# Life Annuity Portfolio

In this section we showcase the SWIM package to conduct sensitivity analysis of a life annuity portfolio.

Model inspired by Olivieri & Pitacco (2003). See also Hari et al. (2009) for an application to a pension scheme. Point of view of an insurer who aims at building up a partial internal model for a portfolio of life annuities.

Types of risks:  
+ The *mortality risk* is originated by the random lifetimes of annuitants. It can be split in process and longevity risk.  
- The *investment risk* originates from the randomness stemming from financial markets in which the insurer invests.

\ Annuitants same age at entry time (when they buy the annuity) single cohort. In every policy holder is deceased. Same immediate life annuity ( annual amount) for every policy holder. \\ Notation:

At time , initial fund: Portfolio fund dynamics: For a fixed solvency level , the initial margin is chosen so that:

The model considers simultaneously age and calendar year .\\ The random variable gives the one-year death probability for an individual aged x+t in calendar year (where is a fixed year). \\ Under the assumptions of homogeneous and indipendent lives, the conditional probability distribution of is Binomial: In , the uncertainty of represents the longevity risk. \\ \\ Model: M7 (Quadratic CBD model with cohort effects) extension of Lee-Carter model. See Cairns et al. (2009).\\ Be careful: notation here is a bit different! (the single cohort of policy holders in the life annuity portfolio allows a simplified notation):

Structure of M7. See GAPC family StMoMo (Villegas et al. (2018): Model M7 is fitted to Italy male mortality data for ages from 55 to 89 and calendar years from 1950 to 2014 M7 is the best.

is defined by the equation: is a Geometric Brownian motion: where is a Wiener process, (percentage drift) and (percentage volatility) are constants. The stochastic differential equation admits analytic solution: hence

variable: The opposite of the fund at time is chosen as output variable to allow adverse scenarios as stress. \\ Risk Factors :

$\\$ We have: where is implicitly defined by the equations in previous sections.\\ Stress: