ECON 4673 Assignment #1

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Problem 1

#	**Statement**	**True/False**
1	Every finite normal-form game has a pure strategy Nash equilibrium.	`false`
2	Every dominant strategy equilibrium is also a Nash equilibrium.	`true`
3	It is possible that there might exist some pure strategy Nash equilibria which did not survive the iterated elimination of strictly dominated strategies.	`true`
4	A strictly dominated strategy can also be a Nash equilibrium.	`false`
5	Every Nash equilibrium is also a dominant strategy equilibrium.	`false`
5	Every rationalizable strategy profile is also a Nash equilibrium.	`false`

Problem 2

		Player 2	
		C	D
Player 1	A	$1, 1$	$2, 0$
	B	$0, 2$	x, y

2.1)

Thse values will lead to a dominant strategy equillibrium of (A, C):

$$x < 2$$

$$y < 2$$

What follows is justification for the solution ...

Player 1's strategy A will be dominant if A has a greater payoff than strategy B for all strategies of Player 2.

$$u_1(A, s_2) > u_1(B, s_2) \ \ \forall \ \ s_2 \in S_2 = \{C, D, \sigma_2\}$$

where σ_2 is a mixed strategy with probbility $0 < p < 1$ that $s_2 = D$.

When $s_2 = C$, it is given that $s_1 = A$ has a higher payoff than $s_2 = B$.

$$u_1(A, C) = 1 > u_1(B, C) = 0$$

To satisfy the condition of dominance same must be true for $s_2 = D$:

$$u_1(A, D) = 2 > u_1(B, D) = x$$

$$x < 2$$

For the mixed strategy case:

$$u_1(A, \sigma_2) = u_1(A, C)(1 - p) + u_1(A, D)p = 1(1 - p) + 2p = 1 + p$$

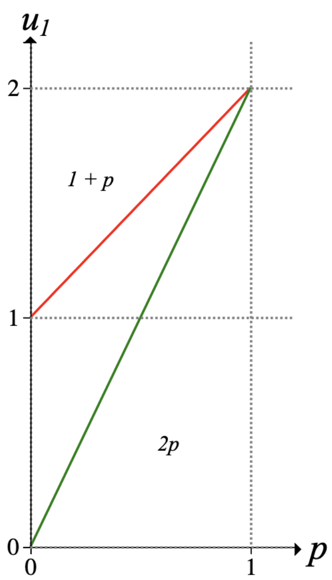
$$u_1(B, \sigma_2) = u_1(B, C)(1 - p) + u_1(B, D)p = 0(1 - p) + xp = xp$$

To satisfy the dominance condition:

$$u_1(A, \sigma_2) > u_1(B, \sigma_2)$$

$$1 + p > xp$$

The relationship is satisfied for $0 < p < 1$ if $x < 2$, which can be seen graphically:



The same analysis for player 2:

$$u_2(s1, C) > u_2(s1, D) \;\; \forall \;\; s_1 \in S_1 = \{A, B, \sigma_1\}$$

$$(\sigma_1 \text{ is a mixed strategy with probbility } 0 < p < 1 \text{ that } s_1 = B)$$

$$u_2(A, C) = 1 > u_2(A, D) = 0$$

$$u_2(B, C) = 2 > u_2(B, D) = y$$

$$y < 2$$

$$u_2(\sigma_1, C) = 1 + p$$

$$u_2(\sigma_1, D) = yp$$

$$u_2(\sigma_1, C) > u_2(\sigma_1, D)$$

$$1 + p > yp \;\; , \;\; 0 < p < 1 \implies y < 2$$

2.2)

These values will make this game into a *Prisoner's Dilemna*:

$$1 < x < 2$$

$$1 < y < 2$$

In a *Prisoner's Dilemna* there are three scenarios that can occur:

Scenario 1: Both criminals defect

The authorities offer each of the criminals the opportunity to testify on the other in exchange for a reduced sentence. In this scenario, both criminals accept the offer and testify against the other (strategy D). They receive a payoff value x_1 representing the reduced sentence.

$$u_1(D, D) = u_2(D, D) = x_1$$

Scenario 2: One criminal cooperates & the other defects

Criminal i decides to testify against the other ($s_i = D$) and is freed, which means this criminal's payoff x_{2i} is higher than in scenario 1.The other criminal $-i$ does not testify ($s_{-i} = C$), which means they get the full sentence for the crime so their payoff x_{2ii} is lower than the payoff from scenario 1.

$$u_i(C, D) = x_{2i} > x_1$$

$$u_{-i}(C, D) = x_{2ii} < x_1$$

Scenario 3: Criminals Cooperation

Each criminal decides not to testify on the other so the authorities are only able to convict them of a minor offense. The sentence is less harsh than the sentence for the main crime, which means the payoff value x_3 is higher than in scenario 1, but not as high as the case where the criminal goes free.

$$x_1 < u_1(C, C) = u_2(C, C) = x_3 < x_{2i}$$

The prisoner's dilemna in normal form:

		Criminal 1	
		D	C
Criminal 1	D	x_1, x_1	x_{2i}, x_{2ii}
	C	x_{2i}, x_{2ii}	x_3, x_3

$$x_{2ii} < x_1 < x_3 < x_{2ii}$$

Substituting the payoff from the game in the problem:

$$x_1 = 1, \;\; x_{2i} = 2, \;\; x_{2ii} = 0$$

		Criminal 1	
		D	C
Criminal 1	D	$1, 1$	$2, 0$
	C	$0, 2$	x_3, x_3

$$0 < 1 < x_3 < 2$$

Problem 3

		Player 2	
		X	Y
Player 1	A	$3, 3$	$0, 1$
	B	$5, 5$	$2, 2$

3.1)

The only pure strategy Nash equillibrium is (B, X).

