

Problem 5

Exercise 32a on page 449.

a)

The data does suggest the population mean differs from 100 using $\alpha = 0.05$

Hypotheses

$$H_0 : \quad \mu = \mu_0$$

$$H_a : \quad \mu \neq \mu_0$$

Rejection region

$$t \geq t_{\alpha/2, n-1} \quad \text{or} \quad t \leq -t_{\alpha/2, n-1}$$

$$t_{\alpha/2, n-1} = t_{0.05/2, 12-1} = 2.201$$

Test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{98.46 - 100}{6.142/\sqrt{12}} = -0.870$$

The null hypothesis should **not** be rejected

$$-2.201 < -0.870 < 2.201 \implies -t_{\alpha/2, n-1} < t < t_{\alpha/2, n-1}$$

Calculations:

```
data <- c(
  105.6, 90.9, 91.2, 96.9, 96.5, 91.3,
  101.1, 105.0, 99.6, 107.7, 103.3, 92.4
);

# calculate test statistic value
n <- length(data);
x_bar <- mean(data);
s <- sqrt(var(data));
mu_0 <- 100;
t <- (x_bar - mu_0)/(s/sqrt(n))

# calculate rejection region
alpha <- 0.05;
t_alpha <- qt(alpha/2, n-1, lower.tail=FALSE)

# check whether should reject
reject_h0 <- t > t_alpha || t < -1*t_alpha # <- FALSE
```