In [ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_validate
```

In [ ]:
```
from ucimlrepo import fetch_ucirepo

# fetch dataset
automobile = fetch_ucirepo(id=10)

# data (as pandas dataframes)
DATA = automobile.data.features

FEATURES_OF_INTEREST = ["wheel-base", "length", "width", "height", "curb-weight", "
"bore", "stroke", "compression-ratio", "horsepower", "peak-rpm", "city-mpg", "highw
TARGET = "price"

x = DATA[FEATURES_OF_INTEREST]
y = DATA[TARGET]


# remove rows with unknown values (for X and Y)
initialRows = x.shape[0]
x = x.dropna()
y = y.dropna()
nonNullIndices = y.index.intersection(x.index)
x = x.loc[nonNullIndices]
y = y.loc[nonNullIndices]
finalRows = x.shape[0]
rowsDeleted = initialRows-finalRows
print("Rows deleted:",rowsDeleted)
print("Remaining Observations:",x.shape[0])
```

```
Rows deleted: 10
Remaining Observations: 195
```

# Cross Validation

In [ ]:
```
num_cv = 40
folds = 5
def cross_validation_analysis(model_features, targets):

    # generate cv results
    predictions = np.zeros((num_cv*folds, targets.shape[0]))
    for i in range(num_cv):

        cv_results = cross_validate(LinearRegression(), model_features, targets, cv
        for i2 in range(folds):
            model = cv_results["estimator"][i2]
            # each row is one model realization
            predictions[i+i2:] = model.predict(model_features)
```

```
        break

    # each row is an observation
    predictions = predictions.T

    # sort observation by target value
    sortY = sorted(zip(targets, predictions), key=lambda i: i[0])
    sorted_targets = np.array([i[0] for i in sortY])
    sorted_predictions = np.array([i[1] for i in sortY])

    # average prediction for each observation
    avg_prediction = np.mean(sorted_predictions, axis=1)

    # plot avgpred vs true
    # plot 20 model preds vs true
    fig,ax = plt.subplots()
    for i in range(20):
        modelx = sorted_predictions[:, i]
        ax.scatter(sorted_targets,modelx,alpha=0.3)

    ax.scatter(sorted_targets,avg_prediction, alpha=0.9, label="avg prediction")
    ax.set_ylabel("Predicted Price")
    ax.set_xlabel("True Price")
    ax.set_title("Predicted vs True Price")
    ax.legend()
    # plt.show()

    # calc mse of each prediction
    mses = np.zeros((200,1))
    for model_idx in range(200):
        mses[model_idx] = np.mean((targets-predictions[:,model_idx])**2)
        break

    expectedMSE = np.mean(mses)
    print("expected MSE:",expectedMSE)

    # calc variance
    variance = 0
    for y_i in range(avg_prediction.shape[0]):
        temp = 0
        for model_l in range(num_cv*folds):
            temp += (sorted_predictions[y_i,model_l]-avg_prediction[y_i])**2

        variance += temp/(num_cv*folds)
    variance /= avg_prediction.shape[0]
    print("variance:",variance)

    print("bias^2 + noise variance:",expectedMSE-variance)
```

# a)

Model 1) $price = 1/cityMpg + peakRpm + height$

E{MSE}: 157710.72873780655

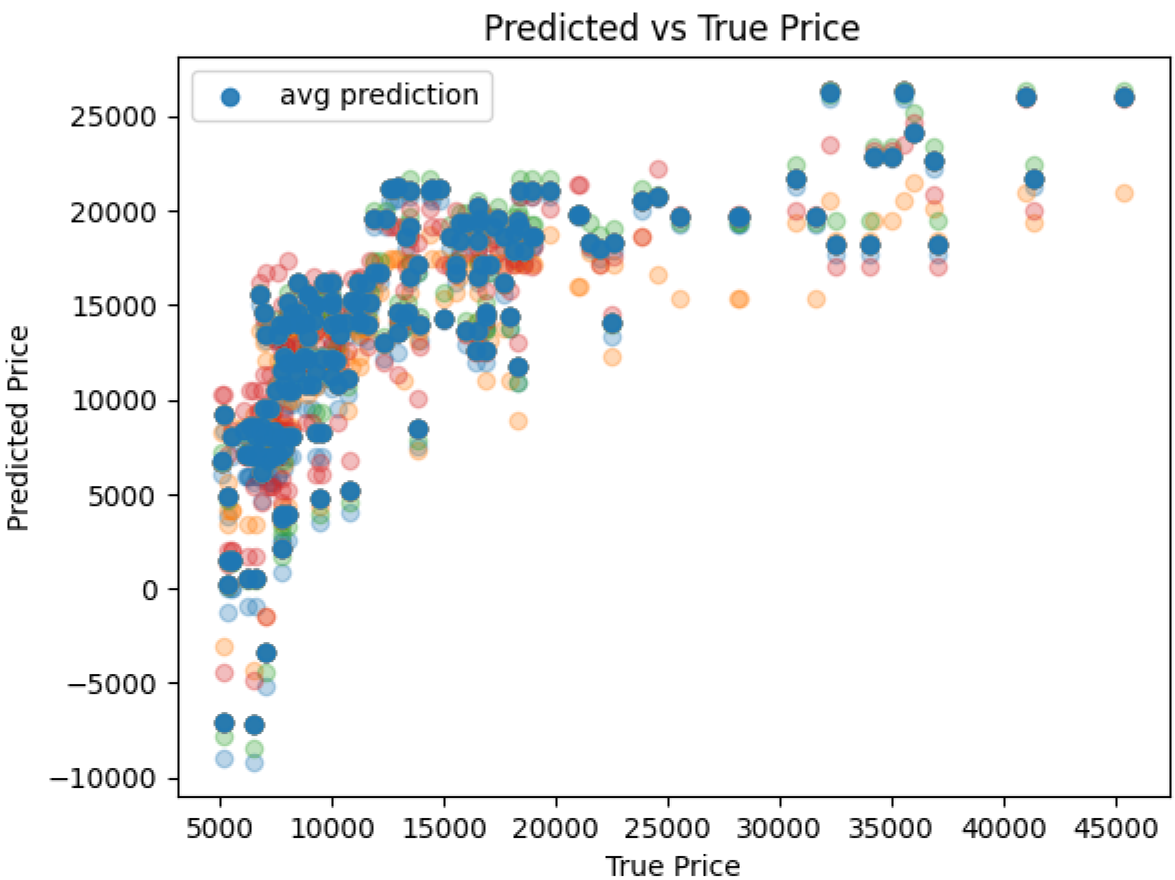
variance: 36323.41642162207

$bias^2$ + noise variance: 121387.31231618449

This model has much more bias compared to variance. This means that this model has greater consistency but more systematic error across data sets.

```
In [ ]: m1_features = x[["city-mpg","peak-rpm","stroke"]]
        m1_features["city-mpg"].transform(lambda i: 1/i)

        cross_validation_analysis(m1_features, y)
```

```
expected MSE: 157710.72873780655
variance: 36323.41642162207
bias^2 + noise variance: 121387.31231618449
```




## b)

Model 2) $price = 1/cityMpg + peakRpm + compressionRatio$

E{MSE}: 135954.56410878792

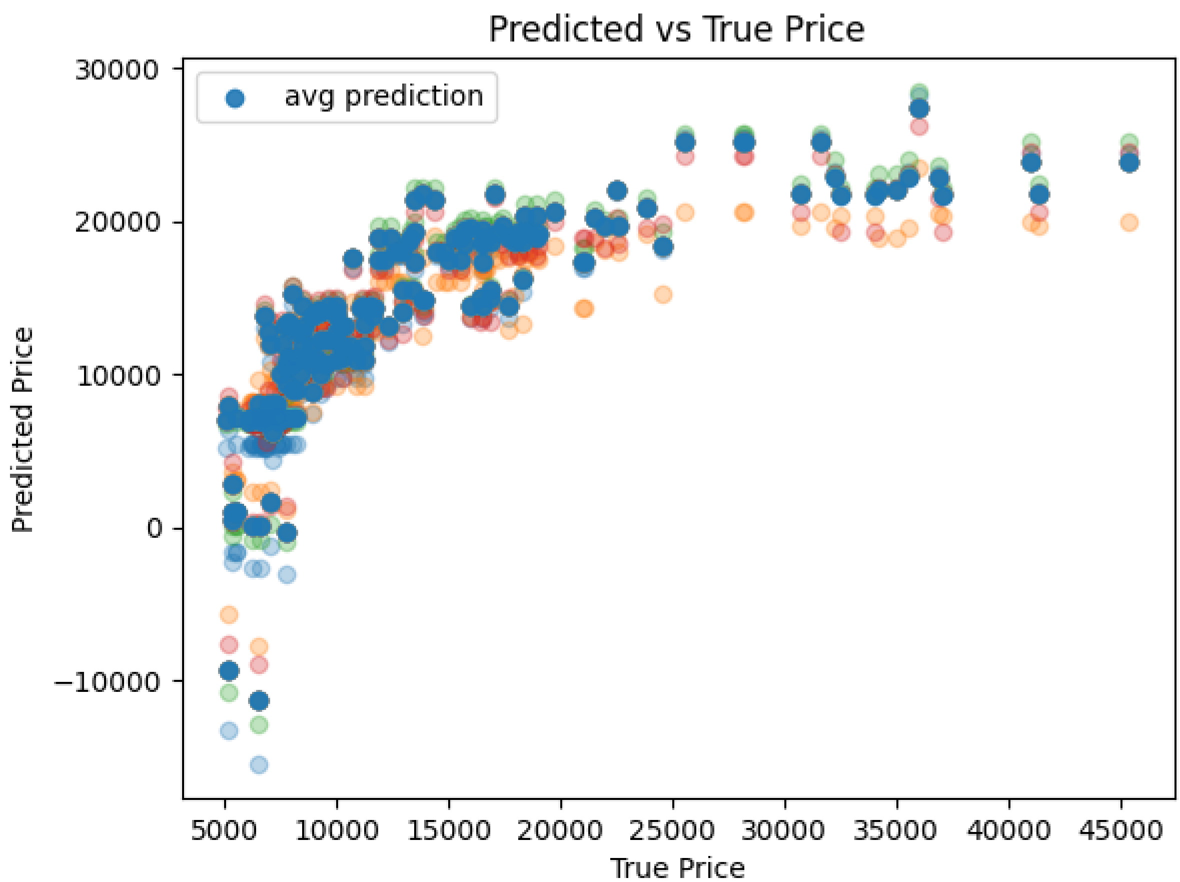
variance: 30485.6260777169

$bias^2$ + noise variance: 105468.93803107101

This model has much more bias compared to variance. This means that this model has greater consistency but more systematic error across data sets.

In [ ]:
```
m2_features = x[["city-mpg","peak-rpm","compression-ratio"]]
m2_features["city-mpg"].transform(lambda i: 1/i)

