Problem 3

Exercise 30, on page 360.

a)

The mles are

$$\hat{\lambda} = \frac{n}{\sum x_i + n \cdot \min(x_i)} \qquad \hat{\theta} = \min(x_i)$$

The joint PMF is

$$L(\lambda, \theta) = f(x_1, ..., x_n; \lambda, \theta) = (\lambda e^{-\lambda(x_1-\theta)}).....(\lambda e^{-\lambda(x_n-\theta)}) = \begin{cases} \lambda^n e^{-\lambda \sum x_i - \lambda n\theta}, & x \geqslant \theta \\ 0 & \text{otherwise} \end{cases}$$

$$ln(L(\lambda, \theta)) = \begin{cases} n \ln \lambda - \lambda \sum x_i - \lambda n\theta, & x \geqslant \theta \\ 0 & \text{otherwise} \end{cases}$$

Set the partial derivative to 0 to get an equation for the maximum likelihood

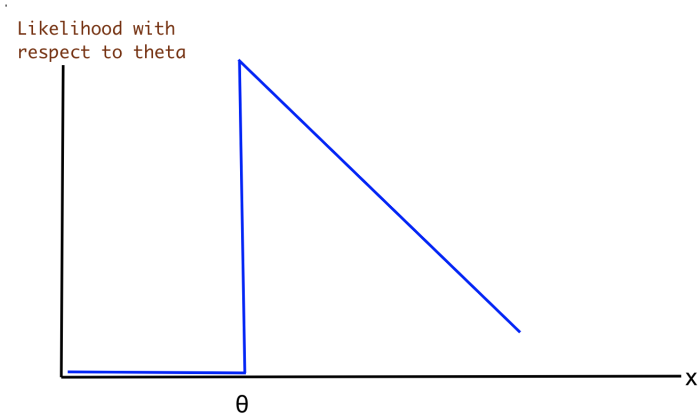
$$\frac{\partial}{\partial \lambda}\left(n \ln \lambda - \lambda \sum x_i - \lambda n\theta\right) = \frac{n}{\lambda} - \sum x_i - n\theta = 0$$

$$\implies \lambda = \frac{n}{\sum x_i + n\theta} \tag{3}$$

Setting the partial derivative with respect to θ would not work well

$$\frac{\partial}{\partial \theta}\left(n \ln \lambda - \lambda \sum x_i - \lambda n\theta\right) = -\lambda n = 0$$

Plotting the likehood with respect to θ, it can be seen that it is maximized by choosing the $\hat{\theta} = \min(x_i)$



Substitute into equation 3 to get the estimator for λ

$$\hat{\lambda} = \frac{n}{\sum x_i + n \cdot \min(x_i)}$$

b)

$$\hat{\theta} = \min(x_i) = 0.64$$

$$\hat{\lambda} = \frac{n}{\sum x_i + n \cdot \min(x_i)} = \frac{10}{55.8 + 10 \times 0.64} = 0.161$$