Hands-On: Polynomial Regression

Introduction

Having collected N data for the variables $y(t)$ and $u(t)$, over the time window $t = 1, 2, \dots N$, we want to compute (if possible) $q+1$ parameters $\vartheta_0, \vartheta_1, \vartheta_2, \dots \vartheta_q$ such that

$$y(t) = \vartheta_q \, u^q(t) + \vartheta_{q-1} \, u^{q-1}(t) + \cdots + \vartheta_1 \, u(t) + \vartheta_0 \quad , \qquad t = 1, 2, \dots N \qquad (+)$$

The stated problem is a polynomial regression problem, given the degree q of the polynomial. Rewriting in a compact form the Eq. (+), the regression problem appears as **a linear in the parameters problem**, so we can solve it by applying the LS approach

$$y(t) = \begin{bmatrix} 1 & u(t) & u^2(t) & \cdots & u^q(t) \end{bmatrix}^\top \cdot \begin{bmatrix} \vartheta_0 \\ \vartheta_1 \\ \vdots \\ \vartheta_{q-1} \\ \vartheta_q \end{bmatrix} \qquad t = 1, 2, \cdots N$$

Important Remark

Obviously, we know **perfectly a-priori** what is the degree q of the polynomial regression function to determine.

```
clear
close all
clc
```

Solving the Polynomial Regression Problem

Let's start selecting the degree q of the polynomial function to estimate

```
Nq = 4; % the degree q of the polynomial curve to be estimated
```

Generate a random polynomial. We will use it to generate the data

```
coeff_v = -10 + 20 * rand(Nq,1); % the "true" polynomial coefficients
```

Tune the variance of the additive noise acting on the data, collected during the simulated experiment

```
sigma2_n = 5.28; % the variance of the additive noise
```

Collecting the Data

How many data?

```
Ndata = 300; % the data we will collect during the simulated experiment
```

Setting the constraints on the variable u

```
Umin = -2; Umax = 3; % the extrema for the variable u
U_i = Umin + (Umax-Umin)*rand(Ndata,1); % the data used for the independent variable/r
```

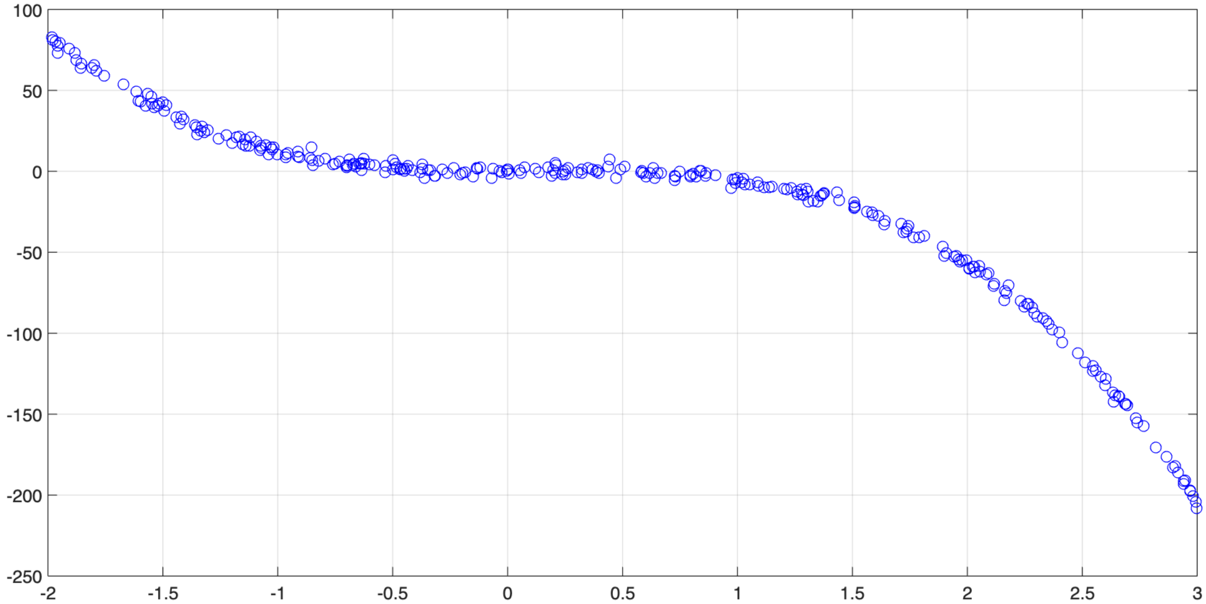
Now let's simulate the collection of the data from the simulated system

```
add_noise = sqrt(sigma2_n) * randn(Ndata,1); % the noise samples

Y_i = polyval(coeff_v, U_i) + add_noise; % the data we collected
```

Plotting the raw data

```
figure('Units', 'normalized', 'Position',[0.1, 0.1, 0.9, 0.75]);
plot(U_i, Y_i,'bo'); grid on;
```



Looking for a Model

Building the regressor matrix Φ_N and the observation vector Y_N

$$\Phi_N \cdot \vartheta = Y_N$$

```
regressorMAT = ones(length(U_i),1); % initialization of the regressor matrix

Y_N = reshape(Y_i, length(Y_i),1); % the vector Y_N (as column vector)

[Us, idx]= sort(U_i);

%degree = Nq; % <-- NB we know it!!
for degree=1:Nq
    regressorMAT = [reshape(U_i, length(U_i),1) .^degree ,regressorMAT];
end
% the matrix Phi_N
```

Solving the equation

$$\hat{\vartheta}_N = \left(\Phi_N^\top \Phi_N\right)^{-1} \Phi_N \, Y_N$$

```
    hat_thetaN = regressorMAT \ Y_N;
```

Comparing the Data with the Model

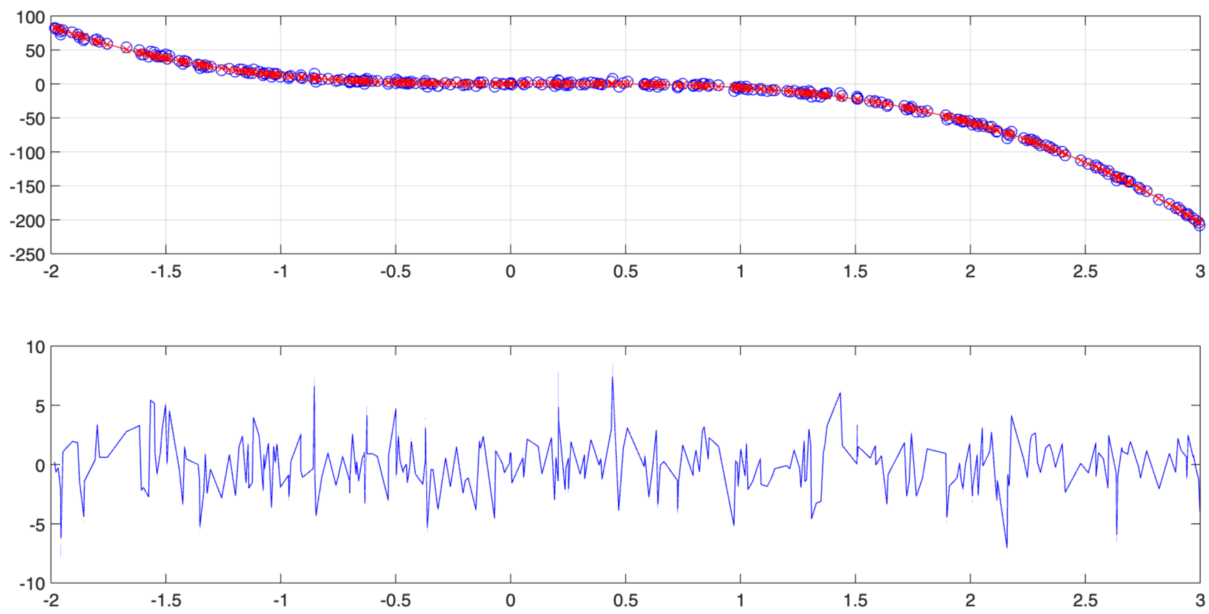
```
model = polyval(hat_thetaN,U_i); % using the model
errors = Y_i - model;

% plotting the results - comparing with the raw data and plotting the
% regression errors
figure('Units', 'normalized', 'Position',[0.1, 0.1, 0.9, 0.75]);

subplot(2,1,1)
plot(U_i, Y_i, 'bo'); % the raw data

hold on;
plot(Us, model(idx), 'x-r'); % the model output
grid on; hold off;

subplot(2,1,2);
plot(Us, errors(idx), '-b');
```



Some statistics about the error:

```
mean(errors)
```

```
ans = 1.3885e-15
```

```
var(errors)
```

```
ans = 4.8947
```