DMO — Final Exam

Student ID: ……

15/12/2023

1 Branch-and-bound (15%)

Solve the following Integer Programme using the branch-and-bound algorithm and following the instructions below.

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{subject to} \quad & 2x_1 + 3x_2 \leq 9 \\ & 4x_1 + 3x_2 \leq 12 \\ & x_1, x_2 \geq 0 \text{ and integer.} \end{aligned}$$

1. Use a depth-first strategy, i.e., explore the open node furthest down in the tree.
2. If following rule 1. results in a tie, explore the open node that was created most recently.
3. If following rules 1. and 2. results in a tie, explore the $\leq$ branch before the $\geq$ one.
4. If more than one variable is fractional in the optimal solution of a continuous relaxation, branch on the one with the lowest index, i.e., branch on x_1 before x_2.
5. Exploit the fact that the objective function coefficients are integer numbers.

2 Multiple knapsacks (25%)

You are given n objects and m knapsacks. As in the 0–1 Knapsack Problem, each object j has weight $w_j \in \mathbb{N}^+$ and profit $p_j \in \mathbb{N}^+$. Furthermore, each knapsack i has capacity $c_i \in \mathbb{N}^+$. Devise an Integer Programme with the objective of maximising the profit of packed objects, such that:

- Each object is packed in at most one knapsack;
- The sum of the weights of the objects packed in each knapsack does not exceed its capacity.

Make sure that you clearly specify which variables you are using, which value they take, over which indices they are defined, and what is their meaning. After writing the Integer Programme, provide a short description of the objective function and of each constraint. Recall that the model must be linear.

3 Fair multiple knapsacks (30%)

You are using the above multi-knapsack problem to distribute candies (the objects) to children who bring their bag to be filled (the knapsacks). To ensure a fair division of the candies, you want to ensure that the knapsack with the lowest packed profit has the highest possible profit. In other words, you want to maximise the minimum profit collected in any of the knapsacks. Change your model from the previous question to achieve this objective. You can add new variables and constraints, change the previous constraints and change the previous objective function if needed. The only requirement is that the new model is still an Integer Programme and that it is linear.