# A Local Search Based Metaheuristic for a Class of Earliness and Tardiness Scheduling Problems

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  - Motivation
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#### Problem

- Class of Just-In-Time scheduling problems
- Penalties if a job is completed before or after its due date
- Usually  $\mathcal{NP}$ -Hard
- Additional features
  - ▶ Nonzero release dates
  - ▶ Multiple machines
  - Sequence-dependent setup times

### Problem definition

- $J = \{1, \ldots, n\}$  is a set of jobs
- $M = \{1, \ldots, m\}$  is a set of **unrelated** parallel machines
- For each job  $i \in J$ :
  - ▶  $p_i^k$ : processing time of j in machine  $k \in M$
  - $\triangleright$   $d_i$ : due date of i
  - $ightharpoonup r_i$ : release date of i
  - $w_i'$ : earliness penalty weight

  - \$\vec{w}\_j\$: tardiness penalty weight
    \$s\_{ij}^k\$: setup time required before starting to process \$j\$ immediately after i in machine k

### Problem definition

- Schedule all the jobs over the machines
- Each machine can process only one job at time
- The jobs are ready to be processed after their release date
- Setup times are required when changing the jobs being processed
- Preemption is not allowed
- Idle time can be inserted between two consecutive jobs
- Objective: minimize the sum of the weighted earliness and tardiness

### Problem definition

#### Objective Function

$$\text{Minimize } \sum_{j \in J} w_j' E_j + w_j T_j$$

- $E_i = \max\{d_i C_i, 0\}$ : earliness
- $T_i = \max\{C_i d_i, 0\}$ : tardiness

#### Classification $(\alpha |\beta| \gamma)$

$$R|r_j, s_{ij}^k| \sum w_j' E_j + w_j T_j$$

# Considered problems

• Particular cases of the  $R|r_j, s_{ij}^k| \sum w_j' E_j + w_j T_j$ 

#### Considered problems

Single	Identical	Uniform	Unrelated
Machine	Parallel Machines	Parallel Machines	Parallel Machines
$1    \sum w_i T_i$	$P  \sum w_i T_i$	$Q  \sum w_i T_i$	$R  \sum w_i T_i$
$1 r_j \sum w_j T_j$	$P r_j \sum_{j}w_j^{*}T_j$	$Q r_j \sum^{v}w_j^{v}T_j$	$R r_j \sum w_j T_j$
$1 s_{ij} \sum w_j T_j$	$P s_{ij} \sum w_j T_j$	$Q s_{ij}^k \sum w_j T_j$	$R s_{ij}^k \sum w_j T_j$
$1 r_j, s_{ij}  \sum w_j T_j$	$P r_j, s_{ij}  \sum w_j T_j$	$Q r_j, s_{ij}^k  \sum w_j T_j$	$R s_{ij}^{\bar{k}} \sum w_j T_j$
$1  \sum w_i' E_j + w_j T_j$	$P    \sum w_i' E_j + w_j T_j$	$Q  \sum w_i' E_j + w_j T_j$	$R \sum w_i' E_j + w_j T_j$
$1 r_j \sum w_j' E_j + w_j T_j$	$P r_j \sum w_j' E_j + w_j T_j$	$Q r_j \sum w_j'E_j+w_jT_j$	$R r_j \sum w_j'E_j+w_jT_j$
$1 s_{ij} \sum w_j'E_j+w_jT_j$	$P s_{ij} \sum w_j'E_j+w_jT_j$	$Q s_{ij}^k \sum w_j'E_j+w_jT_j$	$R s_{ij}^k \sum w_j'E_j+w_jT_j$
$1 r_j, s_{ij}  \sum w_i' E_j + w_j T_j$	$P r_j, s_{ij}  \sum w_i' E_j + w_j T_j$	$Q r_j, s_{ij}^k  \sum w_i' E_j + w_j T_j$	$R r_j, s_{ij}^k  \sum w_i' E_j + w_j T_j$
$1 r_i \sum w_i C_i$	$P  \sum w_i C_i$	$Q  \sum w_i C_i$	$R  \sum w_i C_i$
$1 s_{ij} \sum w_j C_j$	$P r_j \sum_{j}w_j^{\dagger}C_j$	$Q r_j \sum w_j^*C_j$	$R r_j \sum w_j^*C_j$
$1 r_j, s_{ij}  \sum w_j C_j$	$P s_{ij} \sum w_j C_j$	$Q s_{ij}^k \sum w_j C_j$	$R s_{ij}^k \sum w_j C_j$
	$P r_j, s_{ij}  \sum w_j C_j$	$Q r_j, s_{ij}^k  \sum w_j C_j$	$R r_j, s_{ij}^k  \sum w_j C_j$

#### Motivation

- $\mathcal{NP}$ -Hard problems
- Real life problems
- Even harder when idle time is allowed (timing problem)

#### Literature Review

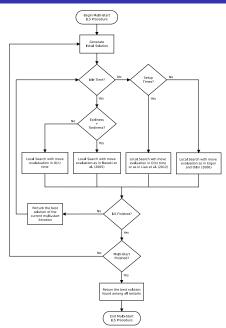
- More than 120 relevant papers in the last 25 years
- Exact approaches (B&B, DP, LR...)
- (Meta)-heuristics Methods (GA, TS, ILS...)
- Efficient local search methods

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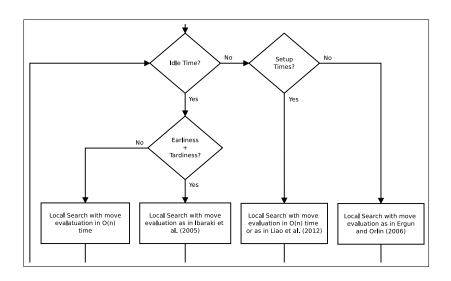
#### Iterated Local Search

- Multi-start local search based meta-heuristic
- High quality results on many kind of problems (e.g. routing)
- Devised to solve a large class of ET scheduling problems
- The method considers specif features of the different problems
- Components:
  - ▶ Initial solutions
  - Local search
  - Perturbation

### Iterated Local Search



#### Iterated Local Search



#### Initial Solution

- Two simple procedures:
  - ▶ Random iterative insertions
  - ▶ EDD based on GRASP constructive phase
- The initial solutions did not significantly affect, on average, neither the final solution quality nor the speed of convergence.

#### Local search

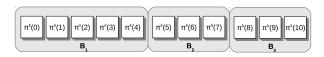
- Performed by the Randomized Variable Neighborhood Descent (RVND) procedure
- RVND consists of:
  - ► List of neighborhoods
  - ▶ Selected at random
  - ▶ If a neighborhood improves the best solution, all the neighborhoods come back to the list
  - ▶ Repeat until the list becomes empty

#### Perturbation mechanisms

- (l, l')-block swap intra-machine: one (l, l')-block swap intra-machine move is performed at random with l and  $l' \in \{2, \ldots, \lfloor n/4 \rfloor\}$ .
- Multiple (1,1)-insertion inter-machine: a job j from a machine k is moved to a machine k', while a job j' from k' is moved to k. Such procedure is repeated one, two or three consecutive times.
- Multiple (l, l')-block insertion inter-machine: this perturbation generalizes the previous one but in this case blocks of jobs of sizes l and l', respectively, are involved in the move with  $l \in \{1, 2\}$  and  $l' \in \{2, 3\}$ .

- Intra machine
  - ▶ l-block insertion
  - $\blacktriangleright$  (l, l')-block swap
- Inter machines
  - ▶ l-block insertion
  - $\blacktriangleright$  (l, l')-block swap

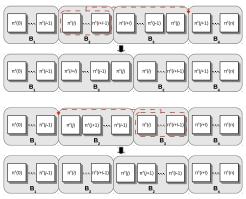
- Solution representation
  - Let  $\pi_k = (\pi_k(0), \pi_k(1), \dots, \pi_k(n))$  be a sequence of n+1 jobs associated to a machine  $k \in M$
  - ▶ A block B consists of subsequence of consecutive jobs



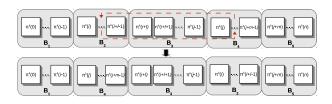
### Neighborhoods<sup>1</sup>

#### *l*-block insertion intra machine

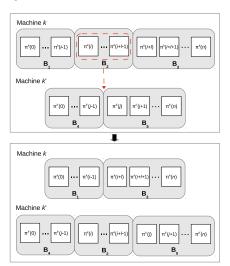
ightharpoonup Consists of moving a block of size l forward or backward in the sequence



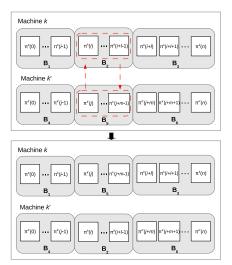
- (l, l')-block swap intra machine
  - ightharpoonup Consists of interchanging a block of size l with another one with size l' of the same sequence



- *l*-block insertion inter machines
  - ightharpoonup Consists of removing a block of size l from a machine k and inserting it in machine k'



- (l, l')-block swap inter machines
  - ▶ Consists of interchanging a block of size l of machine k with a block of size l' of machine k'



- The size(s) of the block(s) in each type of neighborhood is an input parameter
- Size of each neighborhood:  $\mathcal{O}(n^2)$
- For problems without idle time, a straightforward move evaluation can be performed in  $\mathcal{O}(n)$
- Overall complexity of  $\mathcal{O}(n^3)$
- For problems with idle time the timing problem should be solved

#### Move Evaluation

- Without IT: straightforward move evaluation  $\to \mathcal{O}(n)$ . Overall complexity:  $\mathcal{O}(n^3)$
- Without both IT and ST: amortized O(1) → following the ideas of Ergun and Orlin (2006). Overall complexity: O(n²)
- Without IT and with ST: amortized O(1) → following the ideas of Liao et al. (2012). Overall complexity: O(n<sup>2</sup> log n)
- With IT: Timing problem should be solved for each sequence:  $\mathcal{O}(n \log n)$ . Overall complexity:  $\mathcal{O}(n^3 \log n)$ . (Ibaraki *et al.*, 2008) reduce the overall complexity

#### Move Evaluation

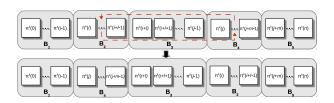
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- Ergun and Orlin (2006) method
  - ▶ Ergun and Orlin (2006) proposed a move evaluation in  $\mathcal{O}(1)$  for the  $1||w_i T_i||$ 
    - Insertion intra machine (1-block insertion intra)
    - Swap intra ((1-1)-block swap intra)
    - Block reverse Twist
  - ▶ We extend to the unrelated parallel machines cases without both idle times and setup times
  - ▶ Intra and inter neighborhoods
  - ▶ This method uses auxiliary functions and data structures

 $W_j^k$ : cost of the block  $B = (\pi_k(0), \dots, \pi_k(j))$  in a sequence  $\pi_k$ 

$$W_{j}^{k} = \begin{cases} \sum_{i=1}^{j} w_{\pi_{k}(i)}'(d_{\pi_{k}(i)} - C_{\pi^{k}(i)}), & \text{if} \quad C_{\pi^{k}(i)} \leq d_{\pi_{k}(i)} \\ \sum_{i=1}^{j} w_{\pi_{k}(i)}(C_{\pi^{k}(i)} - d_{\pi_{k}(i)}), & \text{if} \quad C_{\pi^{k}(i)} \geq d_{\pi_{k}(i)} \end{cases}$$

• Example - (l, l')-block swap intra



• New cost:  $C(B_1) + C(B_4') + C(B_3') + C(B_2') + C(B_5')$ 

#### Penalty function of a job

•  $\rho_j^k(t)$ : penalty of starting to process j in machine k at time t

$$\rho_{j}^{k}(t) = \begin{cases} M(r_{j} - t) + w_{j}'(d_{j} - p_{j}^{k} - t), & t \in [0, r_{j}] \\ w_{j}'(d_{j} - p_{j}^{k} - t), & t \in [r_{j}, d_{j} - p_{j}^{k}] \\ w_{j}(d_{j} - p_{j}^{k} + t), & t \in [d_{j} - p_{j}^{k}, +\infty) \end{cases}$$

• M: big number to allow the  $r_i$  violation

- Characteristics of  $\rho_i^k(t)$ :
  - ▶ Non-negative, convex and piecewise linear function
  - At most three segments defined by:  $r_i$ ,  $d_i$  and  $p_i^k$
  - Without  $r_i \to \text{two segments}$

#### Penalty function of a block of tasks

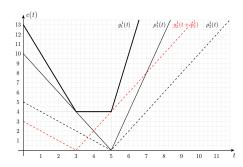
•  $g_j^k(t)$ : cost of a block  $(\pi_k(j), \dots, \pi_k(n_k))$  sequenced in machine k in this order where job  $\pi_k(j)$  starts to be processed at time t

$$g_{j}^{k}(t) = \begin{cases} +\infty, & j = n_{k} + 1, & t \in (-\infty, 0) \\ 0, & j = n_{k} + 1, & t \in [0, +\infty) \\ g_{j+1}^{k}(t + p_{j}^{k} + s_{j, j+1}^{k}) + \rho_{j}^{k}(t), & n_{k} \ge j > 0, & -\infty < t < +\infty \end{cases}$$

- Example  $\rho_k^k(t)$  e  $g_j^k(t)$
- Assuming  $r_i = 0, \forall j \in J$
- Given  $\pi_1 = (0, 1, 2, 0)$

$$\rho_1^1(t) = \left\{ \begin{array}{ll} 10 - 2t, & t \in [0, 5] \\ 4(t - 5), & t \in [5, +\infty) \end{array} \right.$$

$$\rho_2^1(t) = \left\{ \begin{array}{ll} 5 - 1t, & t \in [0, 5] \\ 2(t - 5), & t \in [5, +\infty) \end{array} \right.$$



$$g_2^1(t\!+\!p_2^1) = \left\{ \begin{array}{ll} 3-1t, & t \in [0,3] \\ 2(t-3), & t \in [3,+\infty) \end{array} \right.$$

$$g_1^1(t) = \begin{cases} 13 - 3t, & t \in [0, 3] \\ 4, & t \in [3, 5] \\ 4 + 6(t - 5), & t \in [5, +\infty) \end{cases}$$

- $\rho_j^k(t)$  e  $g_j^k(t)$  can be stored in memory by means of linked lists
- Each element of the list is associated with one of the segments of the piecewise linear function
  - $\triangleright$  b<sub>1</sub>: beginning of the segment's interval;
  - ▶ b<sub>2</sub>: end of the segment's interval;
  - c: value of the function at time  $b_1$ ;
  - $\triangleright$   $\alpha$ : slope of the segment.

$$[b_1, b_2]$$
:  $c - \alpha(t - b_1)$ 

- Cost of a block  $(\pi_k(j), \pi_k(j+1), \dots, \pi_k(n_k))$  starting at time t:
  - Walk through the breakpoints of the corresponding function
  - ▶ Reach the interval  $b_1 \le t \le b_2$
  - ▶ Compute the cost. Given by  $c + \alpha \times (t b_1)$

# Without idle times and without setup times

- Another important data structure
  - ▶ ProcessingList
    - Stores the processing time of a block of size *l*
    - In our case is defined for all blocks of size l from a given sequence  $\pi_k$
    - Must be sorted in descending order of the processing time of the block

#### Algorithm 1 CreateProcessingList

- 1: **Procedure** CreateProcessingList(k, k', l)
- 2:  $p_b \leftarrow 0$
- 3: **for**  $i' = 1, ..., l_i$ **do**
- 4:  $p_b \leftarrow p_b + p_{\pi_k(i')}^{k'}$
- 5: **for**  $i \leftarrow 1, ..., n_k l + 1$  **do**
- 6: ProcessingList.pos[i]  $\leftarrow i$
- 7: ProcessingList.p[i]  $\leftarrow p_b$
- 8:  $p_b \leftarrow p_b p_{\pi_k(i)}^{k'} + p_{\pi_k(i+l)}^{k'}$
- 9: sort(ProcessingList,p) return ProcessingList
- 10: End CreateProcessingList.

# Auxiliary functions and data structures

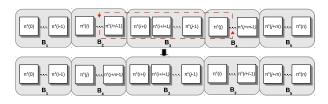
- $W_j^k$ : represents the cost of a block  $B=(\pi_k(0),\dots,\pi_k(j))$
- $\rho_j^k(t)$ : penalty function for a job j starting at time t on machine k
- $g_i^k(t)$ : the cost of a block  $B = (\pi_k(j), \dots, \pi_k(n_k))$
- ProcessingList: ordered list containing the processing times of the blocks of size l

#### Move evaluation

- For a given sequence  $\pi_k$ ,  $g_j^k(t)$  can have at most  $n_k$  breakpoints
- $g_j^k(t)$  are piecewise linear functions, then the complexity of obtaining the cost of a block  $(\pi_k(j), \ldots, \pi_k)$  starting at time t is  $\mathcal{O}(n_k)$
- The complexity of evaluating a move would be  $\mathcal{O}(n_k)$ .
- However, it is **possible** to evaluate a move of the neighborhoods in  $\mathcal{O}(1)$  time
- The costs that depend on  $g_j^k(t)$  are precomputed following a given order (ProcessingList)

#### Move evaluation

- Example (l, l')-block swap intra
  - ▶ Preprocessing:  $\mathcal{O}(n^2)$
  - ▶ Neighborhood complexity:  $\mathcal{O}(\max(l, l')n^2)$
  - Exchanged blocks are evaluated in  $\mathcal{O}(l)$  and  $\mathcal{O}(l')$  since they are small, but can be evaluated in  $\mathcal{O}(1)$  by using  $g_j^k(t)$  functions



$$c(\pi_{ij}^{\prime k}) = W_{i-1}^k + c(B_4^{\prime}) + [g_{i+l}^k(t_1) - g_j^k(t_2)] + c(B_2^{\prime}) + [W_{nk}^k - W_{j+l^{\prime}}^k]$$

$$t_1 = C_{i-1}^k + \sum_{a=j}^{j+l^{\prime}-1} p_{\pi_k(a)}^k \quad t_2 = C_{j+l^{\prime}-1}^k - \sum_{a=i}^{i+l-1} p_{\pi_k(a)}^k$$

# Outline

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- Coded in C++
- Intel Core i7 with 3.40 GHz and 16 GB of RAM
- Ubuntu Linux 12.04

- Problems / instances
  - Publicly available instances
  - ▶ With results reported in the literature
  - We do not report the results found for some single machine problems without sequence-dependent setup times
  - ▶ Well solved by Tanaka et al. (2009); Tanaka and Fujikuma (2012)

Problem	#Instances	J	M
$P  \sum T_j$	2250	20 and 25	2 to 10
$P  \sum w_j T_j$	1125	40  to  100	2, 4, 10, 20 and $30$
$P  \sum w_j' E_j + w_j T_j $	5000	40 to $500$	2, 4, 10, 20 and $30$
$R  \sum w_j T_j$	1440	40 to $200$	2 to 5
$R  \sum w_j' E_j + w_j T_j $	1440	40 to 200	2 to 5

- Neighborhoods parameters
  - $L_{intra} = \{1, 2\}$
  - $L'_{intra} = \{(1,1)\}$
  - $L_{inter} = \{1, 2\}$
  - $L'_{inter} =$

$$\{(1,1),(1,2),(1,3),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)\}$$

- UILS parameters
  - Number of restarts: 10
  - ► Number of consecutive perturbations without improvements: 4n and n (with IT)

- $P||\sum T_j$ 
  - ▶ 2250 instances proposed by Tanaka and Araki (2008)
  - $n = \{20, 25\} \text{ jobs}$
  - $m = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$  identical parallel machines
  - Optimal solutions available
  - The UILS algorithm was capable of finding all optimal solutions
  - ▶ In at least 9 of the 10 runs
  - ▶ Average computational time: smaller than one second

- $P||\sum w_j T_j$ 
  - ► Instances proposed by Rodrigues *et al.* (2008)
  - $n = \{40, 50, 100\} \text{ jobs}$
  - $m = \{2, 4, 10\}$  machines
  - ▶ 25 instances for each (n, m)
  - Heuristics and optimal solutions

	Rod	rigues e	t al.	UILS							
Instance		st run		erage		Best ru	Average				
group	Gap	#BKS	Gap	Time <sup>1</sup>	Gap	#Equal to BKS	#Imp		Time		
	(%)	#10113	(%)	(s)	(%)	to BKS	#-1111p	(%)	(s)		
wt40-2m	0.00	24	0.87	11.4	0.00	25	0	0.01	3.1		
$\rm wt40\text{-}4m$	0.00	25	3.29	38.2	0.00	25	0	0.00	4.0		
$\rm wt50\text{-}2m$	0.00	25	1.85	28.3	0.00	25	0	0.00	6.5		
$\mathrm{wt}50\text{-}4\mathrm{m}$	0.00	25	3.08	96.9	0.00	25	0	0.00	8.2		
Total	-	99	_	-	_	100	0	_			
Avg.	0.00	-	2.27	43.7	0.00	_	_	0.00	5.5		

<sup>&</sup>lt;sup>1</sup> Average of  $30 \times m \times n$  runs in an Intel Xeon 2.33 GHz.

	Pesse	oa et al.(2010)	UILS							
Instance		Best run		Best ru	Average					
group	Gap (%)	#BKS	Gap (%)	#Equal to BKS	#Imp	Gap (%)	Time (s)			
wt100-2m	0.00	21	0.00	22	2	0.00	71.1			
wt100-4m	0.00	16	-0.01	15	5	0.00	94.8			
Total	_	37	-	37	7	-	_			
Avg.	0.00	=	0.00	-	-	0.00	83.0			

- $P||\sum w_j' E_j + w_j T_j$ 
  - ▶ Instances proposed by Amorim (2013)
  - ▶ Heuristics and optimal solutions
  - ▶ 11 or 12 instances for each (n, m)
  - ▶ Idle times not considered

	ILS+PR				GA+LS+PR				ILS-M				UILS				
Instance	Bes	st run	Αv	Average		Best run		Average	Best run		Average		Best run			Average	
Group	Gap	#BKS	Gap	Time <sup>1</sup>	Gap	#BKS	Gap	Time <sup>1</sup>	Gap	#BKS	Gap	Time <sup>1</sup>	Gap	#Equal	#Imp	Gap	Time
	(%)	#BK5	(%)	(s)	(%)	#BK5	(%)	(s)	(%)	#BK5	(%)	(s)	(%)	#Equal to BKS	#Imp	(%)	(s)
wet40-2m	0.00	12	0.00	15.4	0.00	12	0.01	33.6	0.00	12	0.00	13.6	-0.58	11	1	-0.58	5.6
wet50-2m	0.00	12	0.00	41.2	0.00	12	0.02	99.1	0.00	12	0.00	32.2	-0.78	11	1	-0.78	12.6
wet100-2m	0.02	5	0.02	588.0	0.00	10	0.01	3098.2	0.00	11	0.00	533.0	-0.67	10	1	-0.66	168.5
wet40-4m	0.74	11	0.74	68.4	0.74	11	0.75	201.1	0.74	11	0.74	37.0	0.00	12	0	0.00	6.3
wet50-4m	0.00	10	0.00	163.1	0.02	8	0.02	546.8	0.00	10	0.00	73.5	-0.91	10	1	-0.63	14.1
wet100-4m	0.14	2	0.16	3047.1	0.01	4	0.01	22689.9	0.01	8	0.03	1876.4	0.08	5	1	0.15	190.3
wet40-10m	0.00	5	0.00	325.6	0.00	5	0.00	4780.4	0.00	5	0.00	46.9	0.00	5	0	0.00	4.1
wet50-10m	0.03	4	0.03	852.7	0.00	5	0.01	10237.8	0.00	5	0.01	130.0	-0.60	4	1	-0.59	9.3
wet100-10m	1.43	0	1.45	16817.9	1.19	1	1.19	132735.9	1.19	0	1.23	8424.8	-0.01	0	1	0.05	140.4
Avg.	0.26	-	0.27	2435.5	0.22	-	0.22	19380.3	0.22	-	0.22	1240.8	-0.39	-	-	-0.34	61.3
Total	-	61	-	-	-	68	-	_	-	74	-	-	-	68	7	-	-

Average of 3 runs in an Intel i7-3770 3.40GHz with 12 GB of RAM.

- $R||\sum w_j T_j$ 
  - ▶ Instances proposed by Sen and Bülbül (2015)
  - ▶ Heuristic, optimal solutions and lower bounds
  - ▶ 6 groups of instances, each of them containing 60 test-problems

		Sen	and B	ülbül (20	UILS							
Instance	В	est	PΙ	R+Bende	PS		Be	st run		Average		
group	"O-+ Gaplb		#Opt	#Equal	$Gap_{LB}$	Time	#Opt	#Imp	#Equal	$\overline{\text{Gap}_{\text{LB}}}$	$\overline{\text{Gap}_{\text{LB}}}$	Time
	#Opt (%)	(%)	#Opt	to ${\rm BKS}$	(%)	(s)	#Opt	#Imp	to BKS $$	(%)	(%)	(s)
N40_M2	17	1.24	0	13	1.76	1.9	17	20	40	0.76	0.76	3.4
$N60\_M2$	8	1.2	0	40	1.46	6.4	8	41	19	0.63	0.63	11.7
$N60\_M3$	6	4.27	0	35	4.62	3.0	6	37	23	3.13	3.13	15.4
$N80\_M2$	3	1.43	0	55	1.52	13.2	3	55	5	0.66	0.66	29.4
N80_M4	5	4.8	3	48	4.93	43.1	4	48	11	3.83	4.26	41.7
N90_M3	1	3.05	1	56	3.11	8.0	1	56	4	1.18	1.24	57.1

- $R||\sum w_j' E_j + w_j T_j|$ 
  - ▶ Same Instances proposed by Sen and Bülbül (2015)
  - ▶ Heuristic, optimal solutions and lower bounds
  - ▶ 6 groups of instances, each of them containing 60 test-problems
  - ▶ Idle times are permitted

		Sen	and B	ülbül (20	UILS							
Instance Best			PF	R + Bende	ers+SiP	Si		Be		Average		
group	group #Opt	$Gap_{LB}$	#Opt	#Equal	$Gap_{LB}$	Time	#Ont	- 41mm	#Equal	$Gap_{LB}$	$Gap_{LB}$	Time
		(%)	#Opt	to BKS	(%)	(s)	#Opt #Imp	#IIIIp	to BKS	(%)	(%)	(s)
N40_M2	22	0.16	1	1	0.95	52.9	22	13	47	0.13	0.13	21.1
$N60\_M2$	5	0.89	0	42	0.98	109.7	5	50	10	0.43	0.45	126.8
$N60\_M3$	4	0.82	0	20	1.58	120.3	4	37	23	0.38	0.44	105.0
$N80\_M2$	2	0.90	0	56	0.92	134.1	2	57	3	0.36	0.38	476.2
N80_M4	0	4.54	0	58	4.57	228.3	0	60	0	1.59	1.72	312.2
N90_M3	0	2.52	0	58	2.55	153.1	0	59	1	1.27	1.37	574.0

## Outline

- 1 Introduction
  - Problem
  - Motivation
  - Literature
- 2 Method
  - Iterated Local Search
  - Neighborhoods
  - Move Evaluation
- 3 Computational Experiments
- 4 Conclusions and Future Work

### Conclusions and Future Work

#### Conclusions

- Algorithm for a class of problems
- ► Takes into account the particularities of each problem (move evaluation)
- ▶ Generalization of efficient local search methods
- ► The method was capable of finding high quality solutions in competitive CPU times
- Paper currently under second revision
- ▶ More details in: http://arxiv.org/abs/1509.02384

#### Future Work

- Generation of new set of instances for the problems where it was not possible to test the proposed algorithm
- ▶ Development of a general exact method capable of tackling the variants considered in this work
- ▶ Lower bounds for the instances to be created

## Conclusions and Future Work

Thank you

# A Local Search Based Metaheuristic for a Class of Earliness and Tardiness Scheduling Problems

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