Hw1

Monday, February 19, 2024 3:27 PM

① ⓐ $\sigma_1^2 = 4 \quad \mu_1 = 0$

$\sigma_0^2 = 1 \quad \mu_0 = 0$

$$LR = \frac{P(x|H_1)}{P(x|H_0)} = \frac{\frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2}}{\frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma_0}\right)^2}} = \frac{\frac{1}{2\sqrt{2\pi}} e^{-\frac{x^2}{8}}}{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}$$

$$= \frac{1}{2} e^{-\frac{x^2}{8} - \left(-\frac{x^2}{2}\right)}$$

$$= \frac{1}{2} e^{\frac{3x^2}{8}}$$

LR receiver: $\gamma \lessgtr \frac{1}{2} e^{\frac{3x^2}{8}}$

ⓑ $\ln e^{\frac{3x^2}{8}} \lessgtr \ln(2\gamma)$

$\frac{3x^2}{8} \gtrless \ln(2\gamma)$

$x \gtrless \sqrt{\frac{8}{3}\ln(2\gamma)}$

$R_0 < \sqrt{\frac{8}{3}\ln(2\gamma)}$

$R_1 > \sqrt{\frac{8}{3}\ln(2\gamma)}$

ⓒ As $\sigma_1^2 \to \infty$, the performance will decrease because more variance increases P_{miss}

ⓓ more observations decreases variance so performance will increase since P_{miss} and P_{FA} will decrease

②

$$L(\bar{x}) = \frac{P(\bar{x}|H_1)}{P(\bar{x}|H_0)} = \frac{\prod_i^N P(x_i|H_1)}{\prod_i^N P(x_i|H_0)} = \frac{\prod_i^N \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x_i-0)^2}{\sigma^2}}}{\prod_i^N \frac{1}{1-(-1)}}$$

$$=\prod_{i}^{N} \frac{2}{\sqrt{2\pi\sigma^2}} e^{-\frac{x_i^2}{2\sigma^2}} = \left(\frac{2}{\sqrt{2\pi\sigma^2}}\right)^N e^{-\frac{1}{2\sigma^2}\sum^N x_i^2} \quad \text{for } -1 \le x_i \le 1$$

$$L(x) \lessgtr \gamma$$

$$e^{-\frac{1}{2\sigma^2}\sum x_i^2} \lessgtr \gamma \left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N$$

$$\left(-\frac{1}{2\sigma^2}\sum x_i^2 \lessgtr \ln\left[\gamma \left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right]\right)\frac{\sigma^2}{N}$$

$$-\frac{1}{2N}\sum x_i^2 \lessgtr \frac{\sigma^2}{N} \ln\left[\gamma \left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right]$$

$$\frac{1}{N}\sum (x_i - 0)^2 \lessgtr -\frac{2\sigma^2}{N} \ln\left[\gamma \left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right] \quad \text{for } -1 \le x_i \le 1$$

$$R_0 : \mathrm{Var}(\bar{x}) < -\frac{2\sigma^2}{N} \ln\left[\gamma \left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right] \quad \text{for } -1 \le x_i \le 1$$

$$R_1 : \mathrm{Var}(\bar{x}) > -\frac{2\sigma^2}{N} \ln\left[\gamma \left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right] \quad \text{for } -1 \le x_i \le 1$$