CT image reconstruction through a FBP algorithm

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***Abstract*— Filtered back Projection (FBP) is the standard algorithm applied in order to reconstruct biomedical images from projections. In this report the reconstruction performances are discussed, in dependence from parameters like the number of projections, the amount of noise and the type of filter used.**
Moreover, a new FBP algorithm is implemented, and its performance is evaluated on both a 2D and 3D Shepp-Logan phantom.

Index Terms — Filtered back Projection (FBP), Biomedical Images Reconstruction, Filtering, Radon transform, Noise Reduction.

Introduction

Image Reconstruction from projection is a special class of image restoration problem where a 2D object is reconstructed from several 1D projections. Each projection is obtained by radiating a parallel x-ray beam (or other penetrating radiation) through the object.
The Filtered Back Projection (FBP) is an analytic reconstruction algorithm designed to overcome the limitations of conventional back-projection; it applies a high-pass filter to remove the blurring effect intrinsic into the back-projection algorithm. After the filtering, all the projections are rotated by their acquisition angle, and then summed point-by-point.
Back projection has two distinctive limitations, noise and star artifacts. Different sources and types of noise exists; as in this paper the Gaussian type is considered, as an example. It mainly arises during image acquisition (e.g. electronic circuit noise, sensor limitations).
In order to remove this specific noise, in the post-processing steps a Gaussian filter can be applied; it is a linear low pass filter with a circular symmetry in the kernel. An example is reported here:

$$K_{Gaussian} = \frac{1}{16} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Moreover, in order to remove the blurring effect, different types of high-pass filter can be applied.
An evaluation (based on MSE values) on how reconstruction performances varies according to all these parameters/features is reported.
A version of the FBP algorithm has been implemented, reporting a comparison with the *iradon* function (already implemented in MATLAB) applied on the Shepp-Logan phantom.

I. Material and Methods

The analysis is performed in MATLAB, which is a desktop environment tuned for iterative analysis and design processes. The dataset consists of the built-in Shepp-Logan phantom and of a cross sectional slice of a chest CT acquisition.
The following paragraphs report the pipeline of the work:

A. Reconstruction quality over noise, projections and filtering

After loading the Shepp-Logan phantom (256x256 pixels), the parameters over which this discussion is held (number of projections, amount of noise, type of filter) are varied independently. Four conditions of each parameter have been chosen, for a total of $4^3 = 64$ cases.

- Filter implemented in the reconstruction: None, Ram-Lak, Shepp-Logan and Hann
- Projection angles: from 0 to 180° with 20, 40, 70 or 180 values in between
- Noise: Gaussian type with zero-mean and variance of $10^{-2}, 10^{-3}, 10^{-4}, 0$ (a variance of 0 corresponds to absence of noise)

The sinogram of each of the 64 cases is computed and the reconstruction is performed with the *iradon* function. The Mean Squared Error MSE is used to evaluate the quality of the reconstruction, in relation to the parameters.
The MSE is calculated as

$$MSE = \frac{1}{H \cdot W} \sum_{i=1}^{H} \sum_{j=1}^{W} [O(i,j) - R(i,j)]^2$$

where *O* is the original image and *R* the one reconstructed with *iradon*. The best reconstruction corresponds to the minimum MSE value. In *figure 1 (see Appendix)* the reconstructed Shepp-Logan phantom with the lowest MSE is reported.
However, as the MSE is not always the best method to evaluate the performance of the reconstruction, we reported in *figure 2* the reconstructed images.

B. Implementation of a FBP algorithm

We declared the function:

```
rec = fbp(sinog, theta)
```

which applies the Filtered-Back-Projection to the sinogram *sinog*, acquired at the projection angles specified in the vector *theta*.

The Ram-Lak filter is defined in the frequency domain as a column vector with a number of samples equal to the number of rows in the sinogram and is then replicated in the column-direction in order to cover all the projections in the sinogram (this "vectorization" step is done to better exploit the MATLAB power of matrix operations); later, the sinogram is transformed in the frequency domain (column-wise) and then multiplied by the filter matrix.

