Eksperimental Fysik 2 - Øvelse 2
The Michelson-Morley Interferometer

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Introduction

In this experiment we construct the Mach-Zender interferometer and use this setup to examine two distinct scenarios. In the first part we look at the effects of polarizing one of the beams. In the second part we examine the interference pattern as one of the beams passes is passed through a gas with varying pressure.

Theory

Waveplate and Polarization

We shall use the following notation to describe $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{s}}$-polarized waves, where these vectors describe orthogonal planes of polarization. The waves have frequency ω and phase ρ

$$\boldsymbol{E_1} = \|E_1\|\hat{\boldsymbol{p}}\cos(\omega t - \rho), \qquad \boldsymbol{E_2} = \|E_2\|\hat{\boldsymbol{s}}\cos(\omega t - \rho).$$

a and cleary,

$$\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}} = 1, \quad \hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{s}} = 1, \quad \hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{p}} = 0.$$

We can define the angle θ, to describe waves polarized somewhere in between $\boldsymbol{p}$ and $\boldsymbol{s}$.

$$\boldsymbol{E} = \|E\|\left(\hat{\boldsymbol{p}}\cos\theta + \hat{\boldsymbol{s}}\sin\theta\right).$$

We are now interested in measuring the interference pattern two waves, where one wave is held fixed at $\theta = 0$ ($\hat{\boldsymbol{p}}$-polarized) and the other wave is allowed to vary in θ. We assume that the waves have equal amplitude E. They need not however be in phase. We will denote their phases ρ_1, ρ_2. The intensity of this wave is, as always, given by,

$$\begin{aligned} I &= c\epsilon_0\left(\boldsymbol{E_1} + \boldsymbol{E_2}\right)^2 \\ &= E^2\left(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}\right)\cos\left(\omega t - \rho_1\right)^2 + E^2\left(\cos\theta\hat{\boldsymbol{p}} + \sin\theta\hat{\boldsymbol{s}}\right)^2\cos\left(\omega t - \rho_2\right)^2 \\ &\quad + 2E^2\hat{\boldsymbol{p}}\cdot\left(\cos\theta\hat{\boldsymbol{p}} + \sin\theta\hat{\boldsymbol{s}}\right)\cos\left(\omega t - \rho_1\right)\cos\left(\omega t - \rho_2\right) \\ &= E^2\cos\left(\omega t - \rho_1\right)^2 + E^2\cos\left(\omega t - \rho_2\right) \\ &\quad + 2E^2\cos\theta\cos\left(\omega t - \rho_1\right)\cos\left(\omega t - \rho_2\right). \end{aligned}$$

We will only be able to measure the time-average of this intensity. The time average of a periodic function is given by the following function,

$$\langle f\rangle = \frac{1}{\tau}\int_0^\tau f\, d\tau.$$

Before continuing, we notice that the first 2 terms, both integrate to pi over one cuycle,

$$\begin{aligned} \langle I\rangle &= E^2 c\epsilon_0\frac{1}{2\pi}\left(2\pi + \cos\theta\int_0^{2\pi}\cos\left(\omega t - \rho_1\right)\cos\left(\omega t - \rho_2\right)d\left(\omega t\right)\right) \\ &= E^2 c\epsilon_0\left(1 + \cos\theta\cos\left(\Delta\rho\right)\right) \end{aligned}$$

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Which gives us our main result. The amplitude of the intereference should vary sinusoidally in θ.

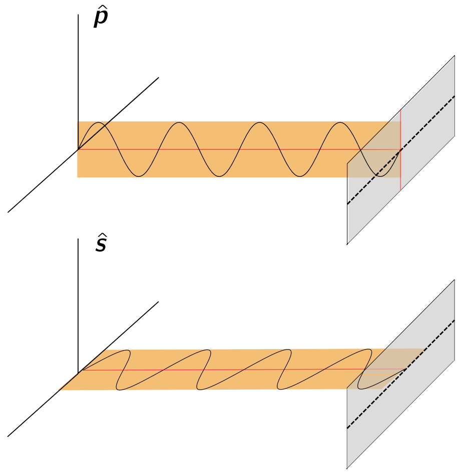

Figure 1: The two orthogonal polarizations of light; $\boldsymbol{s}$ and $\boldsymbol{p}$

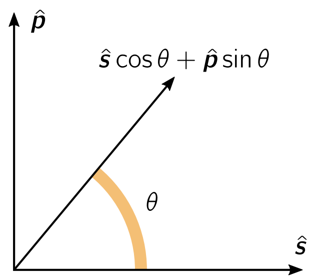

Figure 2: Light may also be polarized somewhere between $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{p}}$. We can describe these polarizations with an angle θ.

Figure 3: waveplate

Waveplate

In the experiment we make use of a $\lambda/2$-waveplate. This is a tool that allows us to rotate the polarization. The waveplate can be set to an angle θ_W, as the light passes through the plate, what happens can be visualized as the light being mirrored in a plane lying at θ_W. Fig **3** illustrates this. In practice, this functions as doubling the polarization angle. So,

$$\theta = 2\theta_W.$$

Combining this, with our knowledge from the previous section, we expect the amplitude of the interference pattern to vary in the following way,

$$A(\theta_W) = k\cos(2\theta_W + \rho).$$

Where k, ρ are constants.

