Problem 1

s_1	s_2	$\bar{X}$	S
0	0	0	0
0	1	0.5	0.7071
0	2	1	1.4142
1	0	0.5	0.7071
1	1	1	0
1	2	1.5	0.7071
2	0	1	1.4142
2	1	1.5	0.7071
2	2	2	0

1.1)

$\bar{X}$	Probability
0	1/9
0.5	2/9
1	3/9
1.5	2/9
2	1/ 9

1.2)

S	Probability
0	3/9
$\sqrt{0.5} = 0.7071$	2/9
$\sqrt{2} = 1.4142$	3/9

calculations were performed using computer [1]

```
const mean = sample =>
  sample.reduce((a, b) => a+b, 0) / sample.length

const stdev = (sample, sm=mean(sample)) =>
  Math.sqrt(sample.reduce((a, b) => a + Math.pow(b - sm, 2), 0))

const means = {}
const stdevs = {}
let samples = []

for (let i of [0, 1, 2]) {
  for (let j of [0, 1, 2]) {
    let sample = [i, j]
    sampleMean = mean(sample)
    sampleStdev = stdev(sample)

    means[sampleMean] = 1 + (means[sampleMean] ?? 0)
    stdevs[sampleStdev] = 1 + (stdevs[sampleStdev] ?? 0)
    samples = [...samples, {i, j, sampleMean, sampleStdev }]
  }
}

console.table(samples)
console.table(
  Object.keys(means).sort()
    .reduce((acc, x) => [
      ...acc, { x, probabilty: `${means[x]}/${samples.length}` }], []))
console.table(
  Object.keys(stdevs).sort()
    .reduce((acc, x) => [
      ...acc, { x, probabilty: `${stdevs[x]}/${samples.length}` }], []))
```

[1] JavaScript with ECMAScript 2020 (ES11) features.

Problem 2

Exercise 14a on page 305.

The approximate probability that the professor has finished grading before the 11:00pm news is 0.6026.

The time the professor spent grading is 250 minutes.

$$6:50 + 10mins = 7:00$$

$$11:00 - 7:00 = 4hours$$

$$4hx \cdot 60\frac{mins}{hour} = 240mins$$

$$10mins + 240mins = 250mins$$

Accoring to Central Limit Theorum

$$\lim_{n\to\infty} P\left(\frac{T_0 - n\mu}{\sqrt{n}\sigma} \leqslant z\right) = \Phi(z)$$

n is large enough, according to rule of thumb

$$n = 40 > 30$$

The values from the problem into the formula

$$\frac{T_0 - n\mu}{\sqrt{n}\mu} = \frac{250mins - 40 \cdot 6mins}{\sqrt{40} \cdot 6mins} = 0.2635$$

The value from table A.3 on page 789 the textbook is used

$$\Phi(0.2635) = 0.6026$$

Problem 3

Exercise 18ac on page 305.

a)

The approximate probability that the amount purchased is at least 12 gallons is 0.8106

$$\lim_{n\to\infty} P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leqslant z\right) = \Phi(z)$$

$$P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leqslant z\right) = P\left(\frac{12-11.5}{4/\sqrt{50}} \leqslant z\right) P(0.8838 \leqslant z) = \Phi(z)$$

$$\Phi(0.8838) = 0.8106$$

c)

The 95th percentile for the total amount purchased by 50 randomly selected customers is approximately 622 gallons.

$$\frac{T_0 - n\mu}{\sqrt{n}\sigma} \leqslant z$$

$$T_0 \leqslant z\sqrt{n}\sigma + n\mu$$

According to table A.3 in the textbook: $\Phi(.95) \approxeq 1.65$

$$T_0 \leqslant 1.65\sqrt{50} \cdot 4 + 50 \cdot 11.5 = 621.6690$$