Exercise 2

Context: given a transactional dataset $\mathbb{T} \in 2^{\{ms:2^{\mathbb{X}} \to \mathbb{N}\}}$ on the universal set of items $\mathbb{X}$, and a two non-empty disjoint set of items $X, Y \subseteq \mathbb{X}$, a rule-quality function $q : 2^{\{ms:\mathbb{X} \to \mathbb{N}\}} \times 2^{\mathbb{X}} \times 2^{\mathbb{X}} \to \mathbb{R}$, let $rule : X \to Y$ let us define the $cpo_{rule,\mathbb{T},q}$ on the set X as $cpo_{rule,\mathbb{T},q}(X') = q(\mathbb{T}, X', Y)$ for any $X' \subseteq X$.

Assignment:
Implement and test a function:

rule-shapley :

$$2^{\{ms:\mathbb{X} \to \mathbb{N}\}} \times \{q : 2^{\{ms:\mathbb{X} \to \mathbb{N}\}} \times 2^{\mathbb{X}} \times 2^{\mathbb{X}} \to \mathbb{R}\} \times 2^{\mathbb{X}} \times 2^{\mathbb{X}} \to \{f : \mathbb{X} \to \mathbb{R}\}$$

Such that for any transactional dataset $\mathbb{T}$, any rule-quality function q, any rule $rule : X \to Y$ the value

$$\textit{rule-shapley}(\mathbb{T}, q, X, Y)(x) = es_{x, cpo_{rule,\mathbb{T},q}}$$

for each $x \in X$.

Where $es_{X,cpo_{rule},\mathbb{T},q}$ is computed as follows ($n_samples$ may be estimated empirically or apriori fixed):

```
Estimate Shapley ( cpo , 𝕏, y, n_samples):
  • pick randomly X_1 .... X_n_samples distinct
    subsets of 𝕏 - {y}
  N, D = 0, 0
  · for i = 1 .... n_samples :
  |   D_i = |X_i|! ( |𝕏| - |X_i| - 1 )!
  |   N += D_i · (cpo(X_i ∪ {y}) - cpo(X_i))
  |   N += D_i ((cpo(𝕏 / X_i) - cpo(𝕏 / (X_i ∪ {y}))))
  |
  |   D += 2 · D_i
  return N / D
```