Data-driven and Learning-based Control

Value-function Reinforcement Learning

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What we know so far?

1 A brief recap

We are now able to solve optimal control problems in the case of dynamics that are:

- Known deterministic (linear or non-linear) or stochastic
  - Dynamic Programming
  - Policy iteration

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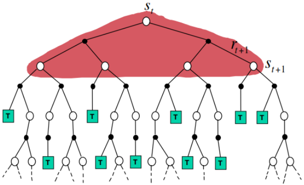
- Unknown deterministic (linear or non-linear) or stochastic
  - Monte Carlo (MC)$\rightarrow$ **sampling**
  - Temporal difference (TD) $\rightarrow$ **sampling & bootstrapping**

What we know so far?

Dynamic Programming updates



$$V\left(x^{(k)}\right) = \mathbb{E}\left[r^{(k+1)} + \gamma V\left(x^{(k+1)}\right)\right]$$

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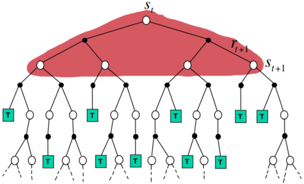

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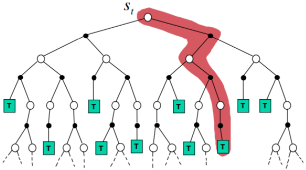
Monte Carlo updates



$$V\left(x^{(k)}\right) = V\left(x^{(k)}\right) + \alpha\left[G_k - V\left(x^{(k)}\right)\right]$$

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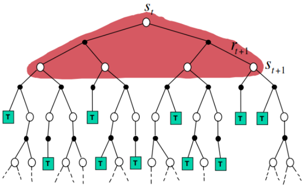

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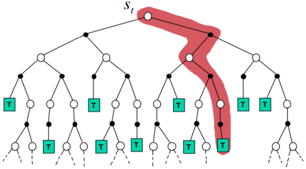

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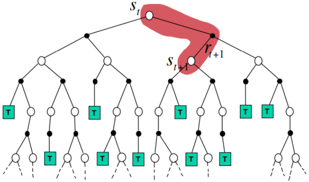
Temporal-difference updates



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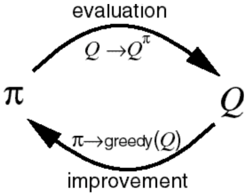
The way in which all these approaches solve optimal control problems is through policy iteration.



- policy evaluation
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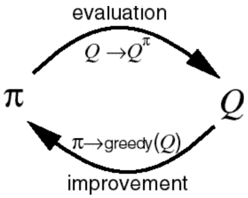

- policy evaluation
- policy improvement

They differ:

1. as already mentioned, in their policy evaluation updates,
2. but also in their policy improvement procedure.

What we know so far?

- **Dynamic Programming improvement**

$$\pi_{i+1} = \arg\max_{\pi} Q_{\pi_i} \rightarrow \textbf{Greedy policy}$$

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$$\pi\left(u^{(k)} | x^{(k)}\right) = \begin{cases} \frac{\epsilon}{|\mathcal{U}|} & u^{(k)} \neq \arg\max_{u^{(k)}} Q\left(x^{(k)}, u^{(k)}\right) \\ 1 - \epsilon + \frac{\epsilon}{|\mathcal{U}|} & u^{(k)} = \arg\max_{u^{(k)}} Q\left(x^{(k)}, u^{(k)}\right) \end{cases} \rightarrow \epsilon\textbf{-greedy policy}$$

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It guarantees an **exploration-exploitation trade off**

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2 Reinforcement Learning taxonomy

Reinforcement Learning taxonomy

We can classify Reinforcement Learning approaches in:

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- **Value-function methods:**
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- **Policy optimization methods:**
  The policy is a parameterized function whose weights are learned in order to maximize the expected cumulative discounted reward $\rightarrow$ **Actor**
- **Actor-critic methods:**
  Merging the two ideas by guiding the actor's learning on the basis of the critic's estimated return

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Value-function methods taxonomy

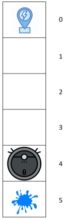
Value function methods assume to work with a discrete compact action set.

Discrete compact set

A set $\mathcal{S}$ is a discrete compact space if it includes h finite number of possible values:

$$\mathcal{S} = \{s_1, s_2, \ldots, s_h\}$$

For example in the robot vacuum cleaner example:



Control input: $u \in \mathcal{U} = \{-1, 1\}$

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We can classify Value-function methods depending on the **state set representation**.

In this case, we can distinguish value function methods in:

- Tabular
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On-policy Value-function methods

On-policy means that the evaluation is based on the **current policy**:

$$Q\left(x^{(k)}, u^{(k)}\right) = Q\left(x^{(k)}, u^{(k)}\right) + \alpha\left[r^{(k+1)} + \gamma Q\left(x^{(k+1)}, u^{(k+1)}\right) - Q\left(x^{(k)}, u^{(k)}\right)\right]$$

At each iteration:

- we apply the **current policy** producing $u^{(k)}$
- we evaluate the **current policy** by producing $u^{(k+1)}$ applying it
- we update the **current policy** according to its evaluation

On policy Value-function methods

An on-policy algorithm is **SARSA:**

Initialization. $Q\,\forall x \in \mathcal{X}\; u \in \mathcal{U}$

Repeat (for each episode)

- Set $x^{(0)}$
- Select an action $u^{(k)}$ with ϵ-greedy policy
- Repeat for each step of the episode until the terminal condition is met
  - Perform the action $u^{(k)}$, observe $x^{(k+1)}$ and $r^{(k+1)}$
  - Select an action $u^{(k+1)}$ with ϵ-greedy policy
  - $Q\left(x^{(k)}, u^{(k)}\right) = Q\left(x^{(k)}, u^{(k)}\right) + \alpha\left[r^{(k+1)} + \gamma Q\left(x^{(k+1)}, u^{(k+1)}\right) - Q\left(x^{(k)}, u^{(k)}\right)\right]$
  - Update $x^{(k)} = x^{(k+1)}$, $u^{(k)} = u^{(k+1)}$

Off-policy means that the evaluation is based on the **greedy policy**:

$$Q\left(x^{(k)}, u^{(k)}\right) = Q\left(x^{(k)}, u^{(k)}\right) + \alpha\left[r^{(k+1)} + \gamma \max Q\left(x^{(k+1)}, u\right) - Q\left(x^{(k)}, u^{(k)}\right)\right]$$

At each iteration:

- we apply the **current policy** producing $u^{(k)}$
- we evaluate by applying the **greedy policy** $\max_u Q\left(x^{(k+1)}, u\right)$
- we update the **current policy** according to the evaluation thus obtained

Off-policy Value-function methods

An off-policy algorithm is the **Q-Learning:**

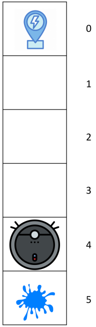
Initialization. $Q \, \forall x \in \mathcal{X} \; u \in \mathcal{U}$

Repeat (for each episode)

- Set $x^{(0)}$
- Select an action $u^{(k)}$ with ϵ-greedy policy
- Repeat for each step of the episode until the terminal condition is met
  - Perform the action $u^{(k)}$, observe $x^{(k+1)}$ and $r^{(k+1)}$
  - $Q\left(x^{(k)}, u^{(k)}\right) = Q\left(x^{(k)}, u^{(k)}\right) + \alpha\left[r^{(k+1)} + \gamma \max Q\left(x^{(k+1)}, u\right) - Q\left(x^{(k)}, u^{(k)}\right)\right]$
  - Update $x^{(k)} = x^{(k+1)}$,

Example

Consider a robot vacuum cleaner that needs to clean a patch on the floor and also needs to recharge the batteries. Set $\gamma = 0.5$.



- **State:** $x \in \mathcal{X} = \{0, 1, 2, 3, 4, 5\}$
- **Control input:** $u \in \mathcal{U} = \{-1, 1\}$
- **State transition function:** unknown
- **Reward function:**

$$\begin{cases} 5 & \text{if } x^{(k)} \neq 5 \text{ and } x^{(k+1)} = 5 \\ 1 & \text{if } x^{(k)} \neq 0 \text{ and } x^{(k+1)} = 0 \\ 0 & \text{otherwise} \end{cases}$$

Example

1. Apply the **SARSA** algorithm for one episode:
   - starting from $x^{(0)} = 2$
   - with $Q_0 =$

0	**1**	**2**	**3**	**4**	**5**
$0; 0$	$0; 0$	$0; 0$	$0; 0$	$0; 0$	$0; 0$

   - and an ϵ-greedy policy with $\epsilon = 0.5$

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2. Apply the **Q-Learning** algorithm for one episode:
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   - and an ϵ-greedy policy with $\epsilon = 0.5$

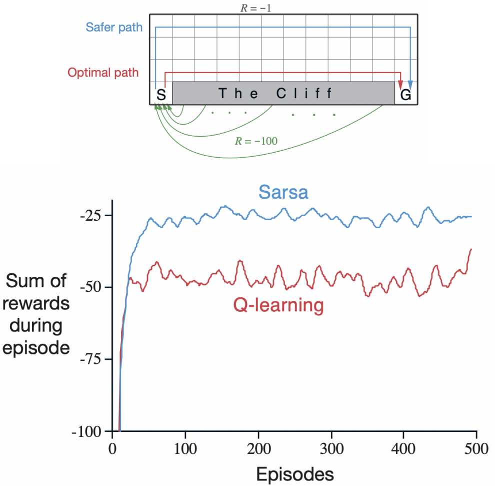
Home exercises

1. Repeat both considering $\epsilon = 0$
2. Repeat both considering $\epsilon = 1$
3. Repeat both considering $\epsilon = \frac{1}{\#\text{of episodes}}$

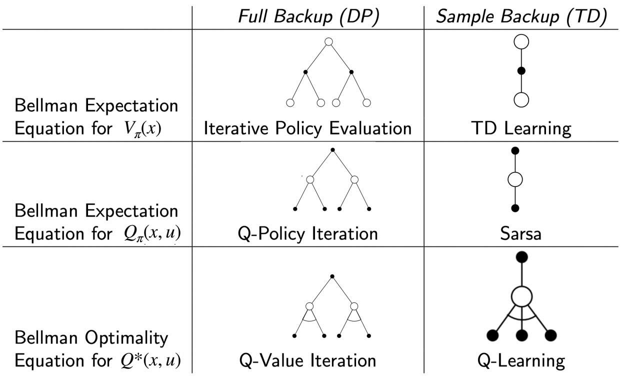
Differences between Sarsa and Q-learning

- SARSA converges to the optimal ϵ-greedy policy, but being on-policy has a better on-line performance

- Q-learning converges to the optimal policy π^* and action-value function Q^* but occasionally fails due to its off-policy nature



Relation between DP and TD



	Full Backup (DP)	*Sample Backup (TD)*
Bellman Expectation Equation for $V_\pi(x)$	Iterative Policy Evaluation	TD Learning
Bellman Expectation Equation for $Q_\pi(x, u)$	Q-Policy Iteration	Sarsa
Bellman Optimality Equation for $Q^*(x, u)$	Q-Value Iteration	Q-Learning

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Tabular means that the $Q\left(x^{(k)}, u^{(k)}\right)$ representation is a lookup table, i.e., one entry for every state/action pair.

Consequently, it assumes to work with a discrete compact state set.

	u_1	u_2	$\dots$	u_h
x_1	$Q\left(x_1, u_1\right)$	$Q\left(x_1, u_2\right)$	$\dots$	$Q\left(x_1, u_h\right)$
x_2	$Q\left(x_2, u_1\right)$	$Q\left(x_2, u_2\right)$	$\dots$	$Q\left(x_2, u_h\right)$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
x_l	$Q\left(x_l, u_1\right)$	$Q\left(x_l, u_2\right)$	$\dots$	$Q\left(x_l, u_h\right)$

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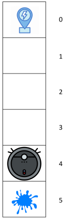

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Tabular Value-function methods

In large MDPs, a lookup table might be prohibitive:

- **Memory demand:** too many actions/states to store
- **Sparsity/Curse of dimensionality:** learning the value of each state/action pair individually might take too long

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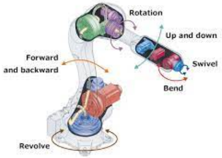
Backgammon: 10^{20} states

Go: 10^{170} states



continuous state space

Function Approximation methods

Function approximation means to define an approximate representation of the value function $\hat{V}(x)$ or of the action-value function $\hat{Q}(x, u)$.

We can recognize two main types of approximators:

- **parametric approximators:** given a vector of parameters $\theta \in \mathbb{R}^t$ a parametric approximator maps from the parameters space $\mathbb{R}^t$ to the value-function $\mathcal{V}$ or action-value function space $\mathcal{Q}$

$$\hat{V} : \mathbb{R}^t \to \mathcal{V} \quad \to \quad V(x) \approx \hat{V}_\theta(x)$$

$$\hat{Q} : \mathbb{R}^t \to \mathcal{Q} \quad \to \quad Q(x, u) \approx \hat{Q}_\theta(x, u)$$

- **non-parametric approximators:** typically still have parameters, but the number of parameters, as well as their values, is determined from data

Function Approximation methods

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If good estimates of Q-values of certain states are available, the algorithm can make reasonable control decisions in the nearby states.

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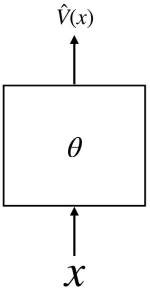
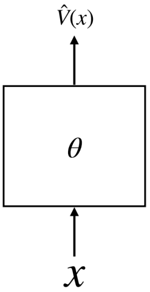
Generalization

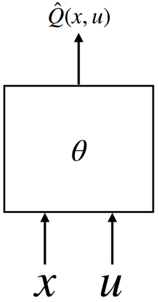
If good estimates of Q-values of certain states are available, the algorithm can make reasonable control decisions in the nearby states.

Parametric functions



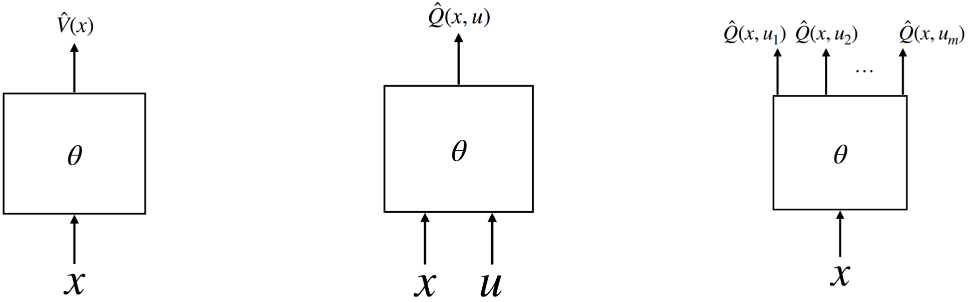
Parametric functions



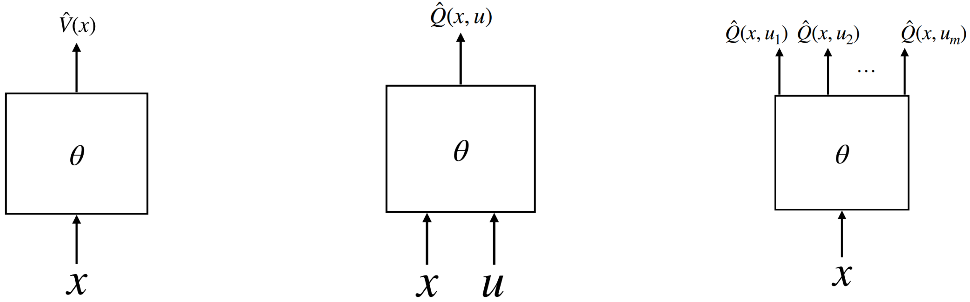


Parametric functions

3 Value-function Reinforcement Learning



Parametric functions



There are many possible function approximators:

- linear function approximation
- neural networks
- ...

Linear function approximation

Basis functions (BF) ϕ are building blocks for creating more complex functions.

In other words, they are a set of h standard functions, combined to estimate another function that is difficult to exactly model.

Examples of BFs are:

- Polinomials
- Fourier Basis
- Radial Basis function

Polynomial Basis Functions

A polynomial basis function is a function defined as a polynomial expression.

The general form of a polynomial basis function given an n-dimensional state vector is given by:

$$\phi_i(x) = \prod_{j=1}^{n} x_j^{c_{i,j}}$$

where:

- x_j is the j-th component of the state vector,
- $c_{i,j}$ is an integer in the set $\{0,\ 1,\ \ldots,\ N\}$ where N is the order of polynomial.

We obtain $h = (1 + N)^d$ basis functions

Fourier Basis Functions

A Fourier basis function is a function defined in terms of cosines.

The general form of a Fourier basis function given an n-dimensional state vector is given by:

$$\phi_i(\boldsymbol{x}) = \cos\left(\pi \boldsymbol{c}_i \cdot \boldsymbol{x}\right)$$

where:

- c_i is a vector of n components whose integer-values belong to $\{0,\ 1,\ \dots,\ N\}$.

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Notice that $c_i \cdot x$ is the element-wise product.

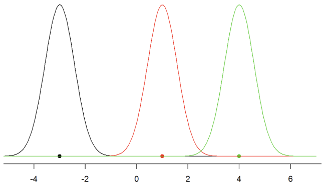
A Radial basis function is a bell-shaped (Gaussian) function.

The general form of a Radial basis function given an n-dimensional state vector is given by:

$$\phi_i(x) = \exp\left(-\frac{||x - c_i||^2}{2\sigma_i^2}\right)$$

where:

- c_i is a vector of n components whose values are the centers of the i-th BF.
- σ_i is a positive real value corresponding to the width of the i-th BF.



Linear function approximation

A linearly parametrized V or Q-function approximator employs h basis function $\phi_1 \ \ldots \ \phi_h : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$ and a h-dimensional parameters vector θ.

Approximated V-values are computed as follows:

$$\hat{V}_\theta(x) = \sum_{l=1}^{n} \theta_l^\top \phi_l(x) = \theta^\top \phi(x)$$

where $\phi(x) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \vdots \\ \phi_h(x) \end{bmatrix}$ and $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_h \end{bmatrix}$

While approximated Q-values are computed as follows:

$$\hat{Q}_\theta(x,u) = \sum_{l=1}^{n} \theta_l^\top \phi_l(x,u) = \theta^\top \phi(x,u)$$

where $\phi(x,u) = \begin{bmatrix} \phi_1(x,u) \\ \phi_2(x,u) \\ \vdots \\ \phi_h(x,u) \end{bmatrix}$ and $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_h \end{bmatrix}$

Linear function approximation

Since in Value-function methods we deal with discrete compact action set, assuming that it includes c values $\{u_1, \ldots, u_c\}$, for approximating Q-values we have the following:

$$\phi(x, u_i) = [0, \ldots, 0, \ldots, \phi_1(x), \ldots, \phi_n(x), 0, \ldots, 0]$$

$$u_1 \quad \ldots \qquad\qquad u_i \qquad\qquad u_c$$

and

$$\theta = \begin{bmatrix} \theta_1 & \theta_2 & \ldots & \theta_{h*c} \end{bmatrix}$$

Function Approximation updates

The goal is to find the parameter vector $\theta^* \in \mathbb{R}^t$ that minimizes the **mean-squared error** (MSE) between the estimated value $\hat{V}_\theta(x)$ and the true value $V_\pi(x)$:

$$\mathbb{E}\left[V_\pi(x) - \hat{V}_\theta(x)\right]$$

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It tries to minimize the MSE, adjusting θ after each step of an episode by a small amount in the direction that would most reduce it:

Function Approximation updates

Gradient

Given a function $f(s)$ of variable s, we denote by $\nabla_s f$ the gradient of the function with respect of s which is defined as follows:

$$\nabla_s f = \frac{\partial f}{\partial s}\vec{s}$$

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Therefore, the SGD performs as follows:

$$\begin{aligned}\theta &= \theta - \alpha \nabla_\theta \mathbb{E}\left[V_\pi(x) - \hat{V}_\theta(x)\right] \\ &= \theta + \alpha\left[V_\pi(x) - \hat{V}_\theta(x)\right]\nabla_\theta \hat{V}_\theta(x)\end{aligned}$$

Function Approximation updates

Do we have all the elements for solving this update?

$$\theta = \theta + \alpha \left[V_\pi(x) - \hat{V}_\theta(x) \right] \nabla_\theta \hat{V}_\theta(x)$$

Function Approximation updates

3 Value-function Reinforcement Learning

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Therefore, we can substitute the TD target in the θ update thus obtaining

$$\theta = \theta + \alpha \left[r^{(k+1)} + \gamma \hat{V}_\theta \left(x^{(k+1)} \right) - \hat{V}_\theta(x^{(k)}) \right] \nabla_\theta \hat{V}_\theta(x^{(k)})$$

The same intuitions hold when we would like to approximate the action value function.

The **mean-squared error** (MSE) between the estimated $\hat{Q}_\theta(x,u)$ and the true $Q_\pi(x,u)$:

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The SGD updating rule used to search for θ^* is

$$\theta = \theta + \alpha\left[r^{(k+1)} + \gamma\hat{Q}_\theta\left(x^{(k+1)}, u^{(k+1)}\right) - \hat{Q}_\theta\left(x^{(k)}, u^{(k)}\right)\right]\nabla_\theta\hat{Q}\left(x^{(k)}, u^{(k)}\right)$$

SARSA with linear function approximation

Initialization. θ choose $\phi(x, u)$

Repeat (for each episode)

- Set $x^{(0)}$
- Select an action $u^{(k)}$ with ϵ-greedy policy
- Repeat for each step of the episode until the terminal condition is met
  - Perform the action $u^{(k)}$, observe $x^{(k+1)}$ and $r^{(k+1)}$
  - Select an action $u^{(k+1)}$ with ϵ-greedy policy
  - $Q\left(x^{(k)}, u^{(k)}\right) = \theta^\top \phi\left(x^{(k)}, u^{(k)}\right)$ and $Q\left(x^{(k+1)}, u^{(k+1)}\right) = \theta^\top \phi\left(x^{(k+1)}, u^{(k+1)}\right)$
  - $\theta = \theta + \alpha\left[r^{(k+1)} + \gamma Q\left(x^{(k+1)}, u^{(k+1)}\right) - Q\left(x^{(k)}, u^{(k)}\right)\right] \nabla_\theta Q\left(x^{(k)}, u^{(k)}\right)$
  - Update $x^{(k)} = x^{(k+1)}$, $u^{(k)} = u^{(k+1)}$

Q-Learning with linear function approximation

Initialization. θ choose $\phi(x, u)$

Repeat (for each episode)

- Set $x^{(0)}$
- Select an action $u^{(k)}$ with ϵ-greedy policy
- Repeat for each step of the episode until the terminal condition is met
    - Perform the action $u^{(k)}$, observe $x^{(k+1)}$ and $r^{(k+1)}$
    - $Q\left(x^{(k)}, u^{(k)}\right) = \theta^\top \phi\left(x^{(k)}, u^{(k)}\right)$ and $\forall u$ $Q\left(x^{(k+1)}, u\right) = \theta^\top \phi\left(x^{(k+1)}, u\right)$
    - $\theta = \theta + \alpha\left[r^{(k+1)} + \gamma \max Q\left(x^{(k+1)}, u\right) - Q\left(x^{(k)}, u^{(k)}\right)\right] \nabla_\theta Q\left(x^{(k)}, u^{(k)}\right)$
    - Update $x^{(k)} = x^{(k+1)}$,

Questions’ time!

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