Coming back to the space domain with the *ifft* function, the high-pass-filtered sinogram is now available. Once the "Filtering" operation is complete, the "Back-Projection" is implemented as follow:

1. Column-wise replication of each column of the filtered sinogram
2. Rotation with the *imrotate* function of an angle equal to the corresponding one in the vector *theta*
3. Stacking of the resulting rotated image in a 3D matrix

The final reconstructed image is the sum of the different projections and is saved into the returned *rec* variable.

A comparison of our FBP algorithm with the *iradon* function is reported in *figure 3*, where in the last right-most image we applied a median filter (with a standard 3x3 kernel) on the image reconstructed through our algorithm.

C. Extension to 3D

This reconstruction can be performed also on 3D data with an iterative implementation over each slice of the available 3D volume. An example is reported in *figure 4*.

II. Results and Discussion

A. Reconstruction quality in relation to noise, projections and filtering

Using the MSE for determining the quality of the reconstruction is not a consistent approach: the MSE varies considerably among different images reconstructed with the same parameters. This is due to the MSE increasing when the reconstruction produces background artifacts, outside the ROI of the image. To compensate this, the image is binarized to segment the foreground, and all the background pixels are set to black. A visual inspection is, consequently, always necessary: however, the MSE always recognizes the best reconstruction as one with the minimum amount of noise and the maximum number of projections – as expected – but the better filter is not always the same.

For the Shepp-Logan phantom in particular, the reconstruction with the Ram-Lak filter is the best, while for a chest CT scan the Shepp-Logan filter performs better. The effect of the filter, consequently, depends on the frequency content of the original image.

B. Implementation of a FBP algorithm

The implemented FBP algorithm show very good results in the quality of the reconstruction, coupled with a relatively low computational time required. For the sake of consistency with the previous discussion, the quality of the reconstruction is evaluated with the MSE.

The MSE values found are:

- $MSE = 0.0028$ between the original image and the one reconstructed with *iradon*
- $MSE = 0.1692$ between the original image and the one reconstructed with the implemented FBP algorithm.

The MSE computed from the reconstruction with *iradon* is very low, and this is due to the additional filtering and interpolation performed by this function, that are not included in the implemented FBP algorithm.

After noticing that the MSE value is highly compromised from background artefacts – that manifests mostly as salt-and-pepper noise – the reconstructed image is postprocessed with median filtering, to reduce the amount of noise outside of the ROI of the image

The MSE after the median filtering is = 0.1667, slightly smaller than the one calculated without the median filtering.

C. Extension to 3D

Once the validity of the FBP algorithm has been estabilished, its performance can be appreciated also on 3D data applying the method slice by slice. The computational time required is, obviously, dependent on the number of slices in the 3D images. An example of the reconstruction is reported in *figure 4*.

III. Conclusion

The filtered back-projection algorithm performs best when a high number of projections is available and when the amount of noise is minimal. An optimal high-pass filter is not defined, as its effect depends on the frequency content of the image to be reconstructed: the reconstruction, moreover, is very similar using any high-pass filter.

The implemented FBP is able to produce high quality reconstructions with reasonable computational time, even for 3D data.

IV. Appendix

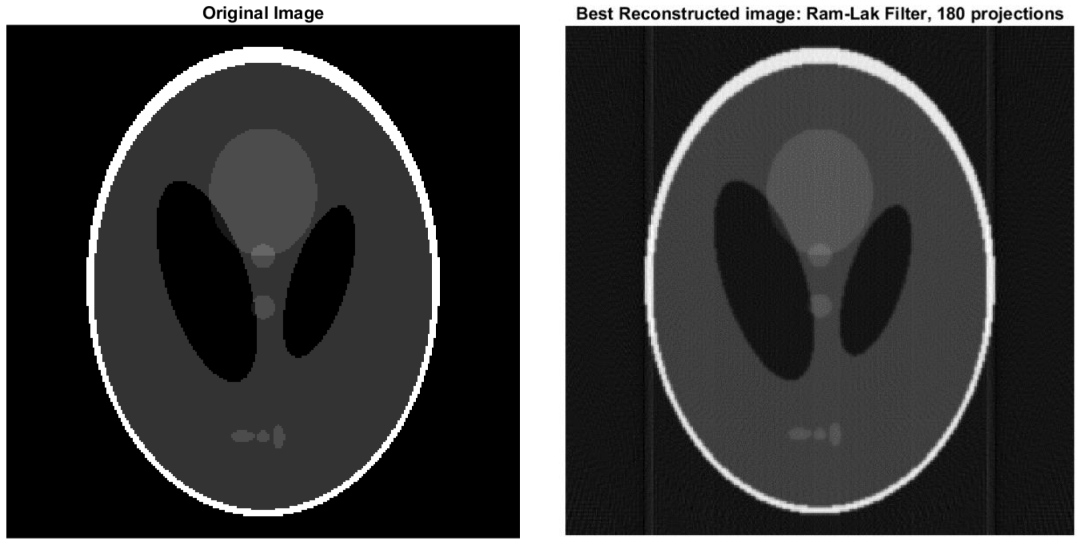

Figure 1: *The best reconstructed image with the* iradon *function compared to the original one.*

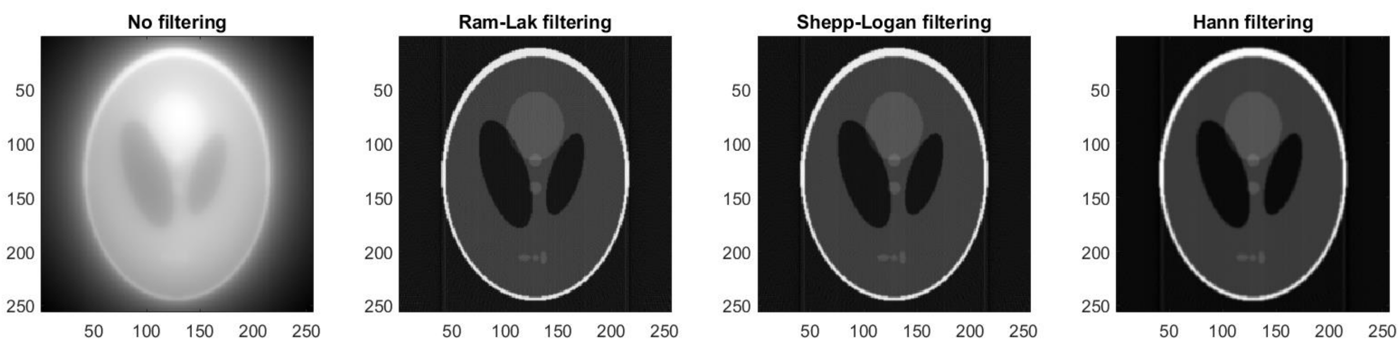

Figure 2: *The best reconstruction of the Shepp-Logan phantom for each type of filter*

