Exercise 1. The only two possibly optimal vertices are $(3, 4)$ and $(4, \frac{7}{3})$. Vertex $(3, 4)$ hase the highest objective value, $\frac{101}{3}$, and is the only optimal vertex.

Exercise 2. For question one, an unbounded primal is:

$$\begin{aligned} \max \quad & x \\ \text{s.t.} \quad & -x \leq -1 \\ & x \geq 0 \end{aligned}$$

Its infeasible dual is:

$$\begin{aligned} \min \quad & -y \\ \text{s.t.} \; & -y \geq 1 \\ & y \geq 0 \end{aligned}$$

For question two, there is no such primal-dual pair. For question three, take the answer to question one and swap primal and dual.

Exercise 3. The problem parameters are:

- Set $T = \{1, \dots, \tau\}$, the time horizon.
- w_0, the workforce at the beginning of the time horizon.
- d_t, the number of employees needed to cover production at month t.
- r, the revenue from an employee who produced widgets during one month.
- m, the penalty to pay if we miss one employee-month worth of demand.
- ℓ, the monthly employee salary (including trainees).

We use the following variables (all indices are $t \in T$):

- $w_t \geq 0$ is the number of employees on the payroll at month t.
- $u_t \geq 0$ is the number of employees producing widgets at month t.
- $x_t \geq 0$ is the number of new hires in month t.
- $y_t \geq 0$ is the demand met at month t.
- $z_t \geq 0$ is the demand missed at month t.

A MIP model is the following:

$$\begin{aligned} \max \quad & \sum_{t \in T} \Big(r y_t - m z_t - \ell w_t \Big) && && (1) \\ \text{subject to} \quad & w_t = w_{t-1} + x_t && \forall t \in T && (2) \\ & u_t = w_t - 2x_t && \forall t \in T && (3) \\ & y_t \leq u_t && \forall t \in T && (4) \\ & y_t \leq d_t && \forall t \in T && (5) \\ & z_t \geq d_t - u_t && \forall t \in T && (6) \\ & w_t, u_t, x_t, y_t, z_t \geq 0 && \forall t \in T. && (7) \end{aligned}$$