Maximizing Revenue with Limited Correlation: The Cost of Ex-Post Incentive Compatibility

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Abstract

In a landmark paper in the mechanism design literature, Cremer and McLean 1985 (CM for short) show that when a bidders valuation is correlated with an external signal, a monopolistic seller is able to extract the full social surplus as revenue. In the original paper and subsequent literature, the focus has been on ex-post incentive compatible (or IC) mechanisms, mechanisms where truth telling is an ex-post Nash equilibrium. This has been viewed as an innocuous assumption due to its sufficiency for full revenue extraction in the CM world. However, more realistic models are unlikely to satisfy the assumptions of CM. In this paper, we explore the implications of Bayesian versus ex-post IC in a correlated valuation setting. We generalize the full surplus extraction result to settings of limited correlation, and we give necessary and sufficient conditions for full surplus extraction that strictly relax the original conditions given in CM. These more general conditions characterize the situations under which requiring ex-post IC leads to a decrease in expected revenue relative to Bayesian IC. We also demonstrate that the expected revenue from the optimal ex-post IC mechanism guarantees at most a $(|\Theta|+1)/4$ approximation to that of a Bayesian IC mechanism, where $|\Theta|$ is the number of bidder types. Finally, by using techniques from automated mechanism design, we are able to show that, for randomly generated distributions, the average expected revenue achieved by Bayesian IC mechanisms is significantly larger than that for ex-post IC mechanisms.

Problem Description

- A monopolistic seller with one item
- A single bidder with type $\theta \in \Theta$ and valuation $v(\theta)$
- An external signal $\omega \in \Omega$ and distribution $\pi(\theta, \omega)$

Definitions

Definition (Ex-Post Individual Rationality (IR))

A mechanism (p, x) is *ex-post individually rational (IR)* if:

$$\forall \theta \in \Theta, \omega \in \Omega : U(\theta, \theta, \omega) \geq 0$$

Definition (Bayesian Individual Rationality (IR))

A mechanism (p, x) is *Bayesian (or ex-interim) individually rational (IR)* if:

$$\forall \theta \in \Theta : \sum_{\omega \in \Omega} \pi(\omega|\theta) U(\theta, \theta, \omega) \geq 0$$

Definition (Ex-Post Incentive Compatibility (IC))

A mechanism (p, x) is *ex-post incentive compatible (IC)* if:

$$\forall \theta, \theta' \in \Theta, \omega \in \Omega : U(\theta, \theta, \omega) \geq U(\theta, \theta', \omega)$$

Definition (Bayesian Incentive Compatibility (IC))

A mechanism (p, x) is *Bayesian incentive compatible (IC)* if:

$$\forall \theta, \theta' \in \Theta : \sum_{\omega \in \Omega} \pi(\omega|\theta) U(\theta, \theta, \omega) \geq \sum_{\omega \in \Omega} \pi(\omega|\theta) U(\theta, \theta', \omega)$$

Theorem: Cremer and McLean (1985)

If bidder beliefs, conditional on the bidder's type, are linearly independent, then there exists a Cremer-McLean mechanism that extracts the full social surplus as revenue in expectation.

