Symbolic Learning and Rule Extraction with Sole.jl

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Where to find these slides

Who are we?

Figure: Dept. of Mathematics and Computer Science, University of Ferrara.

Who are we?

Figure: Applied Computational Logic and Artificial intelligence Lab.

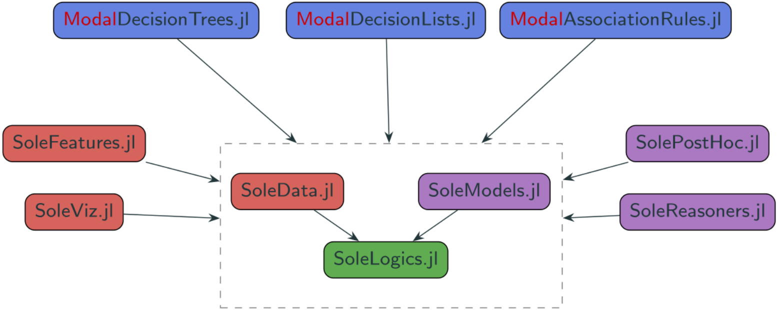

Figure: A general overview of the Sole.jl framework.

Figure: Third Millennium Symbolic Learning with Sole.jl, JuliaCon 2023.

Sole.jl at JuliaCon

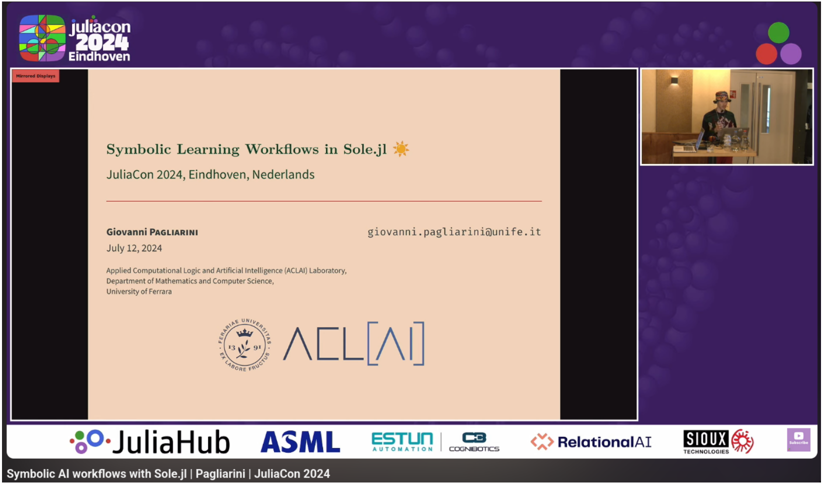

Figure: Symbolic AI workflows with Sole.jl, JuliaCon 2024.

Today's talk

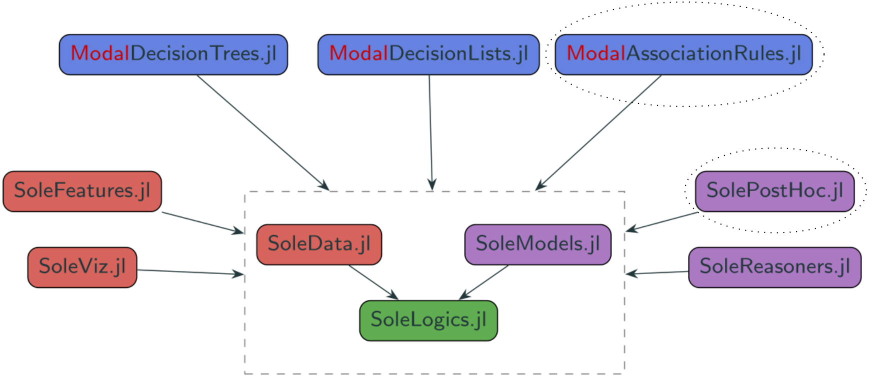

Figure: Today we will focus on ModalAssociationRules.jl and SolePostHoc.jl.