The condition for Nash equilibrium

$$s_i \in BR_i(s_{-1}) \ \ \forall \ \ i$$

Strategy Y is dominated by X, which means $s_2 = X$ is always best response

$$BR_2(s_1) = \{X\} \ \ \forall \ \ S_1$$

$s_1 = B$ is the best response to X

$$BR_1(X) = B$$

3.2)

Let σ_1 be a mixed strategy with probability $0 < p < 1$ that $s_1 = A$
Let σ_2 be a mixed strategy with probability $0 < q < 1$ that $s_2 = X$

A Nash equillibrium will exist $p = 0$ and $q = 1$ which results in the strategy profile (B, X)

Player 1's expected payoff

$$v_1(\sigma_1, \sigma_2) = \sum_{(s_1, s_2)} u_2(s_1, s_2)P(s = (s_1, s_2)), \ \ (s_1, s_2) \in S_1 \bigtimes S_2$$

$$= u_1(A, X)pq + u_1(A, Y)p(1-q) + u_1(B, X)(1-p)q + u_1(B, Y)(1-p)(1-q)$$

$$= 3pq + 0p(1-q) + 5(1-p)q + 2(1-p)(1-q)$$

$$= 3q - 2p + 2$$

It is strictly decreasing with p

$$\frac{\partial}{\partial p} v_1(\sigma_1, \sigma_2) = -2$$

$v_1(\sigma_1, \sigma_2)$ is maximized when p is minimized, so for any given q, $p = 0$ is best.

$$BR_1(\sigma_2) = B, \;\; 0 < q < 1$$

The same analysis to determine player 2's expected payoff

$$v_2(\sigma_1, \sigma_2) = \sum_{(s_1,s_2)} u_2(s_1, s_2)P(s = (s_1, s_2)), \;\; (s_1, s_2) \in S_1 \times S_2$$

$$= u_2(A, X)pq + u_2(A, Y)p(1-q) + u_2(B, X)(1-p)q + u_2(B, Y)(1-p)(1-q)$$

$$= 3pq + 1p(1-q) + 5(1-p)q + 2(1-p)(1-q)$$

$$= -pq - p + 3q + 2$$

It is strictly increasing with q over the interval $0 < p < 1$

$$\frac{\partial}{\partial p} v_2(\sigma_1, \sigma_2) = -p + 3 > 0, \;\; 0 < p < 1$$

Player 2 will maximize the payoff with the largest possible value $q = 1$

$$BR_2(\sigma_1) = X, \;\; 0 < p < 1$$

Problem 4

		Player 2		
		L	M	R
Player 1	T	$1, -1$	$5, 0$	$0, 0$
	C	$0, 5$	$4, 4$	$0, 0$
	B	$0, 0$	$0, 0$	$3, 3$

4.1)

The relationalizable strategies are $\{M, R\} \times \{T, B\}$. Rationalizable strategies are those which can survive the process of iterated dominance. Strategies C and L can be dominated.

Strategy C can be dominated by a mixed strategy σ_1 with of playing T, C and B of

$$4/5 < p_T < 1$$

$$p_C = 0$$

$$p_B = 1 - p_T$$

For example $p_T = 0.85 \implies p_B = 0.15$

$$u_1(\sigma_1, L) = 0.85(1) + 0 + 0 = 0.85 > u_1(C, L) = 0$$

$$u_1(\sigma_1, M) = 0.85(5) + 0 + 0 = 4.25 > u_1(C, M) = 4$$

$$u_1(\sigma_1, R) = 0 + 0 + .15(3) = 0.45 > u_1(C, R) = 0$$

The game with C removed:

Player 2

Player 1		L	M	R
	T	$1, -1$	$5, 0$	$0, 0$
	B	$0, 0$	$0, 0$	$3, 3$

In the new game, strategy L is dominated by a mixed strategy σ_2 of playing L, M and R of

$$p_L = 0$$

$$0 < p_M < p_R < 1$$

For example $p_M = 0.5 \implies p_B = 0.5$

$$u_2(T, \sigma_2) = 0 + 0.5(5) + 0 = 0.25 > u_2(T, L) = -1$$

$$u_2(B, \sigma_2) = 0 + 0 + 3(0.5) = 1.5 > u_2(B, L) = 0$$

The resulting game with L removed:

Player 2

Player 1		M	R
	T	$5, 0$	$0, 0$
	B	$0, 0$	$3, 3$

There are no more dominated strategies.

4.2)

There are two Nash equilibria (T, M) and (B, R).

The condition for Nash equilibrium

$$u_i(s_i, s_{-1}) > u_i(s_i', s_{-1}) \;\; \forall \;\; i, \;\; s_i' \in S_i$$

(T,M) is a Nash equillibrium but (T,R) is not:

$$u_1(T, M) = 5 > u_1(T, R) = 0$$

$$u_2(T, M) = 0 \geqslant u_2(T, R) = 0$$

(B,R) is a Nash equillibrium but (B,M) is not:

$$u_1(B, R) = 3 > u_1(T, R) = 0$$

$$u_2(B, R) = 3 > u_2(B, M) = 0$$

4.3)

There is not a dominant strategy equilibrium for the game.