Hands-On - AR(2) Stochastic Process: The Yule-Walker Equations

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Introduction

Given an **Auto-Regressive stationary stochastic process AR(2),** defined as

$$AR(2): \quad v(t) = a_1\, v(t-1) + a_2\, v(t-2) + \eta(t) \qquad \eta(\cdot) \sim WN(0, \lambda^2)$$

or, equivalently, as

$$AR(2): \quad v(t) = \frac{1}{A(z)} \cdot \eta(t) \qquad A(z) = z^2 - a_1 z - a_2$$

then the initial values $\gamma(0)$, $\gamma(1)$ and $\gamma(2)$ of the autocorrelation function can be determined solving the **Yule-Walker equations**

$$\begin{bmatrix} a_2 & a_1 & -1 \\ a_1 & a_2 - 1 & 0 \\ 1 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} \gamma(0) \\ \gamma(1) \\ \gamma(2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \lambda^2 \end{bmatrix} \quad (\star)$$

For given a_1, a_2 and λ^2 it is possible to compute $\gamma(0)$, $\gamma(1)$ and $\gamma(2)$, solving Eq. $(\star)$ and afterwars proceed in a recursive way

$$\gamma(\tau) = a_1\, \gamma(\tau - 1) + a_2\, \gamma(\tau - 2) \qquad \tau > 0$$

Hands-On Exercise

Given the stochastic stationary AR(2) process

$$y(t) = \frac{1}{3}\, y(t-1) + \frac{2}{9}\, y(t-2) + \epsilon(t) \qquad \epsilon(\cdot) \sim \mathrm{WN}(0,\, 2)$$

- use the Yule--Walker equation to compute the initial values $\gamma(0)$, $\gamma(1)$ and $\gamma(2)$ of the autocorrelation function;
- given the values $\gamma(0)$, $\gamma(1)$ and $\gamma(2)$, evaluate recursively the vallues of the autocorrelation function $\gamma(\tau)$, till the sample correspoding to $|\tau| = 10$;
- compare the results with the values obtained evaluating the expression

$$\gamma(\tau) = \left[\frac{16}{21}\left(\frac{2}{3}\right)^{|\tau|} + \frac{5}{21}\left(-\frac{1}{3}\right)^{|\tau|}\right] \cdot \gamma(0) \qquad \tau \in \mathbb{Z}$$

```
clear variables
close all
clc

a1 = 1/3;
a2 = 2/9;
lambda2 = 2;
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