A maritime version of the Travelling Salesman Problem

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Plan

The Capacitated TSP with Pickup and Delivery

Variant of the travelling salesman problem

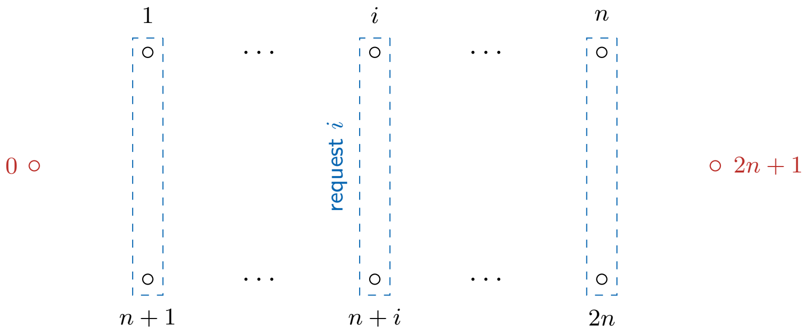
Service n requests

Each request has

- an origin
- a destination
- a quantity of goods going from the origin to the destination

The Capacitated TSP with Pickup and Delivery

Model on a digraph $G = (N, A)$.



The Capacitated TSP with Pickup and Delivery

Nodes

$$N = \{ \underbrace{0}_{\text{start depot}}, \underbrace{1, \ldots, n,}_{\text{origins}} \underbrace{n+1, \ldots, 2n,}_{\text{destinations}} \underbrace{2n+1}_{\text{end depot}} \}$$

Arcs

$$A \subseteq N^2$$

Arc costs

$$c_{ij} \equiv c_a \geq 0 \qquad \forall (i,j) \equiv a \in A$$

The Capacitated TSP with Pickup and Delivery

Demand

$$d_i \geq 0 \qquad \forall i \in \{1, \ldots, n\}$$

And, by convention

$$d_{n+i} = -d_i \qquad \forall i \in \{1, \ldots, n\}$$

$$d_0 = d_{2n+1} = 0$$

The Capacitated TSP with Pickup and Delivery

Feasibility condition

$$Q \geq \max_{i=1,\ldots,n} d_i$$

The Capacitated TSP with Pickup and Delivery

Find a minimal cost hamiltonian path starting in 0 and ending in $2n+1$ such that

- Precedence constraints
  - Every origin i is visited before the destination $n+i$
- Capacity constraints
  - The cargo on the vehicle is always $\leq Q$

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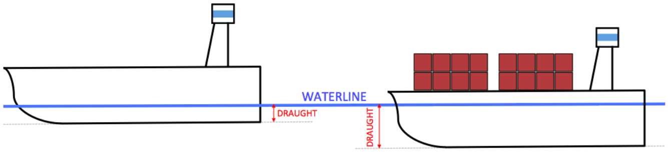
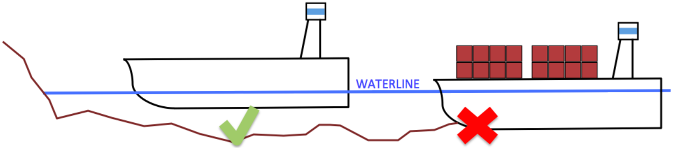
The TSPPDDL

Draught: distance between the waterline and the bottom of the hull of a ship



The TSPPDDL

Each port $i \in \{1, \dots, 2n\}$ has a certain draught $l_i \geq 0$



Draught expressed in the same unit as the demand

The TSPPDDL

Feasibility condition

$$l_i \geq |d_i| \quad \forall i \in \{1, \ldots, 2n\}$$

Plan

Literature

Irina Dumitrescu et al. "The traveling salesman problem with pickup and delivery: polyhedral results and a branch-and-cut algorithm". In: *Mathematical programming* 121.2 (2010), pp. 269–305

- Study of the polytope of the uncapacitated TSPPD
- Facet-defining valid inequalities
- Branch-and-cut with heuristic separation
- Instances with n up to 35

Literature

Stefan Ropke, Jean-François Cordeau, and Gilbert Laporte. "Models and branch-and-cut algorithms for pickup and delivery problems with time windows". In: *Networks* 49.4 (2007), pp. 258–272

Stefan Ropke and Jean-François Cordeau. "Branch and cut and price for the pickup and delivery problem with time windows". In: *Transportation Science* 43.3 (2009), pp. 267–286

- Pickup and Delivery Problem with Time Windows
- Multi-vehicle, homogeneous fleet
- Branch-and-cut and branch-and-cut-and-price

Literature

Jørgen Glomvik Rakke et al. "The traveling salesman problem with draft limits". In: *Computers & Operations Research* 39.9 (2012), pp. 2161–2167

- Ship starts full and only delivers
- Branch-and-cut with variables' bounds tightening
- Instances based on TSP Lib, up to 48 nodes

Literature

Maria Battarra et al. “Exact algorithms for the traveling salesman problem with draft limits”. In: *European Journal of Operational Research* 235.1 (2014), pp. 115–128

- Two branch-and-cut algorithms
- One branch-and-cut-and-price using ng-paths
- Solve all the instances proposed by Glomvik Rakke et al.

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Model

Variables:

- $x_a \in \{0, 1\} \quad \forall a \in A$
  - 1 iff arc a is used
- $y_a \in \mathbb{R} \quad \forall a \in A$
  - Quantity of cargo onboard when travelling along arc a

Model
MIP (1/3)

$$\min \sum_{a \in A} c_{ij} x_{ij} \tag{1}$$

s.t.

$$\sum_{a \in \delta^+(i)} x_a = 1 \qquad \forall i \in \{0, \ldots, 2n\} \tag{2}$$

$$\sum_{a \in \delta^-(i)} x_a = 1 \qquad \forall i \in \{1, \ldots, 2n+1\} \tag{3}$$

$$\sum_{a \in \delta^+(i)} y_a - \sum_{a \in \delta_i^-} y_a = d_i \qquad \forall i \in \{1, \ldots, 2n\} \tag{4}$$

Model
MIP (2/3)

$$\ldots \quad \alpha_a x_a \leq y_a \leq \beta_a x_a \qquad \forall a \in A \qquad (5)$$

$$\sum_{a \in \delta^+(0)} y_a = 0 \qquad (6)$$

where

$$\alpha_{ij} = \begin{cases} d_i & \text{if } i \in \{1, \ldots, n\} \text{ and } j \in \{1, \ldots, n\} \cup \{n+i\} \\ -d_j & \text{if } i, j \in \{n+1, \ldots, 2n\} \\ d_i - d_j & \text{if } i \in \{1, \ldots, n\} \text{ and } j \in \{n+1, \ldots, 2n\} \setminus \{n+i\} \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_{ij} = \min\{l_i + \min\{0, d_i\}, l_j - \max\{0, d_j\}, Q - \max\{0, -d_i, d_j\}\}$$

Model
MIP (3/3)

$$\ldots \quad \sum_{a \in \delta^+(S)} x_a \geq 1 \qquad \forall i \in \{1, \ldots, n\},$$

$$\forall S \subseteq N \;:\; i \in S,\; n+i \notin S \tag{7}$$

$$\sum_{a \in \delta^+(S)} x_a \geq 1 \qquad \forall i \in \{n+1, \ldots, 2n\},$$

$$\forall S \subseteq N \;:\; i \in S,\; 2n+1 \notin S \tag{8}$$

Subtour elimination

- Egon Balas, Matteo Fischetti, and William R Pulleyblank. "The precedence-constrained asymmetric traveling salesman polytope". In: *Mathematical programming* 68.1-3 (1995), pp. 241–265
- Martin Grötschel and Manfred W Padberg. "Lineare Charakterisierungen von Travelling Salesman Problemen". In: *Zeitschrift für Operations Research* 21.1 (1977), pp. 33–64
- Jean-Francois Cordeau. "A branch-and-cut algorithm for the dial-a-ride problem". In: *Operations Research* 54.3 (2006), pp. 573–586

Generalised order

- KS Ruland and EY Rodin. “The pickup and delivery problem: Faces and branch-and-cut algorithm”. In: *Computers & mathematics with applications* 33.12 (1997), pp. 1–13
- Jean-Francois Cordeau. “A branch-and-cut algorithm for the dial-a-ride problem”. In: *Operations Research* 54.3 (2006), pp. 573–586

Capacity (and draught)

- Stefan Ropke, Jean-François Cordeau, and Gilbert Laporte. "Models and branch-and-cut algorithms for pickup and delivery problems with time windows". In: *Networks* 49.4 (2007), pp. 258–272

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Branch-and-cut

Inequalities (7) and (8) are initially relaxed

Exact separation:

- Solve n max-flow problems for (7) (from i to $n+i$)
- Solve n max-flow problems for (8) (from $n+i$ to $2n+1$)

Branch-and-cut

All valid inequalities are separated heuristically

2-cycle elimination inequalities are generated in advance

$$x_{ij} + x_{ji} \leq 1 \qquad \forall (i, j) \in A$$

Plan

Constructive heuristics

Basic idea:

- Start with empty path
- At each iteration insert one origin-destination couple $(i, n+i)$
- Choose where to insert i and where to insert $n+i$

Constructive heuristics

- Two-phases
  - First pass: order the couples
  - Second pass: insert them
- One-phase
  - At each step choose the couple and insert it
- Maximum regret

Constructive heuristics

Criteria to determine what is a good couple

- Minimal (maximal) origin-destination distance
- Minimal (maximal) draught-load difference

Constructive heuristics

Criteria to determine what is a good couple + insertion

- Maximal load over cost
- Minimal load times cost
- Minimal Cost

Refinement heuristics

Simple textbook implementation of tabu search

- Neighbourhood: 3-opt
- Move property in tabu list: length of shortest edge removed

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Instances

`<instance>_<n>_<h>_<k>`

instance: from TSPLIB

n: number of requests

Quantities validated with data from liner operator

Instances

1. Hub: one random port
2. n requests: random origin and destination
3. n requests: random demand $\in [1, 99]$

Instances

4. Capacity: $Q = 50 \cdot h,\ h \in \{2, 3, 4, 5, n, 2n\}$
   - For $h = 2n$: $l_i = Q\ \forall i \in N$ TSPPD
   - For other values of h...

Instances

5. Draught: a fraction of k ports have draught $< Q$, $k \in \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$
   - For $k = 0$: Capacitated TSPPD
   - For other values of k...
6. Draught of i: random between $|d_i|$ and $Q - 1$
   - For those ports i that have draught $< Q$

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Results
Gap %

n = 8

	2	3	4	5	8	16
0	0.00%	0.00%	0.00%	0.00%	0.00%	
0.25	0.00%	0.00%	0.00%	0.00%	0.00%	
0.5	0.00%	0.00%	0.00%	0.00%	0.00%	
0.75	0.00%	0.00%	0.00%	0.00%	0.00%	
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

n = 10

	2	3	4	5	10	20
0	0.00%	0.00%	0.00%	0.00%	1.65%	
0.25	0.00%	0.00%	4.72%	0.00%	0.00%	
0.5	0.00%	0.00%	0.00%	0.00%	0.00%	
0.75	0.00%	0.00%	0.00%	0.00%	0.00%	
1	0.00%	0.00%	3.65%	0.00%	0.00%	0.00%

n = 12

	2	3	4	5	12	24
0	12.54%	13.80%	14.97%	22.00%	7.92%	
0.25	9.52%	14.44%	12.95%	23.93%	3.64%	
0.5	8.74%	12.75%	11.21%	6.89%	0.32%	
0.75	12.10%	14.90%	2.35%	16.48%	5.96%	
1	4.84%	12.93%	2.83%	0.00%	0.00%	0.00%

n = 14

	2	3	4	5	14	28
0	0.00%	0.00%	8.21%	22.19%	21.21%	
0.25	0.00%	7.12%	13.46%	8.63%	11.43%	
0.5	0.00%	7.96%	22.65%	21.29%	2.68%	
0.75	0.00%	14.23%	18.28%	18.38%	0.00%	
1	0.00%	17.39%	23.08%	10.06%	0.00%	0.00%

n = 16

	2	3	4	5	16	32
0	0.00%	2.29%	20.92%	32.82%	27.15%	
0.25	0.00%	15.60%	36.10%	33.72%	18.14%	
0.5	0.00%	8.23%	24.08%	24.44%	17.53%	
0.75	0.00%	27.10%	33.74%	32.45%	10.89%	
1	7.97%	31.10%	28.70%	28.33%	0.00%	0.00%

n = 20

	2	3	4	5	20	40
0	1.66%	11.82%	9.32%	25.22%	24.20%	
0.25	0.01%	10.84%	22.24%	27.07%	4.17%	
0.5	2.44%	16.09%	20.88%	29.18%	13.89%	
0.75	3.53%	17.42%	22.70%	28.25%	22.32%	
1	4.48%	16.22%	20.73%	20.39%	3.46%	1.64%

n = 24

	2	3	4	5	24	48
0	2.42%	12.94%	41.74%	48.05%	39.73%	
0.25	5.73%	28.66%	39.60%	41.16%	29.06%	
0.5	5.73%	24.34%	43.86%	44.49%	23.95%	
0.75	7.69%	39.77%	45.90%	35.40%	41.94%	
1	8.26%	35.99%	38.34%	40.08%	24.18%	23.82%

avg

	2	3	4	5	n	2n
0	2.37%	5.84%	13.60%	21.47%	1.22%	
0.25	2.18%	10.95%	18.44%	19.22%	0.66%	
0.5	2.41%	9.91%	17.53%	18.04%	0.58%	
0.75	3.33%	16.20%	17.57%	18.71%	0.81%	
1	3.65%	16.23%	16.76%	14.12%	0.28%	3.64%

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avg

CTSPPD	2	3	4	5	n	2n
0	2.37%	5.84%	13.60%	21.47%	1.22%	
0.25	2.18%	10.95%	18.44%	19.22%	0.66%	
0.5	2.41%	9.91%	17.53%	18.04%	0.58%	
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TSPPD

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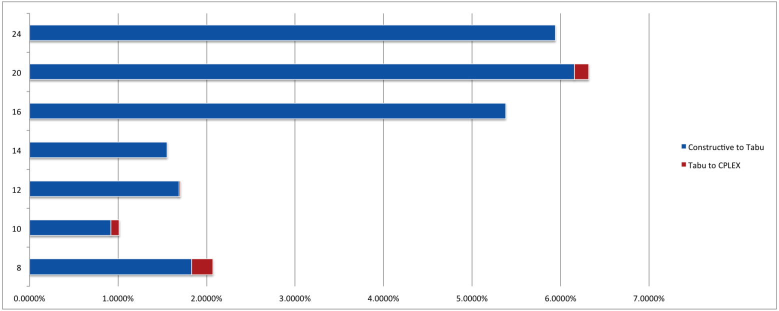
Results
UB improvement



- Quite well on the TSPPD
- Better on the TSPPDDL than the CTSPPD
  - Specific valid inequalities
  - Graph preprocessing

- “Low effort” heuristics work quite well
- The most difficult part is to improve the LB

Thank you