# Problem 1

| $s_1$ | $s_2$ | $\bar{X}$ | $S$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0.5 | 0.7071 |
| 0 | 2 | 1 | 1.4142 |
| 1 | 0 | 0.5 | 0.7071 |
| 1 | 1 | 1 | 0 |
| 1 | 2 | 1.5 | 0.7071 |
| 2 | 0 | 1 | 1.4142 |
| 2 | 1 | 1.5 | 0.7071 |
| 2 | 2 | 2 | 0 |

## 1.1)

| $\bar{X}$ | Probability |
|---|---|
| 0 | 1/9 |
| 0.5 | 2/9 |
| 1 | 3/9 |
| 1.5 | 2/9 |
| 2 | 1/ 9 |

## 1.2)

| $S$ | Probability |
|---|---|
| 0 | 3/9 |
| $\sqrt{0.5} = 0.7071$ | 4/9 |
| $\sqrt{2} = 1.4142$ | 2/9 |

calculations were performed using computer [1]

```
const means = {}
const stdevs = {}
let samples = []

for (let i of [0, 1, 2]) {
  for (let j of [0, 1, 2]) {
    let sample = [i, j]
    sampleMean = mean(sample)
    sampleStdev = stdev(sample)

    means[sampleMean] = 1 + (means[sampleMean] ?? 0)
    stdevs[sampleStdev] = 1 + (stdevs[sampleStdev] ?? 0)
    samples = [...samples, {i, j, sampleMean, sampleStdev }]
  }
}

console.table(samples)
console.table(
  Object.keys(means).sort()
    .reduce((acc, x) => [
      ...acc, { x, probabilty: `${means[x]}/${samples.length}` }], []))
console.table(
  Object.keys(stdevs).sort()
    .reduce((acc, x) => [
      ...acc, { x, probabilty: `${stdevs[x]}/${samples.length}` }], []))
```

[1] JavaScript with ECMAScript 2020 (ES11) features.

## Problem 2

Exercise 14a on page 305.

The approximate probability that the professor has finished grading before the 11:00pm news is 0.6026.

The time the professor spent grading is 250 minutes.

$$6:50 + 10mins = 7:00$$
$$11:00 - 7:00 = 4hours$$
$$4hx \cdot 60\frac{mins}{hour} = 240mins$$
$$10mins + 240mins = 250mins$$

Accoring to Central Limit Theorum

$$\lim_{n\to\infty} P\left(\frac{T_0 - n\mu}{\sqrt{n}\sigma} \leqslant z\right) = \Phi(z)$$

$n$ is large enough, according to rule of thumb

$$n = 40 > 30$$

The values from the problem into the formula

$$\frac{T_0 - n\mu}{\sqrt{n}\mu} = \frac{250mins - 40 \cdot 6mins}{\sqrt{40} \cdot 6mins} = 0.2635$$

The value from table A.3 on page 789 the textbook is used

$$\Phi(0.2635) = 0.6026$$

# Problem 3

Exercise 18ac on page 305.

## a)

The approximate probability that the amount purchased is at least 12 gallons is 0.8106

$$\lim_{n\to\infty} P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leqslant z\right) = \Phi(z)$$

$$P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leqslant z\right) = P\left(\frac{12-11.5}{4/\sqrt{50}} \leqslant z\right) P(0.8838 \leqslant z) = \Phi(z)$$

$$\Phi(0.8838) = 0.8106$$

## c)

The 95th percentile for the total amount purchased by 50 randomly selected customers is approximately 622 gallons.

$$\frac{T_0 - n\mu}{\sqrt{n}\sigma} \leqslant z$$

$$T_0 \leqslant z\sqrt{n}\sigma + n\mu$$

According to table A.3 in the textbook: $\Phi(.95) \approxeq 1.65$

$$T_0 \leqslant 1.65\sqrt{50} \cdot 4 + 50 \cdot 11.5 = 621.6690$$

# Problem 4

Exercise 1ab on page 346, but use this subset of data:

## a)

For this point estimate, $\bar{X}$ is used:

$$\hat{\mu} = \bar{X} = \frac{X_1 + ... + X_n}{n} = 113.7$$

The estimated standard error is the sample standard deviation divided by $\sqrt{n}$

$$\hat{\sigma}_{\bar{X}} = \frac{s}{\sqrt{n}} = 13.0$$

```
const IQ = [
   82,  96, 102, 103, 106, 107, 108, 108, 108, 109, 110, 110, 111,
  113, 113, 113, 115, 115, 118, 119, 121, 122, 127, 136, 140, 146,
]
const xBar = mean(IQ)
const stdErr = stdev(IQ) / Math.sqrt(IQ.length)
console.table([{ xBar, stdErr }])
```

## b)

The estimate of the value that separates the lowest 50% from the highest 50%, is the sample median.

$$\hat{m} = \tilde{X} = \frac{113 - 111}{2} = 112$$

The estimated standard error determined by bootstrapping method was 2.5

```
const boostrapSample = sample => {
  return sample.map(() => {
    const r = Math.floor(Math.random() * sample.length)
    return sample[r]
  })
}

const B = 200;
const bootstrapMedians = []
for (let i = 0; i < B; i++) {
  bootstrapMedians.push(median(boostrapSample(IQ)))
}
const SB = Math.sqrt((1/(B - 1) * variance(bootstrapMedians)))
console.log(SB)
```

# Problem 5

Exercise 8a on page 348.

## a)

The estimate of $p$, the proportion which are not defective

$$\hat{p} = 1 - \frac{X}{n} = 1 - \frac{12}{80} = 0.85$$

The estimate standard error

$$\hat{\sigma}_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.85 \cdot 0.15}{80}} = 0.0399$$

# Problem 6

Exercise 12 on page 348.

For an unbiased estimator

$$E(\hat{\theta}) - \theta = 0 \implies \theta = E(\hat{\theta})$$

For the estimator in the problem

$$\begin{aligned} E(\hat{\sigma^2}) &= E\left(\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}\right) \\ &= \frac{1}{n_1+n_2-2} E\left((n_1-1)S_1^2 + (n_2-1)S_2^2\right) \\ &= \frac{1}{n_1+n_2-2}\left(E((n_1-1)S_1^2) + E((n_2-1)S_2^2)\right) \\ &= \frac{(n_1-1)E(S_1^2) + (n_2-1)E(S_2^2)}{n_1+n_2-2} \end{aligned}$$

In example 7.6 of the textbook, it is shown that

$$E(S^2) = \sigma^2$$

In the problem it states both types of fertilizer have the same variance

$$E(S_1^2) = E(S_2^2) = \sigma^2$$

Substitute into above to show the estimate is unbiased

$$E(\hat{\sigma^2}) = \frac{(n_1-1)\sigma^2 + (n_2-1)\sigma^2}{n_1+n_2-2} = \frac{(n_1-1+n_2-1)\sigma^2}{n_1+n_2-2} = \frac{(n_1+n_2-2)\sigma^2}{n_1+n_2-2} = \sigma^2$$