Hw1

Monday, February 19, 2024 3:27 PM

1 (a) $\sigma_1^2 = 4 \quad \mu_1 = 0$

$\sigma_0^2 = 1 \quad \mu_0 = 0$

$$LR = \frac{P(x \mid H_1)}{P(x \mid H_0)} = \frac{\frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2}}{\frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma_0}\right)^2}} = \frac{\frac{1}{2\sqrt{2\pi}} e^{-\frac{x^2}{8}}}{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}$$

$$= \frac{1}{2} e^{-\frac{x^2}{8} - \left(-\frac{x^2}{2}\right)}$$

$$= \frac{1}{2} e^{\frac{3x^2}{8}}$$

LR receiver: $\gamma \lessgtr \frac{1}{2} e^{\frac{3x^2}{8}}$

(b) $\ln e^{\frac{3x^2}{8}} \lessgtr \ln(2\gamma)$

$\frac{3x^2}{8} \gtrless \ln(2\gamma)$

$x \gtrless \sqrt{\frac{8}{3}\ln(2\gamma)}$

$R_0 < \sqrt{\frac{8}{3}\ln(2\gamma)}$

$R_1 > \sqrt{\frac{8}{3}\ln(2\gamma)}$

(c) As $\sigma_1^2 \to \infty$, the performance will decrease because more variance increases P_{miss}

(d) more observations decreases variance so performance will increase since P_{miss} and P_{FA} will decrease

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$$L(\bar{x}) = \frac{P(\bar{x} \mid H_1)}{P(\bar{x} \mid H_0)} = \frac{\prod_i^N P(x_i \mid H_1)}{\prod_i^N P(x_i \mid H_0)} = \frac{\prod_i^N \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x_i-0)^2}{\sigma^2}}}{\prod_i^N \frac{1}{1-(-1)}}$$

$$= \prod_{i}^{N} \frac{2}{\sqrt{2\pi\sigma^2}} e^{-\frac{x_i^2}{2\sigma^2}} = \left(\frac{2}{\sqrt{2\pi\sigma^2}}\right)^N e^{-\frac{1}{2\sigma^2}\sum^N x_i^2} \quad \text{for } -1 \le x_i \le 1$$

$$L(x) \lessgtr \gamma$$

$$e^{-\frac{1}{2\sigma^2}\sum x_i^2} \lessgtr \gamma\left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N$$

$$\left(-\frac{1}{2\sigma^2}\sum x_i^2 \lessgtr \ln\left[\gamma\left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right]\right)\frac{\sigma^2}{N}$$

$$-\frac{1}{2N}\sum x_i^2 \lessgtr \frac{\sigma^2}{N}\ln\left[\gamma\left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right]$$

$$\frac{1}{N}\sum (x_i - 0)^2 \lessgtr -\frac{2\sigma^2}{N}\ln\left[\gamma\left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right] \quad \text{for } -1 \le x_i \le 1$$

$$R_0 : Var(\bar{x}) < -\frac{2\sigma^2}{N}\ln\left[\gamma\left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right] \quad \text{for } -1 \le x_i \le 1$$

$$R_1 : Var(\bar{x}) > -\frac{2\sigma^2}{N}\ln\left[\gamma\left(\frac{\sqrt{2\pi\sigma^2}}{2}\right)^N\right] \quad \text{for } -1 \le x_i \le 1$$

