below.

2.1 Model Overview

The model is structured to operate over discrete time intervals, encompassing a total of 400 periods (years), denoted as $t \in \{0, 1, \ldots, 399\}$. The demographics of the model is defined by 65 distinct age groups, $a \in \{0, 1, \ldots, 64\}$, representing individuals aged from 25 to 90 years. This age range proposes 43 years of work, culminating in a retirement age of 68 years. This retirement age is reflective of the projected Danish retirement age for individuals retiring between 2030 and 2035 (STAR 2023). The model assumes a stable demographic profile, characterized by a constant birth rate in each period, normalized to 1, and is mirrored by an equivalent death rate, ensuring demographic equilibrium. Notably, the mortality rate increases progressively with age, starting from the retirement age, and culminates in a certainty of death at age 90.

Figure 2.1: Diagram of the BabyMAKRO Model for Real Variables



Note: This diagram is a simplified representation of the economic model. It is designed for clarity and ease of understanding, and does not include every detail or interaction to avoid visual clutter.

The Households in the BabyMAKRO model are characterized by heterogeneity, comprising both hand-to-mouth and Ricardian households. They earn income from The latter category of households exhibits perfect foresight, enabling them to optimize their lifetime utility. This optimization takes into account various factors, including consumption, a warm-glow bequest motive, and the probability of death.

The Firms within the model encompass 4 distinct entities: A labor agency, a capital agency, production firms, and repacking firms. The production firms employ CES-technology, utilizing effective labor and capital sourced from the agencies to generate domestic output. This output is subsequently processed by the repacking firms, which integrate foreign inputs to produce final goods. These final goods may be private consumption goods, government consumption goods, investments, and exports destined for foreign markets. The agencies and production firms maximize their profits intertemporally, discounting future profits. All firms are price takers.

The Government plays a less dynamic role in the model, consistently purchasing a fixed amount of public goods. This expenditure is financed through taxation imposed on households. To maintain fiscal balance, the government dynamically adjusts the tax rate, aiming to neutralize any government deficit or surplus over time.

The Foreign Economy is treated as an exogenous entity within the model, characterized by fixed prices for foreign inputs. The export dynamics are governed by an Armington demand function (Armington 1969), which is influenced by historical export volumes and the relative price of exports to the foreign price level. The foreign economy also provides a pool of assets that may be bought at a fixed price, yielding 2% annual interest. This interest rate is the only nominal interest rate used in the model, being used to determine the yield on private savings, government debt, and to discount future profits for firms*.

2.2 Model Mechanics and Assumptions

The BabyMAKRO model, including its functional form, derivations, and Python implementation, is attributed to the work of Jeppe Druedahl. Detailed information regarding the specific functional forms and mathematical derivations can be accessed in the comprehensive model documentation (Druedahl 2023b). For the sake of brevity and focus, this thesis will primarily address the core mechanisms of the model, referring to the documentation for more intricate, non-central aspects. Below, I assess the blocks of the model in order of evaluation.

2.2.1 Prices of Final Goods

The repacking firms utilize CES-technology, incorporating domestic output Y_t and sector-specific foreign inputs $\bullet_t^M$ to produce final goods. These goods include Private consumption goods C_t, Government consumption goods G_t, Investment goods I_t, and Export

*Although, a distinct specified nominal rent for each of the sectors is parametized in the model. See table 8.2

goods X_t. Each sector is associated with distinct weights for foreign inputs, denoted as $P_t^{M,C}$, $P_t^{M,G}$, $P_t^{M,I}$, and $P_t^{M,X}$; with the highest weights allocated to exports (40%), imports (35%), private consumption (30%), and government consumption (10%). The total overview of parameters can be found in table 8.2.

The repacking firms are modeled to operate under conditions of total competition, implying the absence of any markup. The demand for domestic output and foreign inputs is expressed as follows:

$$\bullet_t^M = \mu^{M,\bullet} \left(\frac{P_t^\bullet}{P_t^{M,\bullet}} \right)^{\sigma^\bullet} \bullet_t, \tag{2.1}$$

$$\bullet_t^Y = (1 - \mu^{M,\bullet}) \left(\frac{P_t^\bullet}{P_t^Y} \right)^{\sigma^\bullet} \bullet_t \, . \tag{2.2}$$

Furthermore, the pricing of the final goods is determined by the equation:

$$P_t^\bullet = \left(\mu^{M,\bullet} \left(P_{M,\bullet_t} \right)^{1-\sigma^\bullet} + (1 - \mu^{M,\bullet}) \left(P_t^Y \right)^{1-\sigma^\bullet} \right)^{\frac{1}{1-\sigma^\bullet}} , \tag{2.3}$$

$$\bullet \in \{C, G, I, X\}. \tag{2.4}$$

2.2.2 Wage Determination

In the BabyMAKRO model, the determination of the nominal wage, denoted as W_t, is intricately linked to the labor market dynamics. The model sets the steady state wage, W_{ss}, as a baseline, which is parameterized to 1 for simplicity. The nominal wage is then modeled as a function of the ratio of current labor force demand, L_t, to the steady state labor force demand, L_{ss}:

$$W_t = W_{ss} \left(\frac{L_t}{L_{ss}} \right)^{\epsilon_w} \tag{2.5}$$

Given that ϵ_w is assumed to be 1.25, which is greater than 1, the wage-to-labor demand relationship is convex. More labor requires a more than proportionate rise in wages.[†]

2.2.3 Search and Match

The 'Search and Match' block of the BabyMAKRO model is complex and involves numerous equations, making a detailed mathematical description beyond the scope of this summary. For an in-depth understanding, readers are referred to the model documentation[‡]. However, a brief overview of the key aspects is provided here.

In essence, the 'Search and Match' block is built around the premise that all households aged between 25 and 68 actively engage in job searching, irrespective of the income implications. A notable feature of this block is that 10% of employed households are

[†] This block was implemented incorrectly in Python. Has been fixed.

[‡] See (Druedahl 2023b) Section 2.6

modeled to lose their jobs annually. Human capital in this model is a function of work experience and age, typically peaking around the age of 50, and is a modifier of the productivity of a household (see figure 8.5). The matching process between job seekers and available vacancies is determined by the number of individuals searching for jobs and the number of vacancies present.

2.2.4 Labor Agency

The labor agency posts vacancies v_t to hire labor L_t and rents it out as effective labor $L_t H_t$ to production firms, with H_t representing average human capital at time t. The agency faces costs $\kappa^L \cdot v_t$ for these vacancies, resulting in the net effective labor rented out being $\ell_t = L_t H_t - \kappa^v v_t$. The agency, operating with perfect foresight, aims to maximize profits intertemporally by optimizing the amount of labor hired and thus rent charged for effective labor, taking human capital and wages as given. The rent for effective labor (r_ℓ) is priced higher than the wage rate W, paid to households for every unit of effective labor, to offset the vacancy-related costs.[§]

2.2.5 Domestic Production

The production of domestic output in the BabyMAKRO model employs CES-technology, using effective labor ℓ_t and lagged capital K_{t-1} as inputs:

$$Y_t = \Gamma \left(\mu_K^{\frac{1}{\sigma^Y}} K_{t-1}^{\frac{\sigma^Y - 1}{\sigma^Y}} + (1 - \mu_K)^{\frac{1}{\sigma^Y}} \ell_t^{\frac{\sigma^Y - 1}{\sigma^Y}} \right)^{\frac{\sigma^Y}{\sigma^Y - 1}} \tag{2.6}$$

Here, Γ represents total factor productivity, μ_K the capital weight in production, and σ^Y the elasticity of substitution between inputs. The production function exhibits constant returns to scale.

The marginal cost of production, derived similarly to section 2.1.2, is dependent on the rental prices of capital and labor:

$$P_t^{Y,0} = \frac{1}{\Gamma_t} \left(\mu_K \left(r_{K_t} \right)^{1-\sigma^Y} + (1 - \mu_K) \left(r_{\ell_t} \right)^{1-\sigma^Y} \right)^{\frac{1}{1-\sigma^Y}} \tag{2.7}$$

The first order condition establishes the model's first target, indicating the optimal ratio of inputs:

$$\frac{K_{t-1}}{\ell_t} = \frac{\mu_K}{(1 - \mu_K)} \left(\frac{r_{\ell_t}}{r_{K_t}} \right)^{\sigma_Y} \tag{2.8}$$

2.2.6 Phillips curve

In the BabyMAKRO model, domestic production firms operate under a sticky pricing mechanism, incurring quadratic adjustment costs proportional to output and price levels

[§]See (Druedahl 2023b) Section 2.7

with each change in the inflation rate. The objective of the firms is to solve the following maximization problem iteratively, and the result becomes a target for the model:

$$V_t^{\text{intermediary}} = \max_{\{p_{Y_t}\}} (p_{Y_t} - P_{Y,0_t})\, y_t - \vartheta_t + \frac{1}{1+r^{\text{firm}}} V_{t+1}^{\text{intermediary}} \tag{2.9}$$

$$\text{s.t.} \quad \vartheta_t = \frac{\gamma}{2} \left(\frac{p_{Y_t}/p_{Y_{t-1}}}{p_{Y_{t-1}}/p_{Y_{t-2}}} - 1 \right)^2 P_{Y_t} Y_t \tag{2.10}$$

$$y_t = \left(\frac{p_{Y_t}}{P_{Y_t}} \right)^{-\sigma^D} Y_t. \tag{2.11}$$

Here, γ represents the adjustment costs of changing prices, and σ^D is the demand elasticity.

2.2.7 Foreign Economy

In the BabyMAKRO model, the behavior of exports is captured using an Armington demand function (Armington 1969), which integrates elements of an autoregressive process (AR(1)). This function posits that exported goods are not perfect substitutes for foreign goods. Changes in relative prices influence the volume of exports in relation to the elasticity of substitution, denoted by σ_F. The export demand function is formulated as follows:

$$X_t = \gamma_X X_{t-1} + (1-\gamma_X) \cdot \chi_t \left(\frac{P_t^X}{P_t^F} \right)^{-\sigma_F} \tag{2.12}$$

In this equation, γ_X signifies the persistence in export volumes, χ represents the foreign demand shifter (obtained from the steady state), P_t^X and P_t^F are the prices of domestic exports and foreign goods respectively, and σ_F indicates the elasticity of substitution between domestic and foreign final goods.

2.2.8 Capital Agency

The Capital Agency in the model acquires investment goods I_t at price P_I and rents them to production firms at rate r_{t+1}^K. Operating with perfect foresight for profit maximization, it faces quadratic adjustment costs (Ψ), and thus $\iota_t < I_t$. These costs necessitate higher rental rates r_t^K to maintain profitability. The agency's decision-making is encapsulated in the following optimization problem:

$$V_{\text{capital}}^0(K_{t-1}) = \max_{\{K_t\}} \sum_{t=0}^{\infty} \left(\frac{1}{1+r_{\text{firm}}} \right)^t \left[r_t^K K_{t-1} - P_t^I(\iota_t + \Psi(\iota_t, K_{t-1})) \right], \tag{2.13}$$

subject to investment and capital accumulation constraints:

$$I_t = \iota_t + \Psi(\iota_t, K_{t-1}), \quad K_t = (1-\delta_K) K_{t-1} + \iota_t. \tag{2.14}$$

The adjustment cost function is defined as:

$$\Psi(\iota_t, K_{t-1}) = \frac{\Psi_0}{2}\left(\frac{\iota_t}{K_{t-1}} - \delta_K\right)^2 K_{t-1}. \tag{2.15}$$

Solving this problem yields an additional target for the model.

2.2.9 Government

The government allocates 25% of steady state output for public goods, interest on previous period's bonds, and benefits to unemployed and retired individuals. Revenue is derived from labor taxes at rate τ, which also applies to benefits. The tax rate adjusts to maintain a steady state bond level, responding to changes in both the tax base and expenditures. The government's budget constraint is:

$$B_t = (1 + r^B)B_{t-1} + P_t^G G_t + (1 - \tau_t)(W_U W_{ss} U_t + W_R W_{ss}(N_t - N_t^{work})) - \tau_t W_t L_t H_t, \tag{2.16}$$

where $W_U < W_{ss} = 1$ and $W_R < W_{ss} = 1$ represent unemployment and retirement benefits relative to the steady state wage. Benefits are fixed across all periods and age groups. N is the total population Tax rate adjustments are formulated as:

$$\tau_t = \tau_{ss} + \epsilon_B \frac{\tilde{B}_t - B_{ss}}{(W_t L_t H_t + W_u W_{ss} U_t + W_R W_{ss}(N_t - N_t^{work}))}, \tag{2.17}$$

with $\tilde{B}_t$ as the debt level at time t assuming a steady state tax rate, and ϵ_B indicating the rate of bond level correction.

2.2.10 Households Saving and Consumption

Households are categorized as either hand-to-mouth or unconstrained Ricardian, as described in Section 2.1. Their income is uniform, comprising labor wages, unemployment benefits, retirement benefits (after taxes), and equally distributed bequests without an inheritance tax. The income for a specific age group at time t is:

$$\text{inc}_{a,t} = (1 - \tau_t)\left(\frac{W_t L_{a,t} H_{a,t}}{N_a} + \frac{W_U W_{ss} U_{a,t}}{N_a} + W_R W_{ss} \cdot 1_{a \geq 68}\right) + \frac{A_t^q}{N} \tag{2.18}$$

Where bequests A_t^q are given by the sum of assets from passing individuals, dependent on the age-dependent mortality ζ_a, population size at each age N_a, and the respective assets of the passing indviduals:

$$A_t^q = (1 + r^{hh}) \sum_{a=0}^{65} \zeta_a N_a A_{a,t}$$

Hand-to-mouth households consume their entire real income in equivalent private consumption goods:

$$C_{a,t}^{HtM} = \frac{\text{inc}_{a,t}}{P_t^C} \tag{2.19}$$

Consequently, they do not save.

Ricardian households aim to optimize their lifetime utility, which is represented by a modified Constant Relative Risk Aversion (CRRA) utility function. They derive utility from consuming private goods and holding real assets, the latter reflecting a bequest motive or 'warm-glow' altruism. Ricardians, with perfect foresight, decide their consumption and savings pattern in their first year of working life t_0, discounting future utility and accounting for the probability of death. They solve the utility maximization problem:

$$V_{t_0} = \max_{\{C_{a,t}^R\}} \sum_{a=0}^{65-1} \left(\prod_{j=1}^{a} \beta(1-\zeta_{j-1}) \right) \left[\frac{{C_{a,t}^R}^{1-\sigma}}{1-\sigma} + \zeta_a \mu^{Aq} \frac{\left(\frac{A_{a,t}^R}{P_t^C}\right)^{1-\sigma}}{1-\sigma} \right] \tag{2.20}$$

$$\text{s.t.} \quad t = t_0 + a \tag{2.21}$$

$$A_{-1,t}^R = 0 \tag{2.22}$$

$$A_{a,t}^R = (1 + r_t^{hh}) A_{a-1,t-1}^R + \text{inc}_{a,t} - P_t^C C_{a,t}^R. \tag{2.23}$$

The first-order condition of this problem yields the consumption pattern, which depends on the inflation rate in P_t^C, π^{hh}:

$$C_{a,t}^R = \begin{cases} \left(\zeta_a \mu_A q \left(\frac{A_{-1,t}^R}{P_t^C} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}}, & \text{if } a = \# - 1 \\ \beta(1-\zeta_a) \frac{1+r_{ss}^{hh}}{1+\pi_{ss}^{hh}} \left(C_{a+1,ss}^R \right)^{-\sigma} + \zeta_a \mu_A q \left(\frac{A_{a,ss}^R}{P_{ss}^C} \right)^{-\sigma} \Bigg)^{-\frac{1}{\sigma}}, & \text{if } t = T-1 \\ \beta(1-\zeta_a) \frac{1+r_{t+1}^{hh}}{1+\pi_{t+1}^{hh}} \left(C_{a+1,t+1}^R \right)^{-\sigma} + \zeta_a \mu_A q \left(\frac{A_{a,t}^R}{P_t^C} \right)^{-\sigma} \Bigg)^{-\frac{1}{\sigma}}, & \text{else} \end{cases} \tag{2.24}$$

This section yields 2 targets for the model relevant for managing bequests.[¶]

[¶]See (Druedahl 2023b) Section 2.13

2.2.11 Goods Market Clearing

The demand for imported foreign inputs and domestic output, given the prices, is determined as follows:

$$\bullet_t^M = \mu^{M,\bullet} \left(\frac{P_t^\bullet}{P_t^{M,\bullet}} \right)^{\sigma^\bullet} \bullet_t \,, \tag{2.25}$$

$$\bullet_t^Y = (1 - \mu^{M,\bullet}) \left(\frac{P_t^\bullet}{P_t^Y} \right)^{\sigma^\bullet} \bullet_t \,. \tag{2.26}$$

Here, $\mu^{M,\bullet}$ and $\sigma^\bullet$ represent the import share and elasticity of substitution for the good $\bullet \in \{C, G, I, X\}$, respectively. The total imports and market clearing for domestic output are then given by:

$$M_t = C_t^M + G_t^M + I_t^M + X_t^M, \tag{2.27}$$

$$Y_t = C_t^Y + G_t^Y + I_t^Y + X_t^Y. \tag{2.28}$$

2.3 Unknowns and Targets

For the 6 x T unknowns of the model (See table 2.1), a set of 6 x T targets are required to reduce the degrees of freedom sufficiently for an equilibrium path. The targets are partially sourced from the blocks and pertain to optimal behavior and market clearing, and partly sourced from the fulfillment of accounting identities.‖

Table 2.1: Unknowns and Targets in the Model

Unknowns

Unknowns	**Variable name**
A_t^q	Bequests
A_{death}^R	Bequests on death
K_t	Capital
L_t	Labor
r_t^K	Return on Capital
P_t^Y	Price of domestic output

Targets

Targets
Consistency in bequests
Consistency in bequests
Market clearing
Optimal production firm behavior
Optimal capital agency behavior
Phillips curve satisfied

Note: While the same amount of targets and unknowns is required, the order row-wise w.r.t Unknowns and Targets in this table is not relevant to solving the model.

2.4 Steady state

Section 2 frequently references a steady state. This subsection aims to delineate the fundamental aspects through which the model attains this steady state. Detailed pseudocode is not included in this paper but is available in Section 3 of the model documentation. The

‖See (Druedahl 2023b) Equations (32), (33), (37), (51), (52), and (56) attainment of a steady state in the model involves setting specific endogenous variables to predetermined values, under the assumption of intertemporal stability and the fulfillment of all model objectives. The steady state is characterized by:

- Stability of all variables, eliminating the need for time subscripts
- Achievement of all predefined targets

The following endogenous variables and their corresponding values constitute the foundation of the steady state:

- Wages are normalized to unity: $W_{ss} = 1$
- All prices are normalized to unity: $P^*_{ss} = 1$
- Inflation is set to zero: $\pi^{hh}_{ss} = 0$
- Job finding rate for job seekers is 75% per period: $m^{ss}_s = 0.75$
- Job filling rate for vacancies is 75% per period: $m^{ss}_v = 0.75$
- Government debt is *initially* set to zero: $B_{ss} = 0$
- Household interest rate is established at 2%: $r^{hh}_{ss} = 0.02$

Additionally, the model calculates the exogenous total factor productivity (Γ) and the foreign demand shifter (χ), which adjust to support the steady state.

2.5 Solving the Model

The process of solving the model when it is not in a steady state, such as when an external shock like a doubling of the foreign price level occurs at $t = 0$, is intricate. The task involves finding a set of unknown variables that, when evaluated through the model's blocks, satisfy all the set targets.

This process is akin to calculating a numerical derivative in a simple model with one target and one unknown: by slightly adjusting the unknown from its steady state value and observing the effect on the target. Using numerical solvers like Newton's method and such a numerical derivative, the equilibrium can be approximated for a well-behaved target. In this model, which involves multiple unknowns and targets, the equivalent concept is the Jacobian matrix, further discussed below.

2.5.1 The Jacobian Matrix

The Jacobian matrix is composed of the partial derivatives of each target with respect to each unknown, numerically approximated by evaluating small perturbations to the unknowns from the steady state. Conceptually, the Jacobian matrix acts as the gradient of a vector-valued function, mapping the directional changes in the target equations in response to changes in the unknowns. These perturbations are based on changes in the steady-state unknowns, and the Jacobian thus illustrates the transitional dynamics towards the steady state in its vicinity. The Jacobian matrix is represented as:

$$J^0(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Given the model's complexity, with multiple periods and targets, the Jacobian matrix becomes a large 2400 x 2400 matrix, representing a significant computational challenge. This is why Broyden's method, a more efficient alternative to Newton's method, is employed, as detailed in the next section.

2.5.2 The Broyden Solver

Broyden's method (Broyden 1965), a quasi-Newton approach, is utilized for numerically solving the model and determining the equilibrium path. This method is similar to Newton's but employs approximations of the Jacobian matrix, reducing the need for its complete recalculation in each iteration. Broyden's method is preferred for its efficiency, especially given the computational demands of recalculating the extensive Jacobian matrix required in Newton's method.

In Newton's method, each iteration recalculates the Jacobian as follows:

$$x^{n+1} = x^n - J^n(x^n)^{-1} f(x^n) \tag{2.29}$$

In contrast, Broyden's method updates the Jacobian matrix using rank-one updates, significantly reducing computational complexity. The updated formulation in Broyden's method is (in equivalent notation):

$$J^{n+1} = J^n + \frac{dy_n - J^n dx_n}{dx_n^T dx_n} dx_n^T \tag{2.30}$$

$$x^{n+1} = x^n - J^n(x^n)^{-1} f(x^n) \tag{2.31}$$

Here, $dy_n = f(x_{n+1}) - f(x_n)$ and $dx_n = x_{n+1} - x_n$ represent the changes in function values and variables, respectively. The solver starts with an initial guess of the steady state values x^0 and an approximated Jacobian around the steady state, J^0, iteratively computing the solution using these rank-one updates to enhance the model-solving process's efficiency.

2.6 Model Modification

The model employed in this study has undergone modifications in three critical areas to better align with the dynamics of supply shocks and fiscal policy. These modifications include adjustments to the bond steady state, elasticity of substitution in repacking firms and lagged government responses.

2.6.1 Bond Steady-State

The model sets the steady-state bond level at 33% of the steady state domestic output. This figure is in line with the EMU's guidelines for gross government debt, encompassing the debts of both central and local governments, currently at 29.8% of GDP (DST 2023a). It is important to note that the actual net government debt may be lower due to substantial government holdings in Nationalbanken (*What is government debt?* 2023). Despite this, a ratio of one-third of the output is considered a reasonable approximation, especially in light of the EMU debt's current 10-year low. This adjustment results in a higher steady-state tax rate, which, in turn, reduces the disposable income within the economy. This is because domestic entities in the model are neither investors in nor owners of government bonds.

$$B_{ss} = 0 \quad \text{is revised to} \quad B_{ss} = 33\% \cdot Y_{ss}$$

2.6.2 Elasticity of Substitution in Repacking Firms

In the updated BabyMAKRO model, the substitution elasticity for repackaging firms is recalibrated to more accurately reflect real-world conditions. Initially set at 1.5, indicating a high substitutability between foreign and domestic inputs, this parameter is less representative of scenarios involving less substitutable inputs like oil. Consequently, the model's substitution parameter for repackaging is reduced to 1.05.

$$\sigma^* = 1.5 \quad \text{is revised to} \quad \sigma^* = 1.05 \quad \text{for} \quad * \in \{C, G, I, X\}$$

2.6.3 Fiscal Interventions and Lags

In the realm of fiscal policy, adjustments to government spending (G) or taxation (τ) are not instantaneous and often entail a lagged response due to administrative and legislative processes. The same goes for monetary policy (Consolo and Favero 2009). In contrast to the original model's assumption of an immediate government reaction to economic shocks where tax rates are contemporaneously adjusted to maintain the targeted bond stock in a steady state, this thesis introduces a modification to encapsulate the inherent delays in fiscal policy implementation.

The government does not change either government spending (G) or taxation (τ) in the first period, but in the second period, the government is allowed to move in lockstep with the economy again - having obtained information on the shock path, just like the other entities in the economy, i.e:

$$G \text{ and } \tau \text{ cannot change for } 1 \text{ period after a shock is initialized}$$

2.7 Calibration of the model

The modifications to the model do not necessitate a recalibration of the model, as evidenced by the Impulse Response Function (IRF) comparison shown in figure 8.6. The modified model remains closely aligned with the original BabyMAKRO IRF's in comparison with "Matching af impuls responser og øvrige kortsigtsmomenter: MAKRO ift. empirien" (DREAM 2021). Notably, figure 8.6 illustrates that in scenarios of positive demand shocks, the modified model predicts lower consumption levels. This is attributed to the delayed government response, which does not immediately decrease despite an expanded tax base.

