Problem 1

Exercise 46 on page 411.

The 99% confidence interval for the standard deviation is:

Lower limit:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}} = \sqrt{\frac{(19-1)7.234}{38.58}} = \mathbf{1.887}$$

Upper limit:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}} = \sqrt{\frac{(19-1)7.234}{6.844}} = \mathbf{4.481}$$

This confidence interval is **not valid** whatever the nature of the distribution. It is only valid for the normal distribution. The formula used to compute the interval relies on the fact that the distribution of a squred normal distribution is a chi-squared distribution (from page 315 of textbook).

<u>Calculations:</u>

```
alpha <- 0.01
vals <- c(
  19.75, 21.25, 21.5, 22.50, 23.25, 23.5, 24.00, 24, 24.25,
  24.5,  25.00, 26.0, 26.25, 26.25, 27.0, 27.75, 28, 28.00,
  28.25, 30
)
n <- length(vals)
s2 <- var(vals)

chi_high <- qchisq(alpha/2, df=n-1)
chi_low  <- qchisq(1 - alpha/2, df=n-1)

lower <- (n-1)*s2/chi_low
upper <- (n-1)*s2/chi_high

lower_s <- sqrt(lower) # lower limit
upper_s <- sqrt(upper) # upper limit
```

Problem 2

Exercise 8 on page 435.

The null hypothesis H_0 is that the average warpage of the special laminate μ_s will be equal to the warpage of the regular laminage μ_r.

$$H_0: \quad \mu_s = \mu_r$$

The alternative hypothesis H_a is that the warpage of the special laminate μ_s is less than the warpage of the regular laminate μ_r.

$$H_a: \quad \mu_s < \mu_r$$

A **type I error** in this context would be determining that the average warpage of the special laminate is less than the average warpage of the regular laminate when it really is not.

A **type II error** in this context would be determining that the average warpage of the special laminate is not less than the warpage of the regular laminate when it really is.

Problem 3

Exercise 12a, b, c on page 436.

a)

The parameter of iterest μ is the real average breaking distance at 40mph using the new design.

$$H_0 : \quad \mu = 120 \text{ feet}$$

$$H_a : \quad \mu < 120 \text{ feet}$$

b)

The appropriate regection region is R_2. We want to reject the only if there is a reduction in braking distance so the regection region should be lower tailed.

c)

The signifigance level is:

$$\alpha = \Phi\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right) = \Phi\left(\frac{115.20 - 120}{10/\sqrt{36}}\right) = \mathbf{1.9884 \times 10^{-3}}$$

To acheive $\alpha = 0.001$:

$$\Phi(z) = 0.001 \implies z = -3.090$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \implies \bar{x} = \frac{z\sigma}{\sqrt{n}} + \mu$$

$$\bar{x} = \frac{-3.090 \cdot 10}{\sqrt{36}} = \mathbf{114.85}$$

Calculations:

```
sigma <- 10;
n <- 36;
mu <- 120;
x <- 115.20;
z <- (115.20 - 120) / (sigma/sqrt(n))
alpha <-pnorm(z)
z2 <- qnorm(0.001)
x2 <- z2 * sigma / sqrt(n) + mu
```

Problem 4

Exercise 28 on page 449.

The data suggests the true average lateral recumebcy time is less than 20 mins.

Hypotheses

$$H_0 : \quad \mu = \mu_0 = 20 \text{ mins}$$

$$H_a : \quad \mu < \mu_0$$

Rejection region

$$z \leqslant -z_\alpha$$

$$-z_\alpha = -z_{0.1} = -1.28$$

Test statistic

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{18.6 - 20}{8.6/\sqrt{73}} = -1.39$$

The null hypothesis should be rejected

$$-1.39 < -1.28 \implies z < -z_\alpha$$

Calculations:

```
n <- 73;
x_bar <- 18.6;
S <- 8.6;
mu_0 <- 20;
alpha <- 0.1;
z_alpha <- qnorm(0.1)
z <- (x_bar - mu_0)/(S/sqrt(n))
reject_h0 <- z < z_alpha # TRUE
```

Problem 5

Exercise 32a on page 449.

a)

The data does suggest the population mean differs from 100 using $\alpha = 0.05$

Hypotheses

$$H_0 : \quad \mu = \mu_0$$
$$H_a : \quad \mu \neq \mu_0$$

Rejection region

$$t \geqslant t_{\alpha/2,n-1} \quad \text{or} \quad t \leqslant -t_{\alpha/2,n-1}$$
$$t_{\alpha/2,n-1} = t_{0.05/2,12-1} = 2.201$$

Test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{98.46 - 100}{6.142/\sqrt{12}} = -0.870$$

The null hypothesis should **not** be rejected

$$-2.201 < -0.870 < 2.201 \implies -t_{\alpha/2,n-1} < t < t_{\alpha/2,n-1}$$

Calculations:

```
data <- c(
  105.6,  90.9, 91.2,  96.9,  96.5, 91.3,
  101.1, 105.0, 99.6, 107.7, 103.3, 92.4
);

# calcuate test statistic value
n <- length(data);
x_bar <- mean(data);
s <- sqrt(var(data));
mu_0 <- 100;
t <- (x_bar - mu_0)/(s/sqrt(n))

# calculate rejection region
alpha <- 0.05;
t_alpha <- qt(alpha/2, n-1, lower.tail=FALSE)

# check whether should reject
reject_h0 <- t > t_alpha || t < -1*t_alpha # <- FALSE
```