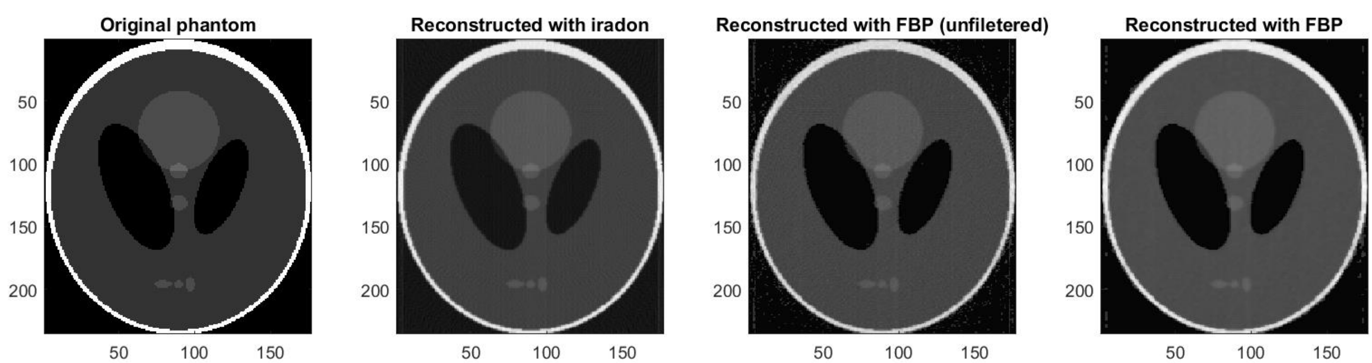

Figure 3: *Shepp-Logan phantom reconstructed with different approaches*

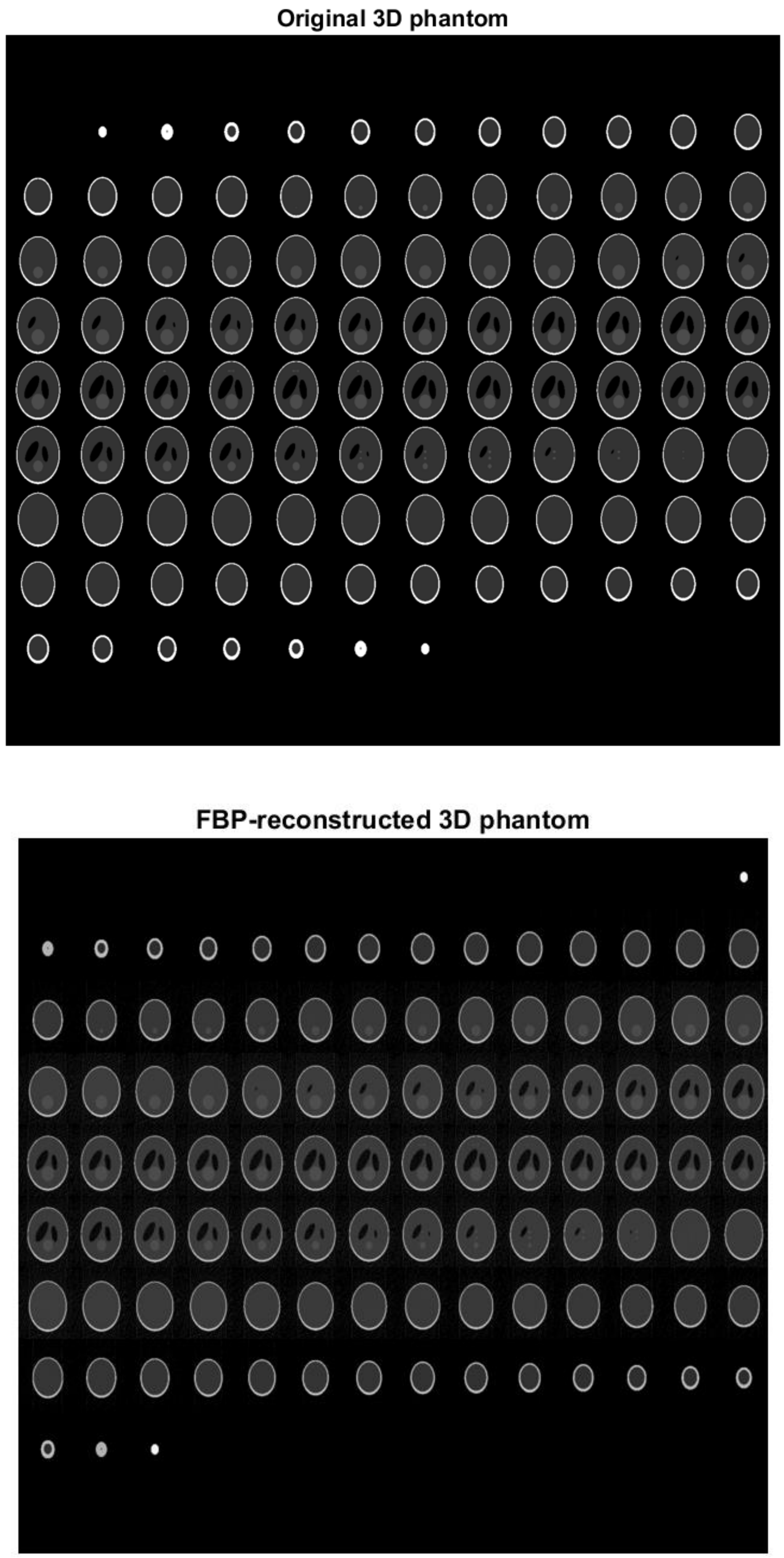

Figure 4: *Slices of the original 3D phantom and its reconstruction*