Cremer-McLean Revenue $\leq$ *Bayesian Revenue*

Full Surplus Extraction

Our Contribution

Cremer-McLean Revenue $\leq$ *Bayesian Revenue*

Full Surplus Extraction

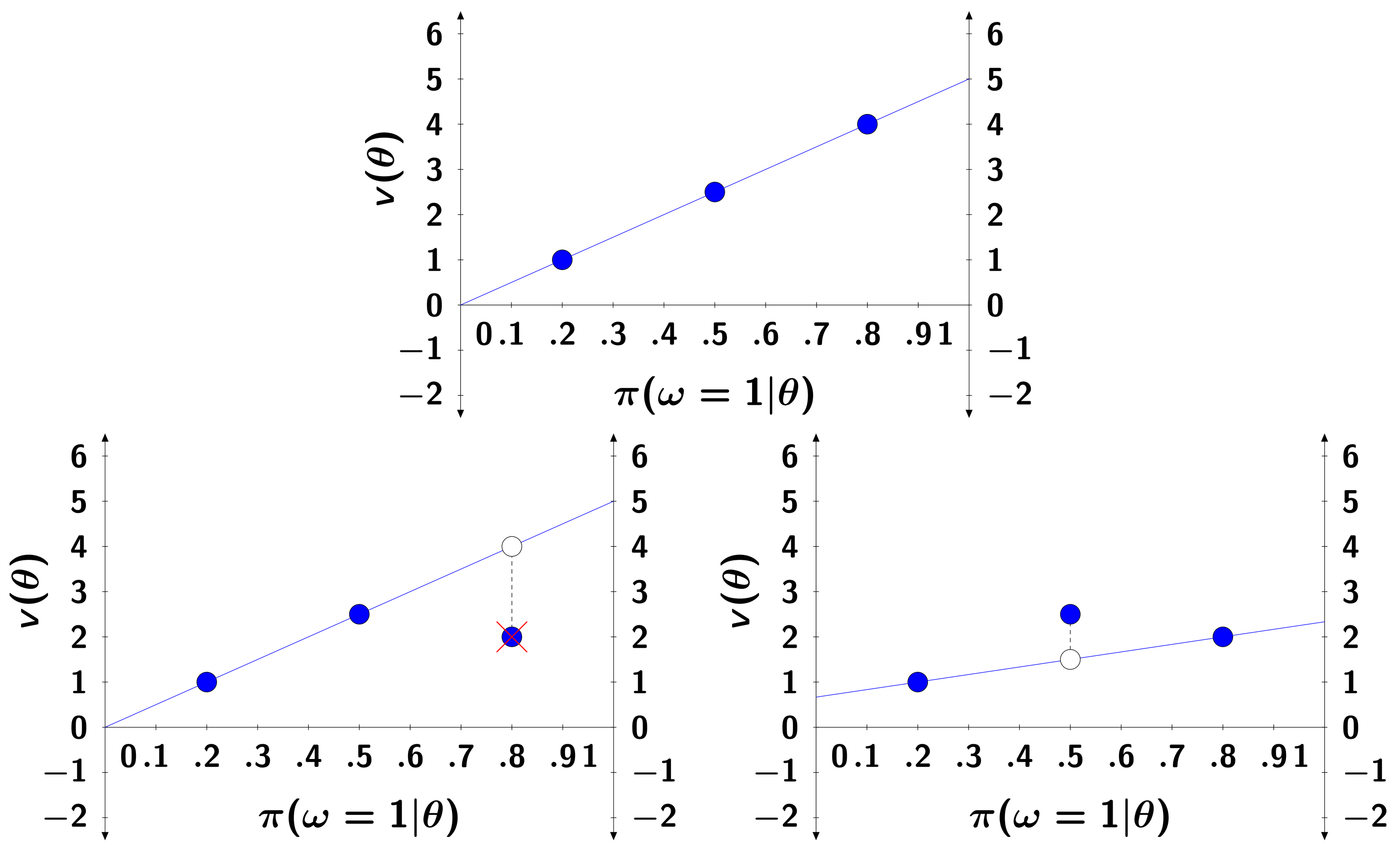
In many situations:

Cremer-McLean Revenue $\ll$ *Bayesian Revenue*

Theorem 1: Full Revenue with Bayesian IR and Ex-Post IC

For a given (π, Θ, Ω), full surplus extraction is possible for a Cremer-McLean mechanism if and only if there exists a *linear* function $G : \mathbb{R}^{|\Omega|} \rightarrow \mathbb{R}$ such that $G(\pi(\bullet|\theta)) = v(\theta)$.