cross_validation_analysis(m2_features, y)
```

```
expected MSE: 135954.56410878792
variance: 30485.6260777169
bias^2 + noise variance: 105468.93803107101
```




## c)

Model 3) $price = 1/cityMpg + curbWeight^2 + stroke$

E{MSE}: 92649.62806278672

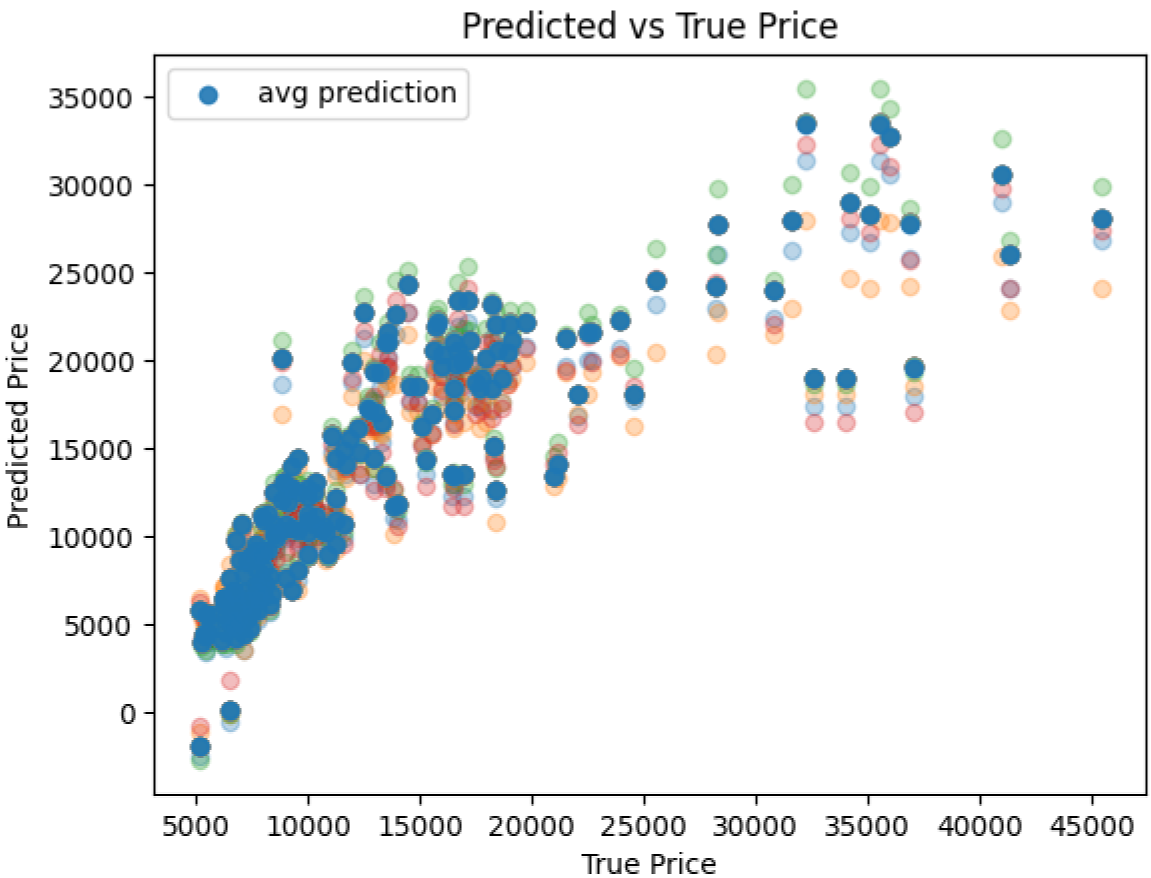
variance: 30132.15237409223

$bias^2$ + noise variance: 62517.475688694496

This model has much more bias compared to variance. This means that this model has greater consistency but more systematic error across data sets.

```
In [ ]: m3_features = x[["city-mpg","peak-rpm","curb-weight"]]
        m3_features["city-mpg"].transform(lambda i: 1/i)
        m3_features["curb-weight"].transform(lambda i: i**2)

        cross_validation_analysis(m3_features, y)
```

```
expected MSE: 92649.62806278672
variance: 30132.15237409223
bias^2 + noise variance: 62517.475688694496
```




## d)

All my models had similar variances of 36323, 30485, and 30132. The third model had lower expected MSE by about 30% so its bias was a lot lower than the first two. To compare the biases and variances I compared the proportion of bias and variance. Model 1 had 3.34 times more bias, Model 2 had 3.46 times more bias, and Model 3 had 2.07 times more bias. Thus, Model 3 appears to be the most complex as it has proportionally more bias than the other models. Model 3 also has the lowest variance overall. The next most complex model appears to be Model 1 since it has 3.34 times more bias instead compared to Model 2 which had 3.46 times more bias. In conclusion, the order of the models from least to most complex is Model 2, Model 1, and finally Model 3.

## e)

I think I will still select Model 3. This is the same decision as the one I made in Homework 1. All of the models had similar variances but this model had about 30% less bias + noise variance. This means when predicting new values, this model will likely be more generalizable to new data compared to the other 2 models.

# Mini Project check in

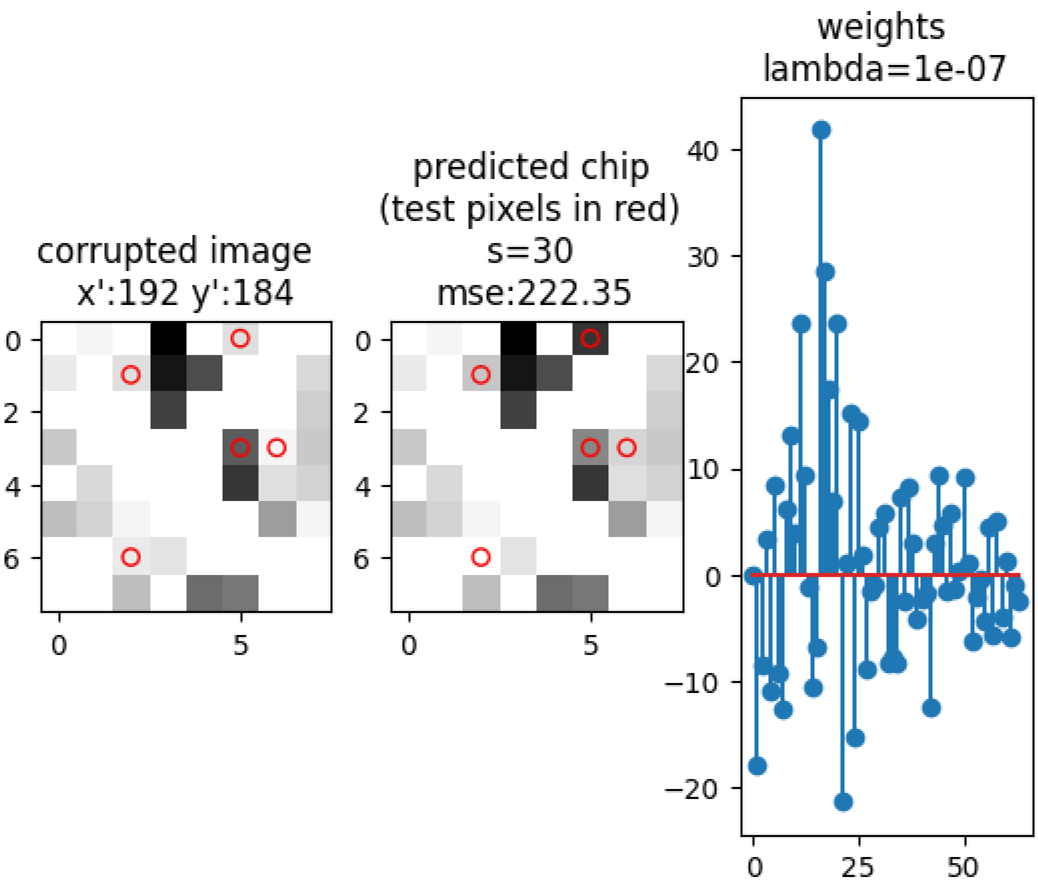
In [ ]:
```
import matplotlib.pyplot as plt
import numpy as np
from PIL import Image
import math
from scipy import signal


