Vibration Analysis and Vibroacoustic

Assignment 3: Modal parameter identification

Homework 3 Report

Students

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Experimental Frequency Response Functions

It is useful to first visualise the data collected in the "Data.mat" file

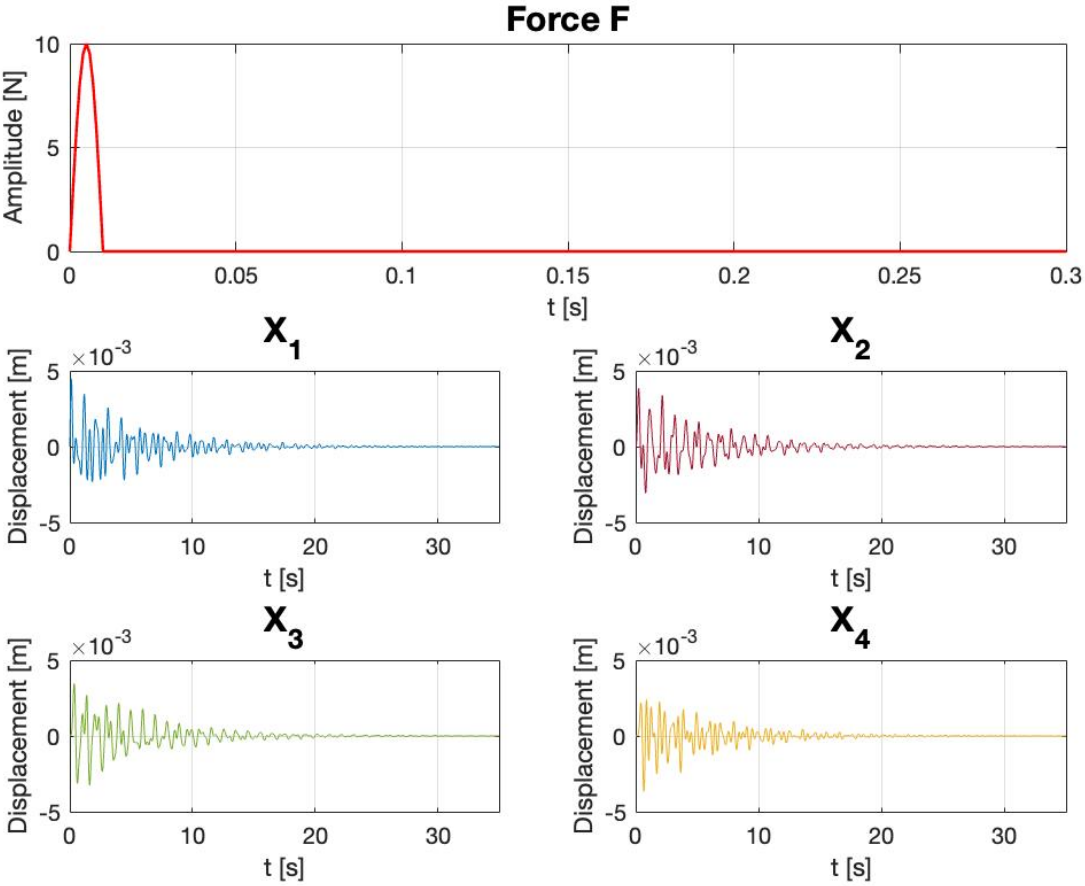

Figure 1: Force F and displacements in four distinct points

It is clear that the force is an impulsive one and so all the responses are different impulse responses at different points of the structure. The frequency response function is now computed for every point. First of all it is necessary to compute the Fourier transform of the force and the four time responses, easily achieved by the function *fft()* of MATLAB. They will be called $F_1(\Omega)$ and $X_i(\Omega)$ where i refers to the point of measurement. Then the ratio between the Fourier Transforms of the displacements and of the force will be computed

$$H_{11}(\Omega) = \frac{X_1(\Omega)}{F_1(\Omega)} \quad H_{21}(\Omega) = \frac{X_2(\Omega)}{F_1(\Omega)}$$

$$H_{31}(\Omega) = \frac{X_3(\Omega)}{F_1(\Omega)} \quad H_{41}(\Omega) = \frac{X_4(\Omega)}{F_1(\Omega)}$$

Remembering that the force is applied in the same location of the first point of measurement, the first FRF $H_{11}(\Omega)$ is the co-located FRF of point 1.
The results are shown below. It is clear that all the four FRFs have four peaks at the same frequencies, which means that we are studying a four degrees of freedom system.

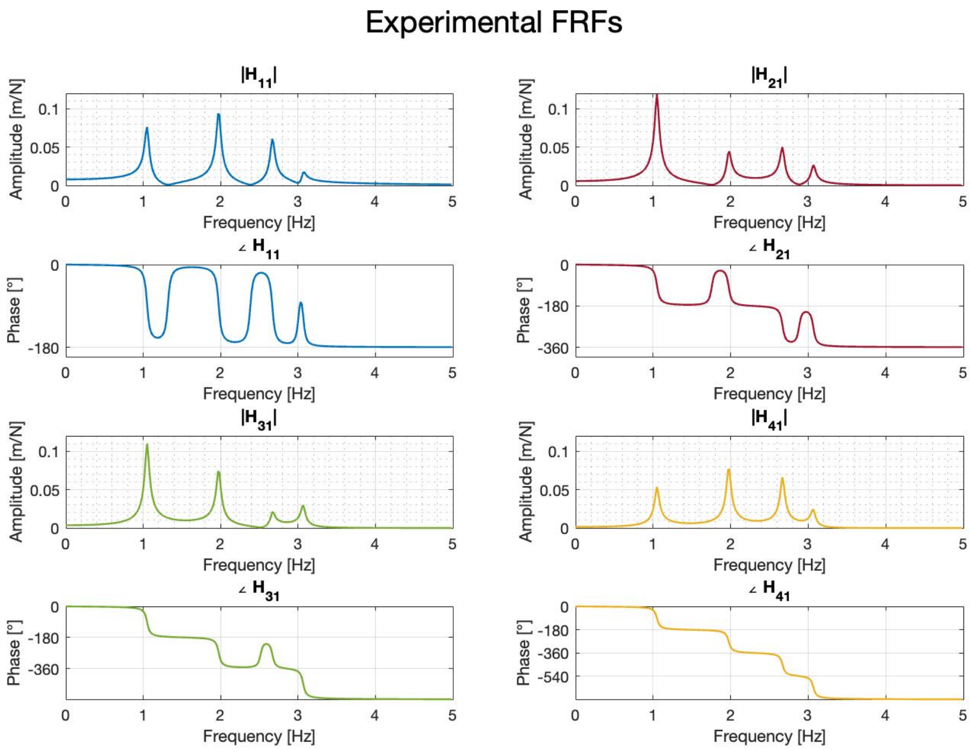

Figure 2: Experimental FRFs of the structure

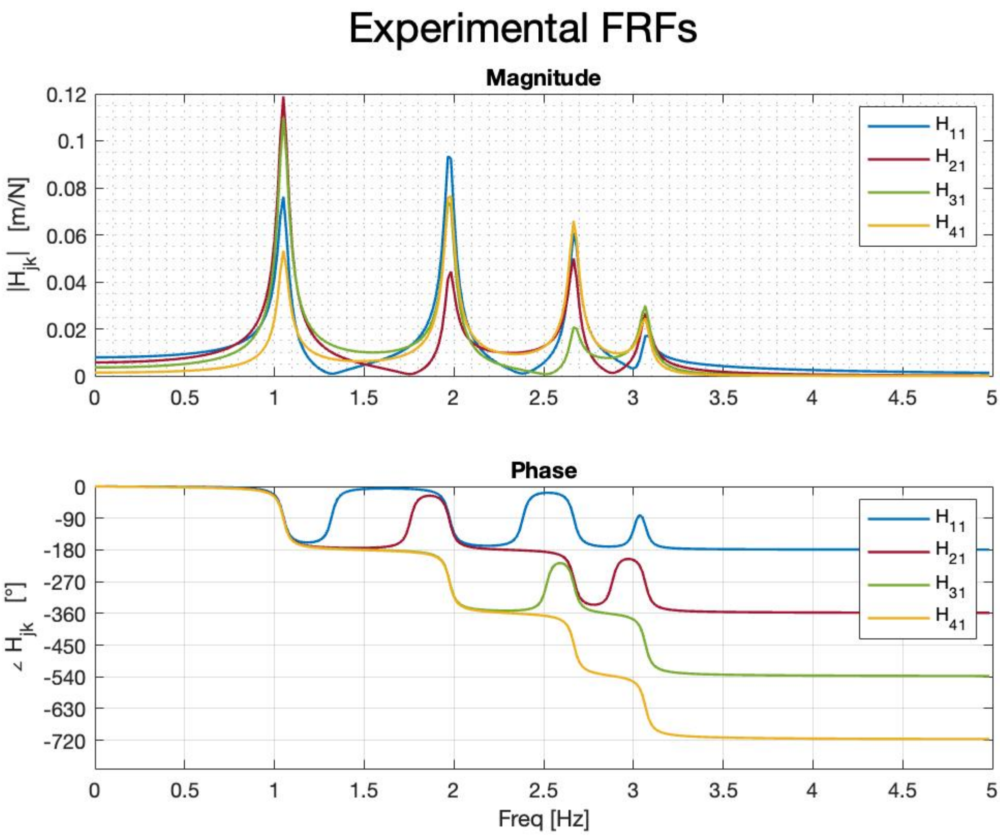

Figure 3: Superimposition of the Experimental FRFs

Estimation of the Parameters through Simplified Methods

To evaluate the parameters of the system through the following simplified method it is first considered, for every FRF, one peak at a time, as if the peaks at lower and higher frequencies have no effect on it. So every FRF is divided in ranges of frequencies centered in the peaks. In other words it is possible to say that it is assumed that the point of measurements are located in the modal coordinates.
It is also assumed that the system is lightly damped, so that the positions of the peaks correspond to the natural frequencies of the system.
Under this assumptions, the FRF of the k-th peak of the j-th point of measurement can be expressed as:

$$H_{j1}^{(k)}(\Omega) = \frac{X_j^{(k)}\ X_1^{(k)}}{-\Omega^2 m_{qkk} + i\Omega c_{qkk} + k_{qkk}}$$

To evaluate the natural frequencies of the system finding the location of the peaks is needed. This is possible by using the MATLAB function *findpeaks()*.
The procedure is done for every FRF, as it will be done for the following parameters evaluations.

Resonance frequencies [Hz]

x_1	x_2	x_3	x_4
1.0500	1.0500	1.0500	1.0500
1.9666	1.9833	1.9666	1.9833
2.6666	2.6666	2.6666	2.6666
3.0833	3.0666	3.0666	3.0666

The adimensional damping ratio through the phase derivation method consists to evaluate the adimensional damping ratio h_k using the derivative of the phase at $\Omega = \omega_{0k}$, by inverting the following equation:

$$\frac{\partial H_{j1}^{(k)}(\omega_{0k})}{\partial \Omega} = -\frac{1}{h_k\ \omega_{0k}}$$

The results are shown below:

Csi computed through the Phase Derivation Method

x_1	x_2	x_3	x_4
0.0286	0.0283	0.0283	0.0282
0.0161	0.0160	0.0160	0.0158
0.0113	0.0113	0.0115	0.0111
0.0144	0.0099	0.0097	0.0096

To compute the adimensional damping ratio through the half power point method, we have to find the frequencies ω_{1k} and ω_{2k} corresponding to the points in which the value of the FRF is

$\frac{1}{\sqrt{2}}$ the value of the considered peak.
Then we apply the following formula:

$$h_k = \frac{\omega_{2k}^2 - \omega_{1k}^2}{4\omega_{0k}^2}$$

The results are shown below:

Csi computed through the Half Power Method

x_1	x_2	x_3	x_4
0.0236	0.0317	0.0317	0.0317
0.0169	0.0168	0.0169	0.0168
0.0094	0.0125	0.0126	0.0125
0.0163	0.0109	0.0109	0.0108

Mode Shapes

Now it is possible to derive the following:

$$H_{j1}^{(k)}(\omega_{0k}) = \frac{X_j^{(k)}\ X_1^{(k)}}{i\omega_{0k}c_{qkk}}$$

It is possible to invert this equation and by posing $X_1^{(k)} = 1$ and $m_{qkk} = 1$ we can evaluate the value of $X_j^{(k)}$.
The results are shown below:

Mode Shapes

x_1	x_2	x_3	x_4
0.1563	0.4564	0.3205	0.1370
0.3284	0.2130	-0.3449	-0.2141
0.3037	-0.3590	-0.1449	0.2389
0.1471	-0.3804	0.4605	-0.1948

Results

The tables in the last paragraphs show the results computed with MATLAB using the previous methods. For natural frequencies and adimensional damping ratios the rows corresponds to different natural frequencies and the columns to different point of measurements.
As we can see the columns are quite similar, which means that for every FRFs the results are almost the same.
The mode shapes are represented by the columns of the last table. We can tell that the results are quite good by looking at the ratio between the same peak in different point of measurements.

Residual Minimization Technique

In this technique it is no longer assumed that the effect of the other peaks is negligible, but it is also to be considered the effect of the lower frequencies peaks as a constant, while considering the effect of the higher ones as inversely proportional to Ω^2. So the expression of the FRF of the k-th peak of the j-th point of measurement becomes:

$$H_{j1}^{NUM(k)}(\Omega) = \frac{A_j + iB_j}{-\Omega^2 m_{qkk} + i\Omega c_{qkk} + k_{qkk}} + C_j + iD_j + \frac{E_j + iF_j}{\Omega^2}$$

It is now needed to find the following parameters (m_{qkk} is assumed equal to 1):
$[c_{qkk}, k_{qkk}, A_j, B_j, C_j, D_j, E_j, F_j]$
To do so, the error will be minimized ϵ between $H_{j1}^{NUM(k)}(\Omega)$ and the experimental FRF, in the range of frequencies considered, that is going to be called $H_{j1}^{EXP(k)}(\Omega)$.

$$\epsilon = \Re e\left(H_{j1}^{NUM(k)}(\Omega) - H_{j1}^{EXP(k)}(\Omega)\right)^2 + \Im m\left(H_{j1}^{NUM(k)}(\Omega) - H_{j1}^{EXP(k)}(\Omega)\right)^2$$

By putting as initial values of the unknowns the one found in the previous section $\left[c_{qkk}, k_{qkk}, X_j^{(k)}, 0, 0, 0, 0, 0\right]$ and by using the function of MATLAB *fminsearch()* to find them. These are the results.