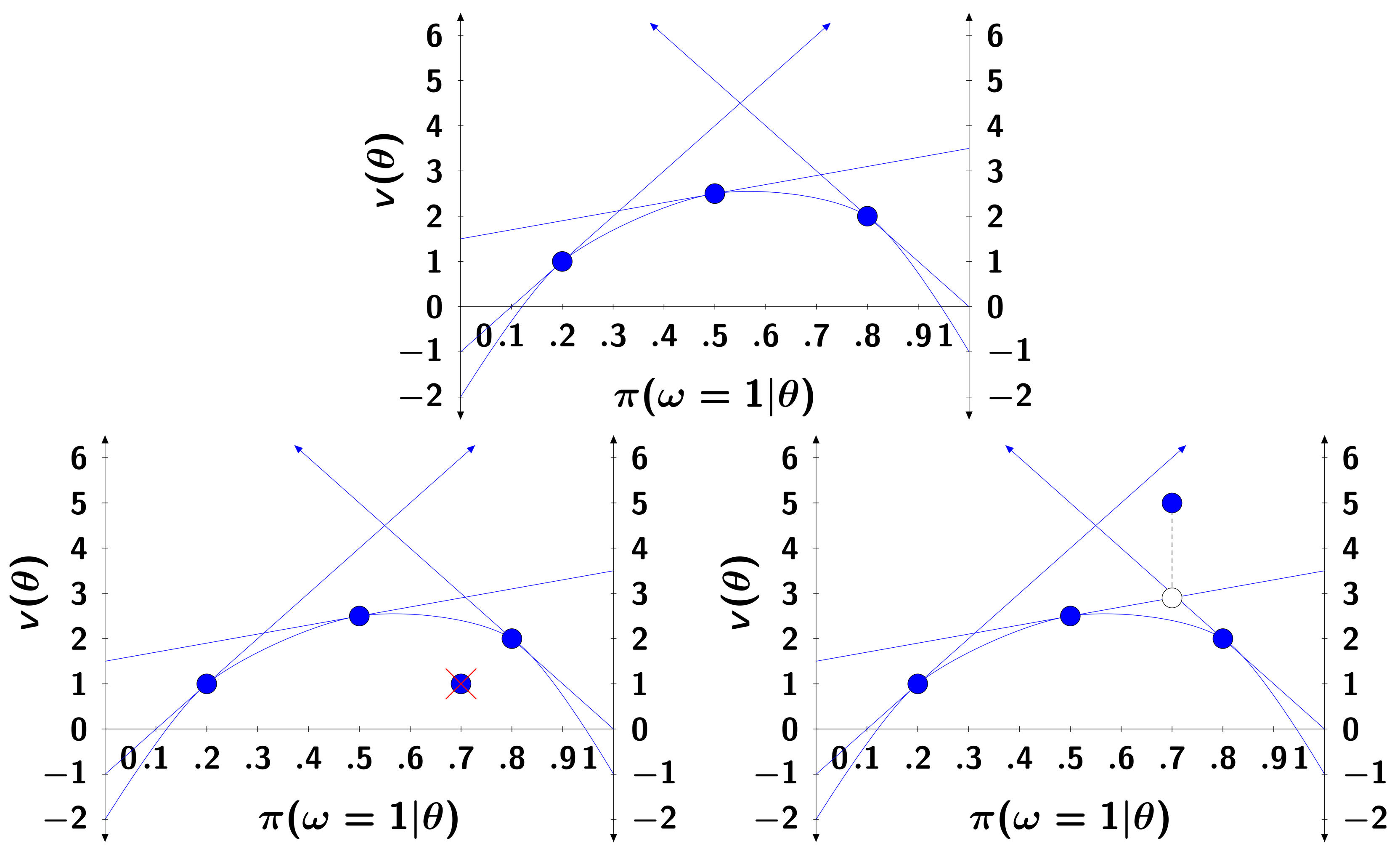
Intuition for Proof of Theorem 1



Theorem 2: Full Revenue with Bayesian IR and Bayesian IC

For a given (π, Θ, Ω), full surplus extraction is possible for a Bayesian mechanism if and only if there exists a *concave* function $G : \mathbb{R}^{|\Omega|} \rightarrow \mathbb{R}$ such that $G(\pi(\bullet|\theta)) = v(\theta)$.

Intuition for Proof of Theorem 2



Theorem 3: Inapproximability of the Optimal Bayesian Mechanism

The expected revenue generated by an optimal ex-post mechanism guarantees at most a $(|\Theta|+1)/4$ **approximation** to the expected revenue generated by an optimal Bayesian mechanism

Experimental Results

- Randomly generated distributions with specified correlation
- Unless otherwise specified parameters are: $|\Theta| = 30$, $Corr = .1$, and $|\Omega| = 5$
- We average over 100 randomly generated distributions