Pressure dependency

When a light beam passes through an object with a refractive index n, the optical path length of that beam is given by:

$$OP = n \cdot L.$$

Where OP is the optical path length and L is the geometrical path length. If the object is filed with a gas the index of refraction is affected by the pressure of the gas, which then also affects the optical path length.

$$\Delta OP = \Delta n \cdot L.$$

When it comes to air the refractive index is very close to one and for low pressures it is proportional to the change in pressure.

$$n_{air} = 1 + kp.$$

where k is a constant. This can be shown be considering the Lorentz-Lorenz equation and the ideal gas equation.

$$\frac{n^2 - 1}{n^2 + 2} = \frac{4}{3}\pi \frac{n_{mol}}{V}\alpha_{mol}$$
$$pV = nRT$$

Here n_{mol} is the amount of molecules, α_{mol} is the polarisability of the molecules, P is pressure, V is volume, R is the universal gas-constant and T is the temperature. n^2 is very close to 1 for many gases, so we can rewrite the equations as:

$$n \approx \sqrt{\frac{\pi p \alpha_{mol}}{RT} + 1}.$$

In this experiment it is only the pressure which varies and the change is very small, so a Taylor expansion of this equation with respect to pressure results in a linear relationship.
We can now justify the change in the refractive index to be:

$$\Delta n = k \Delta p.$$

The change in pressure is then related to the optical path length by.

$$\Delta OP = k \Delta p \cdot L.$$

This can then be related to the phase shift.

$$\Delta \rho = \frac{2\pi}{\lambda} k \Delta p \cdot L.$$

The wavelength in the media is also dependent on the index of refraction but it such a small factor so an approximation is made.

$$\Delta \rho \approx \frac{2\pi}{\lambda_0} k \Delta p \cdot L.$$

Experimental Method

For the experiment we built a Michelson-Morley method. We were then only required to make minor adjustments to switch between each part of the experiment. First, the laser is passed through a beamsplitter, each beam then hits a mirrror. On one of the mirrors is mounted a piezo-electric. This allows us to change the path-length in a systematic fashion. The rays are then collected by another beam-splitter and aimed towards a detector. A sketch of this setup can be seen in the figure.
For the first part of the experiment we placed a waveplate in front of one the mirrors. We were then able to change the polarization for one beam of light. We could then take measurements at different angles.
For the second part of the experiment, we had the piezoelectric turned off. We did this, as we were measuring a frequency change rather than intensity. We had a transparent gas-chamber placed in front of the beams of light. This gas-chamber was connected to a bicycle pump with which we could control the air-pressure. We then took a measurement where we varied the pressure throughout.

Results

Waveplate

We took measurements for different angles of the waveplate. For each angle we measured the intensity of the light beam striking the detector. We then plot these against one another and fit the data to a sinusoidal function.

We expected the data to have a frequency of 2. Our data is displayed in figure, Our data clearly follows a sinusoidal function, however the measured frequency $b = 4$ is twice what we would expect. Furthermore, our estimated error in this measurement is 0.001, so we can quite certain that our data has a frequency in the vicinity of 4 times the angle indicated on the waveplate.

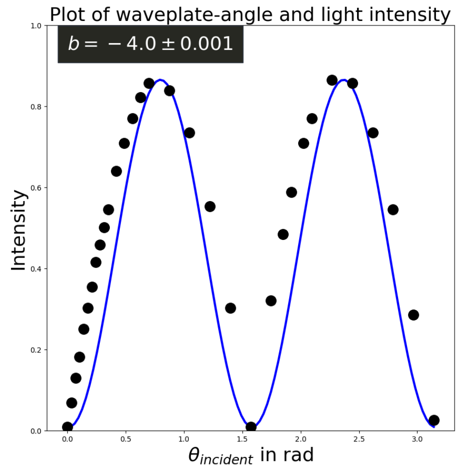

Figure 4: The data collected from the waveplate. Our datapoints are marked with black. The error on these points is small enough to be contained in the points. The fit is marked with the blue line. The error in the fit is small enough to be contained in the line

Discussion

We first discuss the waveplate-experiment. Overall, this experiment went very well. We were able to obtain datapoints, that had very little error. The nature of the experiment also allowed us to take many datapoints, thereby giving us a detailed picture of the sinusoidal behavior we were measuring. In spite of this we did end up measuring a frequency twice as large as expected. The small error in this frequency (0.001), suggests a systematic error. We believe this error be one of two possibilities. Either the waveplate is mislabelled or we have misread it. Our datas strong adherence to the sinusoidal function, is however a great indication that the theory is correct.

Conclusion

Overall, this was a succesful experiment. The waveplate experiment demonstrated the strength of the setup, as we were able to minimize