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

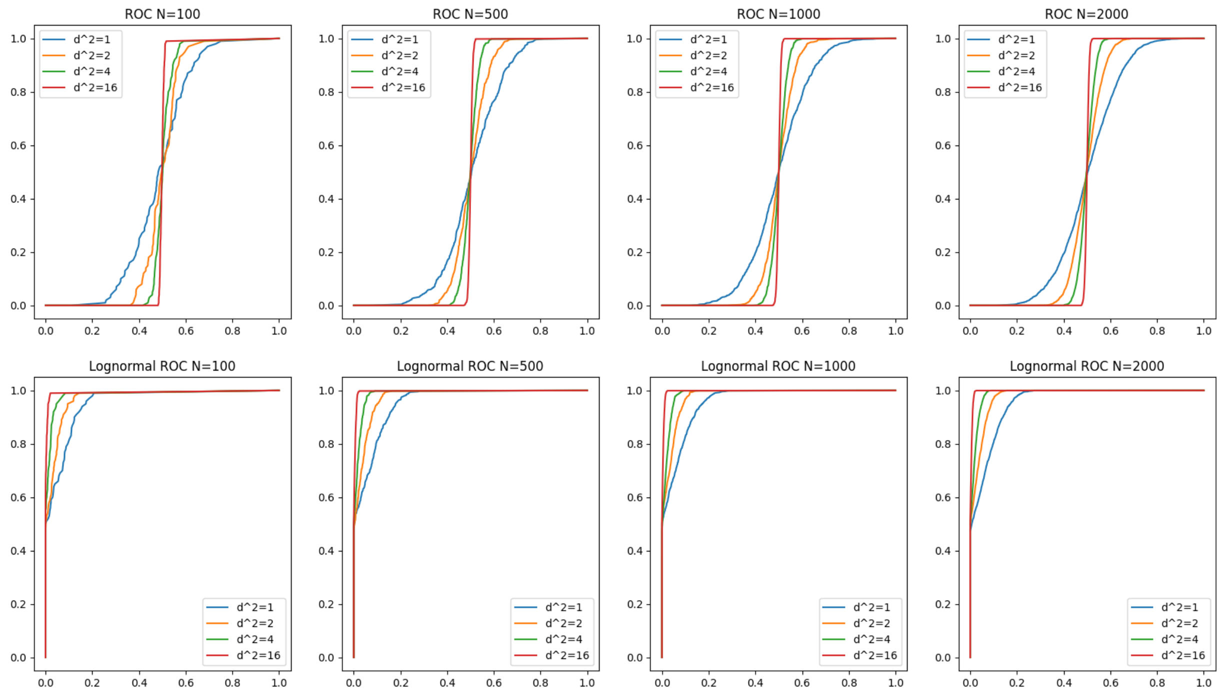
In [ ]:
```
Mu1MinusMu0 = 0-(1/2)

def calcPdPfa(data):
    numdata = data.shape[0]
    detection = numdata
    pd = [1]
    pfa = [1]
    for i in data:
        if i < -1:
            pfa.append(1)
        elif i >= -1 and i <= 1:
            pfa.append(1 - 0.5 * (i+1))
        else:
            pfa.append(0)
        detection -= 1
        pd.append(detection/numdata)
    pd.append(0)
    pfa.append(0)
    return pd,pfa

def plotRoc(ax, n, lognormal=False):
    # d^2 = (Mu1MinusMu0)**2/sigmasq
    for i,d2 in enumerate([1,2,4,16]):
        sigma_sq = (Mu1MinusMu0**2)/d2
        if lognormal:
            data = np.random.lognormal(0, sigma_sq, n)
        else:
            data = np.random.normal(0, sigma_sq,n)
        data.sort()
        pd,pfa = calcPdPfa(data)
        ax.plot(pfa,pd, label=f"d^2={d2}")


# normal ROC
fig,ax = plt.subplots(1,4,figsize=(20, 5))
for i,n in enumerate([100,500,1000,2000]):
    plotRoc(ax[i], n, lognormal=False)
    ax[i].legend()
    ax[i].set_title(f"ROC N={n}")
plt.show()

# lognormal ROC
fig,ax = plt.subplots(1,4,figsize=(20, 5))
for i,n in enumerate([100,500,1000,2000]):
    plotRoc(ax[i], n, lognormal=True)
    ax[i].legend()
    ax[i].set_title(f"Lognormal ROC N={n}")
plt.show()
```


ROC N=100
ROC N=500
ROC N=1000
ROC N=2000
Lognormal ROC N=100
Lognormal ROC N=500
Lognormal ROC N=1000
Lognormal ROC N=2000
d^2=1
d^2=2
d^2=4
d^2=16

In [ ]: