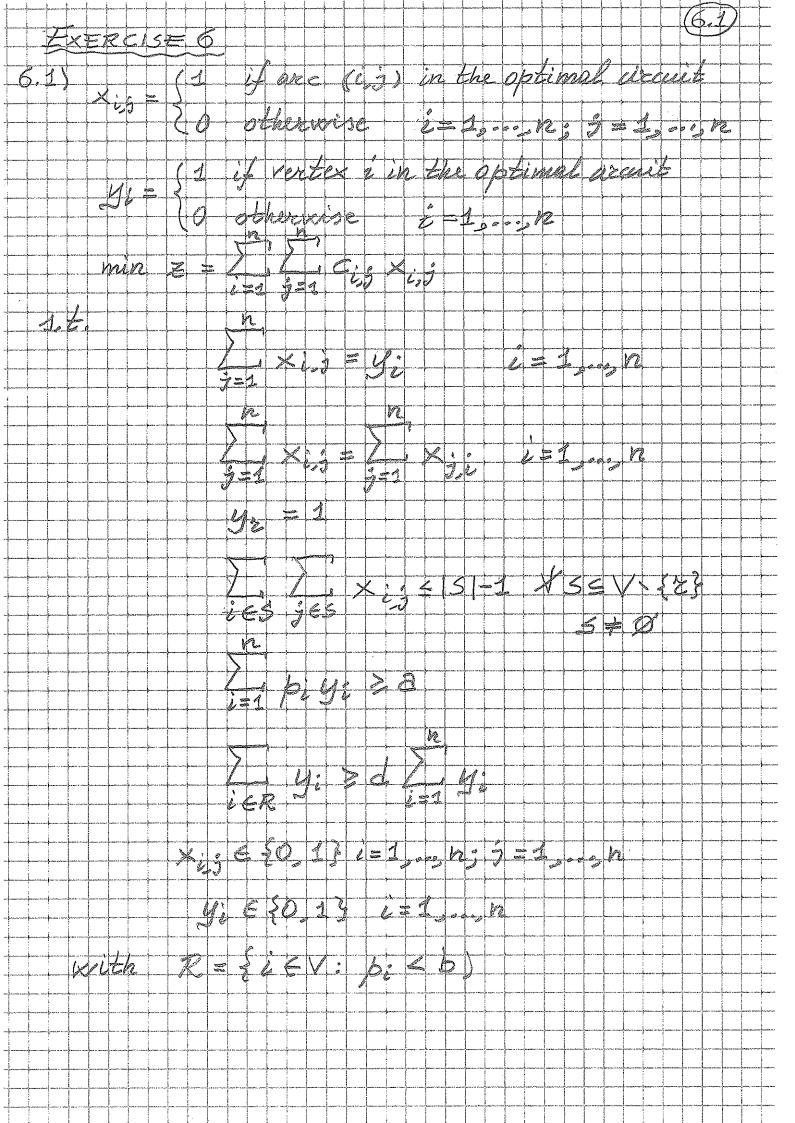
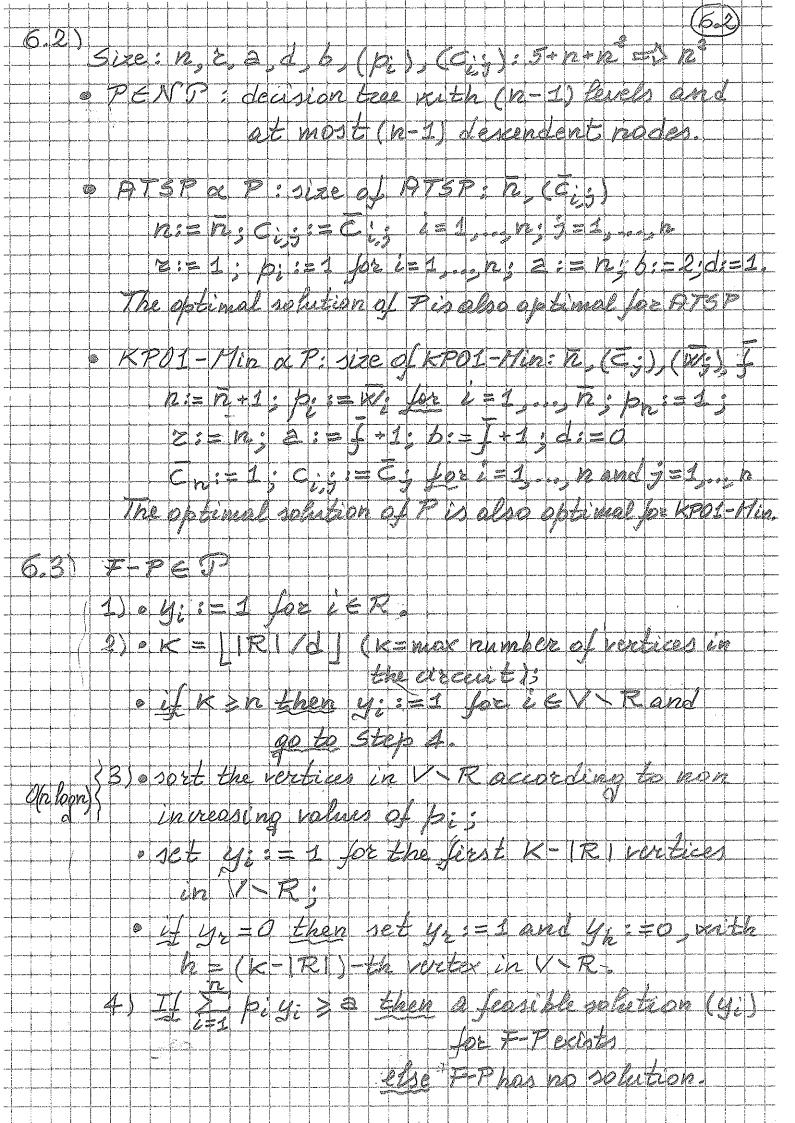
Given a "complete directed graph" G = (V, A), with |V| = n. A positive "cost"  $c_{i,j}$  (with  $c_{i,i}$  = infinity for each vertex i of V) is associated with each arc (i,j) of A. A positive "prize"  $p_i$  is associated with each vertex i of V

Given a vertex r of V, determine an "elementary circuit" of G visiting vertex r and such that:

- a) the sum of the costs of the arcs of the circuit is minimum;
- b) the sum of the prizes associated with the vertices of the circuit is not smaller than a given value a;
- c) the number of vertices of the circuit having a prize smaller than a given value b is not smaller than a given percentage d of the number of vertices of the circuit (with d between 0 and 1).
- 1)- Define a Linear Integer Programming model for the considered problem.
- 2)- Prove that the problem is NP-hard. (determine two NP-hard problems reducible to the considered problem).
- 3) Define the complexity of the problem for determining a feasible solution for the considered problem.
- 4)- Define the complexity of the problem in the case in which:  $c_{i,j} = K$  for each arc (i, j) of G (with K given value).

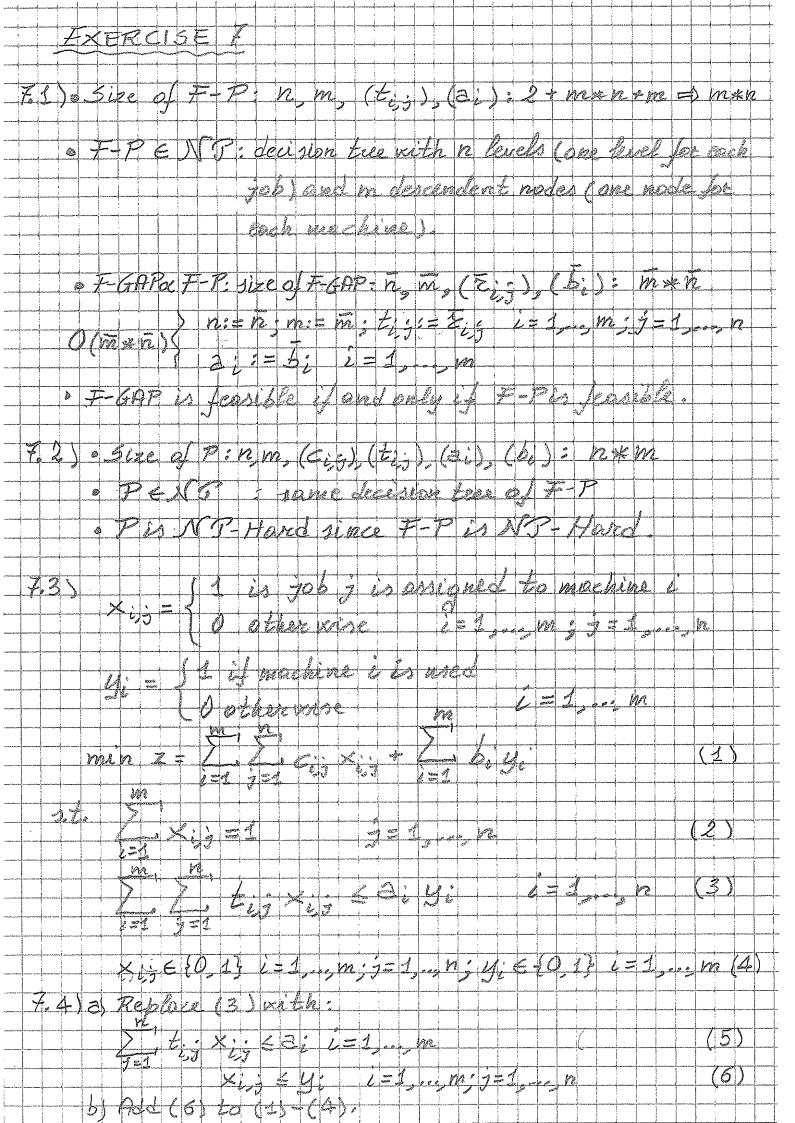




Given n "jobs" and m "machines". The "cost" per processing job j (j = 1, ..., n) on machine i (i = 1, ..., m) is given by  $c_{i,j}$ , while the corresponding "processing time" is given by  $t_{i,j}$ . If machine i (i = 1, ..., m) is used, there is an additional cost equal to  $b_i$  (this cost is equal to zero if machine i is not used). The values  $c_{i,j}$ ,  $t_{i,j}$  and  $b_i$  are positive integers.

Assign each job to one and only one machine so that:

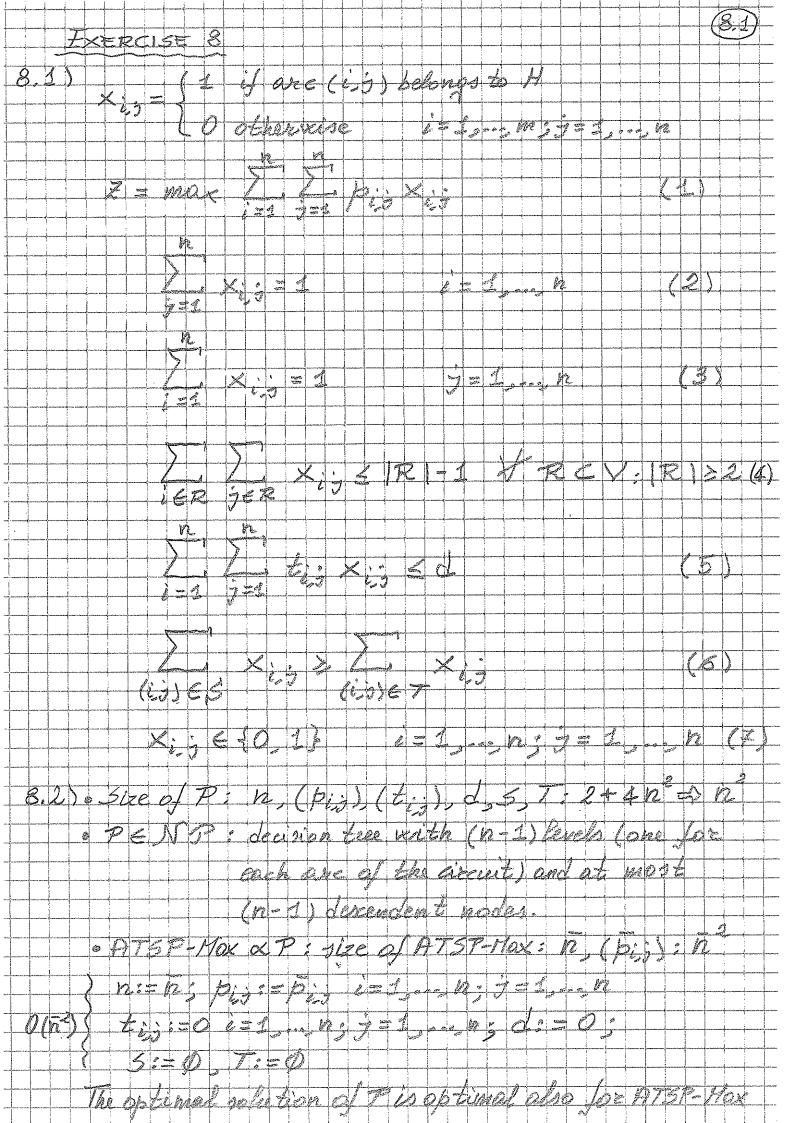
- a) the global processing time for each machine i (i = 1, ..., m) is not greater than a given value  $a_i$  (with  $a_i$  positive and integer);
- b) the "global" cost is minimum.
- 1)- Define the complexity of the problem for determining a feasible solution for the considered problem.
- 2)- Prove that the considered problem is NP-hard.
- 3)- Define a Linear Integer Programming model for the considered problem, so as to minimize the number of constraints.
- 4)- Define additional Linear Integer Programming models which, by using a larger number of constraints, can produce "lower bounds" (obtained with the continuous relaxation of the model) better than those which can be obtained with the continuous relaxation of the model defined at point 3).

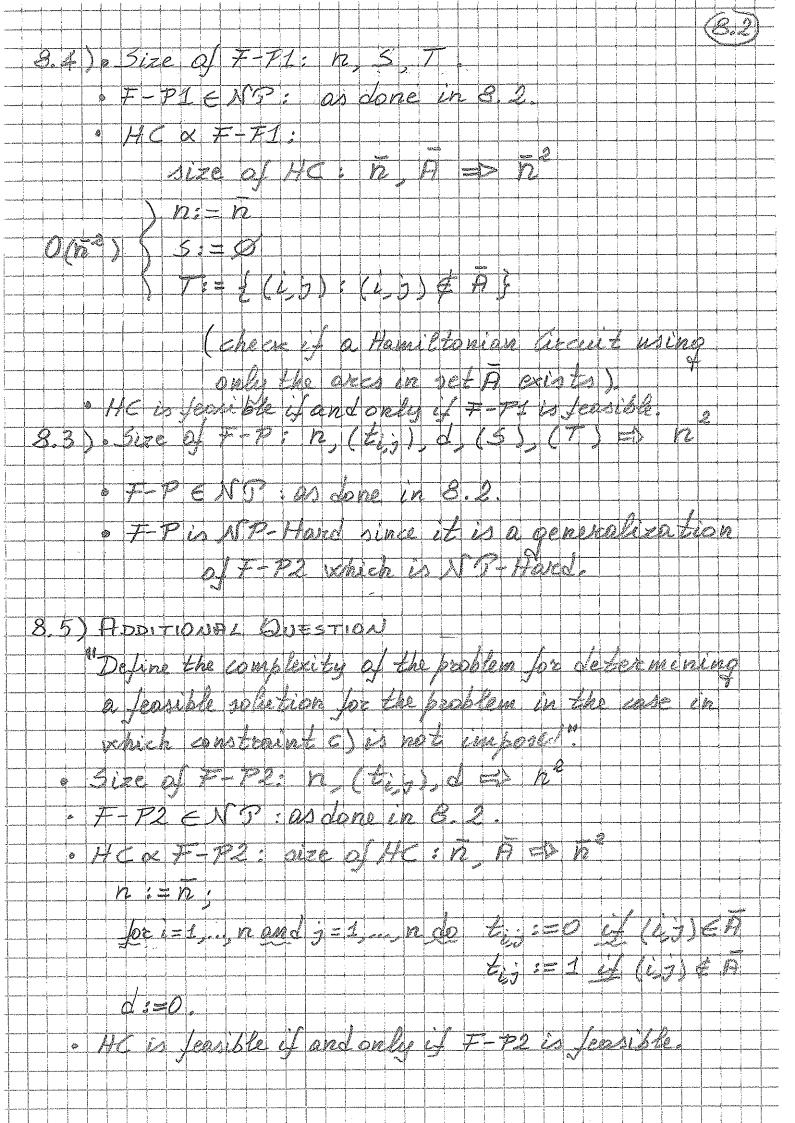


Given a "complete directed graph" G = (V, A), with |V| = n: a "weight"  $p_{i,j}$  and a non-negative "time"  $t_{i,j}$  are associated with each arc (i, j) of A. Two disjoint subsets S and T are also given (with S and T contained in A).

Determine a "Hamiltonian circuit" H of G so that:

- a) the sum of the weights of the arcs of H is maximum;
- b) the sum of the times of the arcs of H is not greater than a given value d;
- c) the number of arcs of H belonging to subset S is not smaller than the number of arcs of H belonging to subset T.
- 1)- Define a Linear Integer Programming model for the considered problem.
- 2)- Prove that the considered problem is NP-hard.
- 3) Define the complexity of the problem for determining a feasible solution for the considered problem.
- 4)- Define the complexity of the problem for determining a feasible solution for the problem in the case in which constraint b) is not imposed.





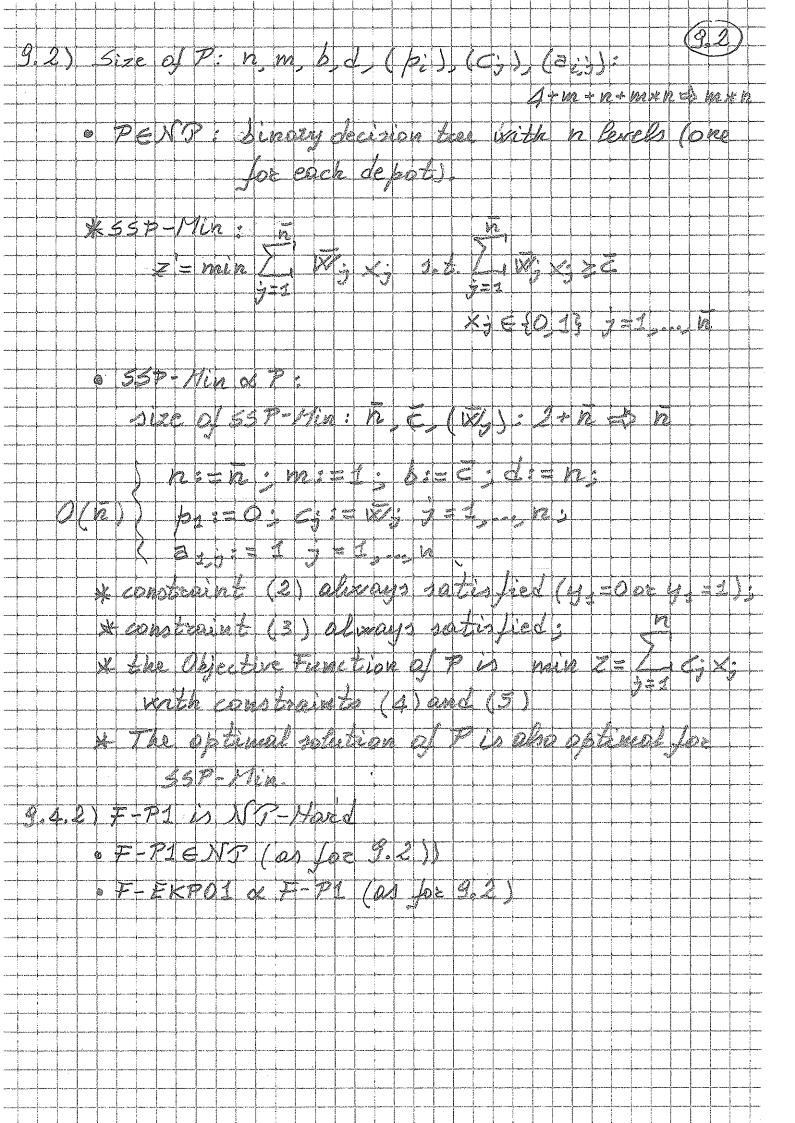
Given n "depots" and m "customers": each customer i (i = 1, ..., m) has a non-negative "potential profit"  $p_i$ . Each depot j (j = 1, ..., n) has a non-negative "cost"  $c_j$  and is able to "serve" a subset of the m customers. In particular, a binary matrix  $(a_{i,j})$  is given, such that for each pair [depot j, customer i] (with j = 1, ..., n and i = 1, ..., m)  $a_{i,j} = 1$  if depot j is able to serve customer i, and  $a_{i,j} = 0$  otherwise.

For each subset S of the n depots, the corresponding "global profit" is given by the difference: (sum of the profits of the customers which can be served by the depots of S) - (sum of the costs of the depots of S).

Determine a subset S\* of the n depots so that:

- a) S\* contains at most d depots (with d given value greater than 0 and smaller or equal to n);
- b) the global profit of S\* is maximum;
- c) the total cost of the depots of S\* is not smaller than a given non-negative value b.
- 1)- Define a Linear Integer Programming model for the considered problem.
- 2)- Prove that the considered problem is NP-hard.
- 3) Define the complexity of the problem for determining a feasible solution for the considered problem.
- 4)- Define the complexity of the problem for determining a feasible solution for the problem in the following cases:
  - 4.1) subset S\* must contain exactly d depots;
  - 4.2) the global cost of the depots of subset  $S^*$  is equal to b.

Q. I EXERCISES 9.1 if depot j is med (i.e. JES\*) other kise 13 = 1 if customer i is served by at least one Y otherwise 4=19 the pige (11) 打造 (=1, -, aib Xs 3 yi (3) 7 (4) (5) Xje (O) 931 77 607 a(n) . in constraint (2) set y; = 0; (2) always to tistied; sort the n depoto excerding to non increasing values of the costo (C) O(n logn) 4,4,4,0,4,7,44,,1 0 (12) 0 Fil dix > = then a trasiple solution (x) exists 0(n) o else no jessible relation exists for F-P. 9.4.1) As for question 9.3).

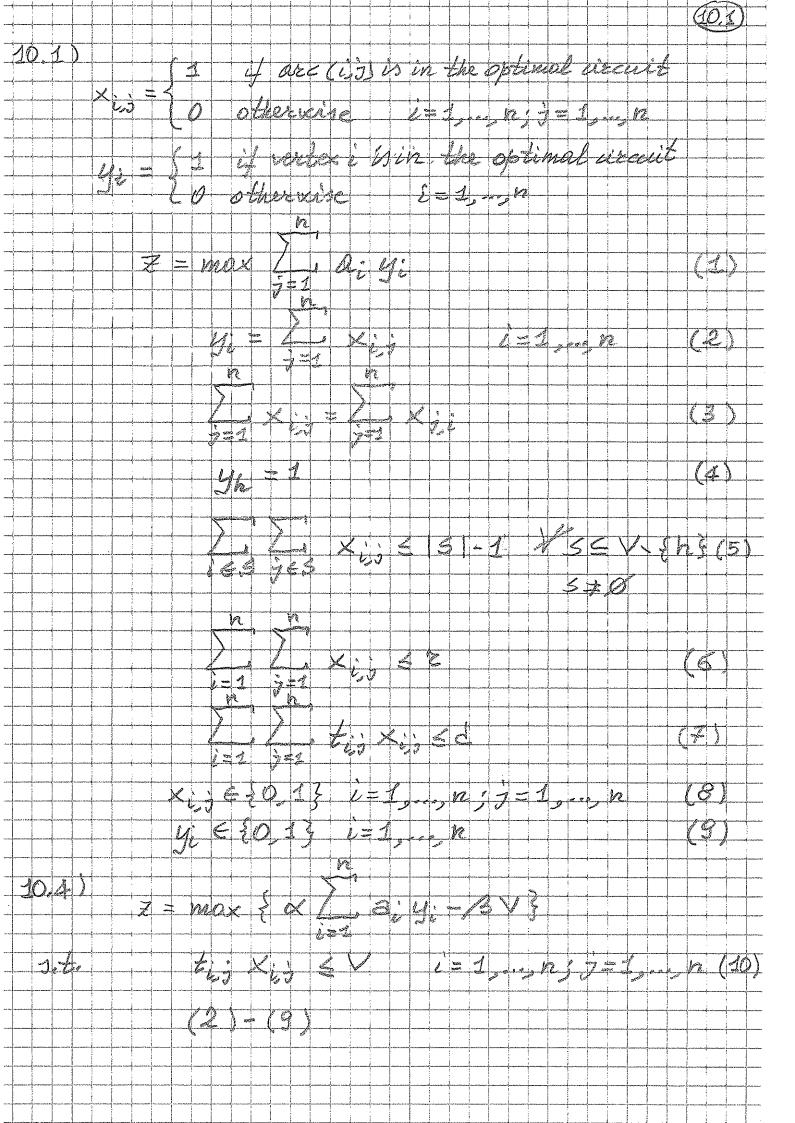


Given a "complete directed graph" G = (V, A), with |V| = n. A non-negative "time"  $t_{i,j}$  (with  $t_{i,i}$  = infinity for each vertex i of V) is associated with each arc (i, j) in A (with the times satisfying the triangularity condition, i.e.:  $t_{i,k} + t_{k,j} \ll t_{i,j}$  for each triple (i, j, k) of vertices of V). A positive profit  $a_i$  is associated with each vertex i of V.

Given a vertex h of V, determine an "elementary circuit" of G visiting h and such that:

- a) the number of arcs of the circuit is not greater than a given value r (with r between 2 and n);
- b) the sum of the times of the arcs of the circuit is not greater than a given value d;
- c) the sum of the profits associated with the vertices of the circuit is maximum.
- 1)- Define a Linear Integer Programming model for the considered problem.
- 2)- Prove that the considered problem is NP-hard.
- 3) Define the complexity of the problem for determining a feasible solution for the considered problem.
- 4) Define a Linear Integer Programming model for the variant of the considered problem in which the objective function (to be maximized) is given by:

alpha \* (sum of the profits associated with the vertices of the circuit) – beta \* (maximum time of the arcs of the circuit) with alpha and beta non-negative given values.



30,2) Size of P: 12, 12, 2, 4 (Eij) (Bi): 4+10+10 10 · PENT: décision tree with (2-1) tents lone for each are of the execut and at most (n-1) descendent nodes \* KPOLOKIA olze al 14 PO1 n = (W:) (A;) = n:=n+1. 芦ルキキュ Wy is I n; j=1,...n: d 3=2+1. 2: - 12: h = h. the optimal rolution of Pinalso optimal for KPDI 10,3) F-PEJ \* circuit starting from h and returning to h within the minimum time: · determine vertex is such that FART FRA = 1949 7 FALL + FRA : 4 EM Ship) O(h) } thurth & d then a fearible so Entron exists sizeset hokoh else no feasible solution exidia