3 Methodology

This study utilizes a two-step simulation approach to assess fiscal interventions in response to a supply-side shock. Initially, a baseline scenario is established to analyze the impact of the global supply shock. Following this, simulations that incorporate tax and government spending measures are performed to evaluate the trade-offs involved in mitigating the shock's effects.

3.1 The Shock

The DREAM model's shock analysis typically includes an oil price shock. However, considering Denmark's significant role as an oil producer since the second oil shock of 1979 (Eurostat 2023), this approach is overly complex for the scope of this modeled economy. For an illustration of how the model's impulse response functions (IRFs) would look in this scenario, refer to figure 8.7.

Instead, this study applies a straightforward 1% shock to foreign input prices. This magnitude is selected for its simplicity and to facilitate a clear understanding of the underlying mechanisms. While minor adjustments to the shock can be made ad-hoc, they are expected to produce similar qualitative outcomes, as the model exhibits predominantly close-to-linear effects (refer to table 8.2 for parameters).

To maintain a connection with empirical data when determining the exogenous shock, this study consults DST (DST 2023b) for information on the years involving the two oil shocks and the recent coronavirus pandemic**. The 1970s oil shocks are not directly applicable to this model, as Denmark did not have a pegged currency then, but these shocks are associated with a simultaneous decrease in output and exports. A similar pattern is observed during the coronavirus pandemic, driven mainly by a decline in service exports. Therefore, it is reasonable to model the shock in a manner that results in a decrease in exports. To achieve this, the increase in the foreign price level in the Armington demand function must be less than 0.4%:

$$X_t = \gamma_X X_{t-1} + (1 - \gamma_X) \cdot \chi_t \left(\frac{P_t^X}{P_t^F} \right)^{-\sigma_F} \tag{3.1}$$

Given a 0.4 weight on foreign input goods in the production of exports (X_t) within the domestic economy and a positive elasticity of substitution in the repacking firms ($\sigma^X > 0$), a 1% increase in foreign input prices will lead to at least a 0.4% rise in P_X. This results from a shift towards domestic output goods, which also increases P_Y. Thus, to reflect the global nature of the supply crisis while allowing for a decline in X_t, the corresponding shock to the foreign price level is set between 0 and 0.4 percent. A quarter of a percent is chosen for this study.

**According to (E Castro, Duarte, and Brinca 2020), the pandemic was primarily a supply shock.

The baseline scenario, therefore, simulates an MIT-shock comprising:

- A 1% increase in all foreign input prices.
- A 0.25% increase in the foreign price level.

Both parameters have a persistence factor of 0.6, to represent a brief supply shock, like the coronavirus pandemic. This results in a shock with a substantial immediate effect at t=0, which diminishes to just 8% of its initial magnitude by period t=5.

3.2 The interventions

In the second part of the analysis, the baseline scenario is extended to include a government intervention, implemented with a one-period lag. The persistence and magnitude of the shock are in lockstep with the foreign input price shock. The extent of the intervention, for government spending changes, is contingent on the steady state G and the actual price shock:

$$G_{t>0} = G_{ss} \cdot (1 \pm 0.01 \cdot 0.6^t) \tag{3.2}$$

Finally the effects of direct tax-measures are analyzed. Like the government, the change in τ_t is contingent on the steady state tax-level and the actual price shock:

$$\tau_{t\in\{1,2,3,4\}} = \tau_{ss} \cdot (1 \pm 0.01 \cdot 0.6^t) \tag{3.3}$$

$$\tau_{t\notin\{1,2,3,4\}} = \tau_t^* \tag{3.4}$$

Where τ_t^* is the model specified tax-rate. Taxes are still the factor responsible for maintaining the bond-steady state, and therefore the intervention must stop at some point. I choose t=5 to be the time-slot where the intervention has ended. Selecting these magnitudes might result in a response from the government that is either higher or lower than expected. Nonetheless, for the sake of simplicity and ease of interpreting the results, it is deemed appropriate to align the percentage magnitude of the government's response with that of the steady-state price shock.

3.3 Shock Time Relevance

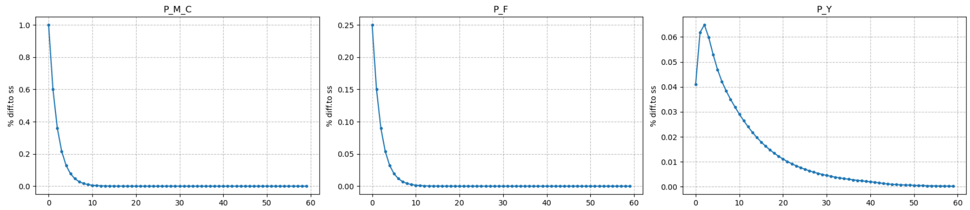
The model introduces the price shock at the initial period, $t = 0$, at which point the economy is presumed to be in a complete steady state for any $t < 0$. This sudden and unexpected price shock aligns with the characteristics of an "MIT shock" (Boppart, Krusell, and Mitman 2018), which is not anticipated by economic agents, thus eliminating any foresight that could potentially alter their behavior in advance of the shock. The assumption of no prior anticipation is critical as it ensures that the shock's immediate impacts are captured without the confounding effects of preemptive adjustments by agents within the economy.

In contrast, the subsequent government intervention, implemented with a delay, does not constitute an MIT shock. Entities with perfect foresight are expected to adjust their behavior in anticipation of the forthcoming intervention. Although it may be beneficial to include governmental response inefficiencies to model reality (Culbertson 1960), it is imperative to keep in mind the anticipatory behavior that such policy changes will provoke in the model.

4 Baseline Shock Model & Analysis

Figure 4.1 presents the time-schedule of the price shock and its impact on output prices. A comprehensive model output can be found in figure 4.2 below this section.

Figure 4.1: Impulse Response Functions for Shock Variables



Note: 0.6% increase and decrease in G for $t > 0$ with persistence 0.6. Figure has been re-indexed to reflect percentage change w.r.t the baseline shock outlined in section 4, see 4.2. All $P_{M,*}$ are equivalent to P_{M_C} and are thus not shown.

Time-zero effect summary: The increase in foreign input prices triggers a substitution in the CES-production function from foreign to domestic inputs, leading to an initial rise in output ($\uparrow Y_0$). This shift necessitates more effective labor ($\uparrow \ell_0$), resulting in higher wages ($\uparrow W_0$). The wage growth does not outpace the rise in consumption goods prices ($\uparrow P_0^C > \uparrow W_0$), but due to higher employment, the real aggregate income rises ($\uparrow inc_0^{real}$). Consequently, hand-to-mouth workers consume more ($\uparrow C_0^{HtM}$), and Ricardian consumers augment their savings in anticipation of higher real rent by the impending deflation ($\uparrow A_0$).

4.1 Central mechanisms

In this part, I will analyze the central mechanisms related to the path of the price shock. Additional details are left out due to space constraints. The central mechanisms in the model of this paper relate primarily to the path of the price shock. First and foremost, the price shock has an immediate effect on the repacking firms' prices:

4.1.1 Impact on Repacking Firms

The price shock's impact on repacking firms is analyzed through two pathways: the direct effect on the final good's price and the indirect effect stemming from increased domestic output prices. Given the positive elasticity of substitution in the model ($\sigma^* = 1.05 > 0$), the direct effect of the shock manifests as a less-than-proportional increase in the final good's price relative to the supply shock, factoring in the weight of foreign inputs.

$$\uparrow P_0^*, \quad * \in \{C, G, I, X\} \tag{4.1}$$

During the transition phase, repacking firms gradually revert to utilizing more foreign inputs as their prices decrease, eventually exceeding the steady-state level of foreign input usage due to the stickiness of P^Y. This results in a monotonic deflation in final goods prices:

$$\downarrow P^{*}_{(t>0)}, \quad * \in \{C, G, I, X\} \tag{4.2}$$

4.1.2 Impact on Non-Repacking Firms

The substitution by repacking firms towards domestic output ($\uparrow Y$) increases the demand for effective labor ($\uparrow \ell$) and capital (K). In response, the labor agency posts more vacancies ($\uparrow v$), and hires additional labor, leading to higher wages ($\uparrow W$) and a decrease in the job-filling rate. Consequently, the required return on effective labor ($\uparrow r_\ell$) is heightened.

Capital, being predetermined at $t = 0$ due to prior investments, sees an initial rise in its marginal product and return ($\uparrow r_K$) due to the increased labor force utilized. However, anticipating a future demand drop and facing higher investment goods prices ($\uparrow P^I$), the capital agency reduces its investments ($\downarrow I \Rightarrow \downarrow \iota \rightarrow \downarrow K_{+1}$). This effect is modified with a high quadratic adjustment cost, and the agency would reduce investments much more if the adjustment costs were less significant.

These dynamics result in elevated marginal costs for production firms:

$$\uparrow P_{Y,0_t} = \frac{1}{\Gamma_t} \left(\mu_K \cdot (r_{K_t})^{1-\sigma^Y} + (1 - \mu_K) \cdot (r_{\ell_t})^{1-\sigma^Y} \right)^{\frac{1}{1-\sigma^Y}} \tag{4.3}$$

Consequently, the price of the domestic output good increases ($\uparrow P_{Y_t}$). Inflation peaks at $t = 2$, and the marginal production costs exceed the initial rise in P_Y - both effects a result of sticky pricing.

In the transition phase, price adjustments of final goods converge faster than the price of domestic output, as foreign inputs become relatively cheaper than domestic output ($\downarrow \frac{P_{M,\cdot}}{Y}$). This will reduce the demand for domestic output ($\downarrow Y$) in the medium to long term, and the capital agency lets the capital stock fall further ($\downarrow K$). It is also noteworthy that exports (X_t) remain below their steady state levels throughout the simulation period, as I have required through the shock to P^F. As the price of price of domestic output and foreign inputs converge, so does capital and all other variables.

4.1.3 Government Response to Price Shock

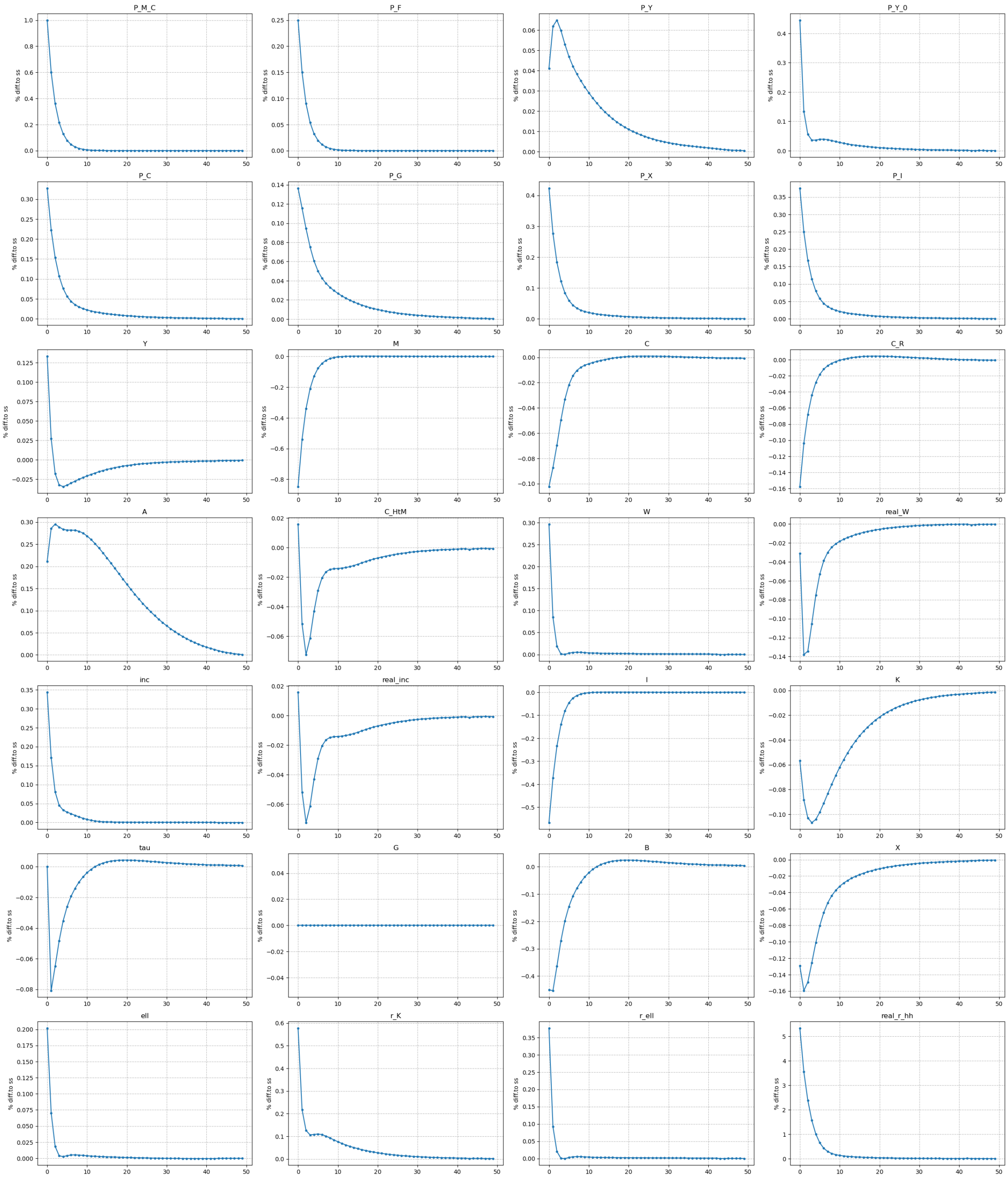
Following the price shock, government goods become more expensive, leading to increased government spending. However, this is offset by a rise in employment due to higher returns on effective labor ($\uparrow \ell$), which reduces overall government expenditure. Initially, this results in a decrease in government spending, enabling a reduction in the tax rate ($\downarrow \tau$). In the transition path, this situation quickly reverses as the employment level stabilizes faster than the prices of public consumption goods.

