Problem 1

Using **method of moments** the result will be $p = 0.1\bar{3}$:

The first population moment of the binomial distribution is

$$E(X) = np = 4p$$

The first sample moment is (m = number of samples)

$$\frac{1}{m}\sum X_i = \frac{8}{15}$$

Set them equal to each other to solve for p

$$p = \frac{8}{4\dot{1}5} = 0.1\bar{3}$$

Using the **method of maximum likelihood** the result will be $p = 0.25$. [1]

$$f(x_1...x_m; p) = L(p) = \prod_{i=1}^{m} \binom{n}{x_i} p_i^x (1-p) p^{n-x_i}$$

$$L(P) = 6\left(\binom{4}{1}p^1(1-p)^3\right) \times 8\left(\binom{4}{0}p^0(1-p)^4\right) \times 1\left(\binom{4}{2}p^2(1-p)^2\right)$$

$$6 \times 8 \times \frac{4!}{3!} \times \frac{4!}{4!} \times \frac{4!}{2!(2)!} \times p^3(1-p)^9 = 1152p^3(1-p)^9$$

$$\ln(L(p)) = 1152\left(3\ln(p) + 9\ln(1-p)\right)$$

$$\frac{d}{dp}L(p) = 1152\left(\frac{3}{p} - \frac{9}{1-p}\right) = 0$$

$$\implies p = \frac{3}{12} = 0.25$$

Comment:
If the business believes that a 10% failure rate is unacceptable, then they should not accept the material for these brake shoes because both methods of point estimation have given results with probability greater than 0.1.

[1] $f(x_1...x_m; p)$ is the notation used in textbook, $L(p)$ is the notation used in class for the joint pmf of the sample.