TSK-TEST-002: Property Tests for Axioms and Invariances

Causal Boolean Integration Project

December 5, 2025

1 Objective

Validate foundational axioms and invariances: φ involution, universe size, band complements, De Morgan laws, ordering invariance for gate index sets, KOFN strictness, and relabelling invariance.

2 Methods

We use package `IntegrationÌndexAlgebra` for set operations and `IntegrationG̀ates` for index sets. For ordering checks, LSB one-sets are mapped to MSB by $\varphi(j, n) = 1 + \text{binrev}_n(j - 1)$. Relabelling invariance permutes bits by a random permutation and compares permuted one-sets to recomputed sets on permuted input indices.

3 Artefacts

- `results/tests/test002/PropertyTests.json`
- `results/tests/test002/Report.txt`
- `results/tests/test002/Status.txt`

4 Results

All cases PASS; Status is OK.

Property	Status
φ involution (n=3,4,5)	OK
Universe size ($\lvert\{1..2^n\}\rvert = 2^n$)	OK
Band complements ($\text{OneBand} \cup \text{ZeroBand} = \mathcal{U}_n$)	OK
De Morgan ($\overline{A \cup B} = \overline{A} \cap \overline{B}$)	OK
Ordering invariance (AND/OR/XOR, n=3, $I_c = \{2, 3\}$)	OK
KOFN strictness ($n = 3, k = 2$)	OK
Relabelling invariance (OR, n=5; XOR, n=6)	OK