ModalAssociationRules.jl

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What:

- find frequent patterns hidden in relational data;
- combine them to generate interesting rules.

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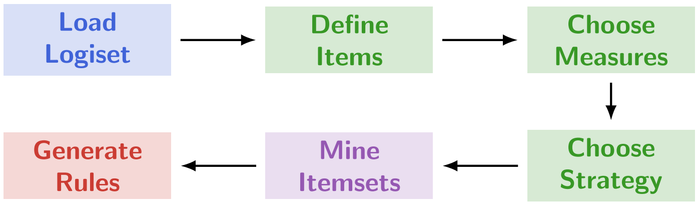
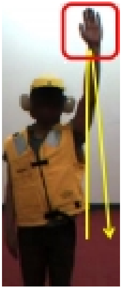
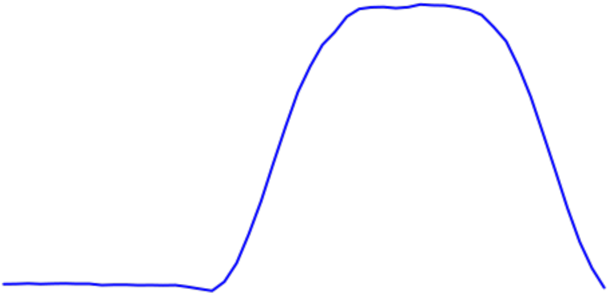


ModalAssociationRules.jl

Let us mine association rules about a specific kind of body movement.

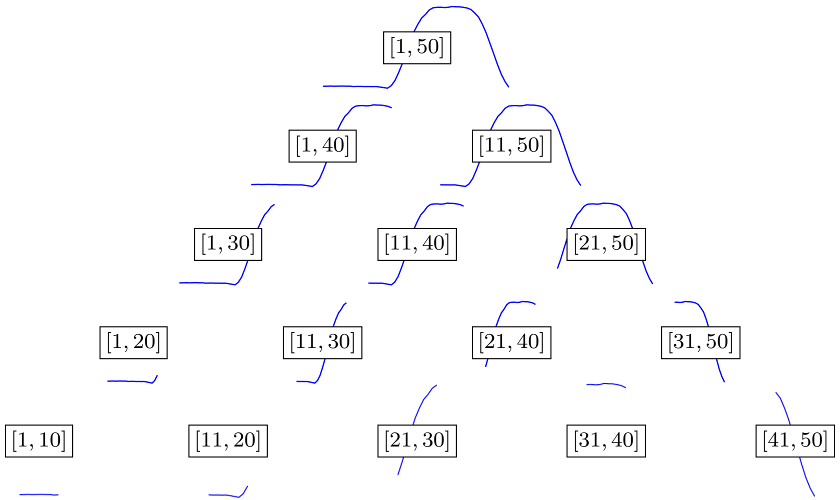




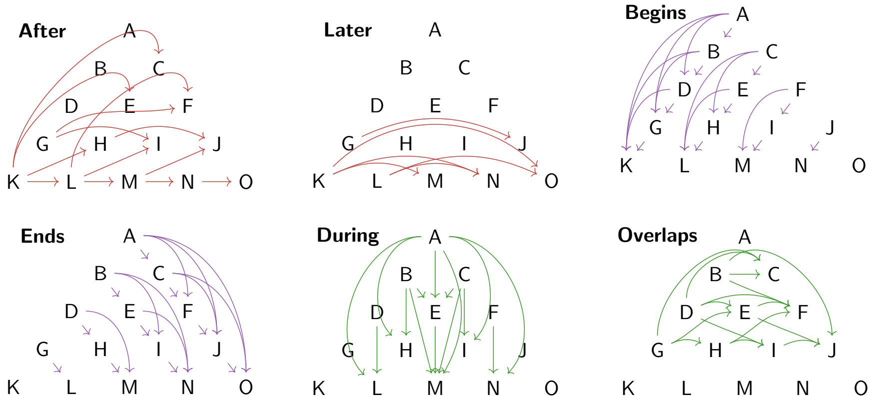
We *dissect* the signal into many intervals, called **worlds**.

A collection of worlds and the relations between them is called **logiset**.

ModalAssociationRules.jl - Logiset Instance



ModalAssociationRules.jl - Logiset Instance



We want to probe data with **items** that can be true or false on each world.

Load Logiset → **Define Items** → Choose Measures

↓

Generate Rules ← Mine Itemsets ← Choose Strategy

p $\coloneqq$ (distance(, mysignal) $\leq$, 1.0) $\triangleright$ Here

r $\coloneqq$ (distance(, mysignal) $\leq$, 3.5) $\triangleright$ After

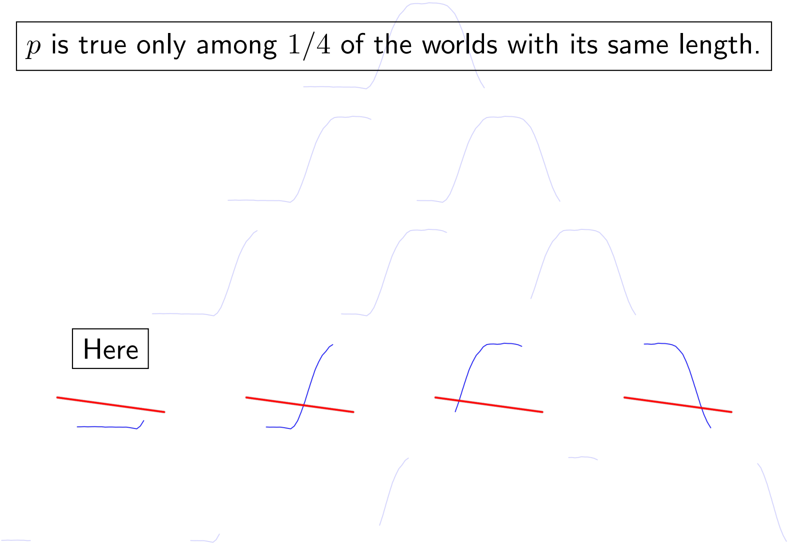
q $\coloneqq$ (distance(, mysignal), $\leq$, 1.7) $\triangleright$ After $\triangleright$ Begins

We need measures to establish if (a set of) items is interesting.

The **support** of an item is its probability to be true on a given world.

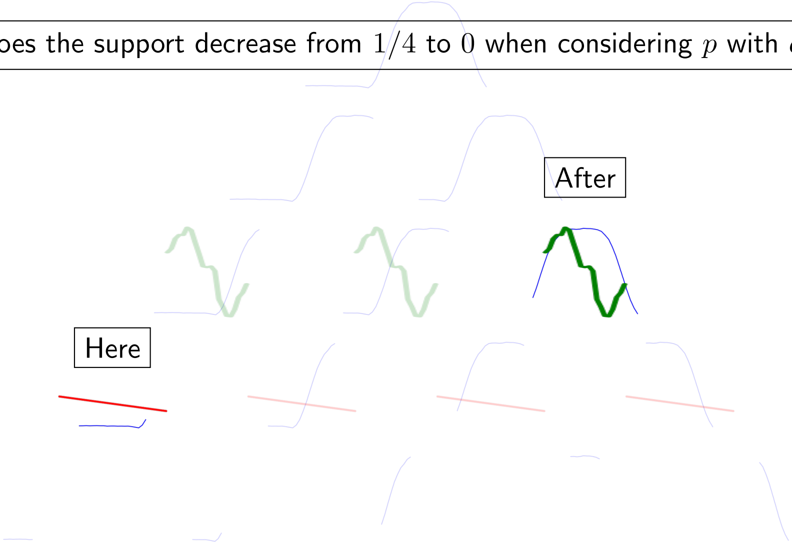
There are many measures (e.g., **confidence**) to assess whether two set of items P and Q might be arranged in an association rule $P \Rightarrow Q$.

p is true only among $1/4$ of the worlds with its same length.