4.1.4 Household Consumption Dynamics

Household consumption patterns are central to the model's mechanism. As the shock occurs, real wages decrease ($\downarrow W_{real}$), but real income increases ($\uparrow inc_{real}$) due to higher employment levels. Notably, 30% of workers, categorized as Hand-to-Mouth, spend their increased real income with a marginal propensity to consume of 1. This behavior has a multiplier effect, raising consumption, output, demanded labor, and thus wages and consumption again, intensifying the price shock's initial impact.

Conversely, Ricardian workers, characterized by perfect foresight and unlimited access to credit and savings, collectively decide to smooth consumption by saving more, anticipating higher real returns on assets ($\uparrow r^{hh}_{real} \Rightarrow \uparrow A$) as the price of consumption goods (P_C) trends back to its steady state. They also save in anticipation of future tax increases, a reflection of Ricardian Equivalence. As the price shock diminishes, these workers increase their consumption, utilizing their accumulated savings to purchase relatively cheaper consumption goods. In the further periods, real income drops ($\downarrow inc_{real}$), as demanded labor and wages fall and the price of consumption is above the steady state.

Figure 4.2: Model output for the baseline shock

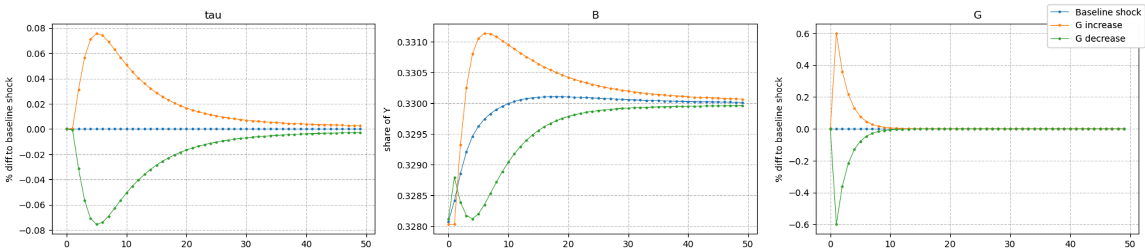


Note: 0.6% increase and decrease in G for $t > 0$ with persistence 0.6. Figure has been re-indexed to reflect percentage change w.r.t the baseline shock outlined in section 4, see 4.2. All $P_{M,*}$ are equivalent to P_{M_C} and are thus not shown.

5 Intervention Simulation

This section explores government interventions to mitigate the impacts from the initial simulation. These include adjustments in government spending and tax interventions, with tax interventions solely for the purpose of calculating utility. A subsequent sensitivity analysis examines the effectiveness of these fiscal strategies under different parameter values. The baseline shock model is set to y=0 in all plots for a clear comparison of intervention effects. Figure 5.1 below illustrates the effects on the government. Figure 5.2 presents a comprehensive overview of the simulation output.

Figure 5.1: Government variables for expansionary and contractionary scenarios



Note: 0.6% increase and decrease in G for $t > 0$ with persistence 0.6. Figure has been re-indexed to reflect percentage change w.r.t the baseline shock outlined in section 4, see 4.2

5.1 Expansionary Fiscal Policy Analysis

This section delves into the ramifications of expansionary fiscal policy on government finances, production dynamics, pricing structures, and household economic behaviors.

Anticipatory Adjustments: With the policy set to initiate at $t = 1$, entities with perfect foresight make preemptive adjustments, a phenomenon commonly known as announcement effects or headline risk, as detailed in Section 3.3. This leads to significant changes from the baseline at $t = 0$. Notably, the capital agency boosts investments ($\uparrow I \Rightarrow \uparrow K_{+1}$), preparing for an expected surge in output demand at $t = 1$.

5.1.1 Government Effects

The government increases its spending by 0.6% at $t = 1$, in tandem with the baseline price shock. Surprisingly, the endogenous tax rate (τ) remains stable initially, as the increased employment effectively offsets the costs of additional government goods purchases. In the short term, including $t = 1$, this policy does not result in a bond stock increase. However, post $t = 1$, tax rates consistently surpass baseline levels ($\uparrow \tau$), necessitated by bond accumulation to fund the intervention, inadvertently suppressing economic demand. This underpins the long-term impacts of the intervention across all economic variables of the model.

5.1.2 Production Effects

To meet the escalated demand for government goods, production ($\uparrow Y$) increases by over 0.1% compared to the baseline in $t = 1$. Government goods, less reliant on imports than private consumption, exert substantial pressure on domestic output prices ($\uparrow P^Y$), leading to an additional (multiplicative) 0.05% price hike. This incrementally affects all final goods prices, with P^X being the least and P^G the most impacted due to their respective domestic production dependencies ($\uparrow P^X, P^I, P^C, P^G$). Both labor and capital agencies respond by acquiring more input factors ($\uparrow K, \ell$).

In the medium to long term, these trends reverse: beyond $t = 2$, output, effective labor, and capital fall below baseline levels, while prices linger higher due to sticky pricing. This reversal stems from the tax-rate dampening effect discussed in Section 5.1.1.

5.1.3 Household Effects

The surge in effective labor demand ($\uparrow \ell$) temporarily boosts real incomes in $t = 1$ ($\uparrow inc_{real}$). For Hand-to-Mouth households, this translates to increased consumption, marking the sole positive impact on aggregate consumption during the simulation. Conversely, Ricardian households opt to save even more and consume less. This behavior aligns with Ricardian Equivalence, as theorized in (Buchanan 1976), suggesting that debt-financed economic stimulation is neutralized by agents who internalize future tax obligations associated with such debts, thereby balancing their assets with future tax liabilities. The model exhibits a complex interplay: while Hand-to-Mouth households' multiplier effect induces a temporary consumption boost in $t = 1$, the Ricardians' savings response counteracts attempts to elevate aggregate demand.

5.2 Contractionary Fiscal Policy and Summary

The exploration of expansionary fiscal policy revealed its ability to provide short-term support for output and consumption. However, it also led to heightened prices in domestic output, inadvertently crowding-out other sectors. Moreover, its medium to long-term effects are notably adverse, with both output and consumption eventually dipping below baseline levels and requiring extended periods to return to steady state.

In contrast, a policymaker might prefer a contractionary approach to maintain lower inflation rates than those in the baseline scenario, potentially circumventing the prolonged consequences of elevated taxes and prices. As depicted in figure 5.2, the contractionary policy's impact appears almost as a mirror image of the baseline scenarios, driven by the same dynamics previously discussed. Therefore, it can be inferred that the outcomes of contractionary measures are essentially the inverse of expansionary actions, at least within the context of the given parameters and shock schedules.

Summary of Contractionary Policy Effects: Rather than reiterating the mechanisms in reverse, I refer to Section 5.1 for a summary of the effects:

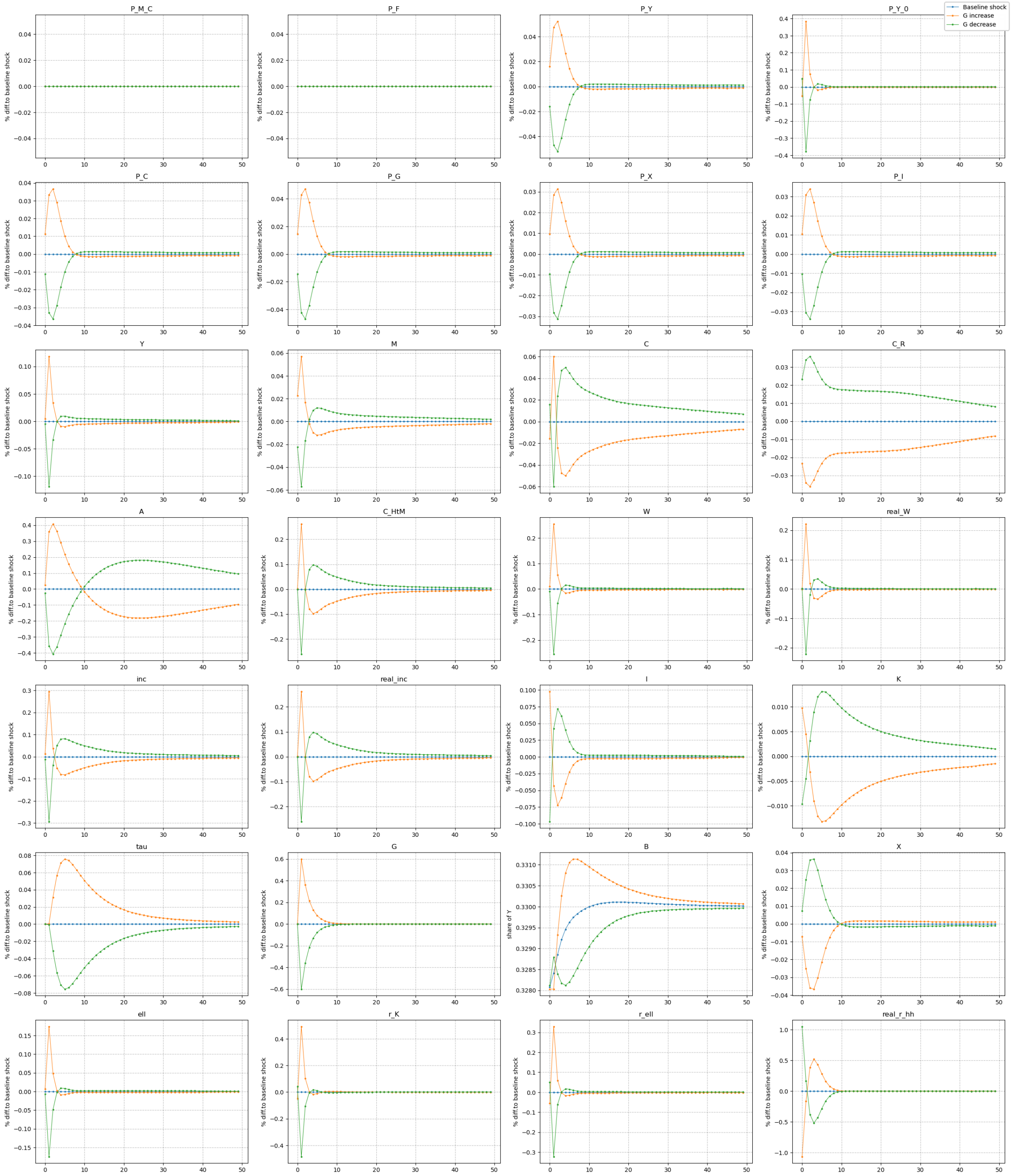
Contractionary fiscal policy initially curtails output by reducing government goods demand, leading to a decrease in output prices and consequently all goods prices.

- This initial periods witnesses a dip in real incomes due to the reduced demand for effective labor and a corresponding decrease in investments at $t = 0$, aligning with the diminished output demand.