# load image
boat = np.asarray(Image.open('fishing_boat.bmp'), dtype=np.float64)

# my section
firstname = len('albert')-1
lastname = len('yuan')-1
x = 8*firstname + 1
y = 8*lastname + 1
K = 8
my_chip = boat[x:x+K,y:y+K]

# basis vector matrix
def basis(u, v, x, y, P, Q):
    alpha = np.sqrt(1/P) if u == 0 else np.sqrt(2/P)
    beta = np.sqrt(1/Q) if v == 0 else np.sqrt(2/Q)
    return alpha*beta*math.cos(math.pi*(2*x-1)*(u-1)/(2*P))*math.cos(math.pi*(2*y-1

def getBasisChip(imgShape, u,v):
    P, Q = imgShape
    img = np.zeros((P,Q))

    # change x and y to be 1-indexed
    # go down by columns
    for y in range(1,Q+1):
        for x in range(1,P+1):
            img[y-1][x-1] = basis(u, v, x, y, P, Q)
    return img

P, Q = my_chip.shape
basisVectorMatrix = np.zeros(((P*Q)*(P*Q)))
i = 0
for x in range(1,P+1):
    for y in range(1,Q+1):
        for v in range(1,Q+1):
            for u in range(1,P+1):
                basisVectorMatrix[i] = basis(u, v, x, y, P, Q)
                i += 1
```

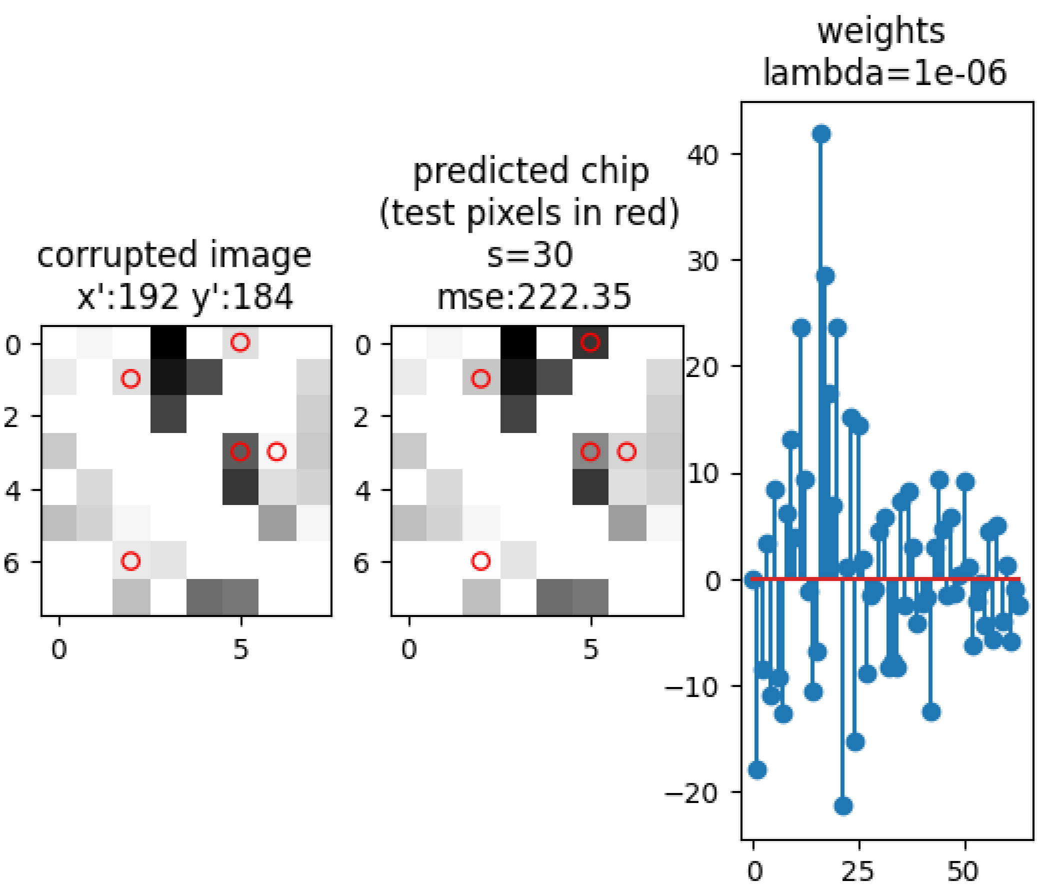
```
basisVectorMatrix = basisVectorMatrix.reshape((P*Q),(P*Q))
```

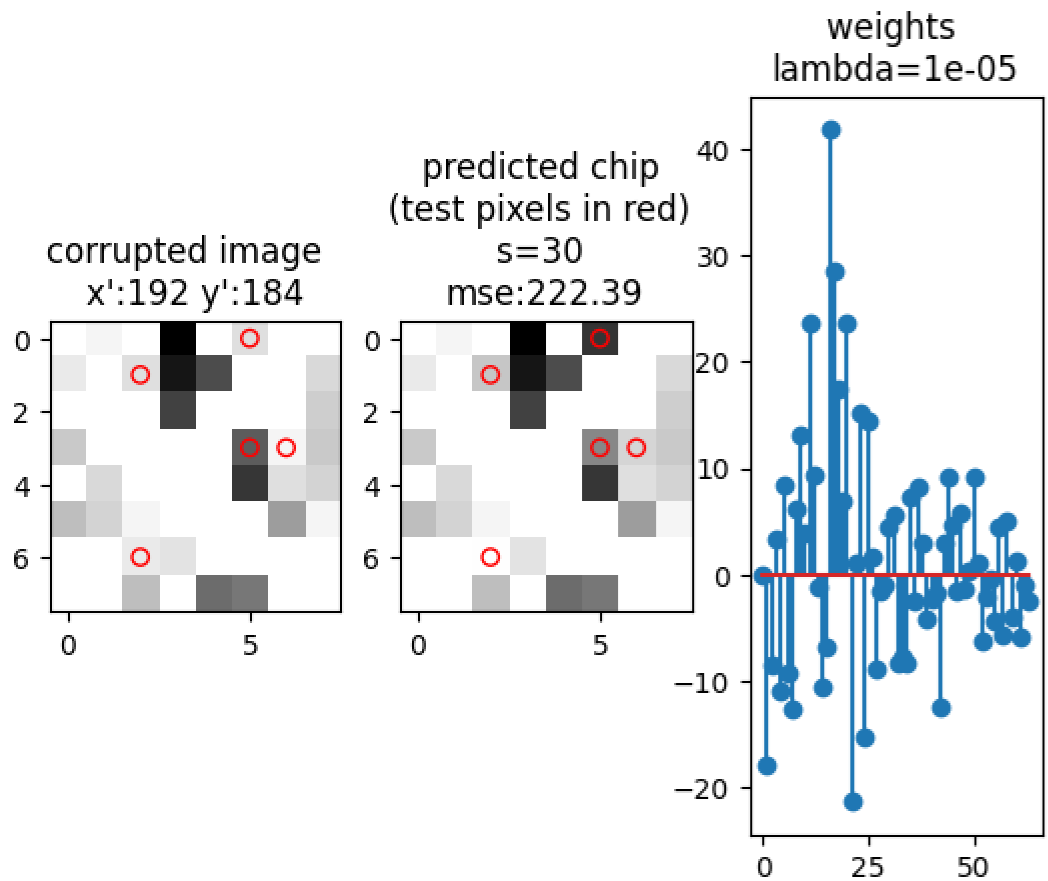
# a)

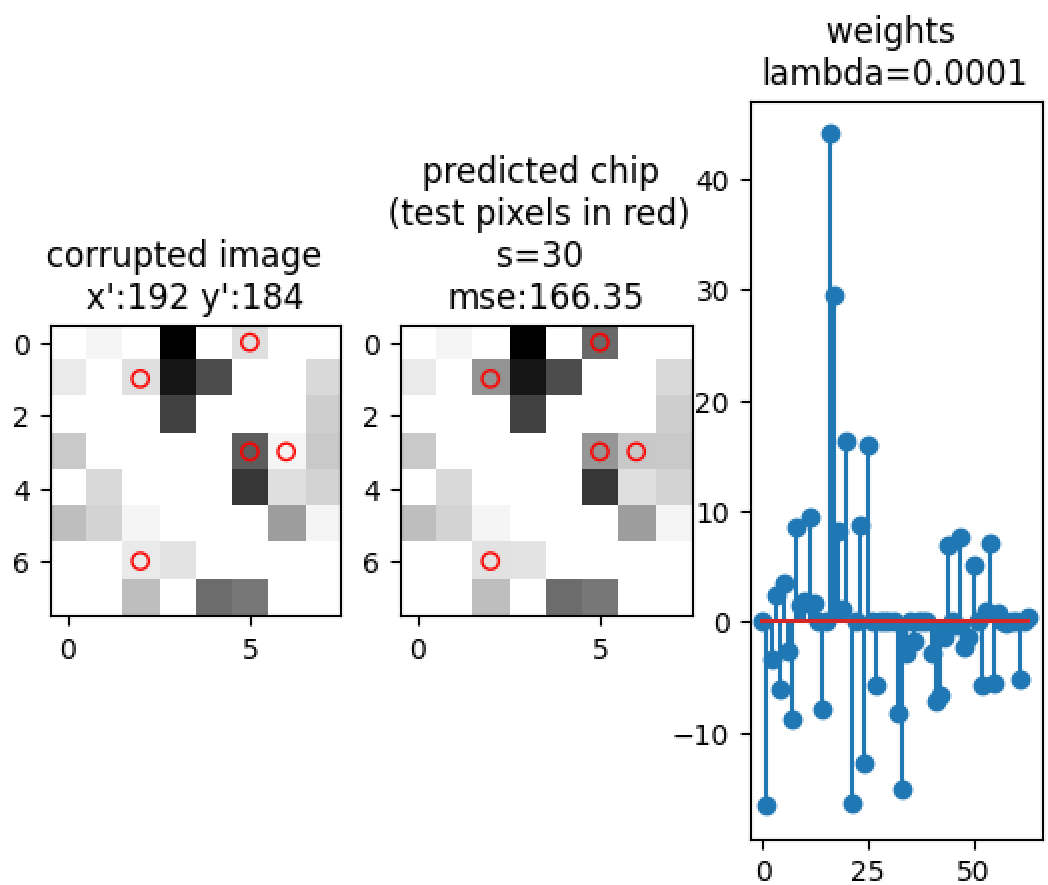
In [ ]:
```
from sklearn.linear_model import Lasso
# make s_30
def makeTrainTestImg(src, s, m):
    totalPixels = src.shape[0] * src.shape[1]
    trainimg = np.full((src.shape[0],src.shape[1]), np.NaN)
    testimg = np.full((src.shape[0],src.shape[1]), np.NaN)
    fullimg = np.full((src.shape[0],src.shape[1]), np.NaN)

