

Q3

Exercise 46 a, b on page 660.

a)

$$n = 20$$

$$\sum x_i = 2817.9$$

$$\bar{x} = \frac{\sum x_i}{n} = 140.9$$

$$\sum y_i = 1574.8$$

$$\bar{y} = \frac{\sum y_i}{n} = 78.74$$

$$x^* = 125$$

$$S_{xx} = 18921.8295$$

$$s = 0.665$$

$$s_{\hat{y}} = s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} = 0.1673$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x^* = 124.24$$

90% Confidence interval

$$\hat{y} \mp t_{0.1/2, n-1} \implies 124.24 \mp 1.729$$

$$(123.9613, 124.5402)$$

b)

$$\text{Prediction Interval } \hat{y} \mp t_{0.1/2, n-1} \sqrt{s^2 + s_{\hat{y}}^2}$$

$$(123.0650, 125.4365)$$

```
Beta_1 <- 0.4103377
Beta_0 <- 72.958547
n <- 20

sum_y <- 1574.8
y_bar <- sum_y/n

x_star <- 125

sum_x <- 2817.9
x_bar <- sum_x / n

S_xx <- 18921.8295
s <- 0.665
s_yhat <- s * sqrt(1/n + (x_star
  - x_bar)^2/S_xx)

y_hat <- Beta_0 + Beta_1 * x_
  star

alpha <- 0.10

t_val <- qt(alpha/2, n-1, lower.
  tail=FALSE)
ci <- c(y_hat + t_val*s_yhat, y_
  hat - t_val* s_yhat)

pi <- c(
  y_hat - t_val * sqrt(s^2 + s_
    yhat^2),
  y_hat + t_val * sqrt(s^2 + s_
    yhat^2)
)
```

It is wider than the confidence interval because the confidence interval preresents the confidence interval for the regression line expected values but the prediction interval is for the actual values of the data.