- By $t = 2$, output, private consumption, capital, and effective labor all surpass baseline levels, while prices remain lower.
- However, in the long term, the boost in private consumption, primarily driven by lower taxes, culminates in output and final goods prices exceeding baseline levels.

This analysis underscores the trade-offs between short-term gains and long-term implications inherent in government interventions channeled using government spending. However, this did not illuminate the welfare of the agents under different scenarios. For this, the analysis turns to tax-scenarios:

Figure 5.2: Model output for the baseline shock with government interventions

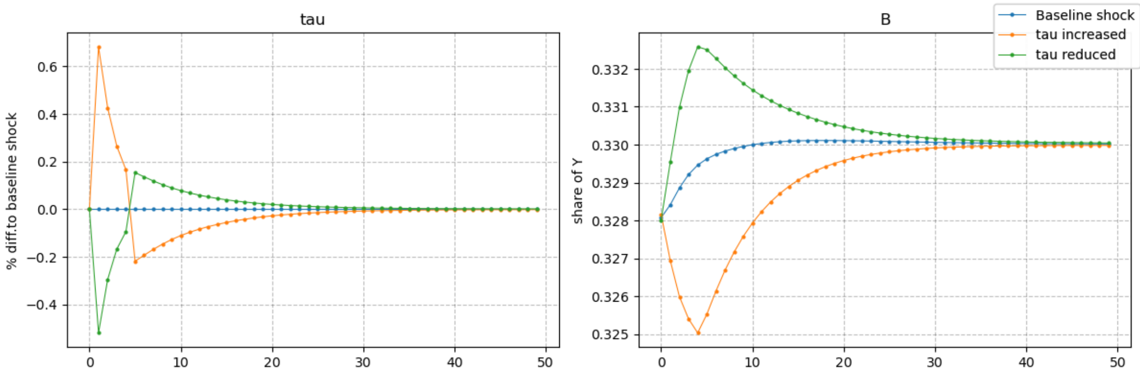


0.6% increase and decrease in G for $t > 0$ with persistence 0.6. Figure has been re-indexed to reflect percentage change w.r.t the baseline shock outlined in section 4, see 4.2. All $P_{M,*}$ are equivalent to P_{M_C} and are thus not shown.

5.3 Tax-Only Scenarios and Utility

In assessing policy impacts, it may be beneficial to consider how different tax interventions – expansionary or contractionary – affect private utility in the economy. It's important to note that the utility of agents is not directly influenced by government spending. This poses a challenge in evaluating the direct benefits of state-funded services like kindergartens and healthcare on agents' utility. To address this, the model focuses on tax scenarios that do not alter the volume of government-procured goods. Figure 5.3 demonstrates the government's response to these scenarios, while Figure 5.5 provides a detailed simulation output.

Figure 5.3: Government IRF's for expansionary and contractionary tax scenarios



Note: 0.6% increase and decrease in τ (from steady state) for $0 < t < 4$ with persistence 0.6. The figure is re-indexed to show percentage changes relative to the baseline shock described in Section 4, see figure 4.2.

The dynamics observed in response to tax adjustments are similar to those seen with direct government spending changes, as illustrated in Figure 5.2. A key difference is that the demand stimulus originates from private consumers – increasing with tax cuts and decreasing with tax hikes. This contrasts with government spending scenarios, where demand changes are secondary to the direct impact of increased government goods demand, thus minimizing crowding out effects. The asymmetry in the graphs is due to the tax interventions being relative to the steady state tax level. The baseline shock's steady state for τ is lower than the steady state level, creating an apparent disparity where tax increases seem more impactful. However, in this model and for this specific shock, this disparity, if it exists, is minimal.

The primary goal in these scenarios is to assess agent utility under varying tax interventions. Figure 5.4 displays the utility, V_t, for private agents, categorized into Ricardians (refer to equation 2.20) and Hand-to-Mouth households. Given that Hand-to-Mouth households' utility is not explicitly modeled, they are assumed to follow the same CRRA-utility function as Ricardians. However, their inability to accumulate assets and lack of forward-looking behavior simplifies their utility function to $\frac{C_{a,t}^{HtM^{1-\sigma}}}{1-\sigma}$ for all (a,t).

Calculating utility at time t follows the same aggregation technique used to calculate aggregate consumption at time t (C_t, C_R, C_HtM), using the utility values instead (here for a cohort born at $t = 0$):

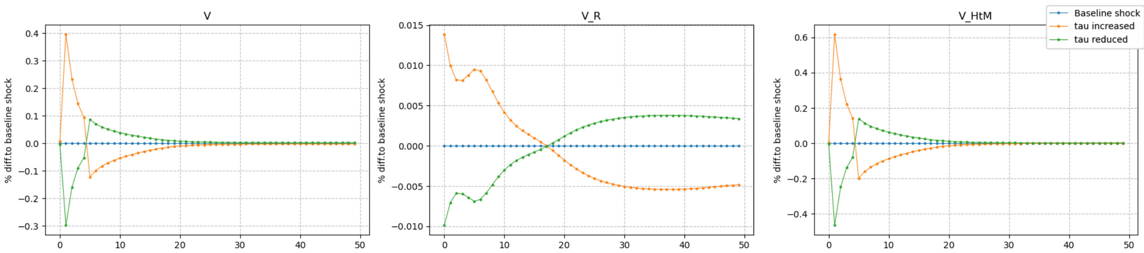
$$V_{0,0}^{R} = \frac{C_{0,0}^{R^{1-\sigma}}}{1-\sigma}, \qquad V_{1,1}^{R} = \beta(1-\zeta_0)\left[\frac{C_{1,1}^{R^{1-\sigma}}}{1-\sigma} + \zeta_a \mu^{Aq} \frac{\left(\frac{A_{1,1}^{R}}{P_1^C}\right)^{1-\sigma}}{1-\sigma}\right]\dots \tag{5.1}$$

$$V_{0,0}^{HtM} = \frac{C_{0,0}^{HtM^{1-\sigma}}}{1-\sigma}, \qquad V_{1,1}^{HtM} = \frac{C_{1,1}^{HtM^{1-\sigma}}}{1-\sigma}\dots \tag{5.2}$$

This assumption of the utility of the Hand-to-Mouth workers implies that later life utility is relatively higher compared to Ricardians, $\beta(1-\zeta_*) = 1$ and will be subject to a sensitivity analysis.

In the short term, utility increases under a tax-cut scenario. Ricardian workers, aware of the government's budget constraints, experience higher utility from both consumption and savings for the initial 18 periods, surpassing the baseline consumption levels. However, subsequent generations of Ricardians, burdened with higher taxes and without the benefits of the initial real income increase, see a different utility trajectory. For Hand-to-Mouth workers, the utility gain is more direct: higher real income translates to increased utility.

Figure 5.4: IRF's for utility in tax scenarios



Note: (1) This is the period-wise utility (2) 0.6% increase and decrease in τ (from steady state) for $0 < t < 4$ with persistence 0.6. The figure is re-indexed to show percentage changes relative to the baseline shock described in Section 4, see 4.2.

It is evident that a utilitarian policymaker with a short time horizon may weigh more towards an expansionary policy. However, it is unclear whether or not the long-term accumulated utility is higher or lower. In table 5.1, I outline the total utilities in the short, medium, and long term.

From this, it can be interpreted that total accumulating utility in this modeling setting is higher for an expansionary fiscal intervention, both in the short, medium, and long term – at least, if it is a tax cut. The added real income in the time when it has been hollowed out by the supply shock is more than sufficient to cover the lower utility for subsequent generations. This is in line with the assumption of CRRA utility with positive $\sigma = 2$, and thus risk-averse agents. In the long term, however, this intervention has a negative effect on total accumulated utility for the Ricardians, who prefer the intervention of raising taxes.

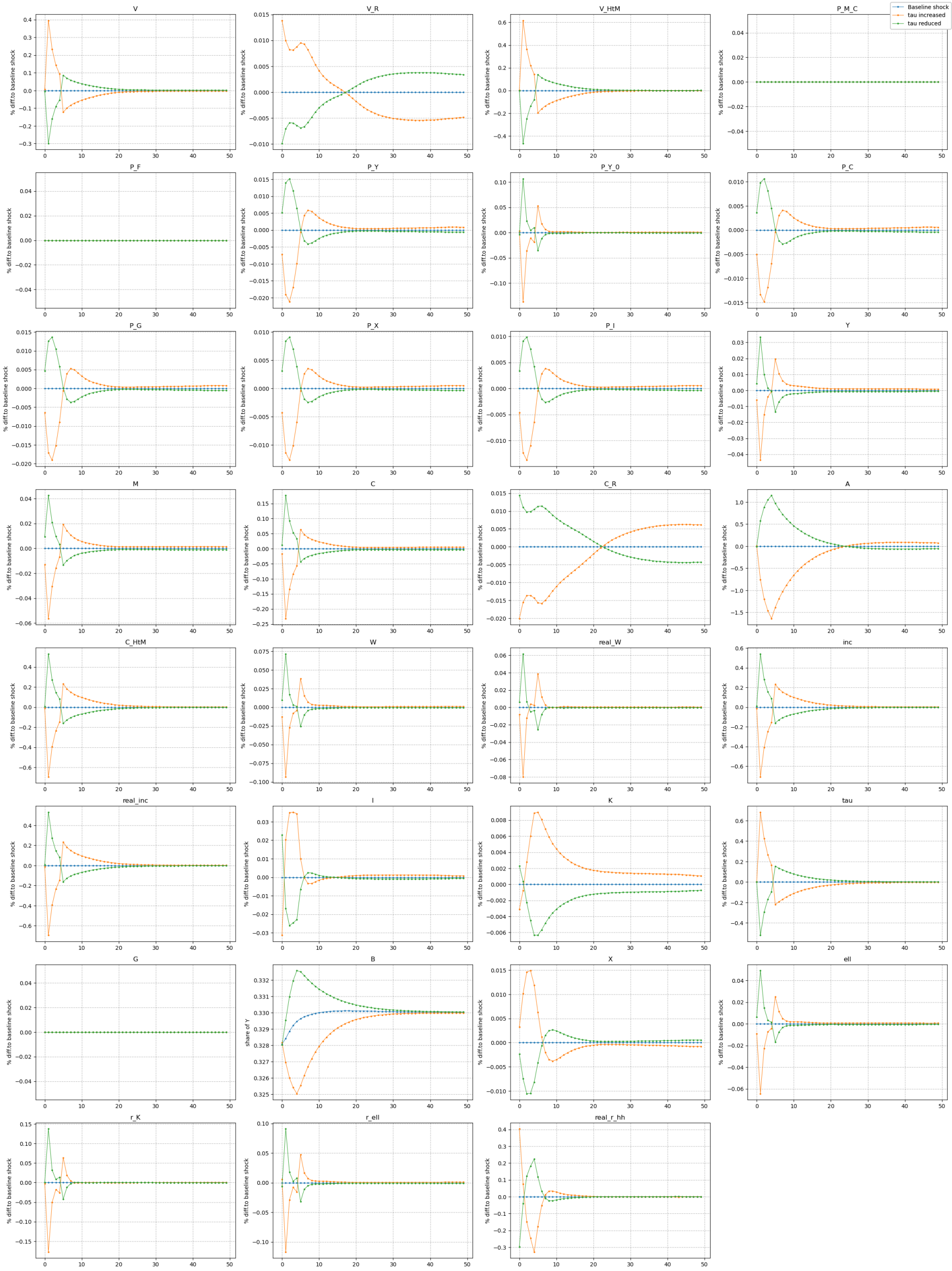
Table 5.1: Utility in scenarios over time, w.r.t changes to τ

Model Type	**Term**	V	V_R	V_{HtM}
Baseline	Short	-131.32	-68.39	-278.17
	Medium	-481.23	-250.60	-1019.38
	Long	-2230.38	-1161.51	-4724.40
τ Raised	Short	-131.60	-68.40	-279.08
	Medium	-481.40	-250.62	-1019.88
	Long	-2230.38	**-1161.50**	-4724.44
τ Reduced	Short	**-131.12**	**-68.38**	**-277.51**
	Medium	**-481.12**	**-250.59**	**-1019.03**
	Long	**-2230.38**	-1161.52	**-4724.38**

Note: This table shows the accumulated utility of workers in the short term (< 3), medium- (< 10) and long term (< 50 years). **Bold** indicates the highest value, either in the tax-reduced or the baseline scenario.

This result depends on how Hand-to-Mouth utility is calculated. In the next section, I delve into the sensitivity of the found results to such key parameters and functional forms.