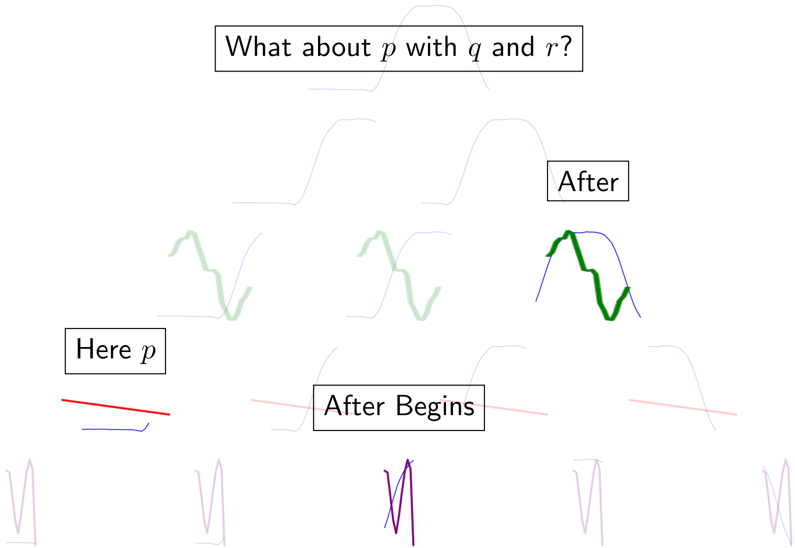


Support of $\{p, q\}$

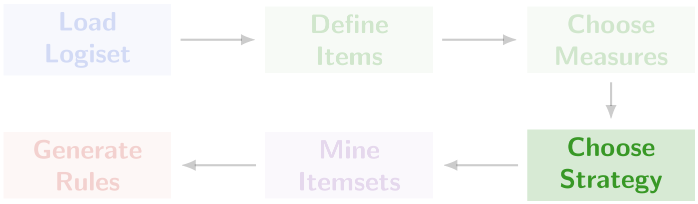
Does the support decrease from $1/4$ to 0 when considering p with q?



Support of $\{p, q, r\}$



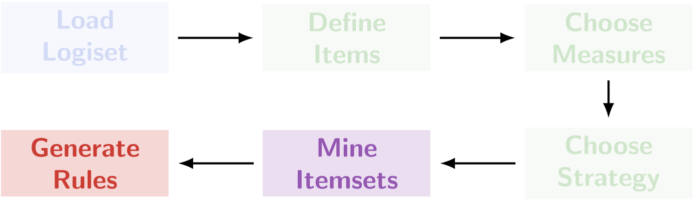
We need a **search strategy** for exploring the space of all the possible set of items ($\{p\}$, $\{q\}$, $\{r\}$, $\{s\}$, $\{p, q\}$, $\{p, r\}$, $\{p, q, r\}$, $\ldots$).



The simplest one, **Apriori**, performs a BFS over the search space.

ModalAssociationRules.jl - Mining and Rules Generation

We are ready to **mine** all the frequent set of items, and combine them into **association rules**.



Both these processes can be constrained with custom **policies**.

```
X = MyDataset |> load |> scalarlogiset

# every item applies the distance between a chunk of data
# coming from a specific variable and a tensor
items = [
    (1, [your floats here]; distance=euclidean) |> Item
    (2, [...]) |> After |> Item
    (3, [...]) |> After |> Begins |> Item
]

# given a fact X, p(X) must be >= 0.1
itemsetmeasures = [(support, 0.1)]

rulemeasures = [ # given a rule X => Y...
    (gconfidence, 0.7) # we want prob(Y|X) >= 0.7
    (lift, 1.3) # X and Y must not be independent
]

# mine the interesting facts, then stream all the rules out
miner |> mine! |> generaterules!
```

SolePostHoc.jl

Overview

Many methods exist to **extract logical rules** from learning models. However, there is currently no unified framework that brings together all these known and modern algorithms under a single approach.

That's exactly what this package aims to provide!

Integrated Algorithms

SolePostHoc.jl integrates a wide range of algorithms for knowledge extraction, including:

- **Surrogate Trees** — *e.g.* Trepan, Refene, Batrees
- **Knowledge Distillation** — *e.g.* RuleCosi+
- **Rule Extraction** — *e.g.* Lumen, Intrees

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Key Innovation

This package's primary purpose is to provide a **uniform interface** for knowledge extraction algorithms, enabling the **comparison** of different post-hoc interpretation methods while maintaining a **coherent and intuitive** user experience.

Practical Example

Consider a machine learning model trained on a generic dataset. For example, let us consider a **Random Forest Classifier** learned on the **Iris dataset** to classify 3 different species of flowers.

We are interested in extracting interpretable rules that explain the model's decision process. **SolePostHoc.jl** offers **two primary methods** for accomplishing this task.

Implementation Approaches

The first approach is to directly call the specific algorithm function. For example:

```
# Extract rules using the LUMEN algorithm directly
extracted_rules = lumen(model, args...)
```

```
# Extract rules using the Intrees algorithm directly
extracted_rules = intrees(model, x_train, y_train, args...)
```

Notice that this approach returns the output in the **original format** defined by the specific algorithm, which may differ significantly between different methods.

Implementation Approaches

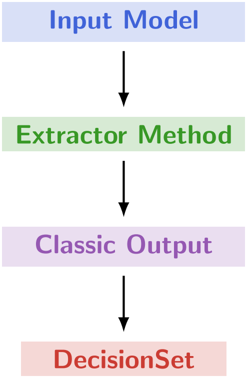
The second approach uses the unified interface through rule extractors:

```
# Extract rules using the unified interface
extractor = LumenRuleExtractor()
decision_set = modalextractrules(extractor, model, args...)
```

```
# Extract rules using the unified interface
extractor = IntreesRuleExtractor()
decision_set = modalextractrules(extractor, model, x_train,
                                          y_train, args...)
```

The key advantage of the second approach is that it not only executes the original algorithm (equivalent to calling `algo(...)` directly) but also converts the output into a **DecisionSet**.

Rule Extraction Process



The unified interface transforms any extractor's classic output into a standardized **DecisionSet** format for consistent interpretation and comparison across different algorithms.

DecisionSet Results

A **DecisionSet** is a vector of propositional logical rules in **Disjunctive Normal Form (DNF)**, with one rule per class/label.

Consider a trained model that classifies flower species.
Using **SolePostHoc.jl**, we might extract the following decision set:

```
Class "Iris-setosa":
    IF (SepalLengthCm < -0.5) AND (SepalWidthCm < 8.2)
    THEN predict "Iris-setosa"


Class "Iris-versicolor":
    IF (SepalLengthCm > 0.5) AND (SepalWidthCm < 3.25)
    THEN predict "Iris-versicolor"


Class "Iris-virginica":
    IF (PetalWidthCm > 2.0)
    THEN predict "Iris-virginica"
```

SoleXplorer.jl

Overview

SoleXplorer.jl: a simple, yet powerful, interactive machine learning framework.

- **Automated Setup**, designed to minimize user effort, with sensible defaults for all parameters;
- **Logics-based**, provides seamless integration with ModalAssociationRules and SolePostHoc packages;
- **Time-series Analysis** built-in support for temporal data, including dataset windowing and temporal feature extraction;
- **GUI-Ready**, designed for upcoming graphical user interface integration.

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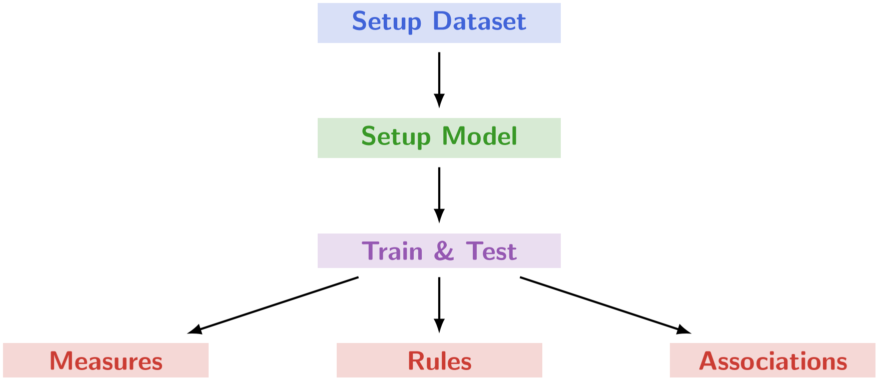
Pipeline



This is the complete pipeline of SoleXplorer.jl, encapsulated in a single function call: `symbolic_analysis()`.

Usage Example: Bronze

We have a dataset composed of a matrix (or dataframe) of measures, and a vector of labels, and we don't know if there's something interesting there, let's check it out...

```
using SoleXplorer, MLJ, JLD2

data_path = joinpath(@__DIR__, "respiratory_pneumonia.jld2")
data  = JLD2.load(data_path)
X = data["X"]
y = MLJ.CategoricalArray{String,1,UInt32}(data["y"])

model = symbolic_analysis(X, y, seed=123);
show_measures(model)
```

Usage Example: Silver

I would like to test various models to see which one suits for my experiment...

```
model = symbolic_analysis(X, y; model=ModalRandomForest(),
    seed=123);
show_measures(model)

model = symbolic_analysis(X, y; model=XGBoostClassifier(),
    seed=123);
show_measures(model)
```

Usage Example: Gold

Now it's time to tweak hyperparameters too find the best setting for the chosen model...

```
range = SoleXplorer.range(:max_depth; lower=1, upper=10)
model = symbolic_analysis(
    X, y;
    model=XGBoostClassifier(),
    seed=123,
    resampling=CV(nfolds=5, shuffle=true),
    tuning=GridTuning(resolution=5, resampling=CV(nfolds=5),
        range=range, measure=accuracy, repeats=5),
    measures=(accuracy, log_loss, confusion_matrix, kappa)
)
show_measures(model)
```

Usage Example: Platinum

It would be nice to dig into rule extraction...

```
range = (
    SoleXplorer.range(:max_depth; lower=1, upper=3),
    SoleXplorer.range(:num_round; lower=1, upper=10))
model = symbolic_analysis(
    X, y;
    model=XGBoostClassifier(),
    seed=123,
    tuning=AdaptiveTuning(range=range, resampling=CV(nfolds
        =5), measure=accuracy, repeats=10),
    extractor=LumenRuleExtractor()
)
```

Extracted Rules:

- `(V3 < 0.0084) && (V2 >= 0.0238) && (V4 >= 0.0031) -> healthy`
- `(V3 >= 0.0087) && (V5 < 0.0045) -> pneumonia`

Usage Example: Diamond

We've selected some rules that sound interesting. Finally, it would be nice to see if there are some associations among them...

```
manual_p = Atom(ScalarCondition(VariableMin(3), >=, 0.0087))
manual_q = Atom(ScalarCondition(VariableMin(5), <, 0.0045))
manual_r = Atom(ScalarCondition(VariableMax(4), <, 0.0031))

symbolic_analysis!(
    model,
    association=FPGrowth(
        Vector{Item}([manual_p, manual_q, manual_r]),
        [(gsupport, 0.1, 0.1)],
        [(gconfidence, 0.2, 0.2)],))
associations(model)
```

- `min[V3] >= 0.0087 => min[V5] < 0.0045`
- `min[V5] < 0.0045 => min[V3] >= 0.0087`
- `min[V5] < 0.0045 => max[V4] < 0.0031`
- `max[V4] < 0.0031 => min[V5] < 0.0045`

Conclusion

SoleXplorer.jl provides a complete pipeline for symbolic machine learning:

- **Measures**: various performance and interpretability metrics, such as accuracy, fidelity, and complexity.
- **Rules**: interpretable rules extracted from the model using `SolePostHoc.jl`.
- **Associations**: association rules mined from the dataset using `ModalAssociationRules.jl`.

Thank you for your attention!

Questions?