

3

$$a) S_p^2 = \frac{m-1}{m+n-2} S_1^2 + \frac{n-1}{m+n-2} S_2^2$$

$$= \frac{(6-1) 11.3^2}{6+8-2} + \frac{(6-1) 8.3^2}{6+8-2}$$

$$= \underline{93.39}$$

$$b) H_0: \sigma_1 = \sigma_2 \quad (\Delta_0 = 0)$$

$$H_a: \sigma_1 \neq \sigma_2 \quad (\Delta_0 \neq 0)$$

$$\therefore \text{rejection region} = t > t_{\alpha/2, m+n-2} \parallel t < -t_{\alpha/2, m+n-2}$$

$$t = \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{S_p^2 \left( \frac{1}{m} + \frac{1}{n} \right)}} = \frac{40.3 - 21.4 - 0}{\sqrt{93.39 \left( \frac{1}{6} + \frac{1}{8} \right)}} = 3.621$$

$$t_{0.05/2, 6+8-2} = -2.17$$

$t > t_{\alpha/2, m+n-2}$   $\therefore$  we should reject the null hypothesis at the given significance level ( $\alpha = 0.05$ ). The evidence suggests the population means are not equal.

### Problem 3

Calculations.

```
# a)
m <- 6
s1 <- 11.3
n <- 8
s2 <- 8.3
sp2 <- (m-1)*s1^2/(m + n - 2) + (n -1)*s2^2/(m + n - 2)

# b)

x_bar <- 40.3
y_bar <- 21.4

t <- (x_bar - y_bar) / sqrt(sp2*(1/m + 1/n))

alpha <- 0.05
t_val <- qt(alpha/2, m + n -2)

reject <- t > t_val || t < -1*t_val
```