    idx_to_keep = np.random.choice(totalPixels,s, replace=False)

    for i in idx_to_keep[m:]:
        x = i // 8
        y = i-(x*8)
        trainimg[x][y] = src[x][y]
        fullimg[x][y] = src[x][y]

    for i in idx_to_keep[:m]:
        x = i // 8
        y = i-(x*8)
        testimg[x][y] = src[x][y]
        fullimg[x][y] = src[x][y]

    return trainimg, testimg, fullimg

def createUnderdeterminedSystem(img):
    sensed_idx= ~np.isnan(img)
    img_noNan = img[sensed_idx]

    underdeterminedBVM = np.zeros((img_noNan.shape[0],basisVectorMatrix.shape[1]))
    i = 0
    P2 = sensed_idx.shape[0]
    Q2 = sensed_idx.shape[1]
    for x in range(P2):
        for y in range(Q2):
            if sensed_idx[x][y]:
                underdeterminedBVM[i] = basisVectorMatrix[x*P2 + y]
                i+=1
    return underdeterminedBVM, img_noNan

lambdas = np.logspace(-7, 2, 10)
S = 30
m = S//6
M = 20

chip_train, chip_test, chip_full  = makeTrainTestImg(my_chip, S, m)
chip_train_BVM, chip_train_noNAN = createUnderdeterminedSystem(chip_train)
MSEs = []
for l in lambdas:
    model = Lasso(l).fit(chip_train_BVM, chip_train_noNAN)
```

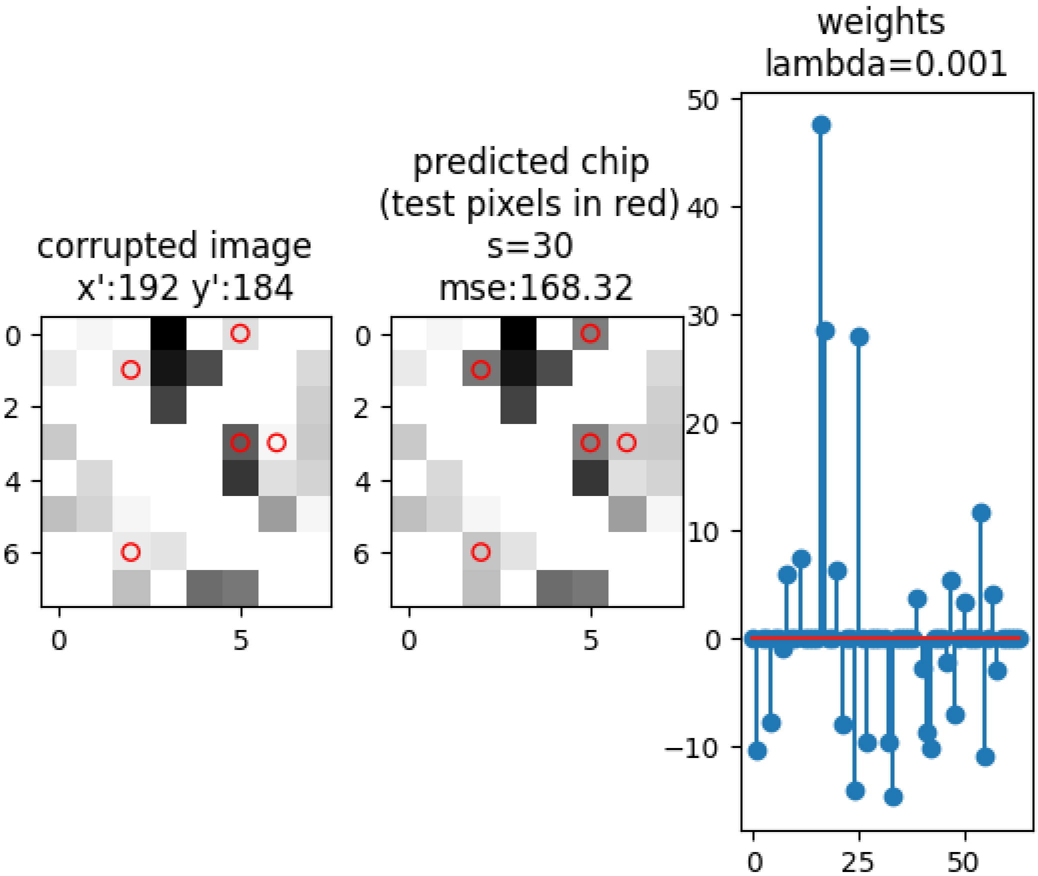
```
    weights = np.reshape(model.coef_, (model.coef_.shape[0],1))
    pred_chip = np.reshape(model.predict(basisVectorMatrix), (P,Q))

    # only calc the chip_test pixels
    test_indices= ~np.isnan(chip_test)
    chip_test_noNan = chip_test[test_indices]
    pred_chip_test_noNan = pred_chip[test_indices]
    mse = np.mean((chip_test_noNan-pred_chip_test_noNan)**2)
    MSEs.append(mse)

    # for displaying diff between true and predicted test pixels
    displaypred = pred_chip
    chipNans = np.isnan(chip_full)
    for i in range(chipNans.shape[0]):
        for j in range(chipNans.shape[1]):
            if chipNans[i][j]:
                displaypred[i][j] = np.NaN
    testpts_x = []
    testpts_y = []
    for i in range(test_indices.shape[0]):
        for j in range(test_indices.shape[1]):
            if test_indices[i][j]:
                testpts_x.append(j)
                testpts_y.append(i)

