

Problem 1

Exercise 46 on page 411.

The 99% confidence interval for the standard deviation is:

Lower limit:

$$\sqrt{\frac{(n-1)s^2}{\chi_{\alpha/2, n-1}^2}} = \sqrt{\frac{(19-1)7.234}{38.58}} = \mathbf{1.887}$$

Upper limit:

$$\sqrt{\frac{(n-1)s^2}{\chi_{1-\alpha/2, n-1}^2}} = \sqrt{\frac{(19-1)7.234}{6.844}} = \mathbf{4.481}$$

This confidence interval is **not valid** whatever the nature of the distribution. It is only valid for the normal distribution. The formula used to compute the interval relies on the fact that the distribution of a squared normal distribution is a chi-squared distribution (from page 315 of textbook).

Calculations:

```
alpha <- 0.01
vals <- c(
  19.75, 21.25, 21.5, 22.50, 23.25, 23.5, 24.00, 24, 24.25,
  24.5, 25.00, 26.0, 26.25, 26.25, 27.0, 27.75, 28, 28.00,
  28.25, 30
)
n <- length(vals)
s2 <- var(vals)

chi_high <- qchisq(alpha/2, df=n-1)
chi_low <- qchisq(1 - alpha/2, df=n-1)

lower <- (n-1)*s2/chi_low
upper <- (n-1)*s2/chi_high

lower_s <- sqrt(lower) # lower limit
upper_s <- sqrt(upper) # upper limit
```