CFG Exercises

Exercise 1

Non-ambiguous CFG for $\{a^n b^n \mid n \geq 0\}$

Solution:

```
S -> aSb |
```

Exercise 2

Non-ambiguous CFG for $\{a^n b^n \mid n > 0\}$

Solution:

```
S -> aSb | acb
```

Exercise 3

Non-ambiguous CFG for $\{a^i b^j \mid i \geq j\}$

Solution:

```
S -> aS | X
X -> aXb |
```

Exercise 4

Non-ambiguous CFG for $\{a^i b^j \mid i \leq j\}$

Solution:

```
S -> Sb | X
X -> aXb |
```

Exercise 5

Non-ambiguous CFG for $\{a^i b^j \mid 2i \leq j\}$

Solution:

```
S -> Sb | X
X -> aXbb |
```

Exercise 6

CFG for $\{a^i b^j | 2i \geq j\}$

Solution:

```
S -> aXb | aS |
X -> Sb |
```

Exercise 7

Non-ambiguous CFG for $\{a^i b^j \mid 2i \geq j\}$

Solution:

```
S -> A | X
A -> aAb | B
B -> aB | a
X -> aXb | Y
Y -> aYbb |
```

Exercise 8

Non-ambiguous CFG for $\{a^i b^j \mid j \leq i \leq 2j\}$

Solution:

```
S -> aSb | X
X -> aaXb |
```

Exercise 9

Non-ambiguous CFG for $\{a^i b^j \mid i \geq j \vee i \leq 2j\}$

Solution:

```
S -> Sb | B
B -> aB |
```

Exercise 10

Non-ambiguous CFG for $\{a^i b^j c^k \mid i = j + k\}$

Solution:

```
S -> aSc | X
X -> aXb |
```

Exercise 11

Non-ambiguous CFG for $\{a^i b^j c^k \mid j = i + k\}$

Solution:

```
S -> AC
A -> aAb |
C -> bCc |
```

Exercise 12

CFG for $\{a^i b^j c^k \mid i = j \vee j = k \vee i = k\}$

Solution:

```
S -> X | Y | Z
X -> Xc | A
A -> aAb |
Y -> aYc | B
B -> bBb | b |
Z -> aZ | C
C -> bCc |
```

Exercise 13

CFG for $\{a^{n_0} b a^{n_1} b ... a^{n_{m-1}} b a^{n_m} \mid m \geq 1 \wedge \exists i \in \{1, ..., m\} : (n_0 = n_i)\}$

Solution:

```
S -> A | B | C
A -> aAa | aBa | b
B -> bPb
C -> Ca | Cb | Ab | Bb
P -> aP | Pa | bP | Pb |
```

Exercise 14

Non-ambiguous CFG for $\{a^{n_0} b a^{n_1} b ... a^{n_{m-1}} b a^{n_m} \mid m \geq 1 \wedge (n_0 = \sum_{1 \leq i \leq m} n_i)\}$

Solution:

```
S -> aSa | Sb | X
X -> b
```

Exercise 15

CFG for $\{a^{n_0} b a^{n_1} b ... a^{n_{m-1}} b a^{n_m} \mid m \geq 1 \wedge \exists i \subseteq \{1, ..., m\} : (n_0 = \sum_{i \in I} n_i)\}$

Solution:

```
S -> A | B | bP
A -> aAa | aBba | Ab | bPb | b
B -> Ba | Bb | Ab
P -> aP | Pa | bP | Pb |
```

Exercise 16

Non-ambiguous CFG for $\{w \in \{a, b\}^* \mid w = w^R\}$

Solution:

```
S -> aSa | bSb | X |
X -> a | b
```

Exercise 17

Non-ambiguous CFG for $\{w \in \{a,b\}^* \mid w = w^R \wedge |w|_{aba} = 0\}$

Solution:

```
S -> aAa | bCb | a | b |
A -> aAa | bBb | a |
B -> bCb | b |
C -> aAa | bCb | b | a |
```

Exercise 18

Non-ambiguous CFG for $\{w \in \{a,b\}^* \mid w = w^R \wedge |w|_a > 0 \wedge |w|_b > 0\}$

Solution:

```
S -> aAa | bBb
A -> aAa | bXb | b
B -> bBb | aXa | a
X -> aXa | bXb | a | b |
```

Exercise 19

Non-ambiguous CFG for $\{w \in \{a,b\}^* \mid w = w^R \wedge |w|_{aba} > 0\}$

Solution:

```
S -> aAa | bBb
A -> aAa | bXb | b
B -> aAa | bBb
X -> aYa | bBb | a
Y -> aYa | bYb | a | b |
```