Figure 5.5: Model IRF's for expansionary and contractionary tax scenarios



Note: 0.6% increase and decrease in τ (from steady state) for $0 < t < 4$ with persistence 0.6. Figure has been re-indexed to reflect percentage change w.r.t the baseline shock outlined in section 4, see 4.2

6 Sensitivity Analysis

This section delves into how varying key parameters influence the outcomes of fiscal policies, specifically in relation to the baseline shock scenario. Due to the limited space, the focus is narrowed to government interventions in the context of the baseline shock. Detailed figures and tables related to this analysis are included in the appendix.

The parameters selected for this sensitivity analysis encompass a range of critical factors. These include the elasticity of substitution in repacking processes (σ^*, applicable to C, G, I, and X), the proportion of Ricardian households in the population (λ), the proportion of foreign inputs in the production of government goods ($\mu^{M,G}$), and the rigidity of tax adjustments (ϵ_B). Additionally, the analysis will explore the implications of assigning a discount value to the utility of Hand-to-Mouth households, assessing how this affects overall outcomes.

6.1 Elasticity of Substitution in Repacking Firms

In this model, the elasticity of substitution for repacking firms is set at 1.05, which moderately restricts the interchangeability between more expensive foreign inputs and domestic output. An increase in this elasticity parameter leads to greater fluctuations in output and an initial surge in output prices. This is due to repacking firms being able to more efficiently substitute towards the cheaper domestic output. As depicted in Figure 8.1, the precise values of these elasticity parameters play a crucial role in determining the model's equilibrium trajectory. With a reduced elasticity of substitution (i.e., $\sigma^* = 0.65$), the immediate consequence is a decrease in demand for domestic output, as it becomes harder for repacking firms to switch between inputs. Thus, the elasticity of substitution is a key factor influencing the model's dynamics, and perhaps would have been fitting for this paper to be even lower.

6.2 Impact of Hand-to-Mouth Household Proportion

The proportion of Hand-to-Mouth households in the economy, while not as pivotal, still influences the response to fiscal interventions. This analysis considers scenarios with $\lambda = 0.2$ and $\lambda = 0.4$ as shown in Figure 8.4. It is observed that a higher λ enhances the multiplier effect, leading to a more pronounced increase in output (Y) and its price (P_Y) following an expansionary fiscal policy. This outcome aligns with expectations, given that the marginal propensity to consume escalates with λ. Despite these variations in magnitude, the fundamental nature of the fiscal policies' effects remains consistent across different shares of Hand-to-Mouth households. Essentially, the proportion of these households enhances the multiplier effects and thus the extent of the macroeconomic variables' reaction to fiscal stimuli.

6.3 Influence of Foreign Input Proportions in Government Goods Production

The proportion of foreign inputs used in the production of government goods plays a crucial role in assessing the impact of fiscal policy. As previously discussed, government spending with a reduced reliance on foreign inputs tends to crowd out private consumption less, leading to a more efficient production process that utilizes fewer foreign resources. This aspect's sensitivity is illustrated in Figure 8.3. Notably, lower dependence on foreign inputs also amplifies the outcomes of expansionary fiscal policies, leading to greater increases in output, employment, and inflation than would occur otherwise. Therefore, the composition of foreign inputs in government goods production emerges as a vital factor in determining the scale and consequences of fiscal interventions.

6.4 Tax Rigidity

The chosen parameter for $\epsilon_B = 0.15$, which represents the government's adjustment of 15% towards bond equilibrium each period, is somewhat arbitrary. A sensitivity analysis, as illustrated in figure 8.2, demonstrates that altering the speed of convergence for outstanding bonds – either halving or doubling – significantly alters the equilibrium path of τ and B. For instance, a higher ϵ_B results in increased volatility and quicker tax-rate convergence, and the opposite is true for a low value. However, with Ricardian Agents smoothing their consumption, this has marginal effects on macroeconomic variables. Notice the grouping of graphs for P_Y, P_C, P_G, Y, and C_R, denoting different fiscal policies and the baseline – not the change in parameter. Therefore, while the tax-rigidity parameter affects the economy's path to steady state, it is not critical within reasonable ranges.

6.5 Utility

I claimed earlier in the analysis that total utility is higher for an expansionary fiscal policy in the form of tax-cuts, while assuming no discounting for Hand-to-Mouth households. The Table 8.1 shows the utility comparison, where the utility of Hand-to-Mouth households are modelled as in equation 2.20:

$$V_t^{HtM} = \sum_{a=0}^{65-1} \left(\prod_{j=1}^{a} \beta(1-\zeta_{j-1}) \right) \left[\frac{{C_{a,t}^{R}}^{1-\sigma}}{1-\sigma} + 0 \right] \tag{6.1}$$

That is, with the same decreasing weight on utility over the lifetime alike the Ricardians. The modelled results yield unanimously that accumulated utility is highest under tax reductions in the short term and medium term, but higher when raising taxes in the long term. The specific discount rate or modeling for utility may thus impact the results of this thesis.

7 Discussion

In this thesis, the sensitivity of the results to the direct response to the supply shock is a critical aspect. The substitution parameters of repacking firms are especially significant, as they fundamentally alter the dynamics of the shock. Within the context of this study, and given the methodology and parameters employed, the trade-offs are evident. A contractionary policy approach initially leads to negative impacts but facilitates a quicker recovery. In contrast, an expansionary policy provides initial stabilization but necessitates an extended period for the economy to return to its steady state.

Extending the utility function to include Hand-to-Mouth workers and considering agents with a high degree of risk aversion $\sigma = 2$, it can be concluded that a utilitarian policymaker with a focus on short to medium-term outcomes would favor an expansionary policy over a contractionary one. However, it's important to note that while an expansionary fiscal policy boosts current utility, this benefit is offset by a burden placed on future generations. This raises concerns about intergenerational inequality, which policymakers must consider.

The discussion will continue with an examination of the modeling of utility for Hand-to-Mouth households. It will delve into the limitations of the current model in capturing more realistic consequences and explore potential enhancements to the model. This includes considering additional factors that could bring the model closer to real-world scenarios and examining the implications of different policy choices on various segments of the population.

7.1 Revisiting the Utility Modeling

In re-evaluating the utility modeling for Hand-to-Mouth households, it's pertinent to consider the possibility of applying a significantly lower discount factor (β) for these households compared to the Ricardians. This perspective is supported by (Aguiar, Bils, and Boar 2020) in their study "Who Are the Hand-to-Mouth?", where they challenge the conventional interpretation of the term "Hand-to-Mouth." Their empirical research reveals ambiguous connections between high marginal propensity to consume and the possession of minimal or no assets. This finding suggests that Hand-to-Mouth households may not necessarily be constrained by credit limitations but could instead be characterized by a high degree of impatience.

Given this insight, it might be more accurate to model Hand-to-Mouth households in a manner similar to Ricardian households but with a markedly lower discount factor (β). This approach would align the model more closely with the empirical evidence suggesting that these households prioritize immediate consumption due to their impatience, rather than being strictly limited by their financial circumstances. Such a modification in the model could provide a more nuanced understanding of the consumption behaviors and utilities, and if implemented, could change the results which, for a utilitarian policymaker, lean towards an expansionary approach.

7.2 Model Simplicity

In considering the impact of expansionary fiscal policy and its associated multiplier effects, one might expect these to significantly stabilize the economy, especially given Denmark's empirical reliance on such policies, as previously mentioned in the introduction. However, the model employed in this thesis, BabyMAKRO, does not seem to capture these effects to their full extent. This limitation can be attributed to certain inherent constraints of the model, of which I will entertain 3 in this discussion.

- **Perfect foresight of agents**:
  - The BabyMAKRO model assumes perfect foresight among agents, an idealized scenario diverging from real-world economic behaviors. While reasonable to assume perfect foresight in the long term, short term perfect foresight maximizes the effects of Ricardian Equivalence. Although not all agents have perfect foresight, much of fiscal policy is thwarted by the Ricardian agents, even with a significant multiplier effect from many Hand-to-Mouth agents.
- **Immediate price flexibility in Repacking Firms**:
  - The CES repacking firms have completely flexible prices, and upon a shock, immediately react and change their demand schedules. This is the reason why the model has higher than steady state output at $t = 0$ in the initial price shock. Although I mentioned that this model and oil price shocks are not completely compatible due to no extraction sector, such sluggish reactions to price changes may initially lower output, and only later let them rise, as output does in the oil price shock from DREAM (See Figure 8.7).
- **Immediate price flexibility in Repacking Firms**:
  - The government's role in BabyMAKRO is simplified, mainly focusing on consumption and transfers. This abstraction overlooks the diverse range of government activities that can significantly influence economic dynamics.
  - The MAKRO model offers a more comprehensive representation of government functions, including public investments and direct employment. These elements are crucial for understanding the government's capacity to influence economic stability and development during supply shocks.

7.3 Future Research Directions and Model Enhancements

Given these differences, future research endeavors could benefit from the following considerations:

- **Incorporating more Passive Fiscal Policy**: An entire study could be made simply discussing and implementing a progressive taxation system in the model. By doing so, the government effectively pushes the model back in steady state in response to demand shocks. The implications in tandem with a supply shock is yet to be studied for BabyMAKRO.
- **Expanding Government Functionality**: By integrating a wider array of government functions, such as public investments and direct employment, future models can offer a more holistic view of the government's role in economic management, especially during periods of economic turmoil.
- **Direct Utility from Government Spending**: Exploring the direct utility impacts of government spending on societal welfare is a complex but essential aspect for comprehensive economic modeling. This inclusion would offer deeper insights into the societal implications of various fiscal policies.

7.4 Concluding Reflections

This thesis, in examining fiscal policy in the face of supply shocks through the lens of the BabyMAKRO model, contributes valuable insights into the nuanced trade-offs involved. Expansionary policy can somewhat stabilize the economy in the short term and contribute to utilitarian social welfare, but comes at the cost of higher prices and inter-generational inequality. However, there is much to be desired from the model as it pertains to resemblance of real world complexities. Many of the features, like incorporation of a more extensive government role, are already featured in the MAKRO model, and could significantly enhance the model's applicability in policy analysis if implemented.

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8 Appendix

Table 8.1: Utility in scenarios over time, w.r.t changes to τ, HtM utility discounted

Model Type	**Term**	V	V_R	V_{HtM}
Baseline	Short	-68.13	-68.39	-67.54
	Medium	-249.70	-250.60	-247.60
	Long	-1157.38	-1161.51	-1147.73
τ Raised	Short	-68.21	-68.40	-67.78
	Medium	-249.76	-250.62	-247.73
	Long	**-1157.37**	**-1161.50**	**-1147.72**
τ Reduced	Short	**-68.08**	**-68.38**	**-67.36**
	Medium	**-249.66**	**-250.59**	**-247.51**
	Long	-1157.38	-1161.52	-1147.73

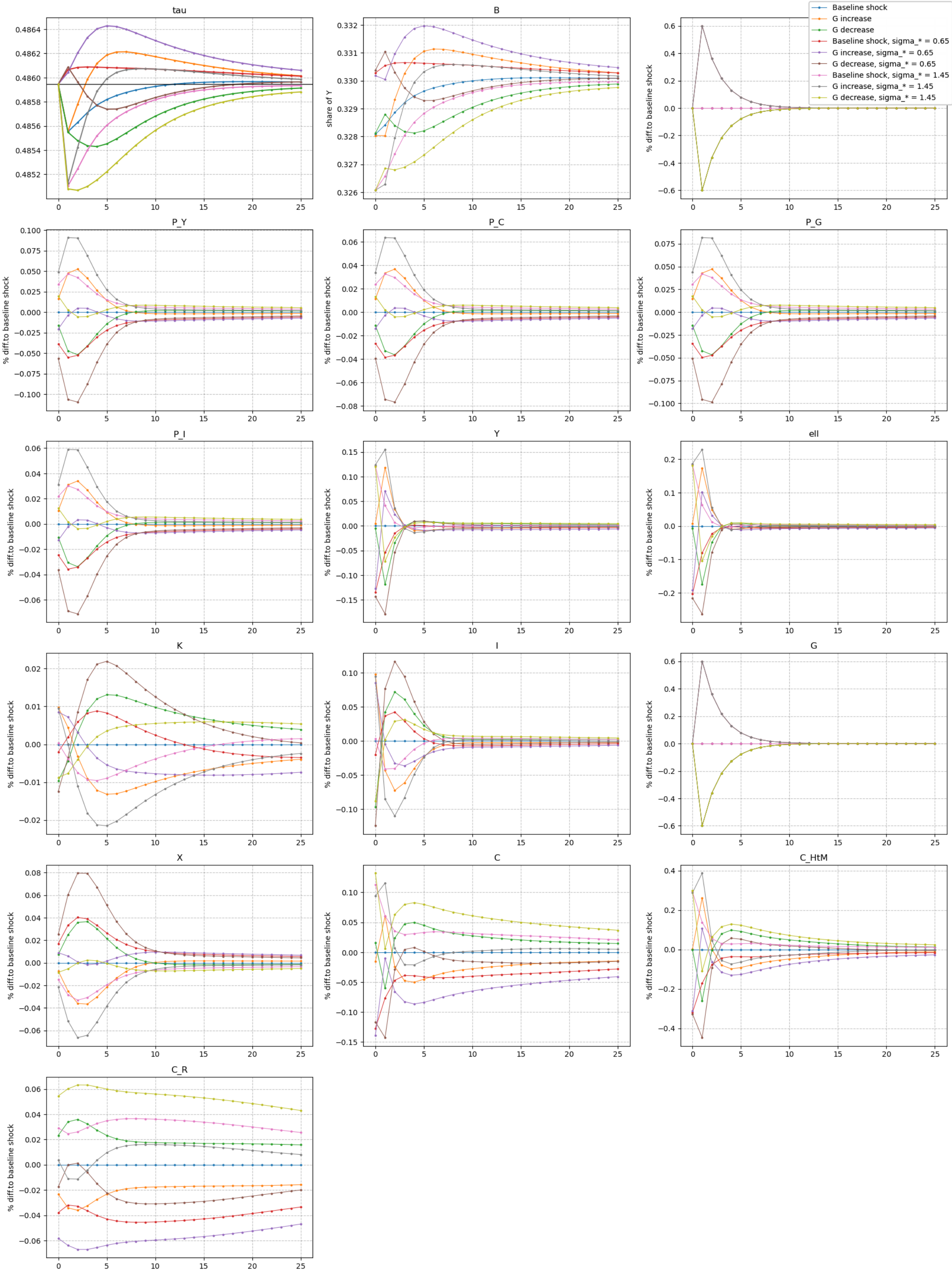
Note:(1) In this table, utility of Hand-to-Mouth consumption is discounted as Ricardians (2) This table shows the accumulated utility of workers in the short term (<3), medium- (<10) and long term (< 50 years). **Bold** indicates the highest value across scenarios.

Table 8.2: Model Parameters

Parameter	Value	Parameter	Value
T	400	age_{ini}	25
$life_span$	65	$work_{\text{life}_{\text{span}}}$	43
ζ	4.0	λ	0.30
β	0.95	σ	2.0
μ_{Aq}	100.0	r_{hh}	0.02
W_{U}	0.80	W_{R}	0.50
$delta_{L_{\text{a}_{\text{fac}}}}$	0.10	ρ_1	0.09
ρ_2	0.0018	Φ	0.6
r_{firm}	0.02	$delta_K$	0.10
μ_K	1/3	σ_Y	1.01
θ	0.1	γ	50.0
κ_L	0.05	Ψ_0	5.0
r_{b}	0.02	ϵ_B	0.15
$G_{\text{share}_{\text{ss}}}$	0.25	μ_{M_C}	0.30
σ_C	1.05	μ_{M_G}	0.10
σ_G	1.05	μ_{M_I}	0.35
σ_I	1.05	μ_{M_X}	0.40
σ_X	1.05	σ_F	1.5
γ_X	0.50	σ_m	M
ϵ_w	1.25	W_{ss}	1.0
$pi_{hh_{\text{ss}}}$	0.00	$m_{s_{\text{ss}}}$	0.75
$m_{v_{\text{ss}}}$	0.75	B_{ss}	M

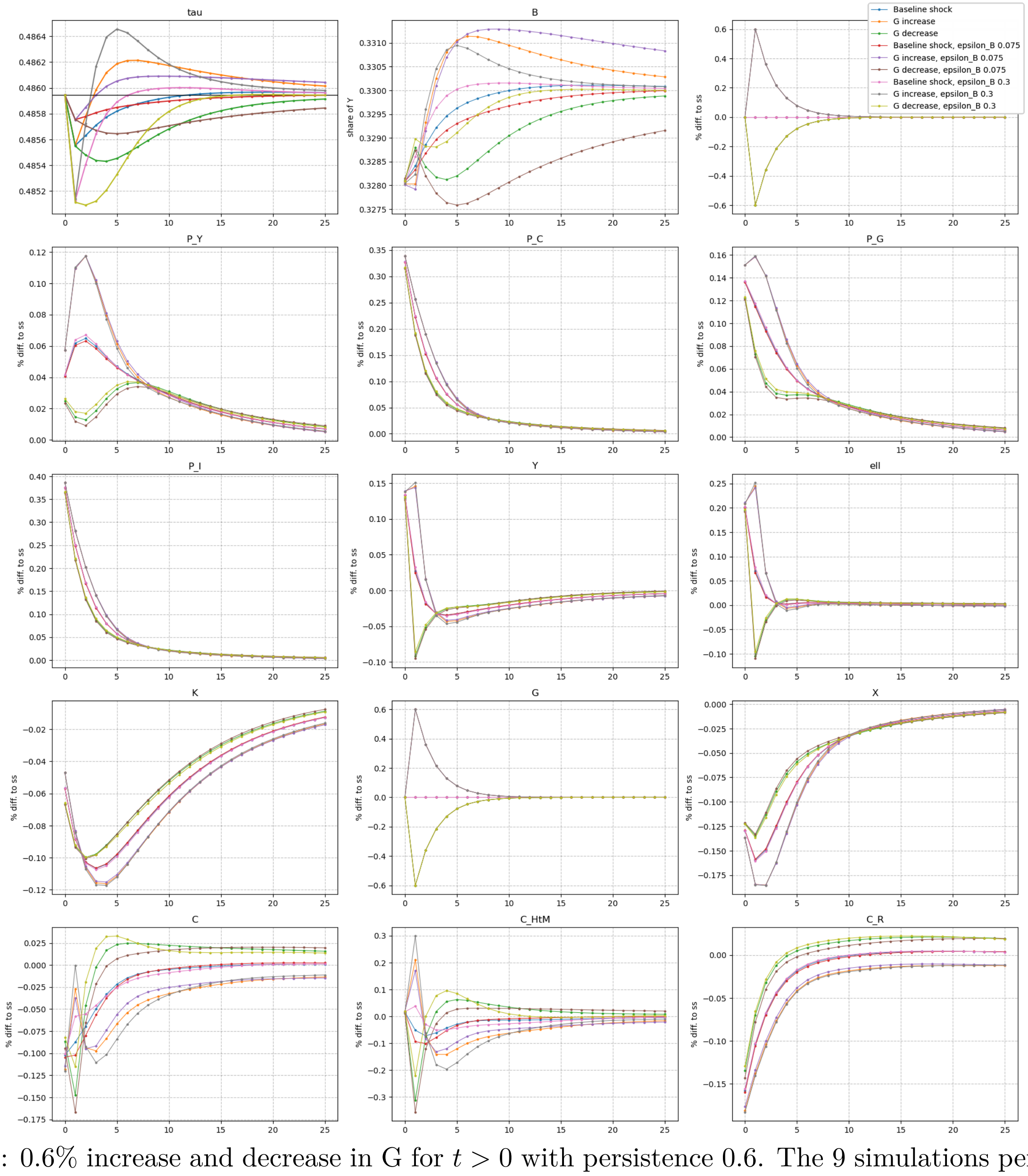
Note: Not all parameters above are discussed in the paper. See the model documentation for reference. ”M” means defined in model. For B, this is $B = 0.33_s s$

Figure 8.1: Sensitivity to σ^* for government interventions



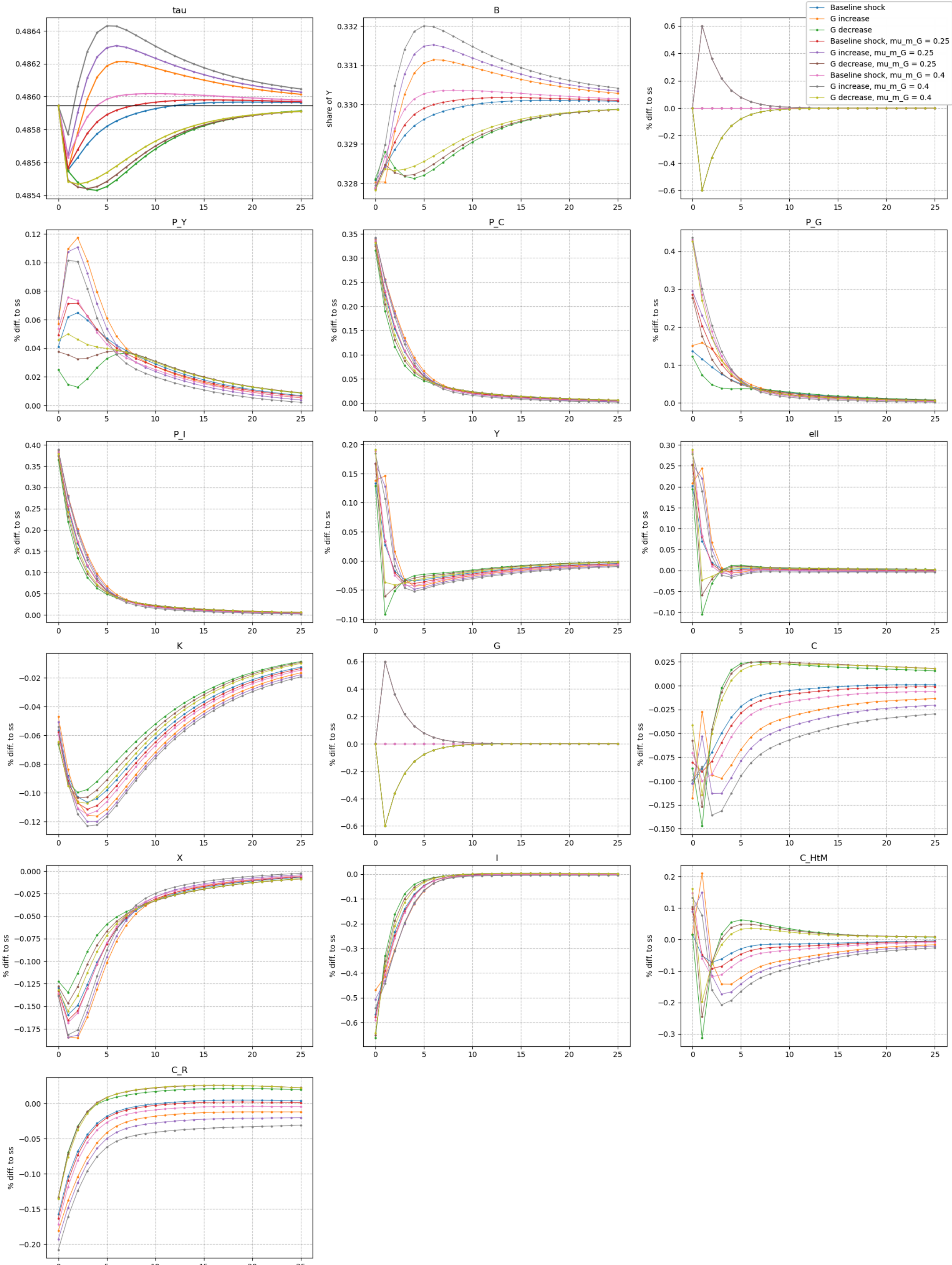
Note: 0.6% increase and decrease in G for $t > 0$ with persistence 0.6

Figure 8.2: Sensitivity to ϵ_B for government interventions



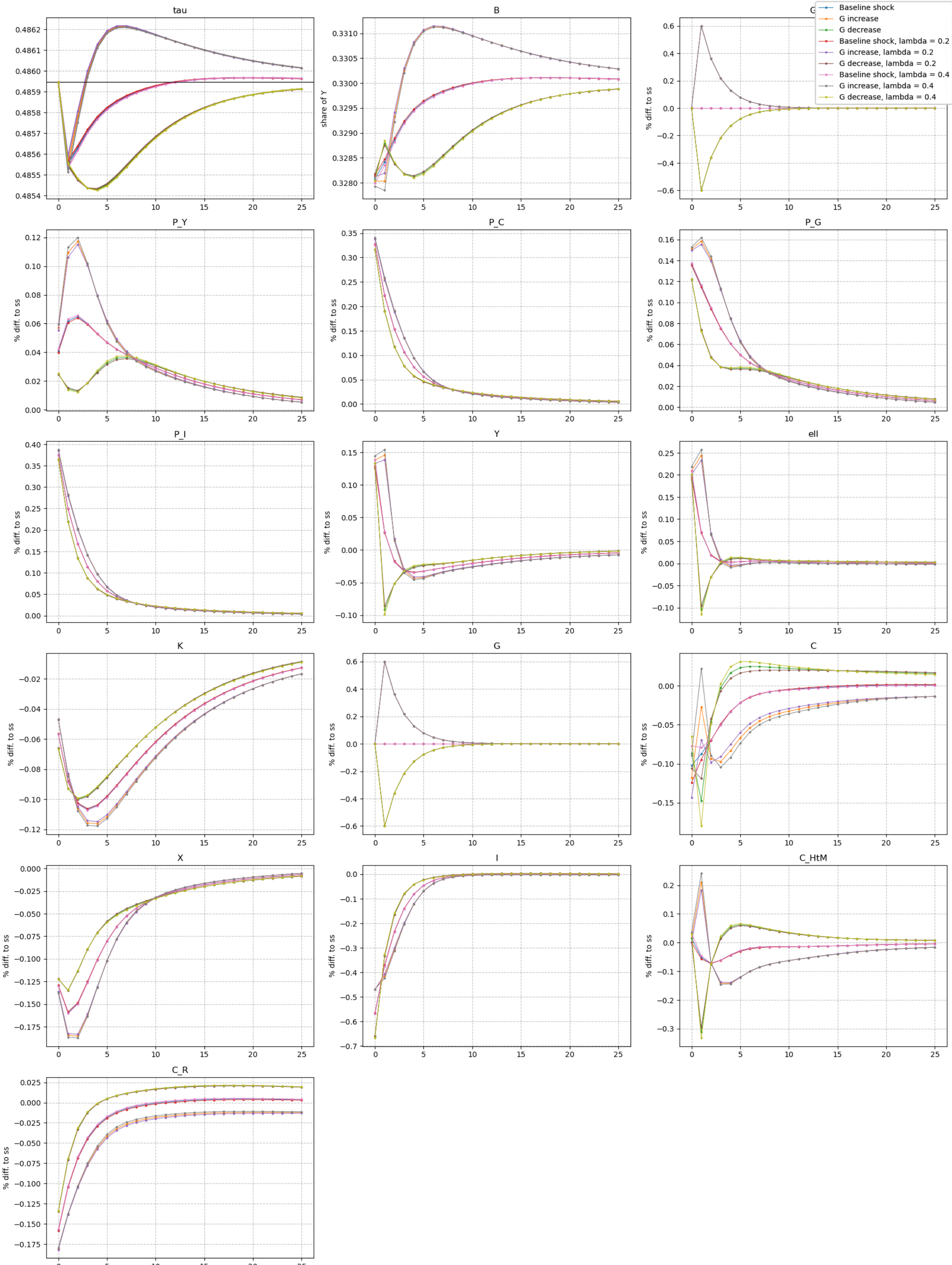
Note: 0.6% increase and decrease in G for $t > 0$ with persistence 0.6. The 9 simulations pertain to $\epsilon_B = 0.075$, $\epsilon_B = 0.15$ (baseline) and $\epsilon_B = 0.30$ respectively.

Figure 8.3: Sensitivity to $\mu_{M,G}$ for government interventions



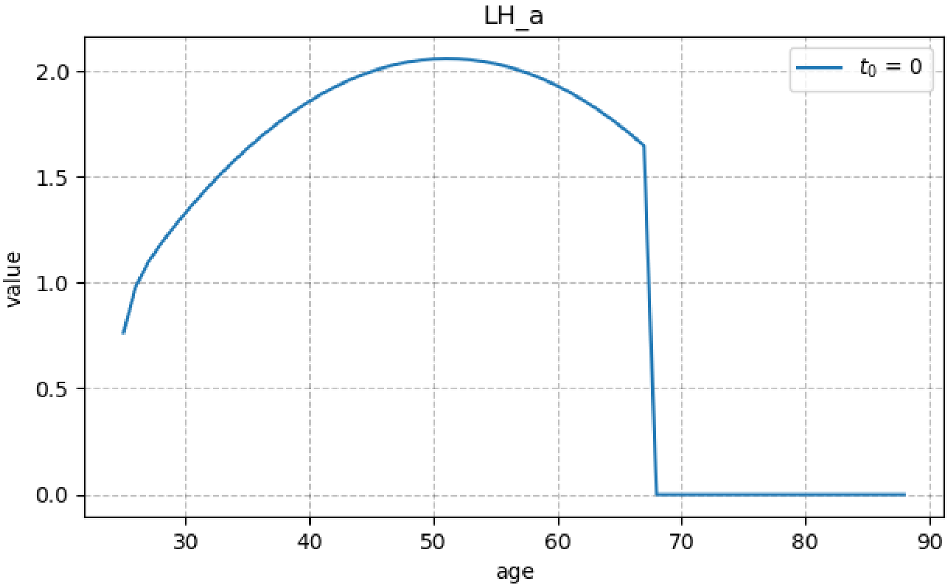
Note: 0.6% increase and decrease in G for $t > 0$ with persistence 0.6

Figure 8.4: Sensitivity to λ for government interventions



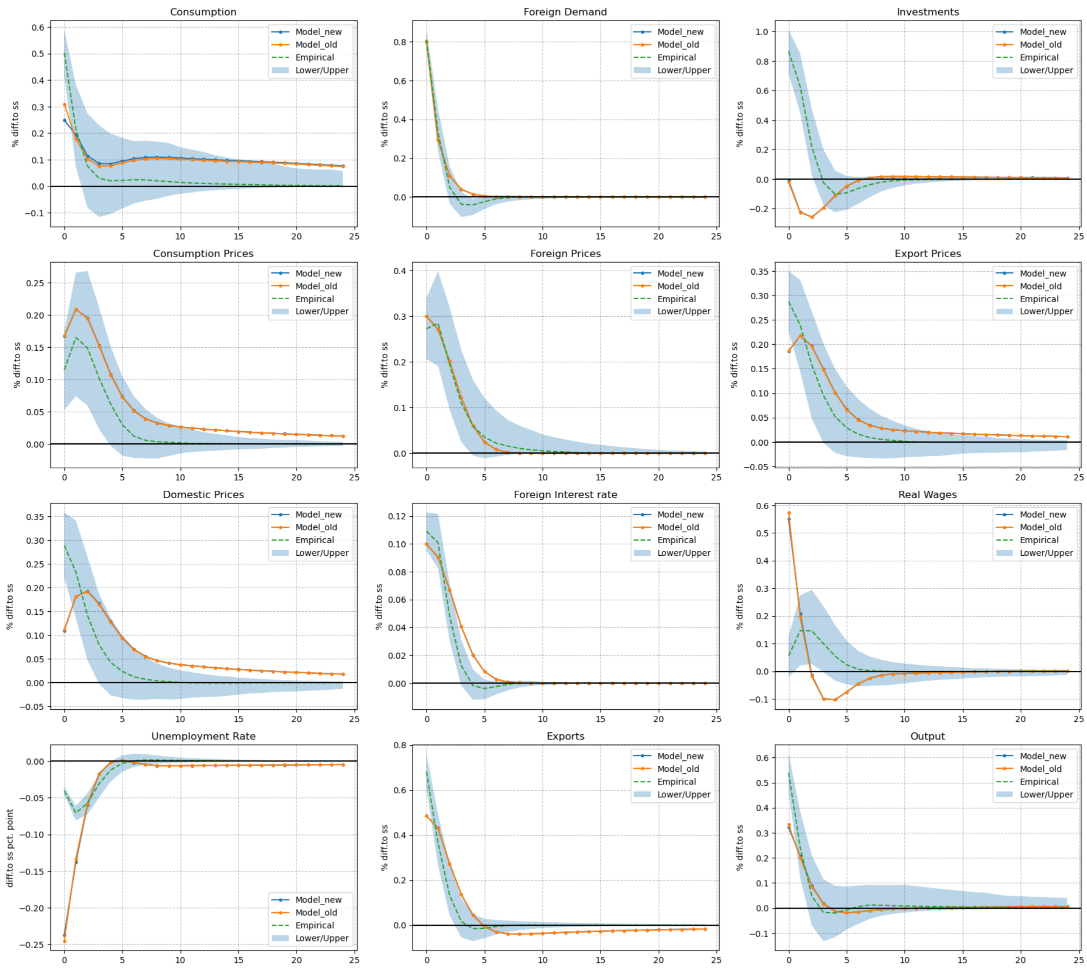
Note: 0.6% increase and decrease in G for $t > 0$ with persistence 0.6

Figure 8.5: Baseline $L_t H_t$ for a given cohort



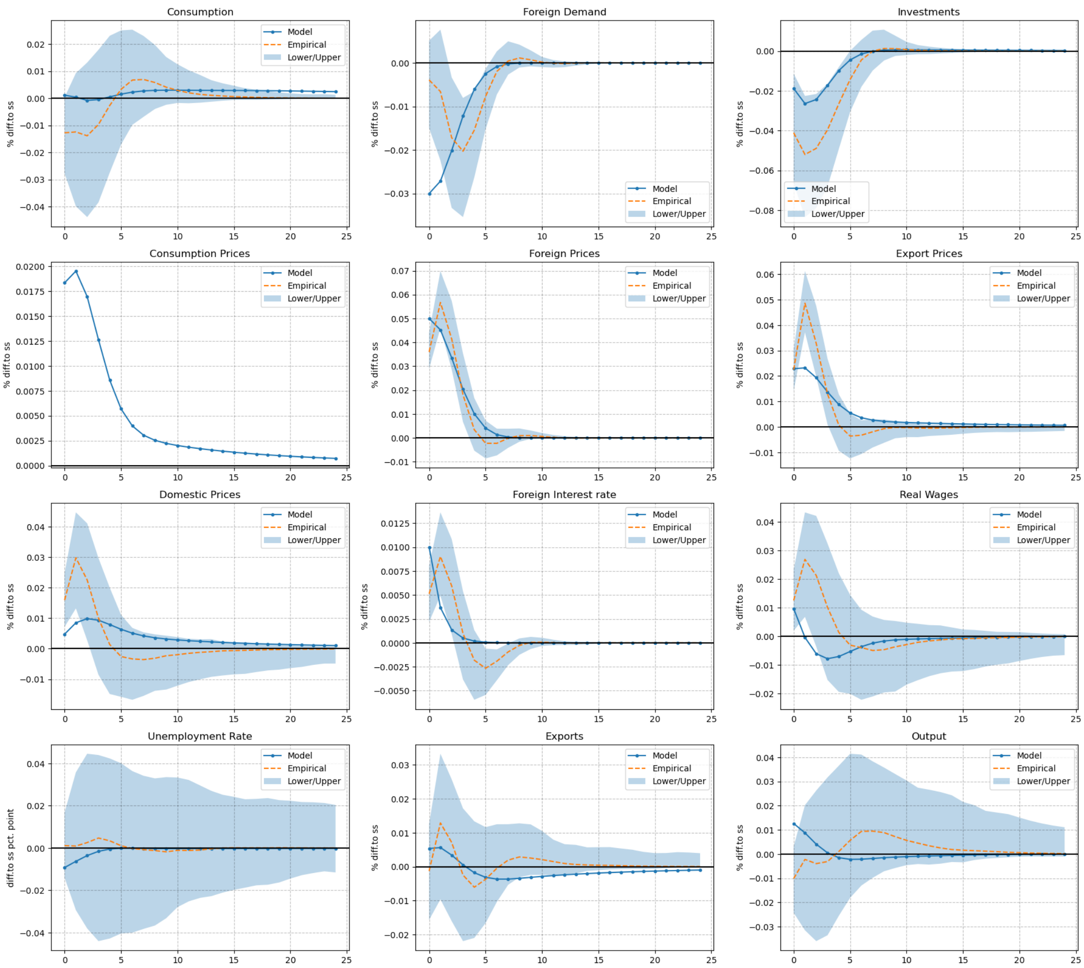
Note: Cohort shown is from steady state

Figure 8.6: Comparison of models w.r.t calibration



Note: This diagram shows the IRF's for the original BabyMAKRO model and the modified version (new) for a foreign demand shock. The shock is a shock to the foreign demand shifter, interest rates, foreign input prices and the foreign price level

Figure 8.7: Model output w.r.t an oil shock



Note: This output was not used in the analysis