Hummingbird

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Contents

1 Problem Formulation

Δ still writing Δ

A hummingbird of mass m, approximated by a blunt body with two wings, has to fly from initial position x_0 to the target position x_f and then hover in that position. The episode runs from $t = 0$ and ends when $t = t_f$.

The system (1D) is governed by the equation: $m\ddot{x} = F_p - F_d - F_g$ where:

$$F_p = c_A\dot{\phi}^2$$

$$F_d = \frac{1}{2}S\rho\dot{x}^2 c_D(\dot{x})$$

$$F_g = mg$$

The system can be written as

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = f(x_1, x_2, \dot{\phi}) = \begin{bmatrix} x_2(t) \\ \frac{1}{m}(F_p(\phi) - F_d(x_2) - F_g) \end{bmatrix}, \tag{1}$$

The hummingbird is approximated as a blunt body with two wings, therefore the drag coefficient $c_d = sign(\dot{x}) * (c_{d,body} + 2c_{d,wing})$ is as follow:

$$c_{d,body} = \frac{24}{Re_{body}} + \frac{6}{1 + \sqrt{Re_{body}}} + 0.4$$

$$c_{d,wing} = \frac{7}{\sqrt{Re_{wing}}}$$

where

$$Re_{body} = |\dot{x}| \frac{L_{body}}{\nu}$$

$$Re_{wing} = |\dot{x}| \frac{S_{wing}/R_{wing}}{\nu}$$

. The parameters of the problem are:

$$\rho = 1.225$$

$$\nu = 1.460e - 5$$

$$L_{body} = 0.1\ ,\ S = ,\ R = ,$$

$$c_A = \frac{0.001}{\sqrt{10}}$$

The action input is related to $\dot{\phi}$, where $\phi = \sum w_i \Phi_i$ is the flapping amplitude and $\Phi_i = e^{\frac{-(x-c_i)^2}{\sigma^2}}$. The relation is discussed later on

To linearize the system around an operative point $[\mathbf{x}^*, u^*]$ the taylor expansion is performed:

$$f(x, \dot{x}, u) = f(x, \dot{x}^*, u^*) + (x - x^*) \frac{\partial f}{\partial x}\bigg|_{\mathbf{x}^*, u^*} + (\dot{x} - \dot{x}^*) \frac{\partial f}{\partial \dot{x}}\bigg|_{\mathbf{x}^*, u^*} + (u - u^*) \frac{\partial f}{\partial u}\bigg|_{\mathbf{x}^*, u^*}$$

where, considering $x = x_1$ and $\dot{x} = x_2$

$$\frac{\partial f}{\partial x_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \frac{\partial f}{\partial x_2} = \begin{bmatrix} 1 \\ \frac{1}{m} \frac{\partial F_d}{\partial x_2} \end{bmatrix} \quad \frac{\partial f}{\partial u} = \begin{bmatrix} 1 \\ \frac{\partial F_p}{\partial u} \end{bmatrix}$$

and $\frac{\partial F_d}{\partial x_2} = \frac{1}{2} S \rho (2 x_2 c_d + \frac{\partial c_d}{\partial x_2} x_2^2)$.

Considering $Re_{body} = \alpha |x_2|$ and $Re_{wing} = \beta |x_2|$:

$$\frac{\partial c_d}{\partial x_2} = \text{sign}(x_2) \left(-\frac{24}{\alpha x_2^2} - \frac{3\alpha}{\left(1 + \sqrt{|x_2|\alpha}\right)^2 \sqrt{|x_2|\alpha}} - \frac{7}{\beta \sqrt{|x_2|\beta}} \right)$$

while $\frac{\partial f}{\partial u}$ =??

2 Solutions

The optimal trajectory is found using ADAM optimizer with the loss function

2.1 Open Loop

2.1.1 Periodic

2.1.2 RBF

The RBF is related to the amplitude $\phi(t) = \sum w_i \Phi_i(t)$. The range of the amplitude is $[-1.1\frac{\pi}{2}, 1.1\frac{\pi}{2}]$ and the action input is proportional to $\dot{\phi}$. I tested as input the average value over every stroke

$$F_p = \frac{c_A}{T_{i+1} - T_i} * \int_{T_i}^{T_{i+1}} \dot{\phi}^2(t)dt$$

where the points $[T_i, T_{i+1}, ...]$ are the maxima and minima of the amplitude $(dot\phi(T_i) = 0)$. I also tested

$$F_p = c_A * \dot{\phi}^2(t)$$

. The loss function per time step k is proportional to

$$r_k = (x_k - x_f)^2 + 0.01(\dot{x}_k)^2$$

the total loss is $\sum_{k=0}^{T} r_k + constraints$ where $constraints$ is used to keep the amplitude and actions within their boundaries and it's proportional to the difference between the actual value and the constraint if not respected:

$$\text{constraint amplitude} = \sum_i (|\phi(t_i)| - 1.1\frac{\pi}{2}) \quad \forall\, t_i : \; |\phi(t_i)| > 1.1\frac{\pi}{2}$$

$$\text{constraint action} = ..$$

.

With the right adjustement in terms of constants and maximum action, it seems that $\phi(t)$ assumes the right shape to approximate the flapping motion [..]

2.2 Closed Loop

2.2.1 RBF

In the closed loop the RBF is dependant on the state $\phi(\mathbf{x}) = \phi(x, \dot{x})$. Now i'm trying to implement the 2D version of the RBF, but im not sure of the results.

$$F_p \propto \frac{d\phi(x(t), \dot{x}(t))}{dt} = \frac{\partial\phi}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial\phi}{\partial \dot{x}}\frac{\partial \dot{x}}{\partial t}$$

(or its integral, but it gets complicated to find the centers in 2D)