Problem 3

Exercise 18ac on page 305.

 \mathbf{a}

The approximate probability that the amount purchased is at least 12 gallons is 0.8106

$$\lim_{n \to \infty} P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leqslant z\right) = \Phi(z)$$

$$P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leqslant z\right) = P\left(\frac{12 - 11.5}{4/\sqrt{50}} \leqslant z\right) P(0.8838 \leqslant z) = \Phi(z)$$

$$\Phi(0.8838) = 0.8106$$

c)

The 95th percentile for the total amount purchased by 50 randomly selected customers is approximately 622 gallons.

$$\frac{T_0 - n\mu}{\sqrt{n}\sigma} \leqslant z$$

$$T_0 \leqslant z\sqrt{n}\sigma + n\mu$$

According to table A.3 in the textbook: $\Phi(.95) \approx 1.65$

$$T_0 \le 1.65\sqrt{50} \cdot 4 + 50 \cdot 11.5 = 621.6690$$