DFA Exercises

Exercise 1

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_a \in \dot{2}\}$

Solution:

	a	b	
q1	q2	q1	+
q2	q1	q2	

Exercise 2

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_a \in \dot{2} \wedge |w|_b \in \dot{2}\}$

Solution:

	a	b	
q0	a	b	+
a	q0	ab	
b	ab	q0	
ab	b	a	

Exercise 3

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_a \notin \dot{2} \vee |w|_b \notin \dot{2}\}$

Solution:

	a	b	
q0	a	b	
a	q0	ab	+
b	ab	q0	+
ab	b	a	+

Exercise 4

Minimum DFA for $\{w \in \{a,b\}^* \mid \exists x \ : \ w = xa\}$

Solution:

	a	b	
q0	q1	q0	
q1	q1	q0	+

Exercise 5

Minimum DFA for $\{w \in \{a,b\}i^* \mid \exists x \ : \ w = xbba\}$

Solution:

	a	b	
q0	q0	b	
b	q0	bb	
bb	bba	bb	
bba	q0	b	+

Exercise 6

Minimum DFA for $\{w \in \{a,b\}^* \mid \exists x \ : \ w = xbabab\}$

Solution:

	a	b	
q0	q0	b	
b	ba	b	
ba	q0	bab	
bab	baba	b	
baba	q0	babab	
babab	baba	b	+

Exercise 7

Minimum DFA for $\{w \in \{a,b\}^* \mid \exists x, y \ : \ (w = xaby \wedge |y| = 1)\}$

Solution:

	a	b	
q0	a	q0	
a	a	ab	
ab	aba	abb	
aba	a	ab	+
abb	a	q0	+

Exercise 8

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x, y \ : \ (w = xay \Rightarrow |x|_b \in \dot{2})\}$

Solution:

	a	b	
q0	q0	b	+
b	a	q0	+
a	a	a	

Exercise 9

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y \ : \ (w = xay \Rightarrow |y|_b \in \dot{2})\}$

Solution:

```
    a   b
q0  a   q0  +
b   pou a
a   a   b   +
pou pou pou
```

Exercise 10

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y \ : \ ((w = xy \wedge |x| \geq 3) \Rightarrow (|x|_a \in \dot{2} \vee |x|_b \in \dot{2}))\}$

Solution:

```
    a   b
q0  a   b   +
a   dp  di  +
b   di  dp  +
di  ap  bp  +
ap  pou dp  +
bp  dp  pou +
dp  bp  ap  +
pou pou pou
```

Exercise 11

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y : ((w = xy \wedge |x| \geq 3) \Rightarrow (|x|_a \in \dot{2} \vee |x|_b \notin \dot{2}))\}$

Solution:

```
        a       b
q0      a       apbi +
a       dp      di   +
di      apbi    pou  +
apbi    di      dp   +
dp      pou     apbi +
pou     pou     pou
```

Exercise 12

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y,z : ((w = xyz \wedge |y| = 3) \Rightarrow (|y|_a \in \dot{2} \vee |y|_b \notin \dot{2}))\}$

Solution:

```
    a   b
q0  a   b   +
a   2a  ab  +
b   ba  2b  +
2a  pou ab  +
2b  pou 2b  +
ba  2a  pou +
ab  ba  pou +
pou pou pou
```

Exercise 13

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y,z : ((w = xyz \wedge |y| = 3) \Rightarrow (|y|_a \in \dot{2} \vee |y|_b \in \dot{2}))\}$

Solution:

```
    a   b
q0  q0  q0  +
```

Exercise 14

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x : (w = bbx \Rightarrow |x|_{aa} = 0)\}$

Solution:

```
    a   b
q0  f   b   +
b   f   bb  +
bb  ba  bb  +
ba  pou bb  +
f   f   f   +
pou pou pou
```

Exercise 15

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{bbb} = 0\}$

Solution:

```
    a   b
q0  q0  b   +
b   q0  bb  +
bb  q0  pou +
pou pou pou
```

Exercise 16

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{bab} = 0\}$

Solution:

	a	b	
q0	q0	b	+
b	ba	b	+
ba	q0	pou	+
pou	pou	pou	

Exercise 17

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{aba} = 0 \wedge |w|_{bab} = 0 \wedge \exists x : w = xaaa\}$

Solution:

	a	b	
q0	a	b	
a	2a	ab	
b	ba	b	
2a	3a	ab	
3a	3a	ab	+
ab	pou	b	
ba	2a	pou	
pou	pou	pou	

Exercise 18

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{abc} \leq 1\}$

Solution:

	a	b	c	
q0	a	q0	q0	+
a	a	ab	q0	+
ab	a	q0	f	+
fa	fa	fab	f	+
fab	fa	f	pou	+
f	fa	f	f	+
pou	pou	pou	pou	

Exercise 19

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x, y, z : (w = xbybz \Rightarrow |y|_a \geq 2)\}$

Solution:

	a	b	c	
q0	q0	b	q0	+
b	ba	pou	b	+
ba	q0	pou	ba	+
pou	pou	pou	pou	

Exercise 20

Minimum DFA for $\{w \in \{a,b\}^* \mid \texttt{value}_2(w) \in \dot{2}\}$

Solution:

	0	1	
q0	q0	1	+
1	q0	1	

Exercise 21

Minimum DFA for $\{w \in \{0,1\}^* \mid \texttt{value}_2(w) \in \dot{3}\}$

Solution:

	0	1	
q0	q0	q1	+
q1	q2	q0	
q2	q1	q2	

Exercise 22

Minimum DFA for $\{w \in \{0,1\}^* \mid \texttt{value}_2(w) \notin \dot{3}\}$

Solution:

	0	1	
q0	q0	q1	
q1	q2	q0	+
q2	q1	q2	+

Exercise 23

Minimum DFA for $\{w \in \{0,1\}^* \mid \texttt{value}_2(w) \in \dot{4}\}$

Solution:

	0	1	
q0	q0	q1	+
q1	q2	q1	
q2	q0	q1	

Exercise 24

Minimum DFA for $\{w \in \{0,1\}^* \mid \texttt{value}_2(w) \notin \dot{4}\}$

Solution:

	0	1	
q0	q0	q1	
q1	q2	q1	+
q2	q0	q1	+

Exercise 25

Minimum DFA for $\{w \in \{0,1\}^* \mid \texttt{value}_2(w) \in \dot{5}\}$

Solution:

	0	1	
q0	q0	q1	+
q1	q2	q3	
q2	q4	q0	
q3	q1	q2	
q4	q3	q4	

Exercise 26

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y,z : ((w = xyz \wedge |y| = 3) \Rightarrow |y|_a = 2)\}$

Solution:

	a	b	
q0	a	b	+
a	aa	ab	+
b	aba	bb	+
aa	pou	ab	+
ab	aba	pou	+
bb	pou	pou	+
aba	aa	pou	+
pou	pou	pou	

Exercise 27

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y : ((w = xy \wedge |x| \notin \dot{2}) \Rightarrow |x|_b = 1 + |x|_a)\}$

Solution:

	a	b	
q0	pou	b	+
b	q0	bb	+
bb	b	pou	+
pou	pou	pou	

Exercise 28

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall x,y : ((w = xy \wedge |y| \notin \dot{2}) \Rightarrow |y|_b = 1 + |y|_a)\}$

Solution:

	a	b	
q0	a	b	+
a	aa	b	
b	a	bb	+
aa	pou	auxb	
bb	auxb	pou	+
auxb	aa	bb	
pou	pou	pou	

Exercise 29

Minimum DFA for $\{w \in \{a,b\}^* \mid \forall y : ((|y| = 2 \wedge |y|_b > 0) \Rightarrow |w|_y > 0)\}$

Solution:

	a	b	
q0	a	b	
a	a	ab	
b	ba	bb	
ab	ba	abb	
ba	ba	bab	
bb	bba	bb	
abb	pou	abb	
bab	ba	pou	
bba	bba	pou	
pou	pou	pou	+

Exercise 30

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{ab} = |w|_{ba}\}$

Solution:

	a	b	
q0	a	b	+
a	a	ab	+
b	ba	b	+
ab	a	ab	
ba	ba	b	

Exercise 31

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{ab} = |w|_b\}$

Solution:

```
    a   b
q0  a   pou +
a   a   q0  +
pou pou pou
```

Exercise 32

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{aba} = |w|_b\}$

Solution:

```
    a   b
q0  a   pou +
a   a   ab  +
ab  a   pou
pou pou pou
```

Exercise 33

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{aba} + 1 = |w|_b\}$

Solution:

```
    a   b
q0  a   b
a   a   ab
b   ba  pou +
ab  a   pou +
ba  ba  b2  +
b2  ba  pou
pou pou pou
```

Exercise 34

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{aba} = |w|_a\}$

Solution:

```
    a   b
q0  pou q0  +
pou pou pou
```

Exercise 35

Minimum DFA for $\{w \in \{a,b\}^* \mid |w|_{aba} + 1 = |w|_a\}$

Solution:

```
        a    b
q0      a    q0
a       pou  ab    +
ab      a    auxb  +
auxb    pou  auxb  +
pou     pou  pou
```

Exercise 36

Minimum DFA for $\{w \in \{a,b\}^* \mid \exists x,y,z : (w = xyz \wedge |y|_b = 3 + |y|_a)\}$

Solution:

```
    a    b
q0  q0   b
b   q0   bb
bb  b    bbb
bbb bbb  bbb  +
```

Exercise 37

Minimum DFA for $\{w \in \{a,b\}^* \mid |x|_a = |y|_a\}$

Solution:

```
    a  b
q1  q2 q1 +
q2  q1 q2
```

Exercise 38

Minimum DFA for $\{w \in \{a,b\}^* \mid |x|_a = |y|_b\}$

Solution:

```
    a   b
q0  q0  q0  +
```

Exercise 39

Minimum DFA for $\{w \in \{a,b\}^* \mid |x|_{aa} = |y|_b\}$

Solution:

```
    a   b
q0  q0  q0  +
```