    fig, ax = plt.subplots(1,3)
    ax[0].imshow(chip_full, cmap='gray')
    ax[0].scatter(testpts_x,testpts_y, color='None',edgecolors="red")
    ax[0].set_title(f"corrupted image  \nx':{x} y':{y}")
    ax[1].imshow(displaypred, cmap='gray')
    ax[1].scatter(testpts_x,testpts_y, color='None',edgecolors="red")
    ax[1].set_title(f"predicted chip \n(test pixels in red) \ns=30 \nmse:{mse.round
    ax[2].stem(weights)
    ax[2].set_title(f"weights \nlambda={l}")
    # ax[3].imshow(chip_test, cmap='gray')
```

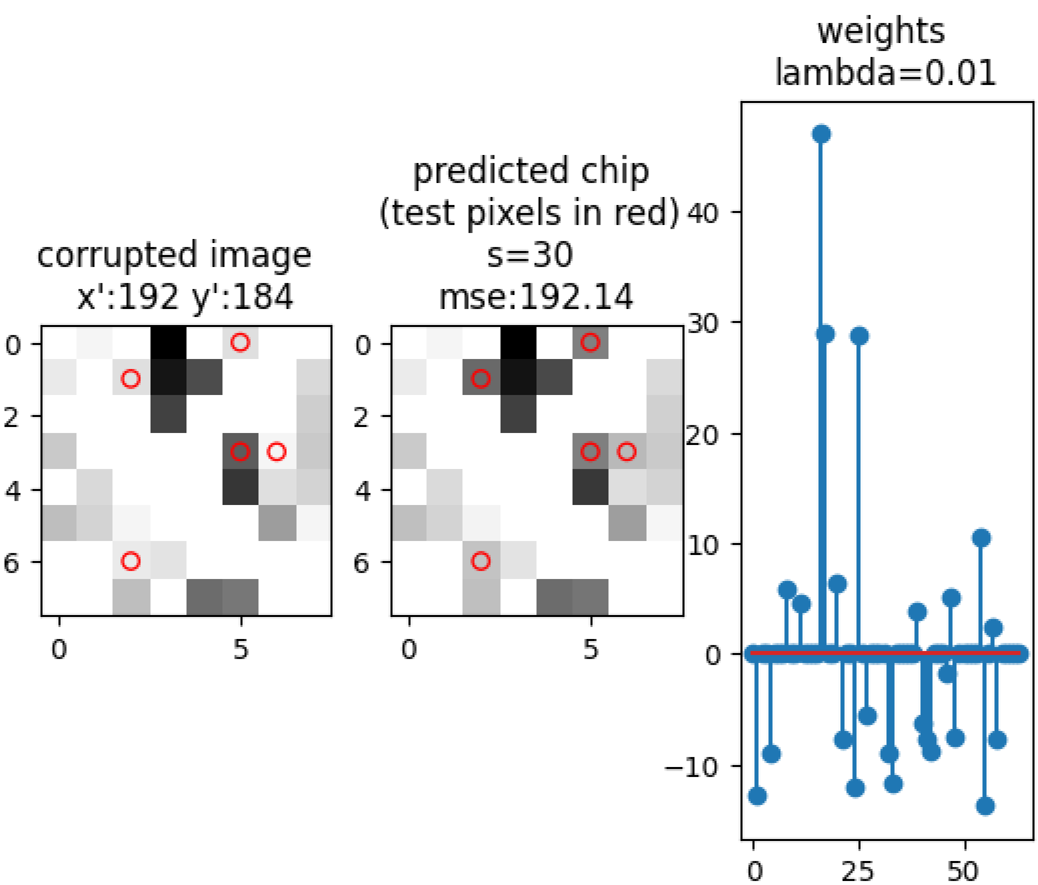

corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:222.35
weights
lambda=1e-07

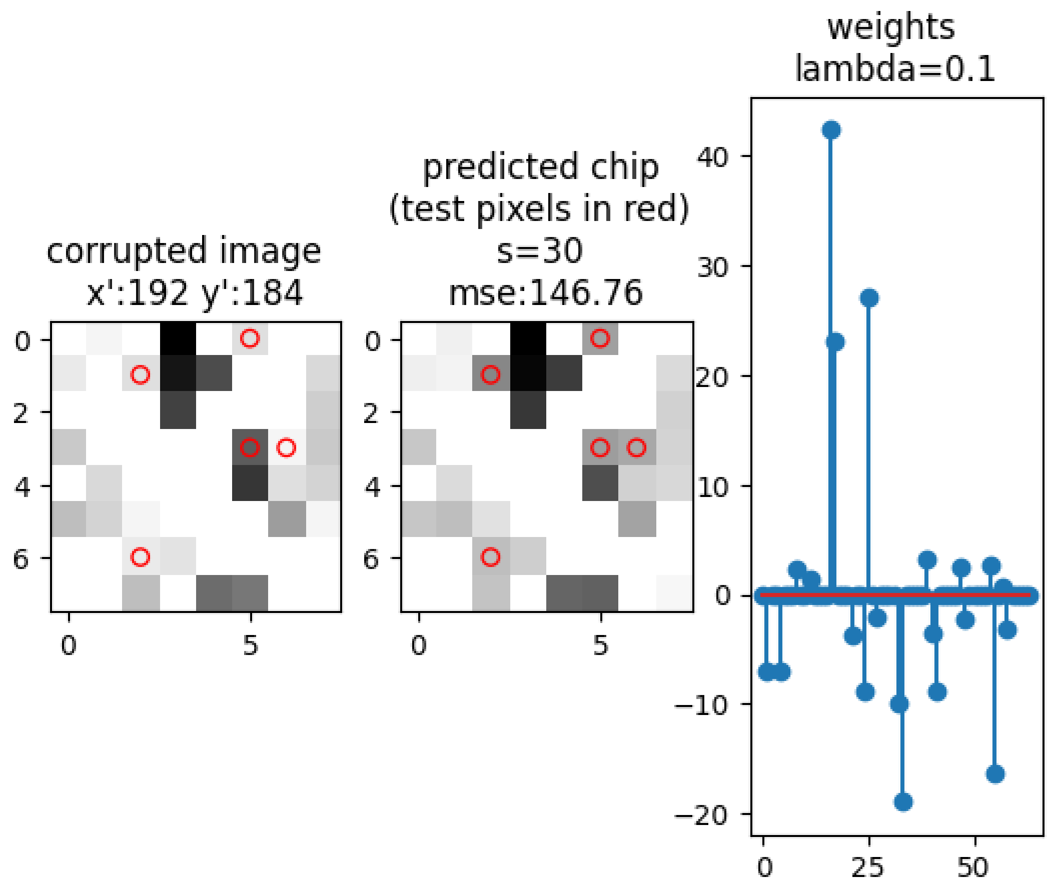

corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:222.35
weights
lambda=1e-06

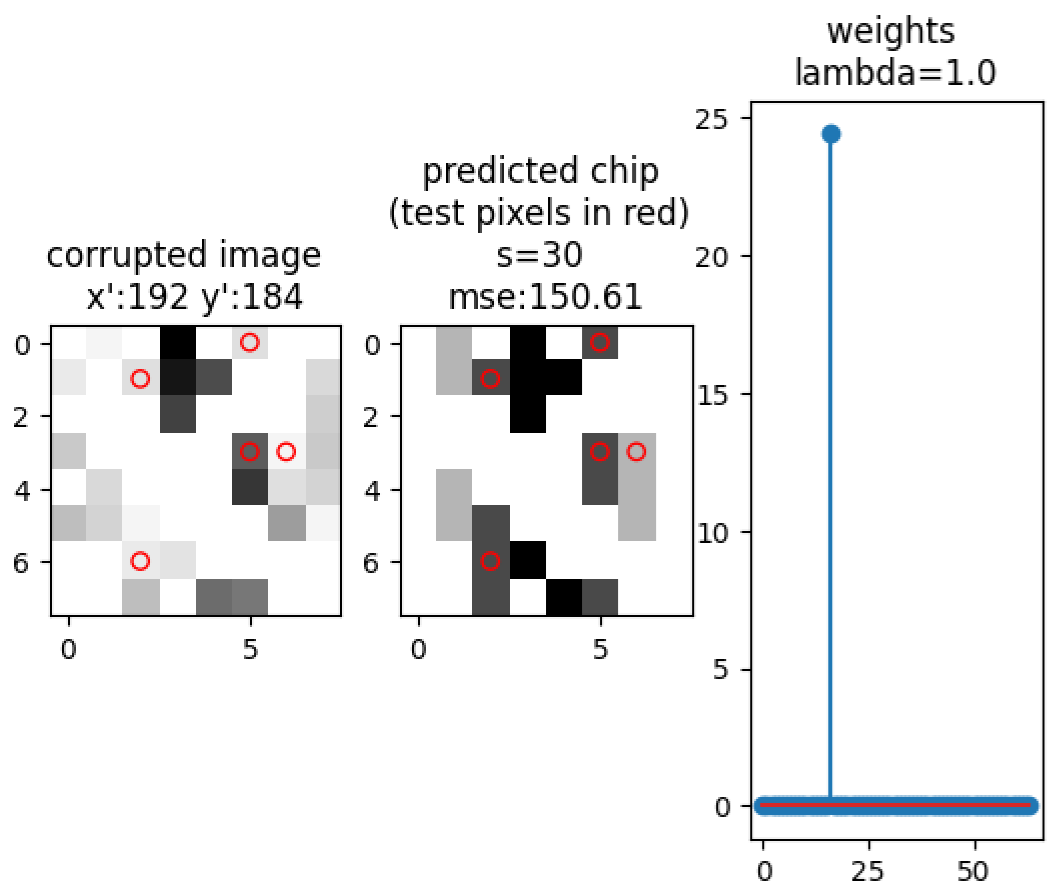

corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:222.39
weights
lambda=1e-05

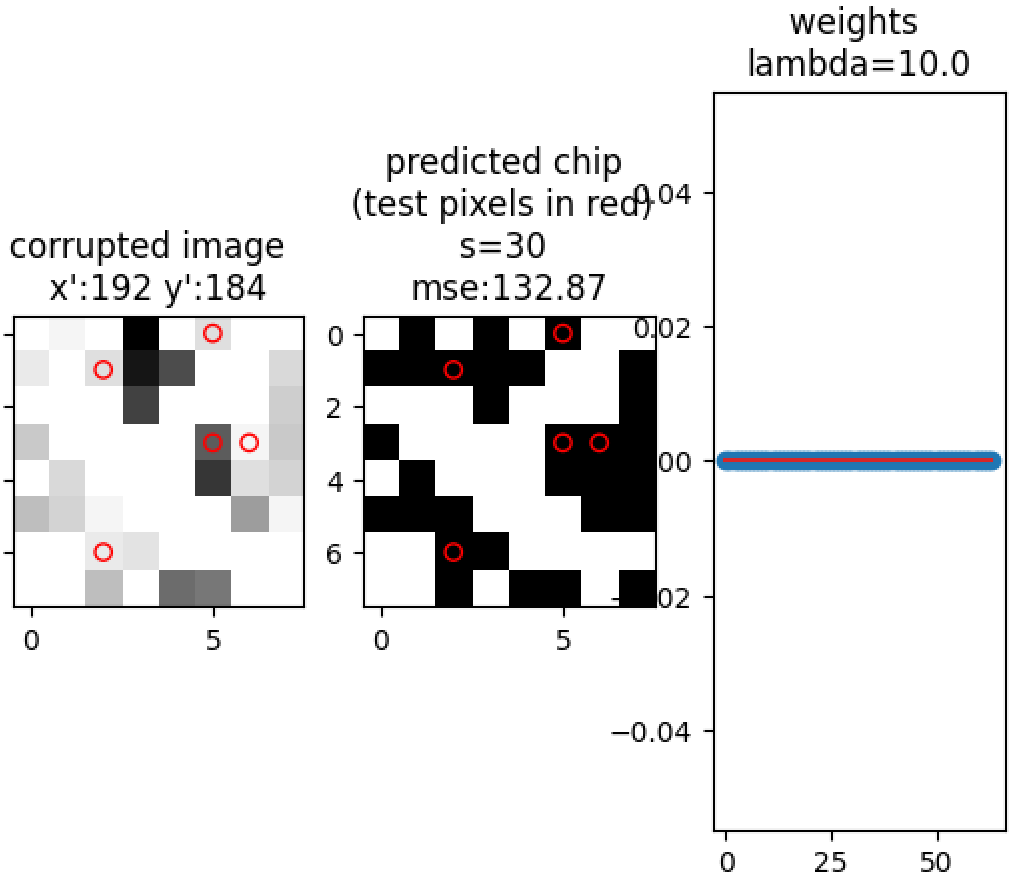

corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:166.35
weights
lambda=0.0001

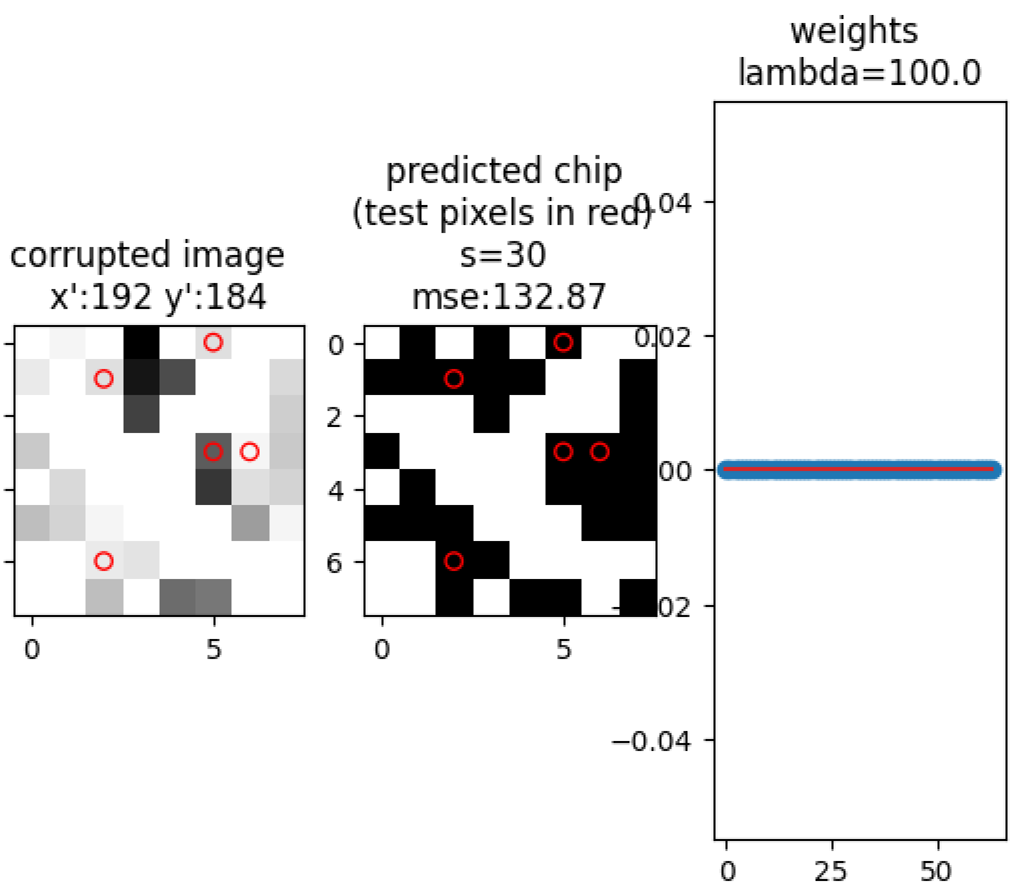

corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:168.32
weights
lambda=0.001
0 2 4 6
0 5
0 25 50
50 40 30 20 10 0 −10


corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:192.14
weights
lambda=0.01
0 2 4 6
0 5
0 25 50
40 30 20 10 0 −10


corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:146.76
weights
lambda=0.1


corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:150.61
weights
lambda=1.0


corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:132.87
weights
lambda=10.0
0.04
0.02
−0.04
0
2
4
6
0
5
0
25
50

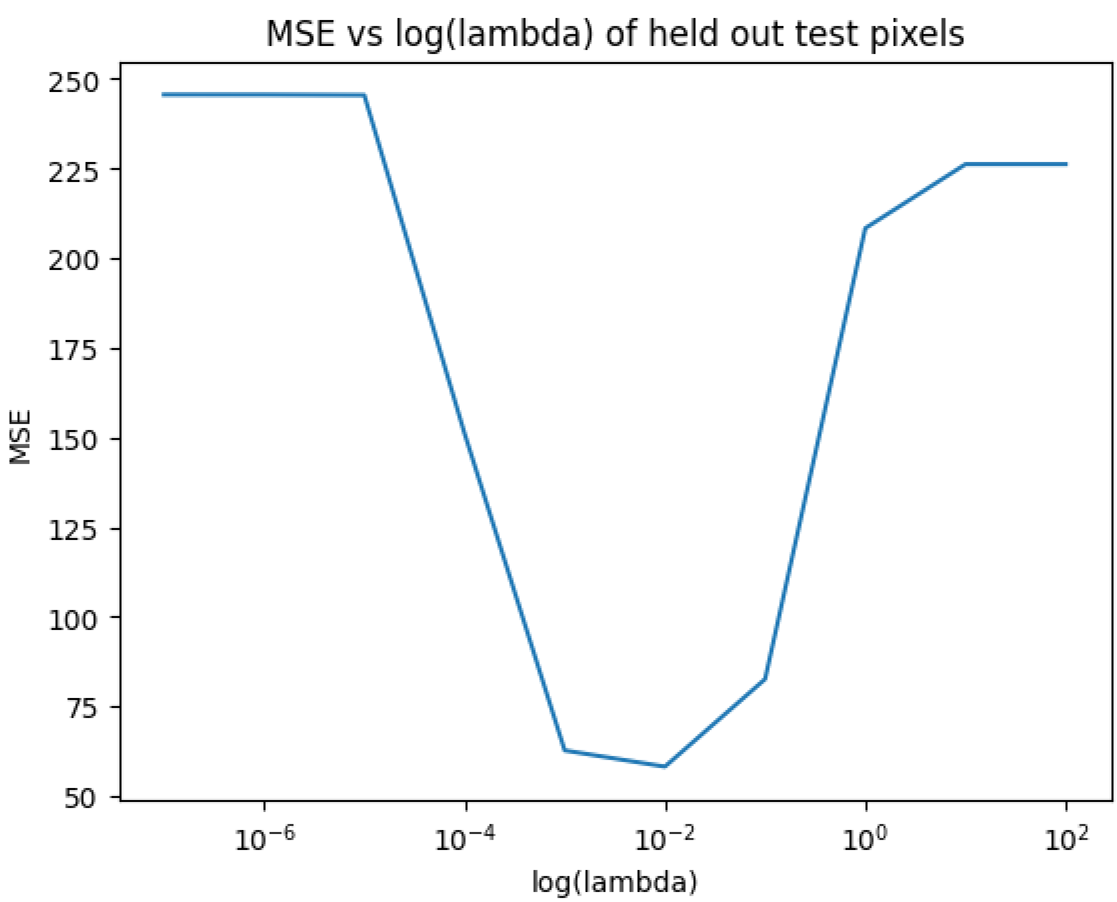

corrupted image
x':192 y':184
predicted chip
(test pixels in red)
s=30
mse:132.87
weights
lambda=100.0
0.04
0.02
−0.04
0
2
4
6
0
5
0
25
50

## b)

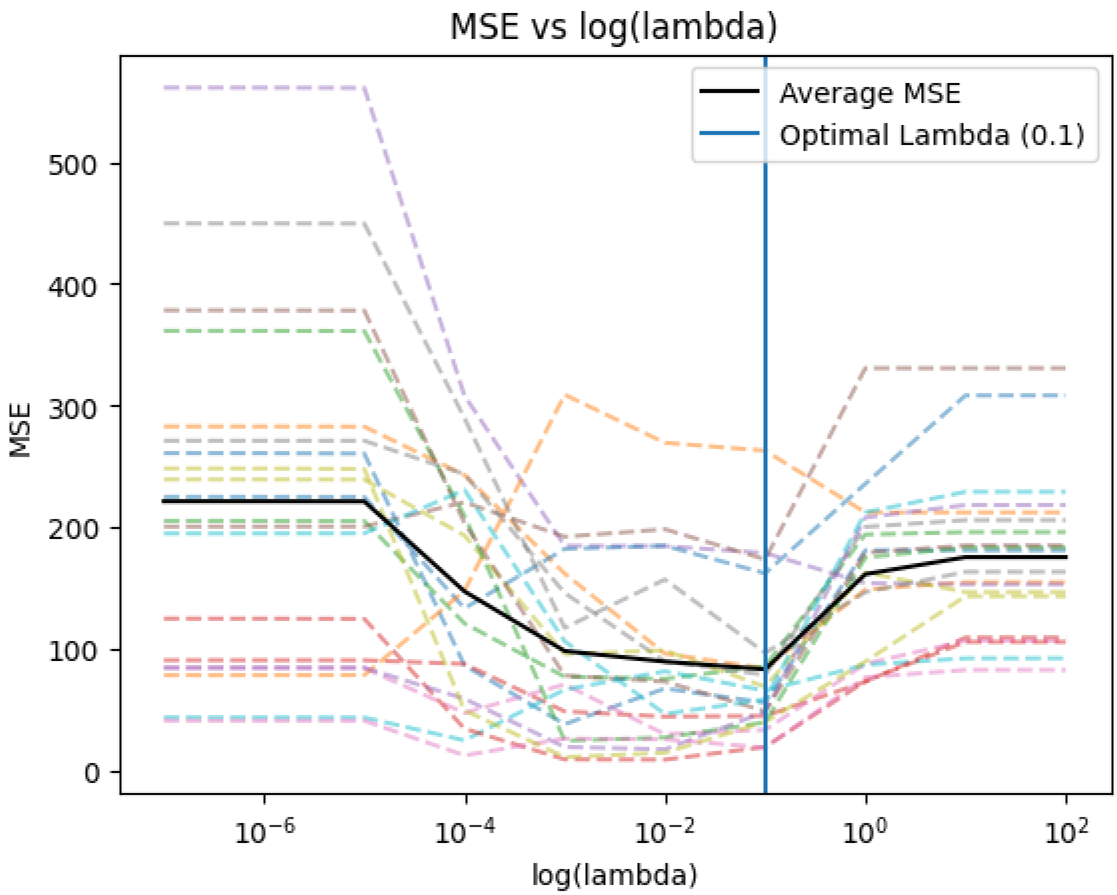
When lambda was $10^0$ or higher, the reconstructed chip was just a black square or a couple of lines. The model weights were all or almost all zero and this makes sense because when lambda is high, the model prioritizes minimizing all weights to be zero. When lambda was $10^{-4}$ or lower, the weights were all pretty much the same. This is because the penalty of non zero weights was so low that it was basically not considered in the objective function. When lambda was $10^{-3}$ to $10^{-1}$ the reconstructed chip was most similar to the original chip. Thus we can conclude that lambdas over $10^{-1}$ are too influential on the objective function and lambda lower than $10^{-3}$ are not influential enough on the objective function.

## c)

In [ ]:
```
fig, ax = plt.subplots()
ax.set_xscale("log")
ax.plot(lambdas, MSEs)
ax.set_title("MSE vs log(lambda) of held out test pixels")
ax.set_xlabel("log(lambda)")
ax.set_ylabel("MSE")

plt.show()
```



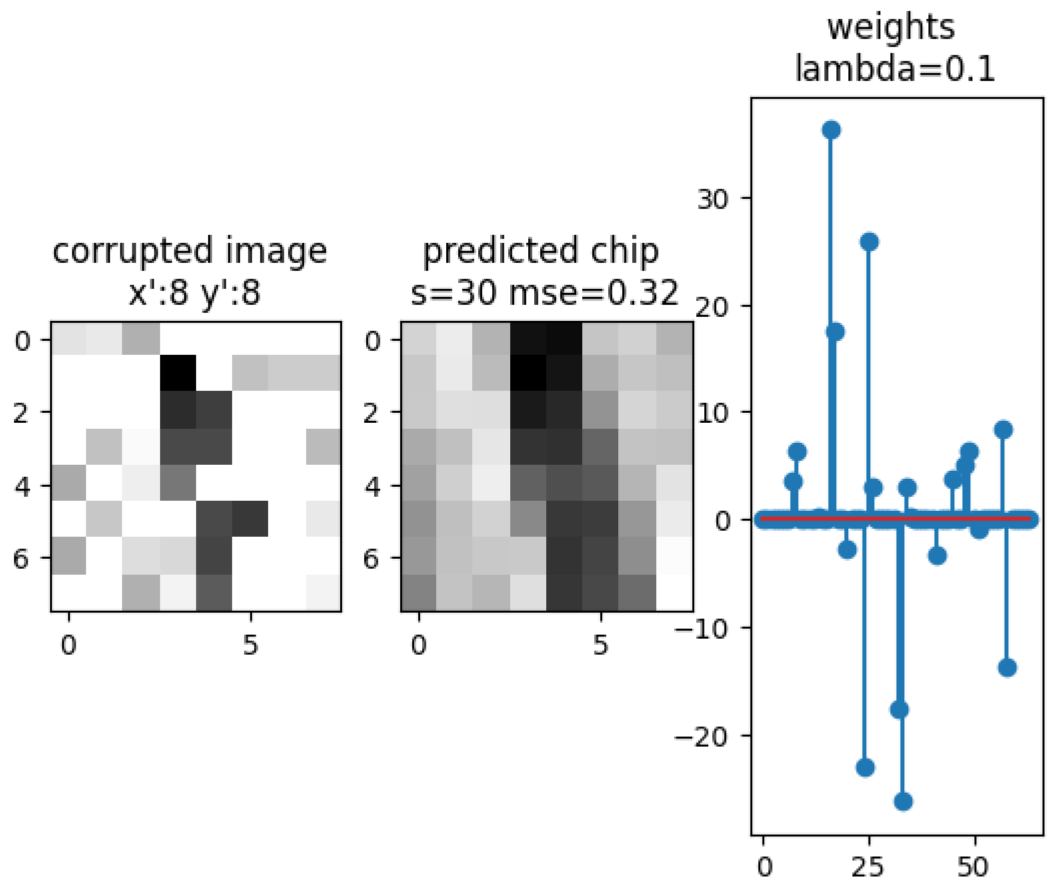
In [ ]:
```
import warnings
warnings.filterwarnings("ignore")

lambdas = np.logspace(-7, 2, 10)
S = 30
m = S//6
M = 20
fig, ax = plt.subplots()
ax.set_xscale("log")
ax.set_title("MSE vs log(lambda)")
ax.set_xlabel("log(lambda)")
ax.set_ylabel("MSE")

avgMSEs = []
for fold in range(M):
    chip_train, chip_test, chip_full  = makeTrainTestImg(my_chip, S, m)
    chip_train_BVM, chip_train_noNAN = createUnderdeterminedSystem(chip_train)
    MSEs = []
    for l in lambdas:
        model = Lasso(l).fit(chip_train_BVM, chip_train_noNAN)
        weights = np.reshape(model.coef_, (model.coef_.shape[0],1))
        pred_chip = np.reshape(model.predict(basisVectorMatrix), (P,Q))

        # calc mse only with test chip
        sensed_idx= ~np.isnan(chip_test)
        chip_test_noNan = chip_test[sensed_idx]
        pred_chip_test_noNan = pred_chip[sensed_idx]

        MSEs.append(np.mean((chip_test_noNan-pred_chip_test_noNan)**2))
    # plot one fold's worth of MSEs
    ax.plot(lambdas, MSEs, linestyle="--",alpha=0.5)
    avgMSEs.append(MSEs)

# plot avg over all the folds
avgMSEs = np.mean(avgMSEs, axis=0)
optimalLambda = lambdas[np.argmin(avgMSEs)]
ax.plot(lambdas, avgMSEs,c="black",label="Average MSE")
ax.axvline(optimalLambda, label=f"Optimal Lambda ({optimalLambda})")
ax.legend()
plt.show()
```




## e)

In [ ]:
```
def reconstruct(chip_x):
    chip_BVM, chip_noNAN = createUnderdeterminedSystem(chip_x)
    model = Lasso(optimalLambda).fit(chip_BVM, chip_noNAN)
    weights = np.reshape(model.coef_, (model.coef_.shape[0],1))
    pred_chip = np.reshape(model.predict(basisVectorMatrix), (P,Q))
    return weights, pred_chip

weights, pred_chip = reconstruct(chip_full)
mse = np.mean((my_chip-pred_chip)**2)/(my_chip.shape[0]*my_chip.shape[1])
fig, ax = plt.subplots(1,3)
ax[0].imshow(chip_full, cmap='gray')
ax[0].set_title(f"corrupted image \nx':{x} y':{y}")
ax[1].imshow(pred_chip, cmap='gray')
ax[1].set_title(f"predicted chip \ns=30 mse={mse.round(2)}")
ax[2].stem(weights)
ax[2].set_title(f"weights \nlambda={optimalLambda}")
```

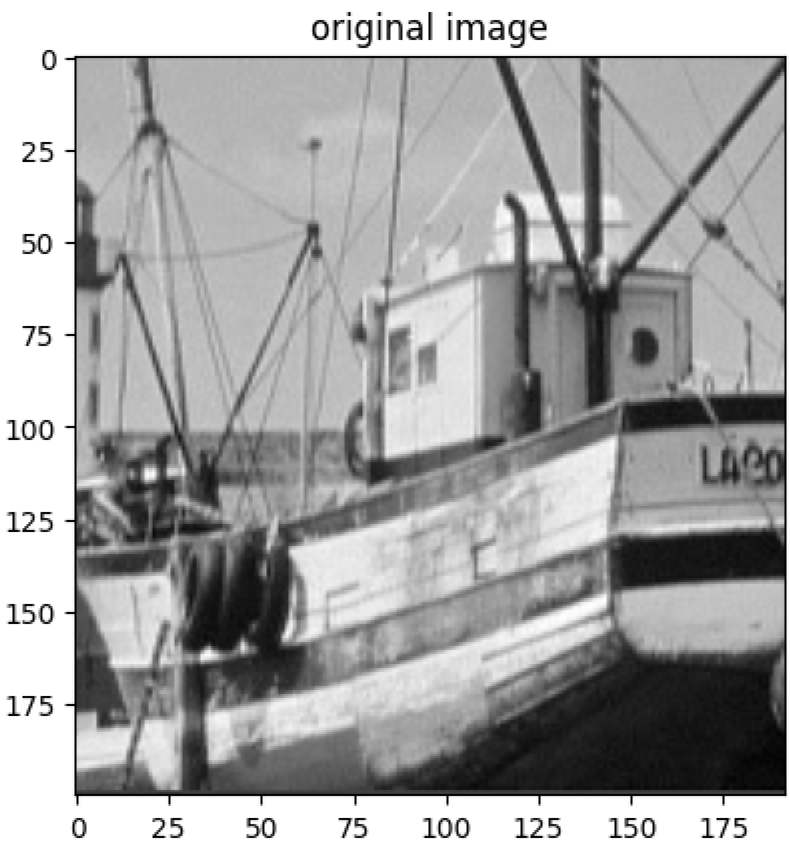
Out[ ]:
```
Text(0.5, 1.0, 'weights \nlambda=0.1')
```



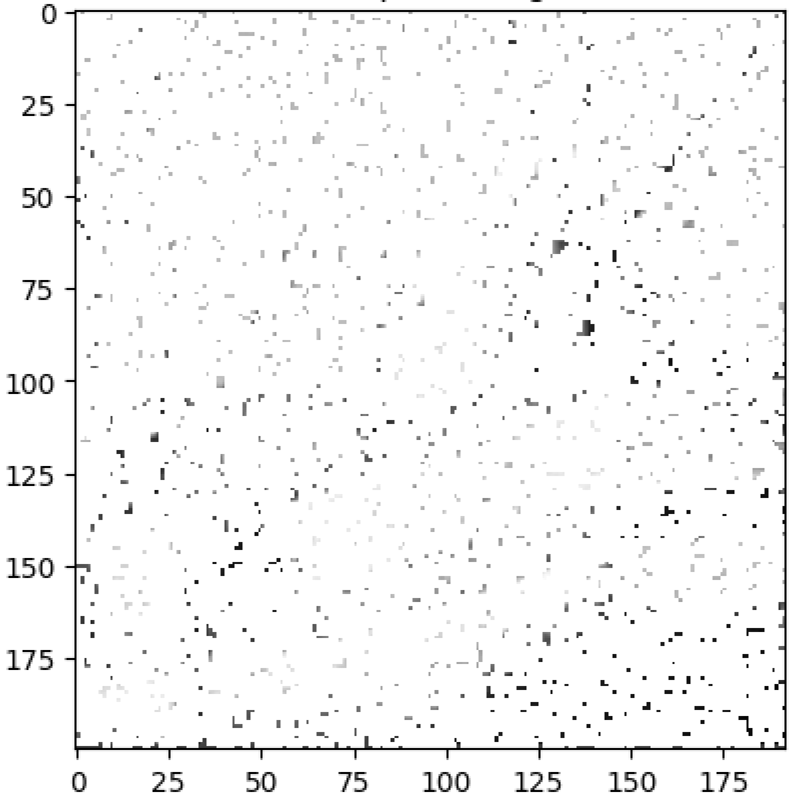

# Whole Image Reconstruction

In [ ]:
```
S = 30
m = S//6
M = 20
K = 8
fig,ax = plt.subplots()
ax.imshow(boat, cmap='gray')
ax.set_title("original image")

corrupted_boat_s30 = np.full((boat.shape[0],boat.shape[1]), np.NaN)
for x in range(0,boat.shape[0],8):
    for y in range(0,boat.shape[1],K):
        chip = boat[x:x+K,y:y+K]
        _, _, chip_full  = makeTrainTestImg(chip, S, m)
        corrupted_boat_s30[x:x+K,y:y+K] = chip_full
fig,ax = plt.subplots()
ax.imshow(corrupted_boat_s30, cmap='gray')
ax.set_title("corrupted image")

reconstructed_boat = np.full((corrupted_boat_s30.shape[0],corrupted_boat_s30.shape[
for x in range(0,boat.shape[0],8):
    for y in range(0,boat.shape[1],K):
        chip = corrupted_boat_s30[x:x+K,y:y+K]
        _, pred_chip = reconstruct(chip)
        reconstructed_boat[x:x+K,y:y+K] = pred_chip
```

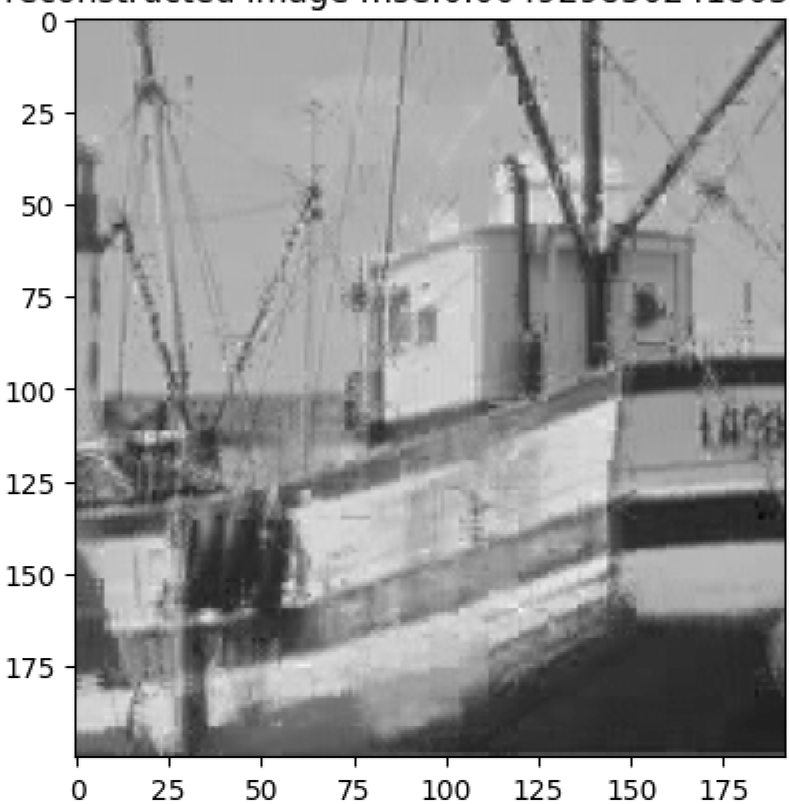
```
mse = np.mean((boat-reconstructed_boat)**2)/(boat.shape[0]*boat.shape[1])
fig,ax = plt.subplots()
ax.imshow(reconstructed_boat, cmap='gray')
ax.set_title(f"reconstructed image mse:{mse}")


reconstructed_boat_medfilt = signal.medfilt2d(reconstructed_boat, kernel_size=3)
mse = np.mean((boat-reconstructed_boat_medfilt)**2)/(boat.shape[0]*boat.shape[1])
fig,ax = plt.subplots()
ax.imshow(reconstructed_boat_medfilt, cmap='gray')
ax.set_title(f"reconstructed image with median filtering mse:{mse}")


plt.show()
```

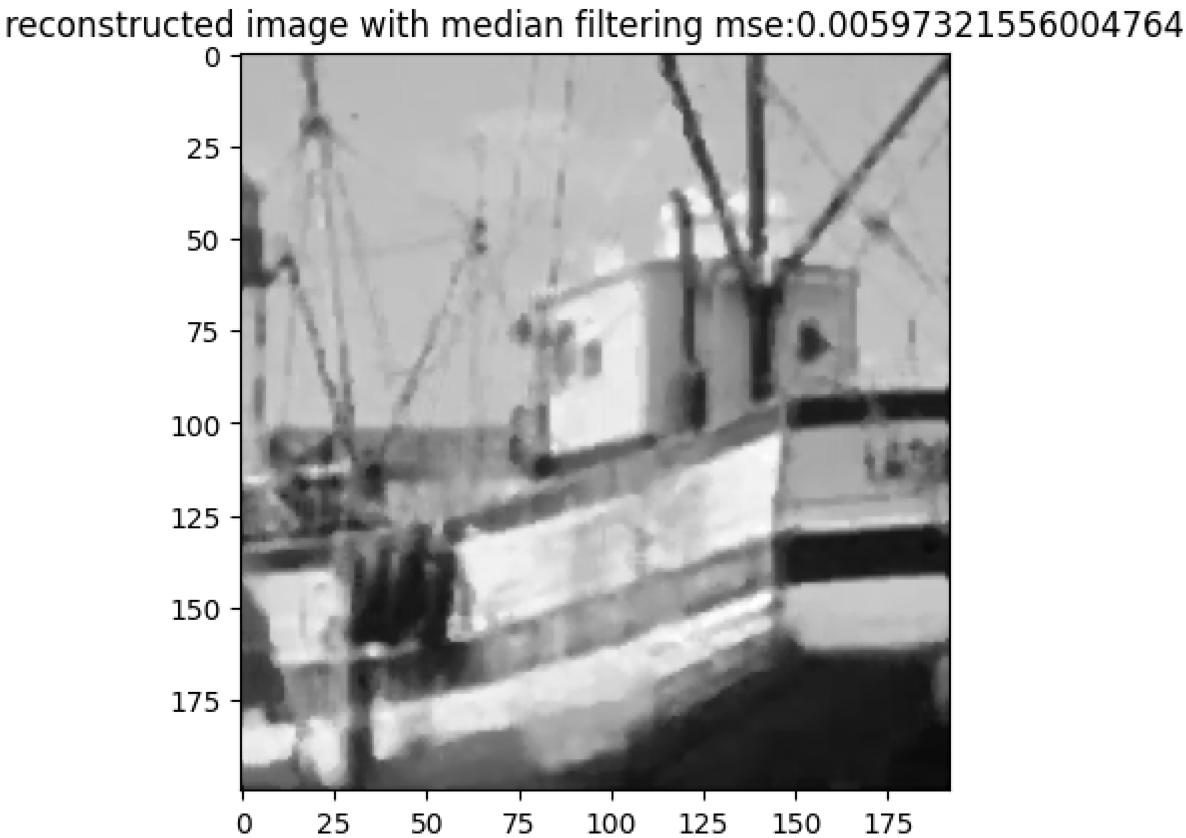


corrupted image




reconstructed image mse:0.004929850241803272




reconstructed image with median filtering mse:0.00597321556004764
0
25
50
75
100
125
150
175
0
25
50
75
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125
150
175

In [ ]: