Discovering the Output Pattern of OR Gates in Boolean Networks

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Abstract

We derive a closed-form, network-aware index-set formula for the OR gate over ordered exhaustive inputs, with explicit ordering policy (LSB-first/MSB-first) and a mapping between orderings. The result is validated against empirical repertoires and presented with examples.

1 Problem Definition

Consider a Boolean network with n nodes and adjacency matrix C. Node k applies OR over its connected inputs $I_c = \{i \mid C_{k,i} = 1\}$. Inputs are the 2^n binary vectors ordered by the binary of $j-1$ (1-based j).

2 Ordering Policy

Two orderings are used:

- LSB-first (code): $v(j) = \text{Reverse}\,\text{IntegerDigits}(j-1, 2, n)$, bit weights 2^{i-1}.
- MSB-first (manuscript): $v(j) = \text{IntegerDigits}(j-1, 2, n)$, bit weights 2^{n-i}.

Define $\varphi(j) = 1 + \text{FromDigits}(\text{Reverse}(\text{IntegerDigits}(j-1, 2, n)), 2)$; φ maps indices between orderings and is an involution.

3 Index-Set Formula (Network-Aware)

For node k and connected set I_c, the one-set indices where OR outputs 1 are

$$J_k = \bigcup_{i \in I_c} \left\{ 1 + w(i) + \sum_{t \in S} w(t) \;\middle|\; S \subseteq \{1, \ldots, n\} \setminus \{i\} \right\},$$

where $w(i) = 2^{i-1}$ for LSB-first and $w(i) = 2^{n-i}$ for MSB-first. Equivalently,

$$J_k = \left\{ j \;\middle|\; \exists i \in I_c : \; v_i(j) = 1 \right\}.$$

Under MSB-first, the index set equals φ-mapped LSB-first indices.

4 Examples

For $n = 3$ and $I_c = \{2, 3\}$, LSB-first weights $(1, 2, 4)$ yield bands $\{3, 4, 7, 8\}$ and {5,6,7,8}, so $J_k = \{3, 4, 5, 6, 7, 8\}$. Under MSB-first the numeric set differs but matches via φ.

5 Validation

Empirical repertoires from `CreateRepertoiresDispatch` match the analytic J_k exactly under the stated ordering. Cross-ordering equality is verified by applying φ.

6 Conclusion

The OR gate admits a band-union index construction that is exact and ordering-aware, with a simple mapping φ between orderings.