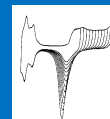


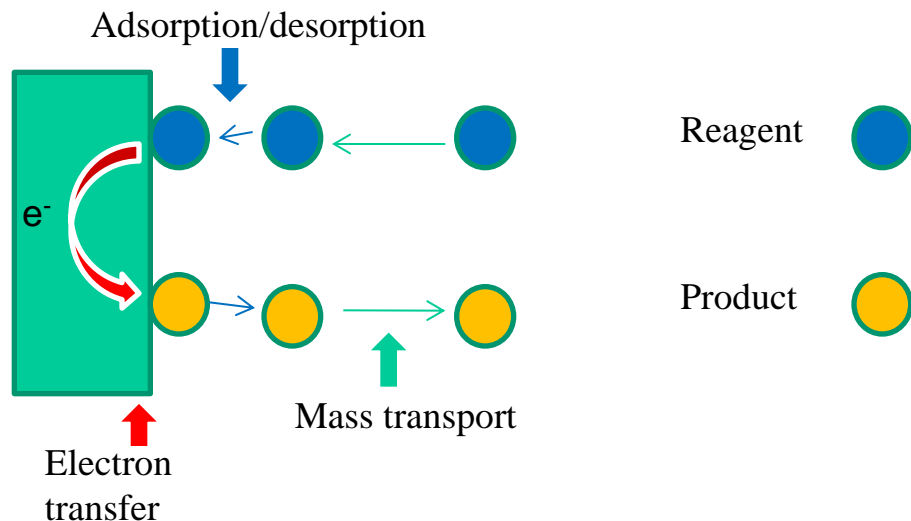
Diffusion in Battery Cells



Solid redox reactions need solid diffusion

SOLUTION

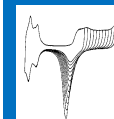
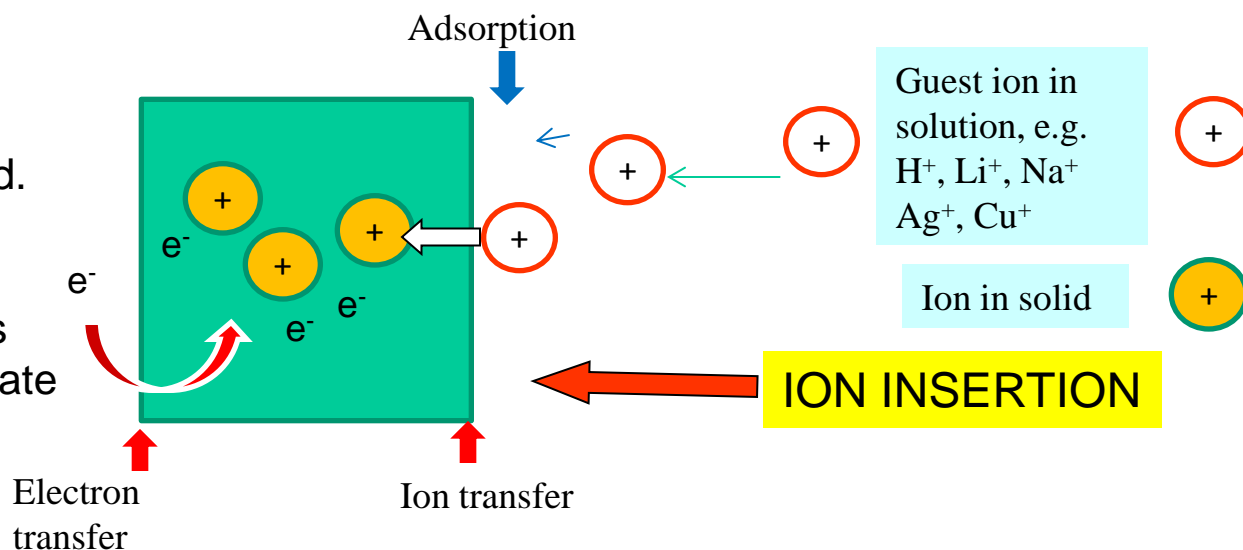
The redox reagent is initially in the solution. It diffuses to the interface, reacts, then returns to the solution,



SOLID

The reagent is a solid.

The guest ion enters the solid to compensate the charge.

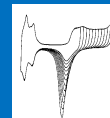


Diffusion is slow, specially in the solid state

time constant = L^2/D

Estimate the time required for lithium ions to diffuse a distance of 100 μm . Use the approximation $t=L^2/D$ and consider $D=10^{-10} \text{ cm}^2/\text{s}$

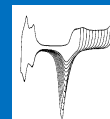
Answer: 10^6 seconds (280 hours)



Diffusion is slow, specially in the solid state

Estimate now the time required for lithium ions to diffuse a distance of 100 nm. Use the approximation $t=L^2/D$ and consider $D=10^{-10} \text{ cm}^2/\text{s}$

Answer: 1 seconds



Diffusion in Battery Cells

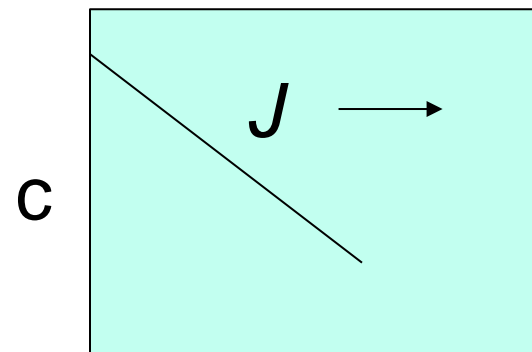
Fick's First Law:

Flux

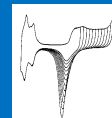
$$J(\text{mol cm}^{-2}\text{s}^{-1}) = -D \frac{\partial c}{\partial x}$$

(with Faraday) Current/area

$$j(\text{A cm}^{-2}) = -nFD \frac{\partial c}{\partial x}$$



Distance, x



Diffusion in Battery Cells

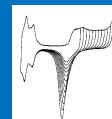
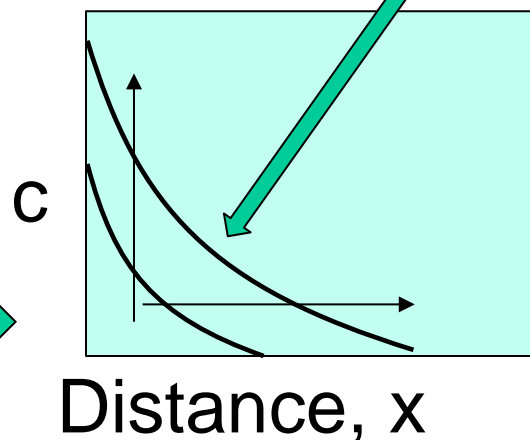
Fick's Second Law:

Positive Curvature accompanies
accumulation

$$D \frac{\partial^2 c}{\partial x^2} = \frac{\partial c}{\partial t}$$

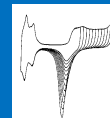
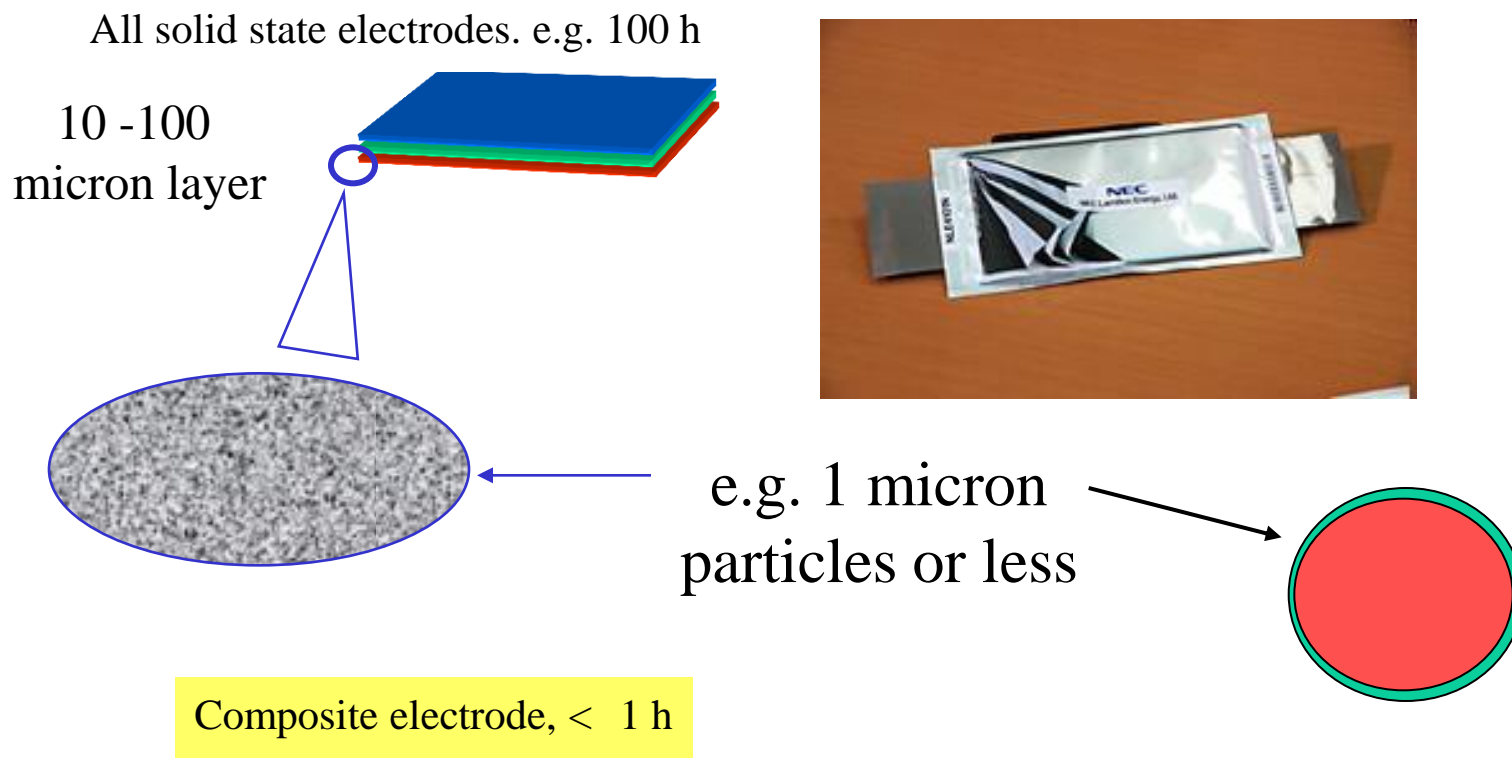
Current in:

$$j \text{ (Acm}^{-2}\text{)} = -nFD \frac{\partial c}{\partial x} (x=0)$$

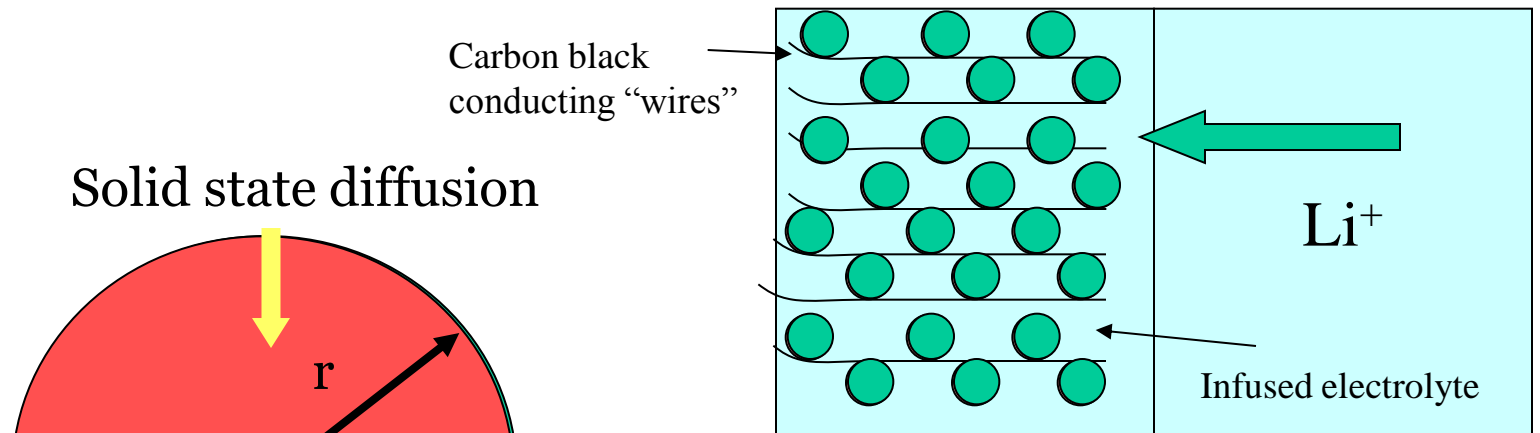


Solution: Composite Electrodes!

Made of active material + conductive additive (carbon) + binder

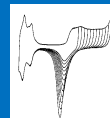
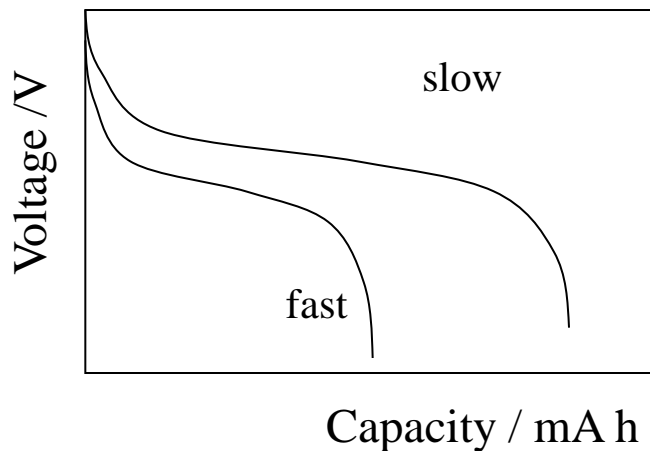


Composite Electrodes



All particles react together and concentration polarisation occurs within the particles.

Discharge profiles:



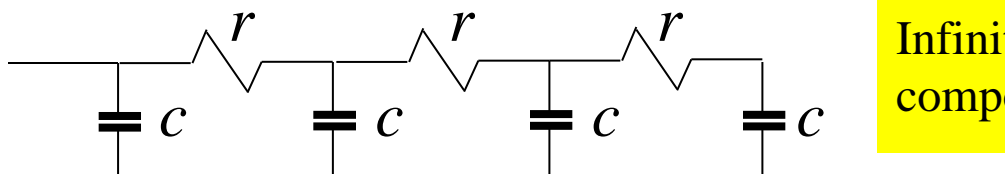
Rough and Porous Electrodes: the De Levie model

electrolyte resistance per unit
electrode length, r

capacitance per unit
electrode length, c

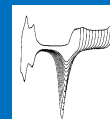


Equivalent
circuit

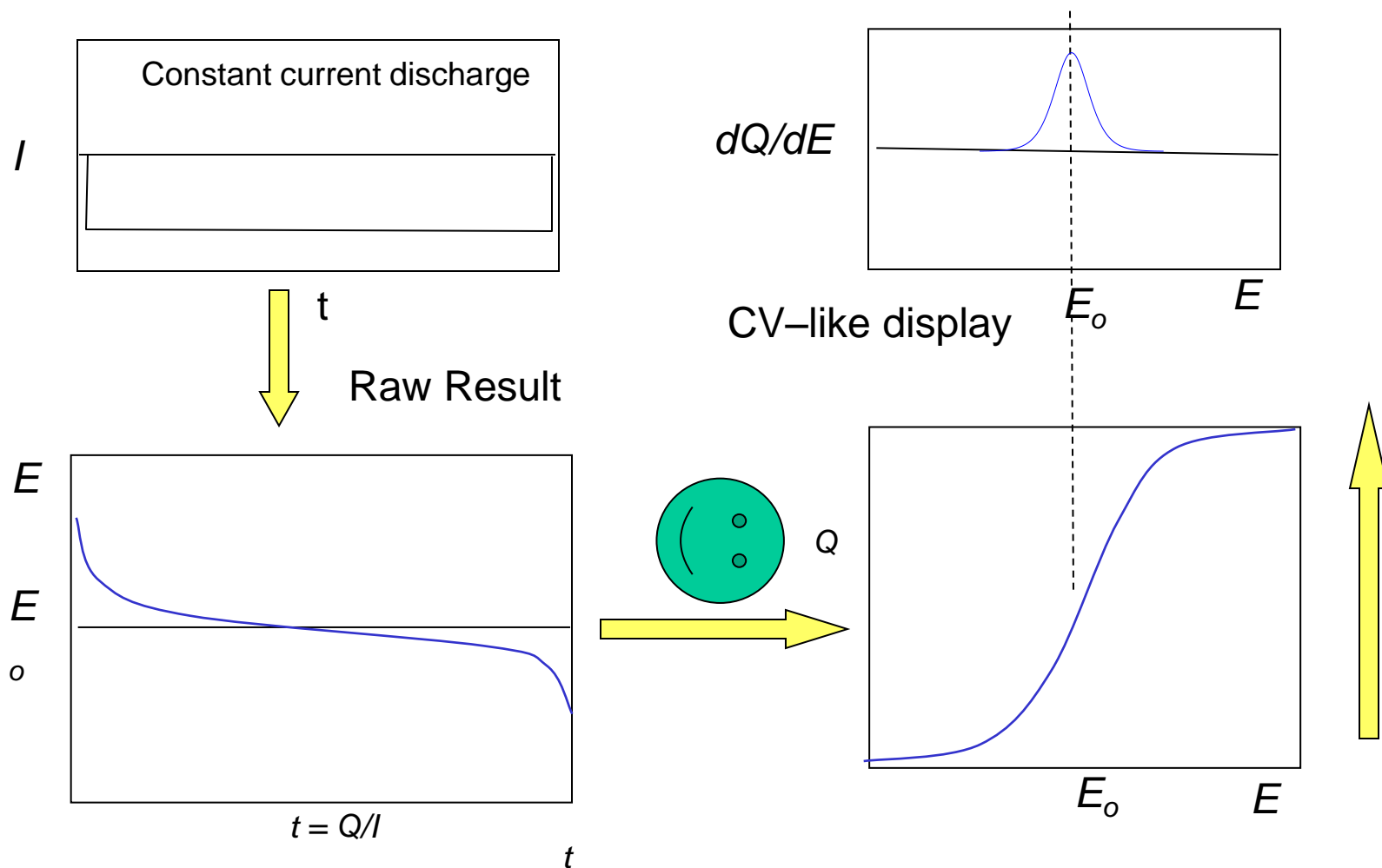


Infinite sum of small
components

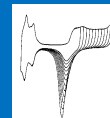
Result:
Effective diffusion coefficient = r^*c



Derivative Chronopotentiometry(DCP)

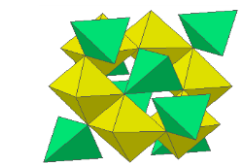


IR drop is constant - does not increase toward peak top

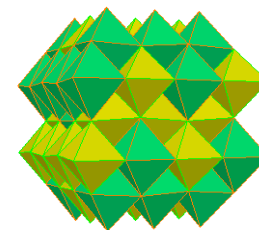
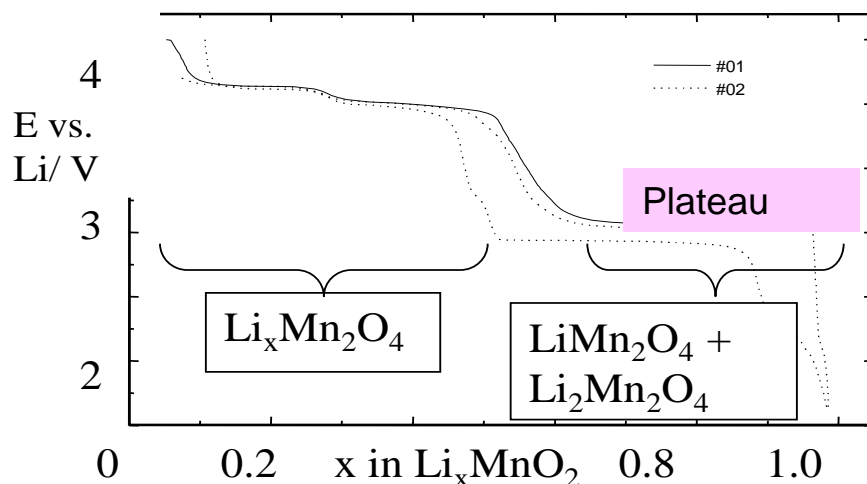


DCP for a Li-ion positive electrode

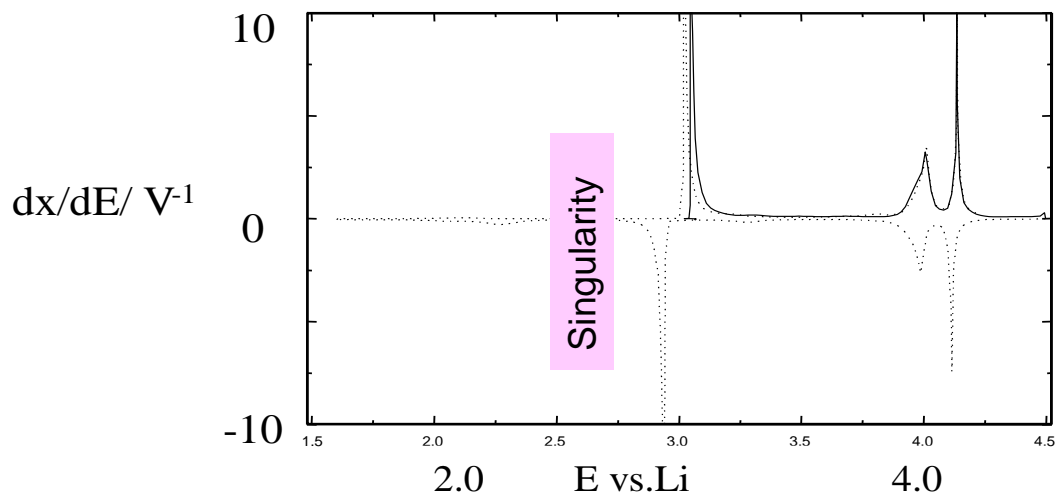
The Spinel $\text{Li}_x\text{Mn}_2\text{O}_4$ lithium cathode shows both 1-phase and 2-phase insertion reactions



Spinel-type
 $\text{Li}_x\text{Mn}_2\text{O}_4$

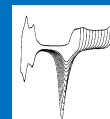


NaCl-type
 $\text{Li}_2\text{Mn}_2\text{O}_4$



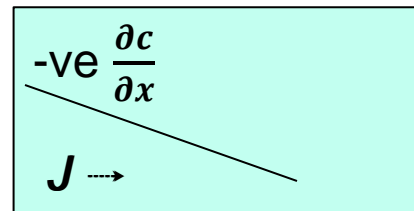
•N.B. plateau in E gives singularity in dx/dE

•Nernst Equation gives $\text{const } E = \Delta G/nF$

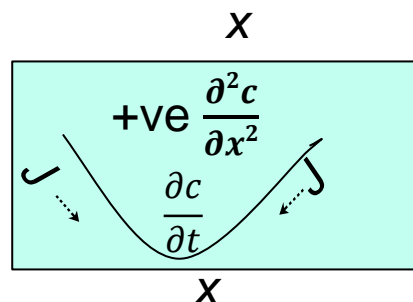


Fick's laws of Diffusion and the Sand Equation for constant current

- 1st law: flux $J_{Li+} = -D \frac{\partial c}{\partial x}$



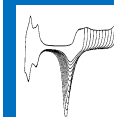
- 2nd law: $\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x} = D \frac{\partial^2 c}{\partial x^2}$



- Current pulse boundary conditions

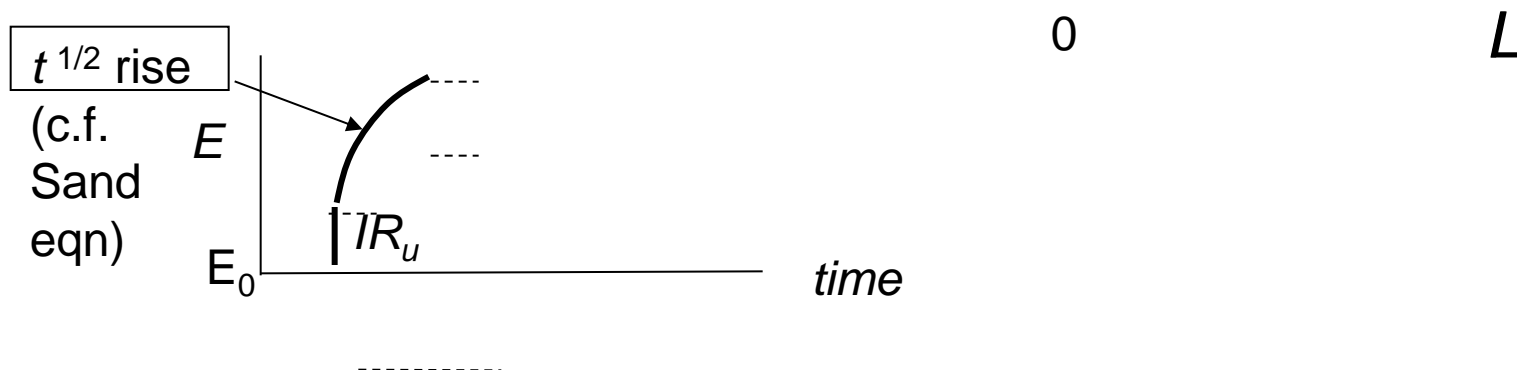
$$J_{Li+}(x=0) = j/F \quad \text{where } j = \text{current} / \text{area}$$

- Result: c increases as $j t^{1/2} / n F D^{1/2} \pi^{1/2}$

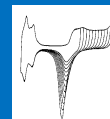


Chronopotentiometry

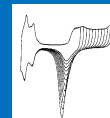
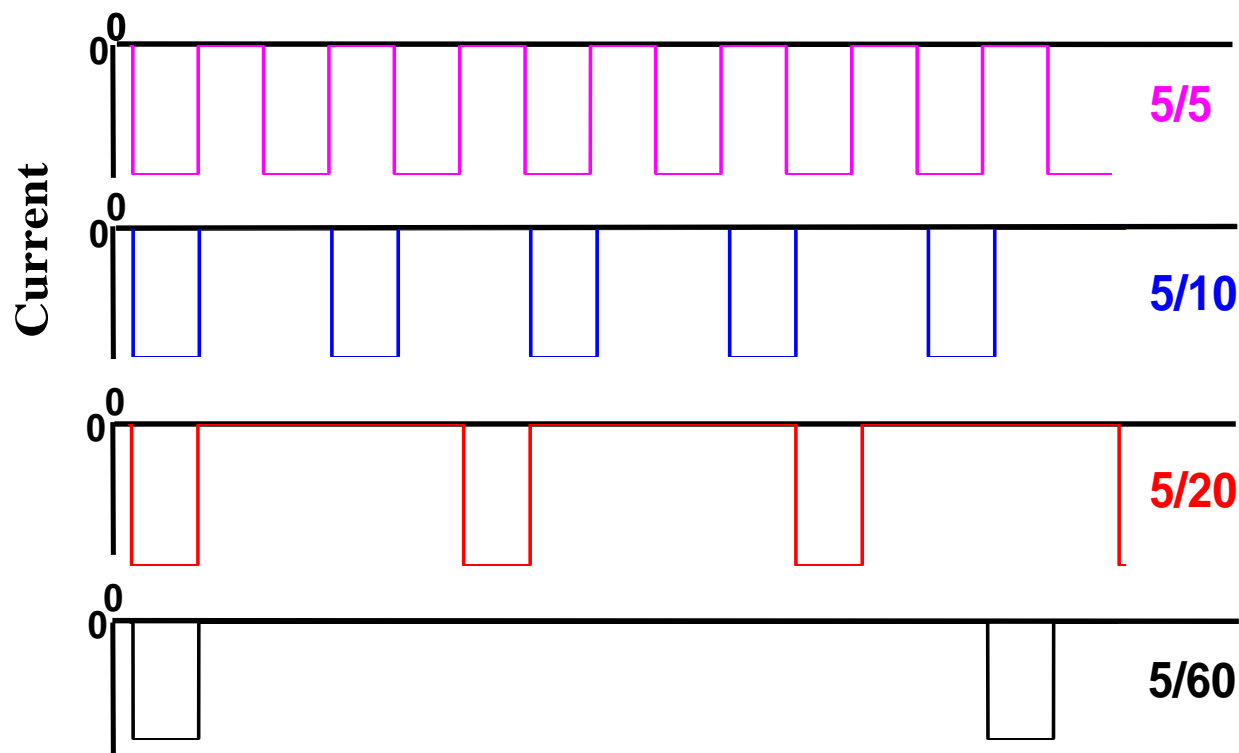
- Constant current pulse
- Measure potential vs time
- Measure equilibrium potential after the pulse



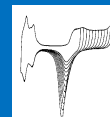
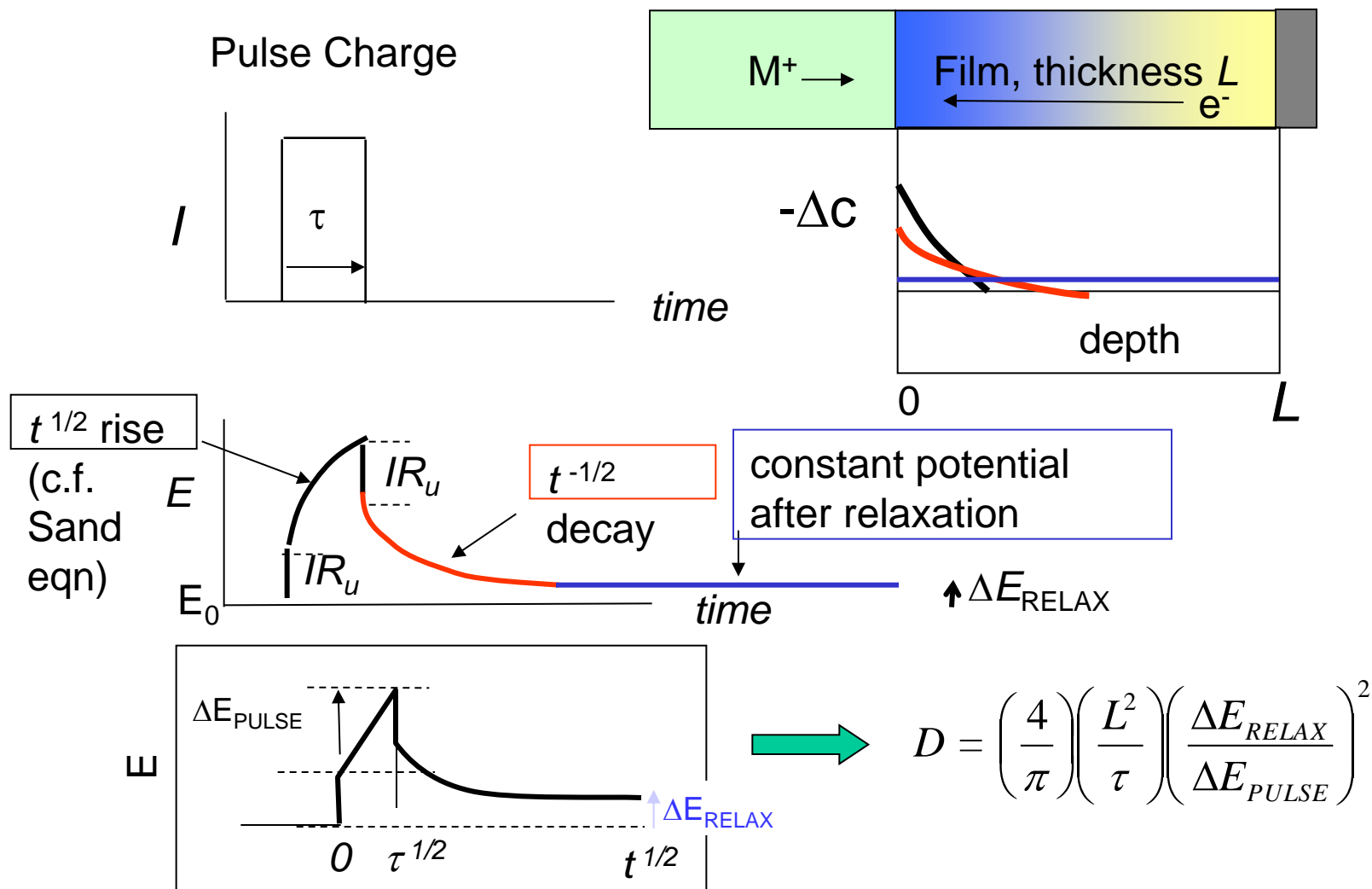
c increases as $j t^{1/2} / n F D^{1/2} \pi^{1/2}$



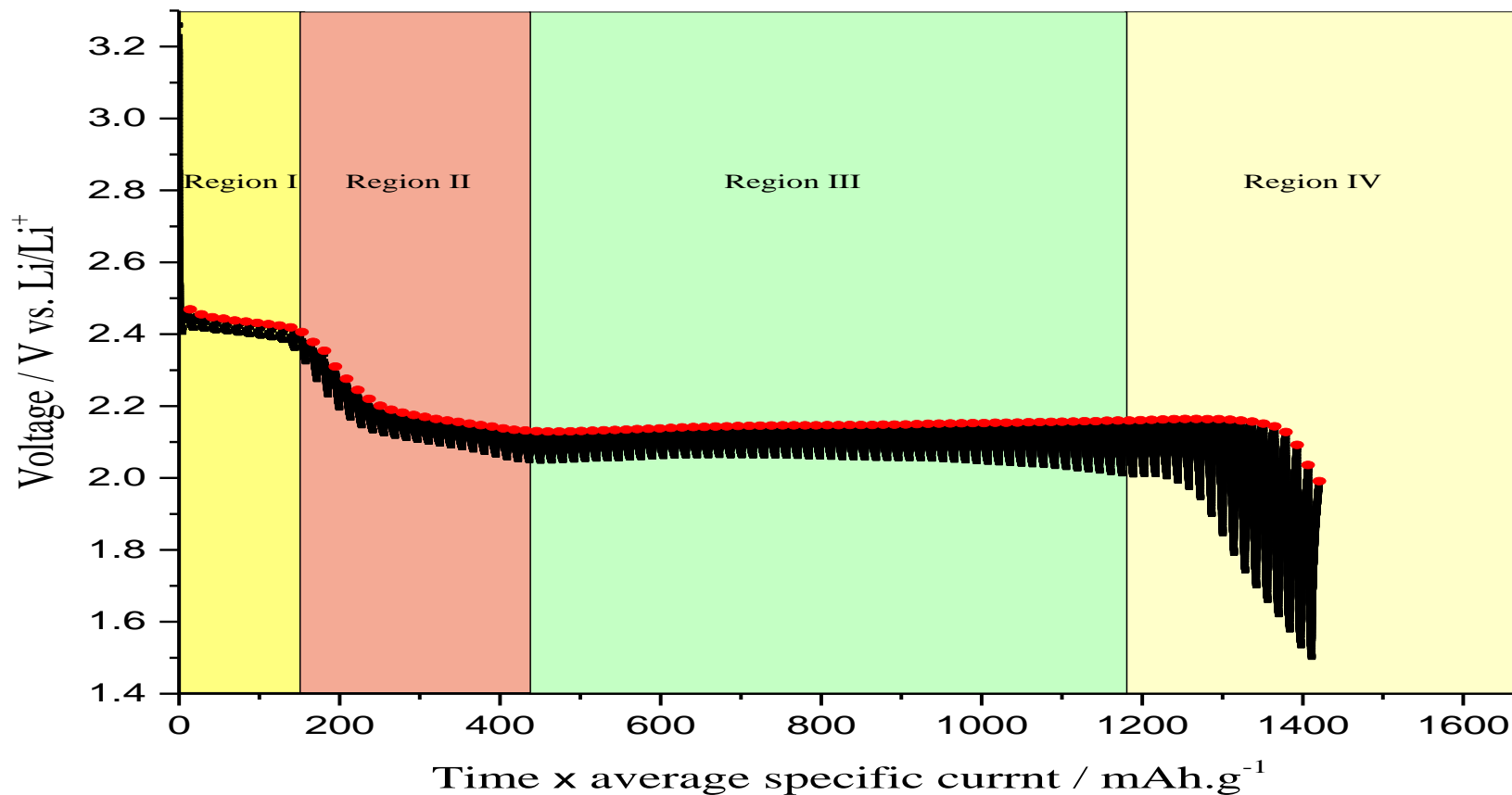
The Galvanostatic Intermittent Titration Technique (GITT)



Current pulse analysis (GITT) to measure diffusion in single phase materials

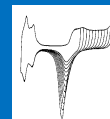


A complete titration

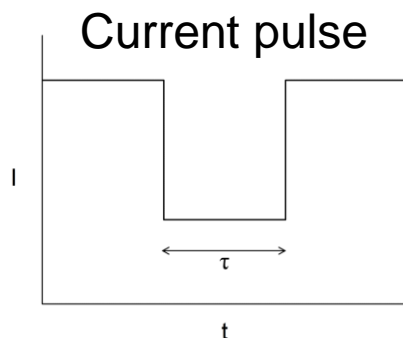


GITT profile for first discharge of Li/S cell using 5 min pulse and 10 min relaxation (5/10), with 1M LiTFSI in TEGDME/DOL. The red circles represent the last data point at the end of each relaxation period at discharge state of Li-S cell.

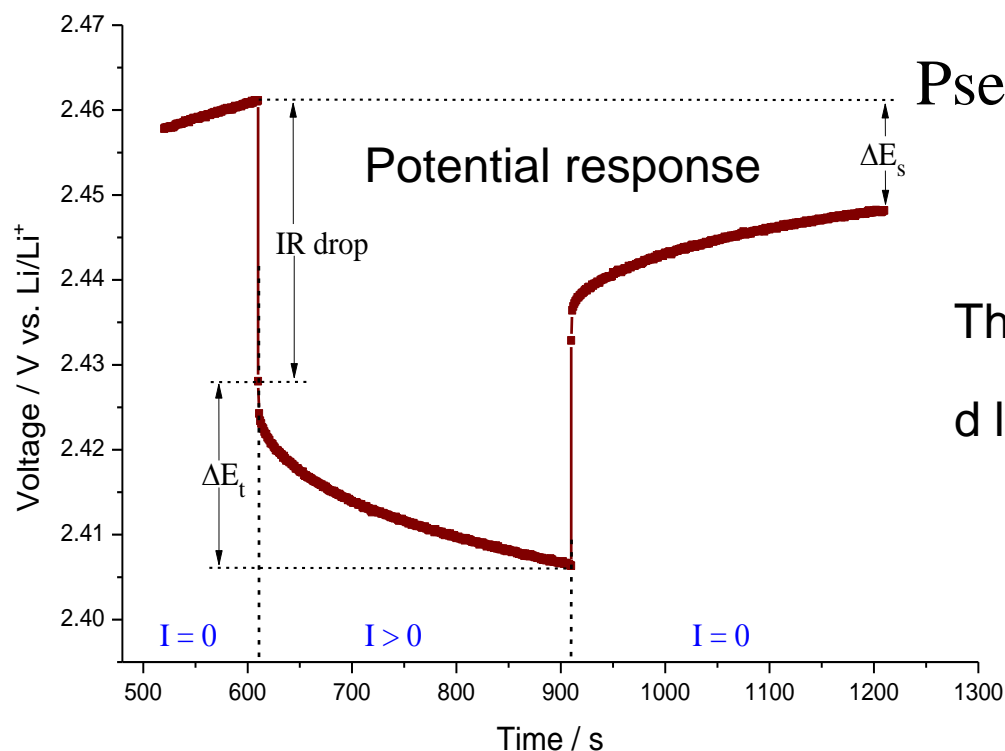
GITT also shows the thermodynamic discharge curve



Further GITT analysis



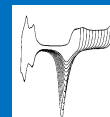
$$D = \left(\frac{4}{\pi} \right) \left(\frac{L^2}{\tau_{pulse}} \right) \left(\frac{dE_s}{dEt} \right)^2$$



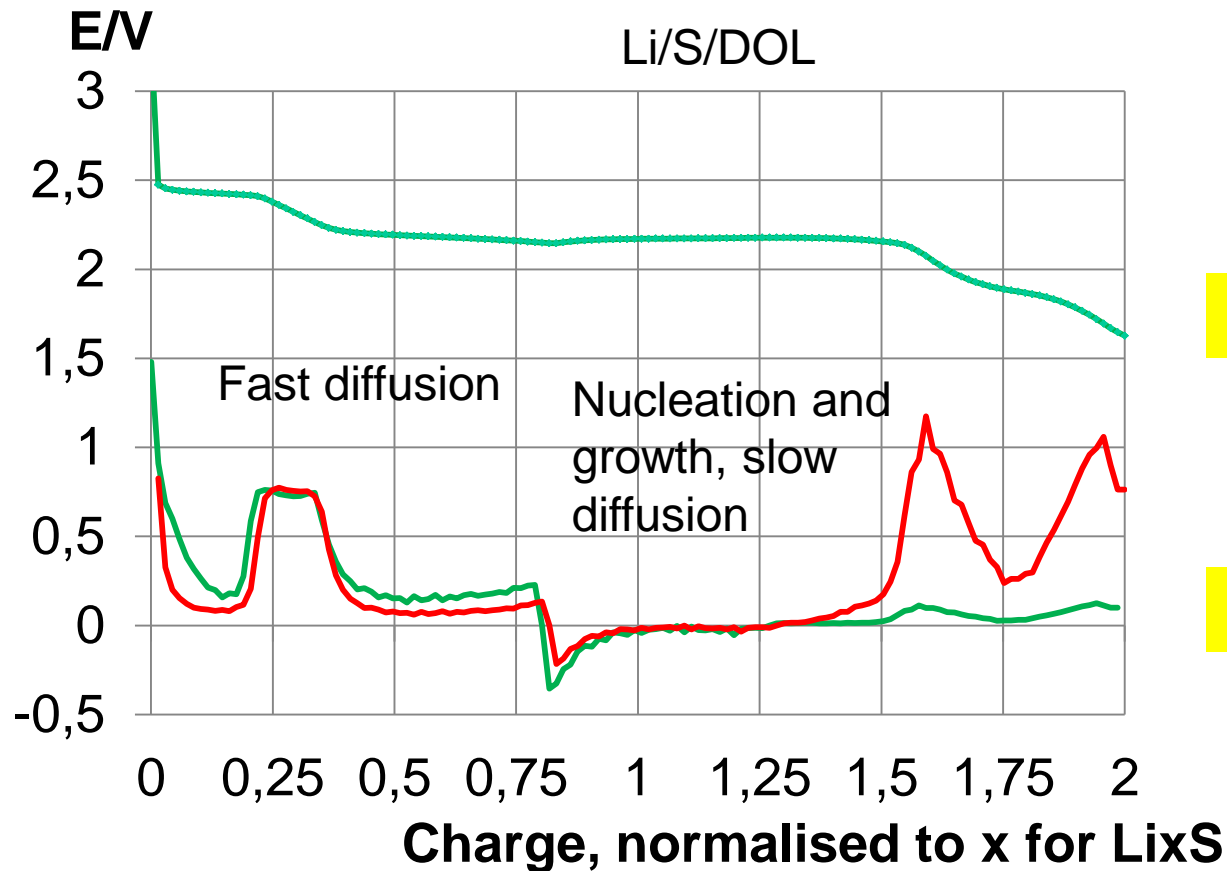
$$\text{Pseudocapacitance, } C_v = \frac{dq_v}{dE_s}$$

Thermodynamic Factor

$$d \ln a / d \ln c = \left(\frac{F^2 c}{RT} \right) \left(\frac{dE_s}{dq} \right)$$



Relaxed Potential, Diffusion Coefficient, Pseudo-capacitance and Thermodynamic Factors from GITT

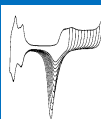


$$T.F. = k \cdot d \ln a / d \ln c$$

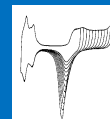
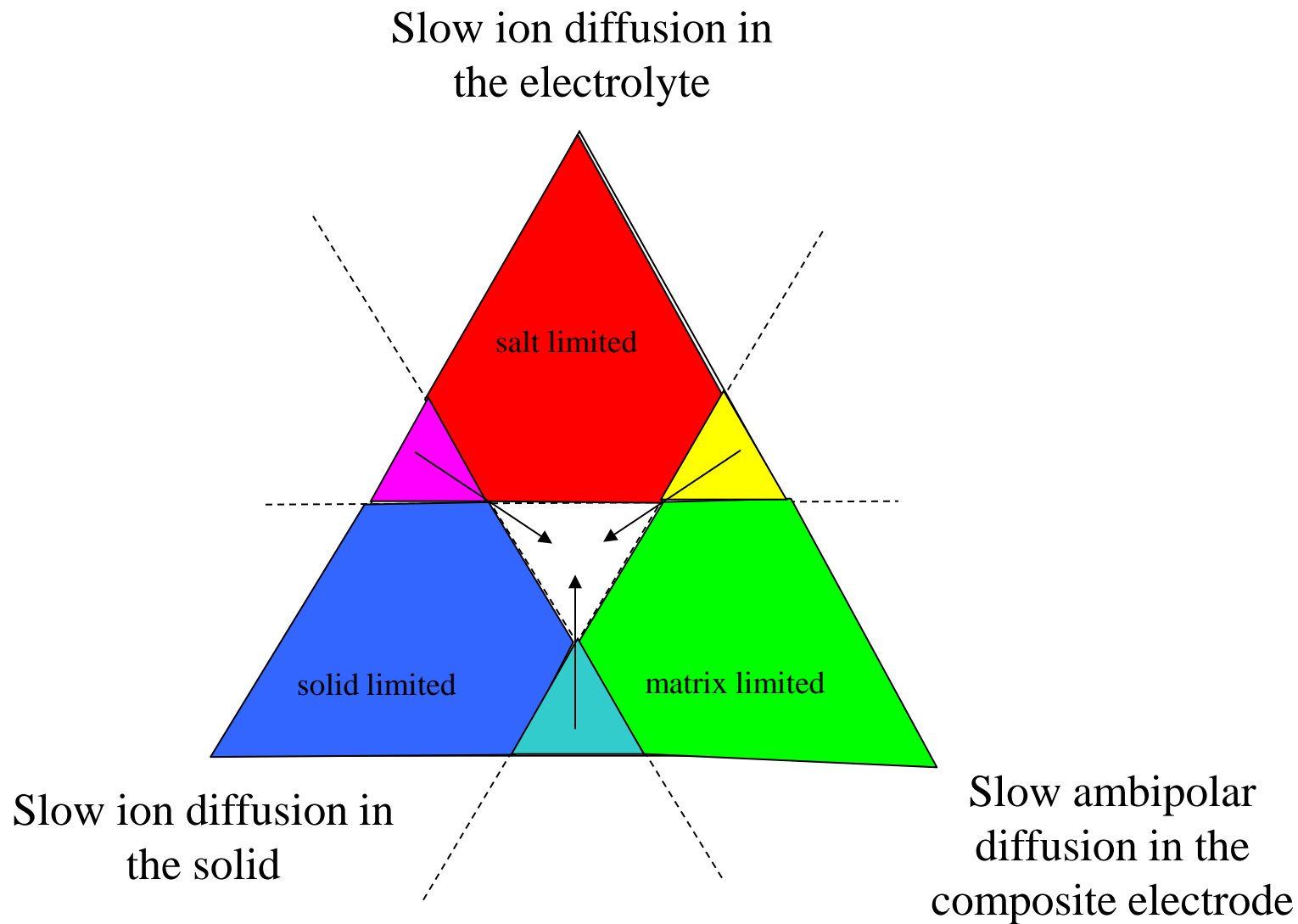
$$4 (dV_s / dV_t)$$

$$D = TF \cdot D_0 \text{ (A.U.)}$$

- D generally follows $T.F.$ except at end point where tortuosity is high.
- Anomalous diffusion at 0.8 followed by $T.F. = 0$.
- Suggests nucleation & growth of a solid phase during which D value is invalid.

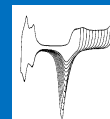


Conclusion – the rate limiting step rules !



Summary

- Diffusion is modelled by Fick's 1st and 2nd Laws
- Slow diffusion in solids is compensated by fine particle composites
- Galvanostatic potentiometry gives $t^{1/2}$ potential profiles, measuring D at a single composition
- GITT gives both Thermodynamic and Diffusion data over a range of compositions,



Diffusion in Battery Cells

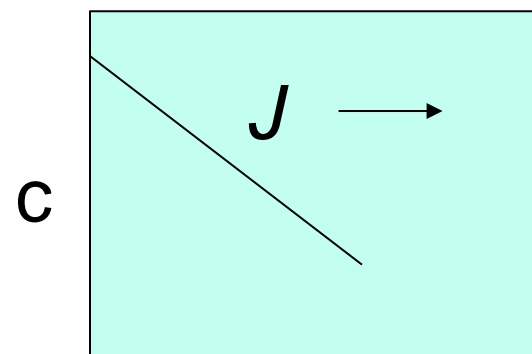
Fick's First Law:

Flux

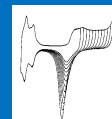
$$J(\text{mol cm}^{-2}\text{s}^{-1}) = -D \frac{\partial c}{\partial x}$$

(with Faraday) Current/area

$$j(\text{A cm}^{-2}) = -nFD \frac{\partial c}{\partial x}$$



Distance, x



Diffusion in Battery Cells

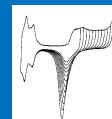
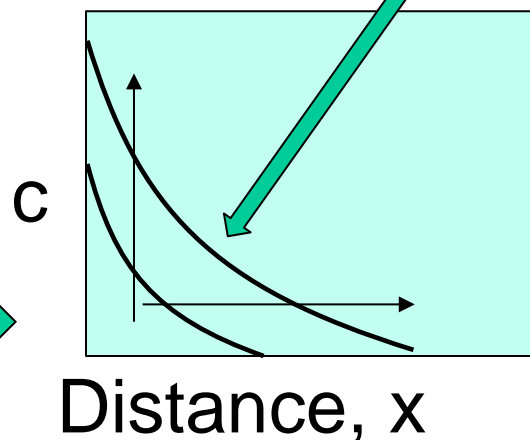
Fick's Second Law:

Positive Curvature accompanies
accumulation

$$D \frac{\partial^2 c}{\partial x^2} = \frac{\partial c}{\partial t}$$

Current in:

$$j \text{ (Acm}^{-2}\text{)} = -nFD \frac{\partial c}{\partial x} (x=0)$$



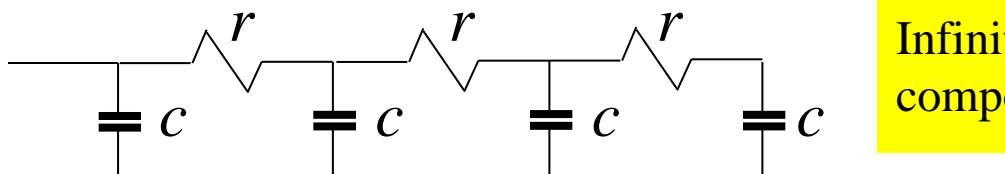
Rough and Porous Electrodes: the De Levie model

electrolyte resistance per unit
electrode length, r

capacitance per unit
electrode length, c



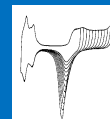
Equivalent
circuit



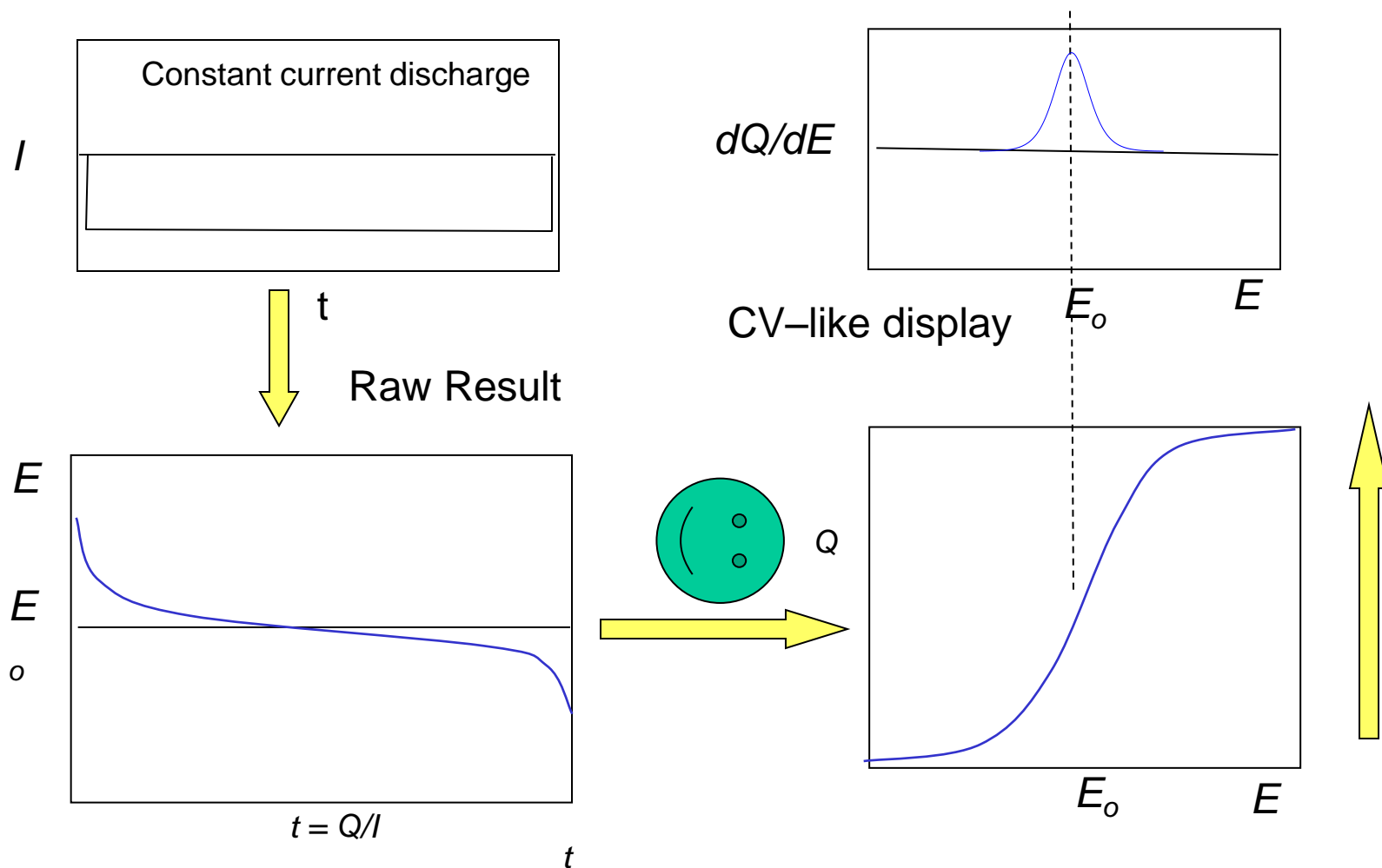
Infinite sum of small
components

Result:

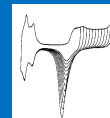
Effective diffusion coefficient = $r \cdot c$



Derivative Coulometric Titration (DCT)

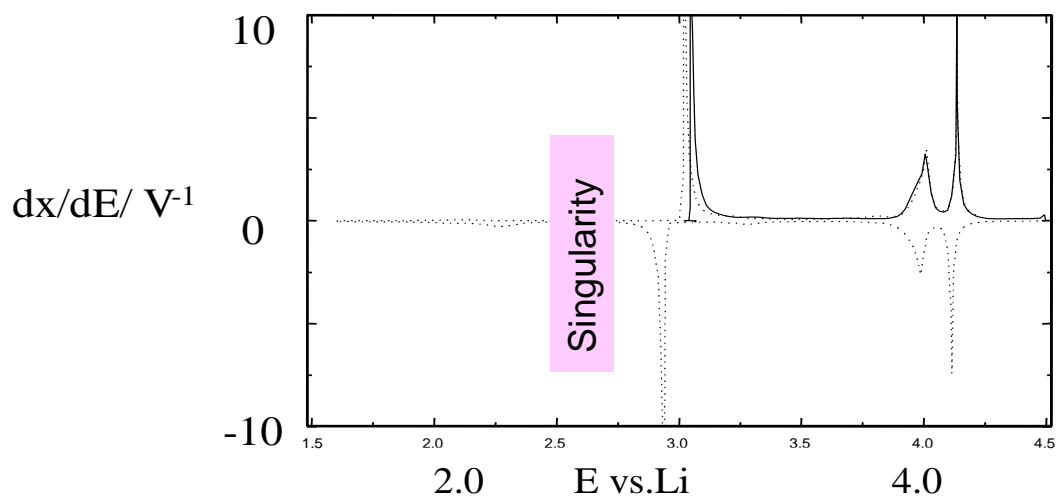
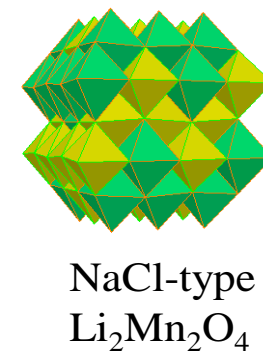
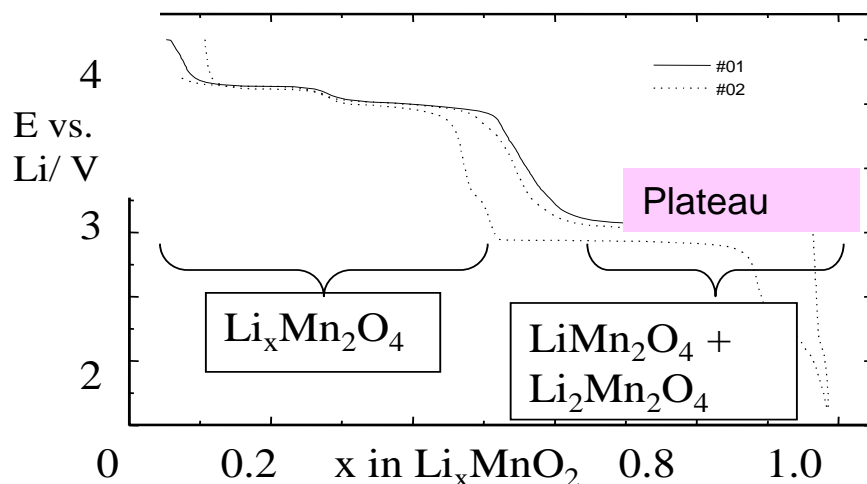
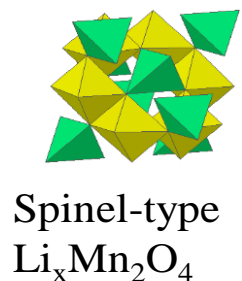


IR drop is constant - does not increase toward peak top



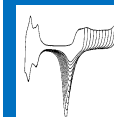
DCT for a Li-ion positive electrode

The Spinel $\text{Li}_x\text{Mn}_2\text{O}_4$ lithium cathode shows both 1-phase and 2-phase insertion reactions

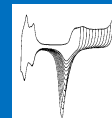
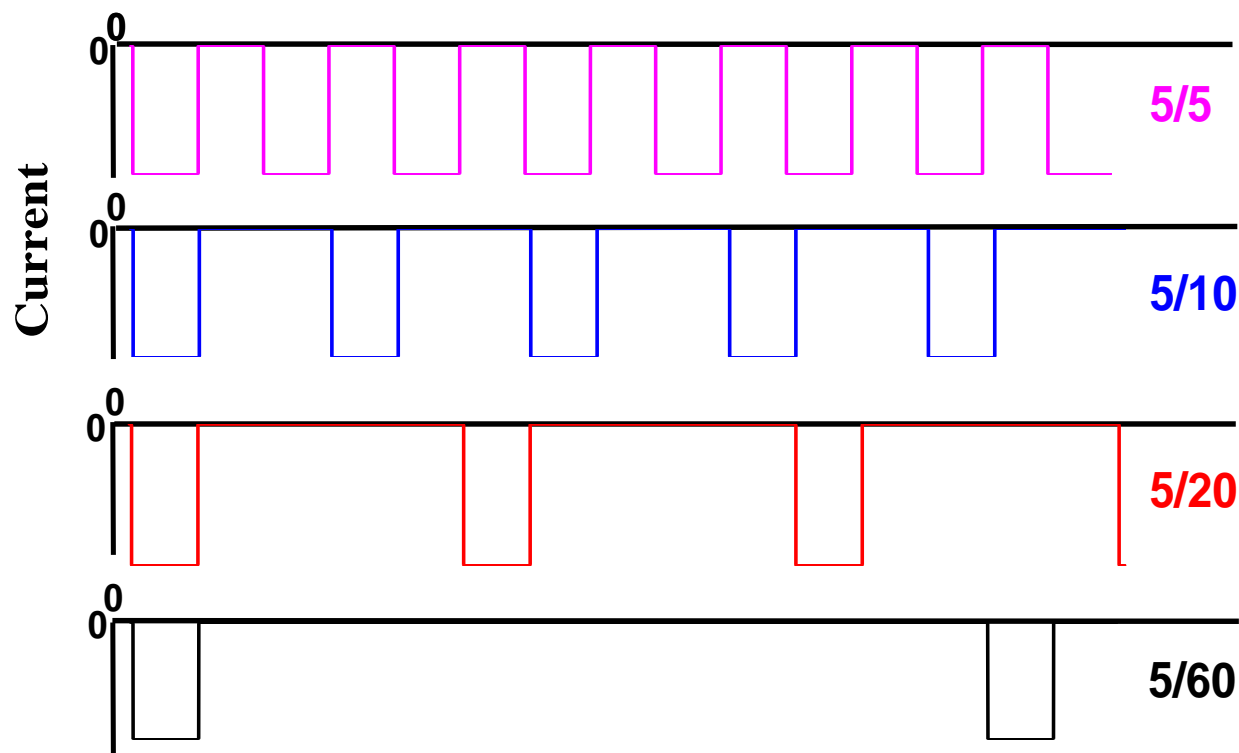


•N.B. plateau in E gives singularity in dx/dE

•Nernst Equation gives $\text{const } E = \Delta G/nF$

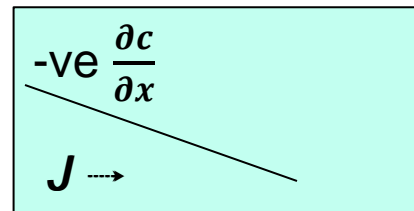


The Galvanostatic Intermittent Titration Technique (GITT)

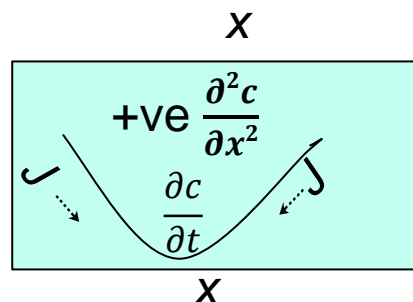


Fick's laws of Diffusion and the Sand Equation for constant current

- 1st law: flux $J_{Li+} = -D \frac{\partial c}{\partial x}$



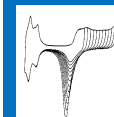
- 2nd law: $\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x} = D \frac{\partial^2 c}{\partial x^2}$



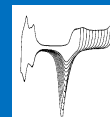
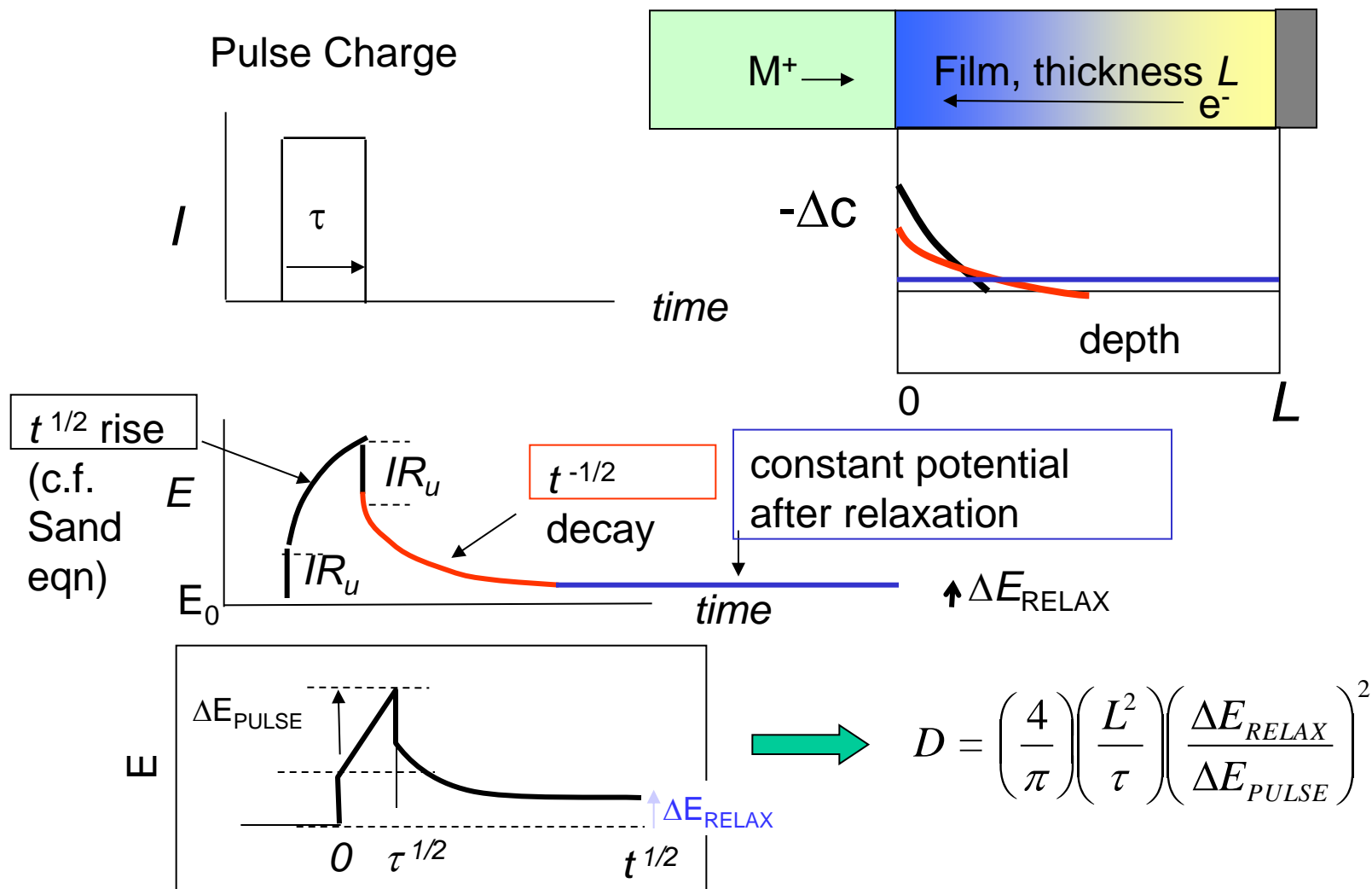
- Current pulse boundary conditions

$$J_{Li+}(x=0) = j/F \quad \text{where } j = \text{current} / \text{area}$$

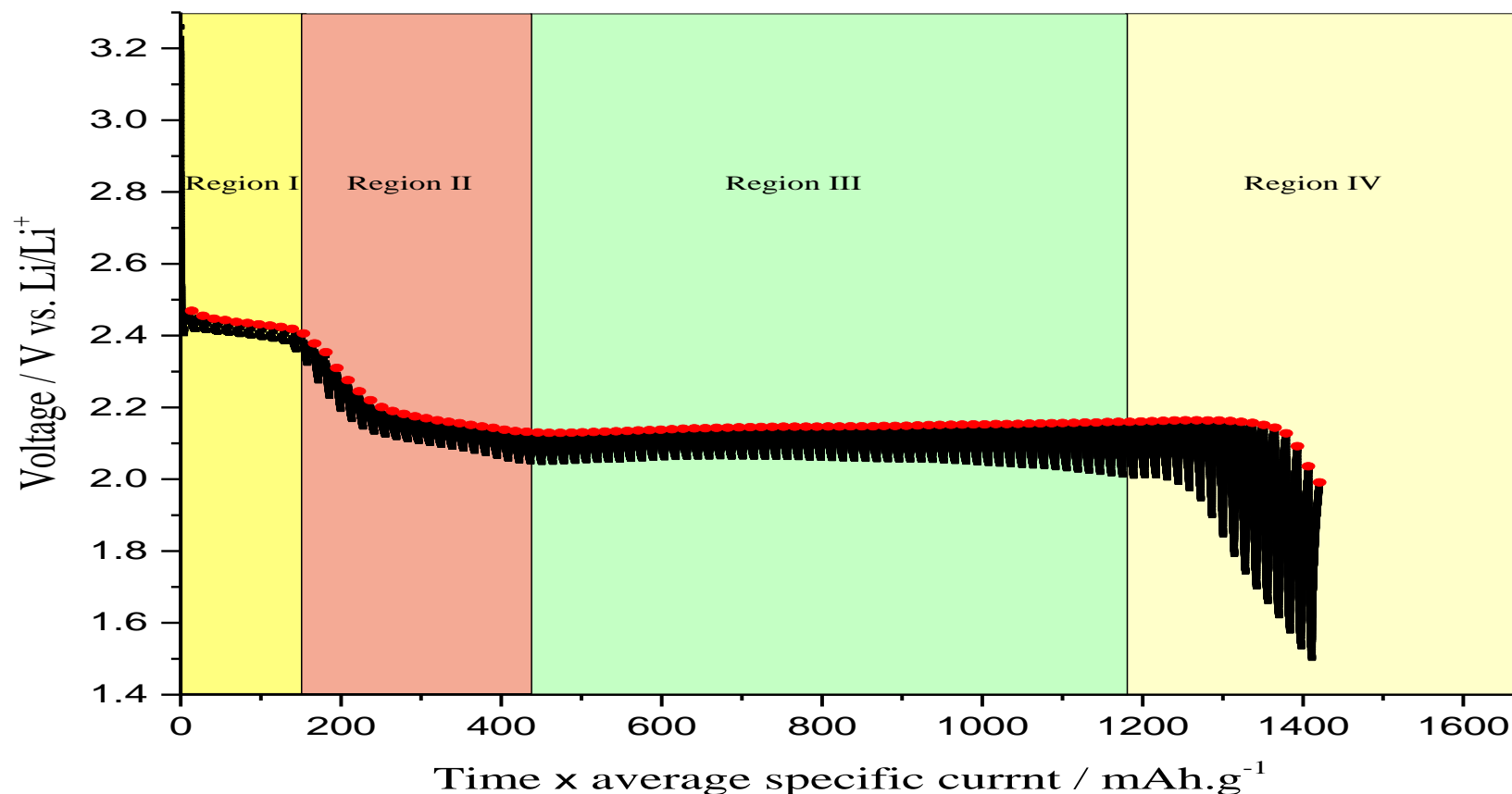
- Result: c increases as $j t^{1/2} / n F D^{1/2} \pi^{1/2}$



Current pulse analysis (GITT) to measure diffusion in single phase materials

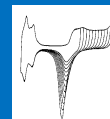


A complete titration

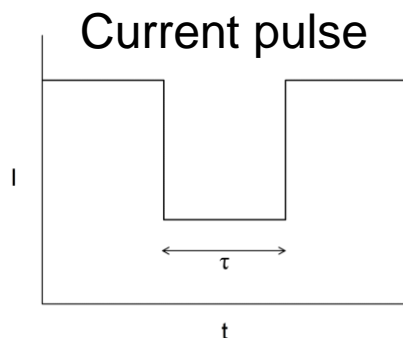


GITT profile for first discharge of Li/S cell using 5 min pulse and 10 min relaxation (5/10), with 1M LiTFSI in TEGDME/DOL. The red circles represent the last data point at the end of each relaxation period at discharge state of Li-S cell.

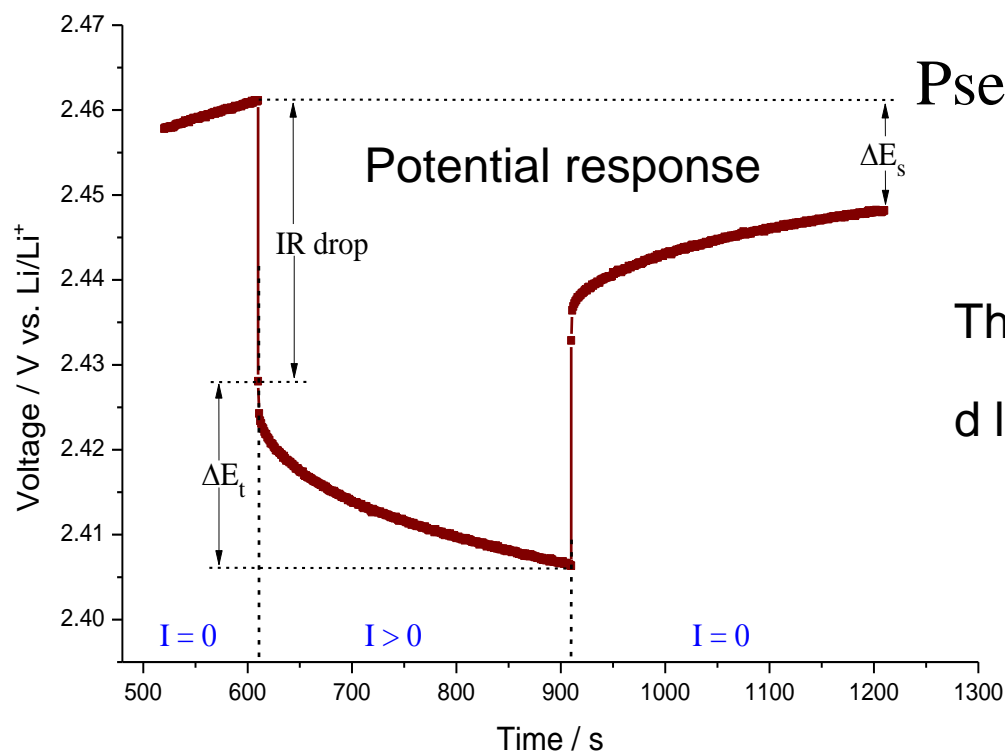
GITT also shows the thermodynamic discharge curve



Further GITT analysis



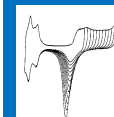
$$D = \left(\frac{4}{\pi} \right) \left(\frac{L^2}{\tau_{pulse}} \right) \left(\frac{dEs}{dEt} \right)^2$$



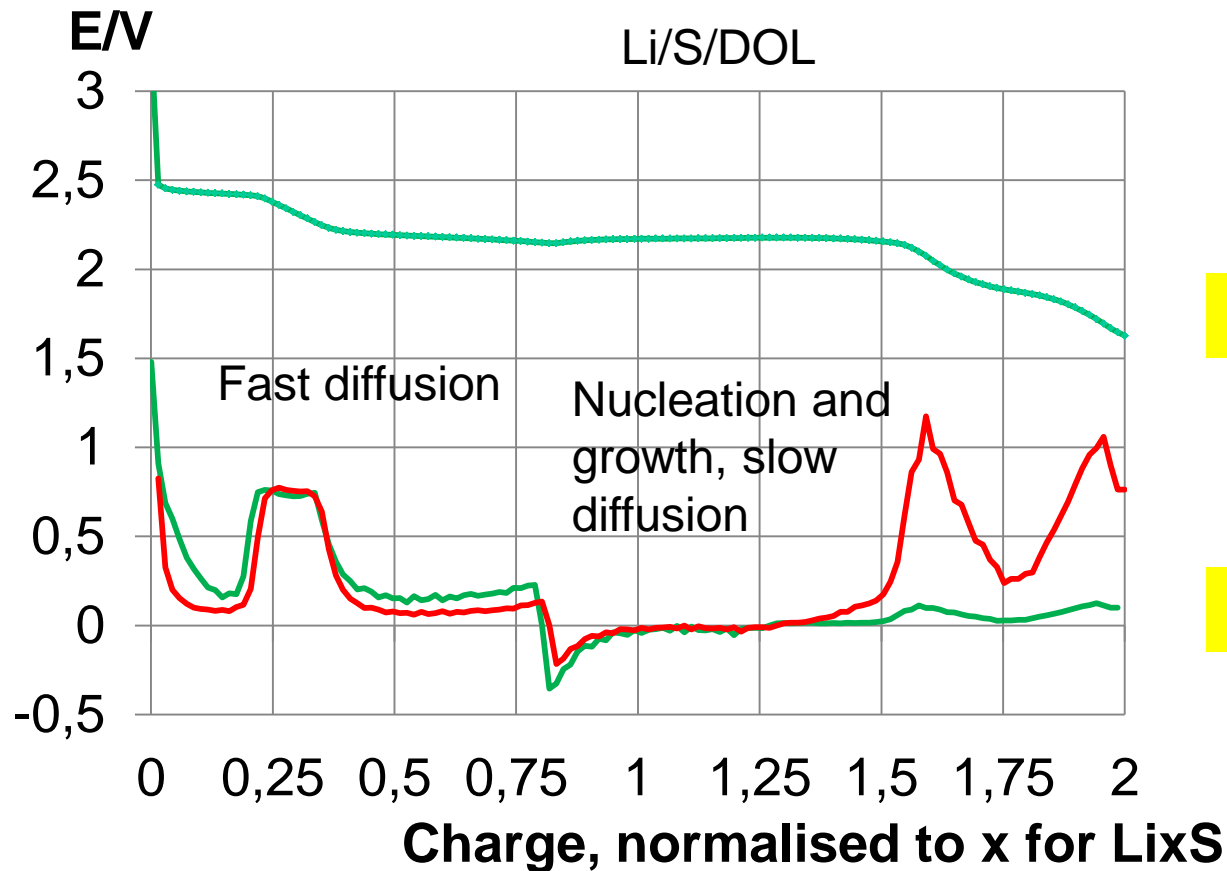
$$\text{Pseudocapacitance, } C_v = \frac{dq_v}{dEs}$$

Thermodynamic Factor

$$d \ln a / d \ln c = \left(\frac{F^2 c}{RT} \right) \left(\frac{dE_S}{dq} \right)$$



Relaxed Potential, Diffusion Coefficient, Pseudo-capacitance and Thermodynamic Factors from GITT

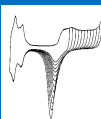


$$T.F. = k \cdot d \ln a / d \ln c$$

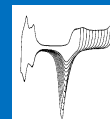
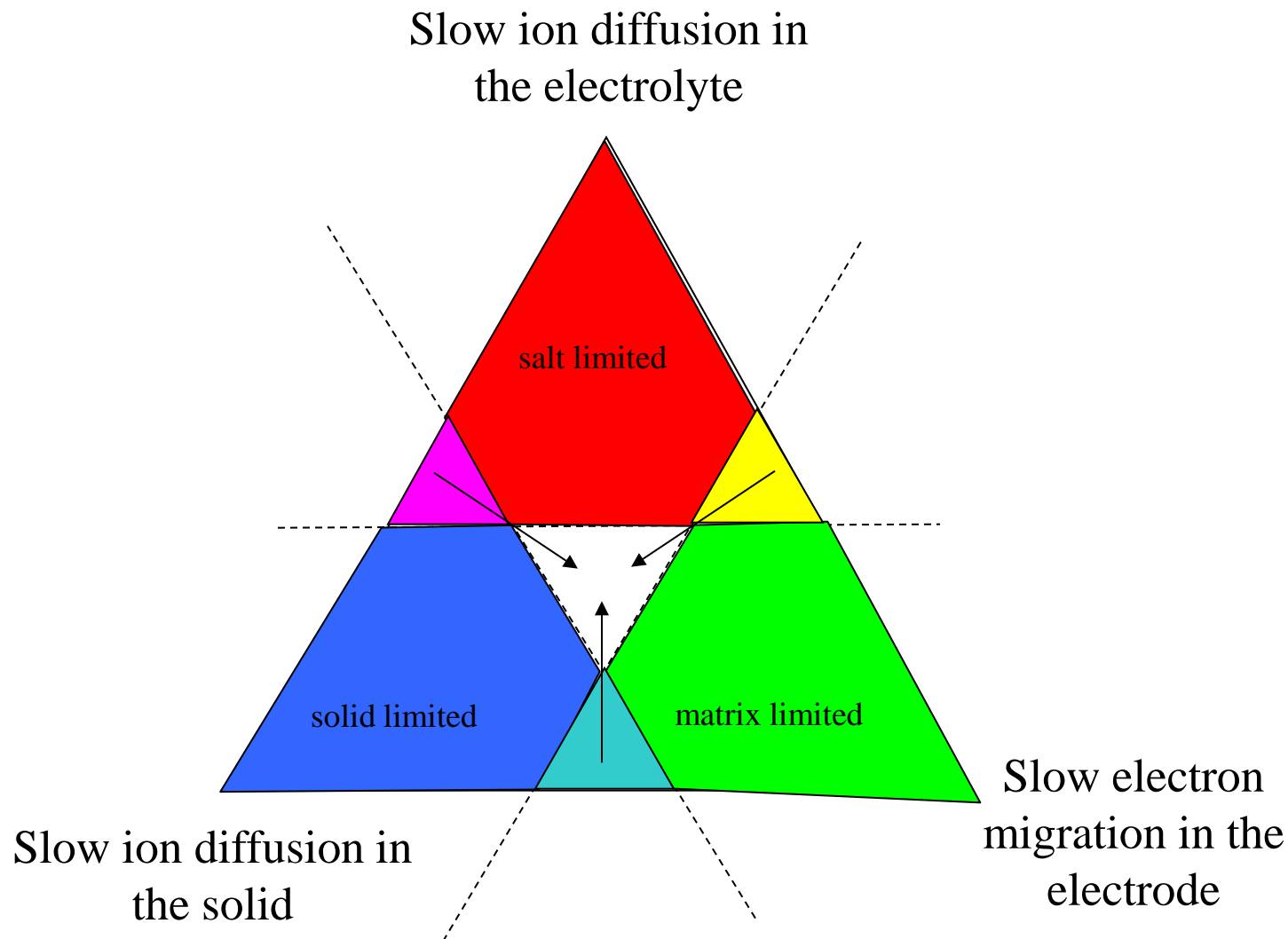
$$4 (dV_s / dV_t)$$

$$D = TF \cdot D_o \text{ (A.U.)}$$

- D generally follows $T.F.$ except at end point where tortuosity is high.
- Anomalous diffusion at 0.8 followed by $T.F. = 0$.
- Suggests nucleation & growth of a solid phase during which D value is invalid.



Conclusion – the rate limiting step rules !



Summary

- Normal cyclic voltammetry does not work well with battery cells
- Slow scan voltammetry gives a good overall picture of redox reactions and the useable potential range
- Derivative Coulometric Titration gives superior results
- GITT gives both Thermodynamic and Diffusion data