Exercise 20

CFG for well-parenthesized words over $\{(,)\}$

Solution:

```
S -> (S)S |
```

Exercise 21

CFG for well-parenthesized words over $\{[,],(,)\}$

Solution:

```
S -> S[S] | S(S) |
```

Exercise 22

Non-ambiguous CFG for well-parethesized words over $\{(,)\}$

Solution:

```
S -> (S)S |
```

Exercise 23

Non-ambiguous CFG for well-parethesized words over $\{[,],(,)\}$

Solution:

```
S -> S[S] | S(S) |
```

Exercise 24

CFG for $\{w \in \{a,b\}^* \mid |w|_a = |w|_b\}$

Solution:

```
S -> SS | bSa | aSb |
```

Exercise 25

CFG for $\{w \in \{a,b,c\}^* \mid |w|_a = |w|_b\}$

Solution:

```
S -> SS | bSa | aSb | cS |
```

Exercise 26

CFG for $\{w \in \{a,b,c\}^* \mid |w|_a + |w|_b = |w|_c\}$

Solution:

```
S -> SS | aSc | bSc | cSa | cSb |
```

Exercise 27

CFG for $\{w \in \{a,b\}^* \mid 2|w|_a = |w|_b\}$

Solution:

```
S -> aAAS | bBS | bCAS |
A -> b | aAAA
B -> bBB | bC
C -> a | bBC
```

Exercise 28

Non-ambiguous CFG for $\{w \in \{a,b\}^* \mid |w|_a = |w|_b\}$

Solution:

```
S -> aAS | bBS |
A -> b | aAA
B -> a | bBB
```

Exercise 29

Non-ambiguous CFG for $\{w \in \{a, b, c\}^* \mid |w|_a = |w|_b\}$

Solution:

```
S -> aAS | bBS | cS |
A -> b | aAA | cA
B -> a | bBB | cB
```

Exercise 30

Non-ambiguous CFG for $\{w \in \{a, b, c\}^* \mid |w|_a + |w|_b = |w|_c\}$

Solution:

```
S -> aXS | bXS | cCS |
X -> c | aXX | bXX
C -> a | b | cCC
```

Exercise 31

Non-ambiguous CFG for $\{w \in \{a, b\}^* \mid 2|w|_a = |w|_b\}$

Solution:

```
S -> aAAS | bBS | bCAS |
A -> b | aAAA
B -> bBB | bC
C -> a | bBC
```

Exercise 32

Non-ambiguous CFG for $\{xcy \mid x, y \in \{a, b\}^* \wedge |x|_a = |y|_b\}$

Solution:

```
S -> aSY | bS | cZ
Y -> Ya | b
Z -> aZ |
```

Exercise 33

Non-ambiguous CFG for $\{xcy \mid x, y \in \{a, b\}^* \wedge |x|_{ab} = |y|_{ba}\}$

Solution:

```
S -> BAabSbaAB | BAcAB
A -> Aa |
B -> Bb |
```

Exercise 34

Solution:

Exercise 35

Non-ambiguous CFG for $\{xcy \mid x, y \in \{a, b\}^* \wedge y^R \text{ prefix of } x\}$

Solution:

```
S -> aSa | bSb | Xc
X -> aX | bX |
```

Exercise 36

Non-ambiguous CFG for $\{xcy \mid x, y \in \{a, b\}^* \wedge y^R \text{ suffix of } x\}$

Solution:

```
S -> XY
Y -> aYa | bYb | c
X -> aX | bX |
```

Exercise 37

Non-ambiguous CFG for $\{xcy \mid x, y \in \{a, b\}^* \wedge |x| = |y| \wedge |x|_{aa} > 0\}$

Solution:

```
S -> aXb | aXa | bSb | bSa
X -> aZb | aZa | bSb | bSa
Z -> aZb | aZa | bZb | bZa | c
```

Exercise 38

CFG for the complement of $\{a^n b^n \mid n \geq 0\}$

Solution:

```
S -> aXb | aPa | bP | a | b
X -> aXb | bP | Pa
P -> aP | bP |
```

Exercise 39

Non-ambiguous CFG for the complement of $\{a^n b^n \mid n \geq 0\}$

Solution:

```
S -> X | Y
X -> aXb | aA | bB
A -> aA |
B -> bB |
Y -> ABbaP
P -> aP | bP |
```

Exercise 40

Non-ambiguous CFG for the complement of $\{w \in \{a, b\}^* \mid w = w^R\}$

Solution:

```
S -> aXa | bXb | aPb | bPa
X -> aPb | bPa | aSa | bSb
P -> aP | bP |
```

Exercise 41

Solution:

Exercise 42

Solution:

Exercise 43

CFG for expression over $\{+, -, *, /, (,), 0, 1, ..., 9\}$

Solution:

```
S -> (S) | O
O -> S+S | S-S | S*S | S/S | N
N -> NN | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Exercise 44

Solution:

Exercise 45

CFG for $\{a^n b^m c^k d^t \mid n = m \vee n = k \vee n = t\}$

Solution:

```
S -> XCD | YD | Z
X -> aXb |
Y -> aYc | B
Z -> aZd | BC
B -> bB |
C -> cC |
D -> dD |
```