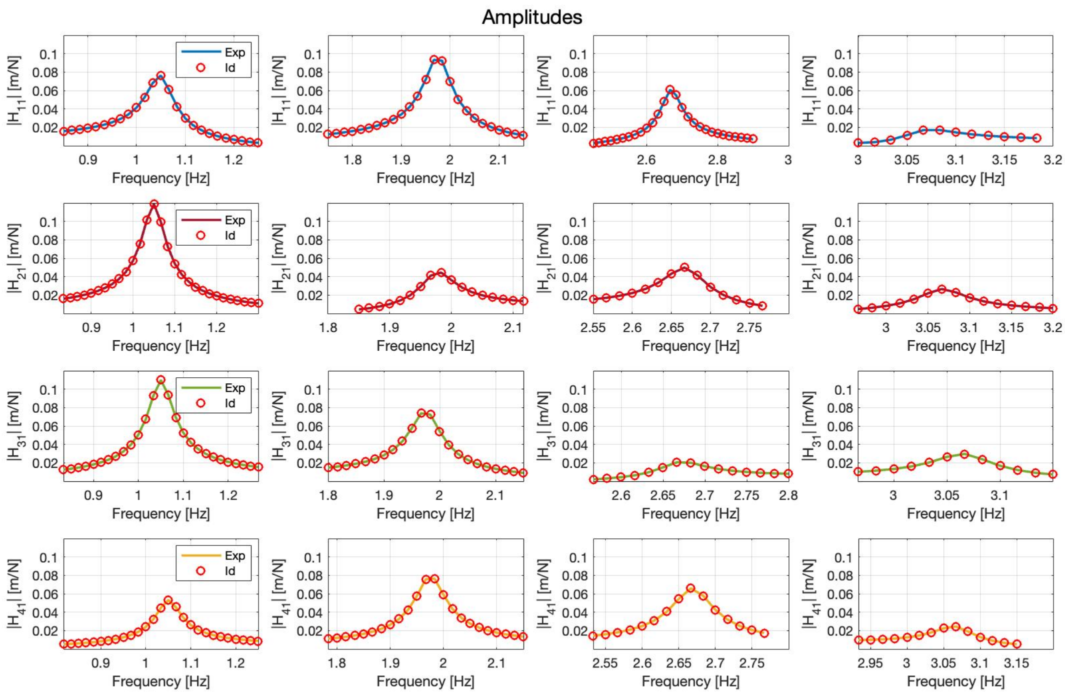

Figure 4: Amplitude of the FRFs

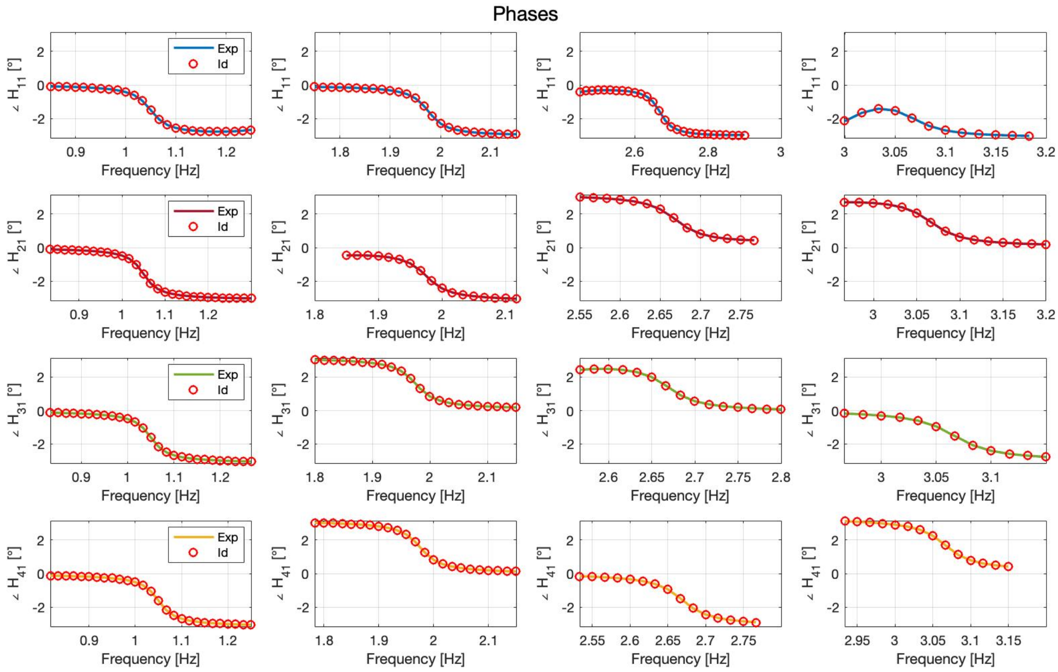

Figure 5: Phase of the FRFs

We can see that the results are almost equal to the experimental FRFs, which means that in this case the residual minimization technique gives us a good approximation of the studied system.

Comparison between Methods

The following tables show the parameters of the system found by using the residual minimization technique.
For the natural frequencies and the adimensional damping ratios we can see a better convergence of the results, compared to the ones obtained with the simplified methods. This is due to the fact that in this case we have considered more than one peak at a time.

Identified frequencies [Hz]

x_1	x_2	x_3	x_4
1.0501	1.0501	1.0500	1.0500
1.9754	1.9753	1.9754	1.9753
2.6691	2.6691	2.6687	2.6689
3.0661	3.0661	3.0664	3.0666

Identified Csi

x_1	x_2	x_3	x_4
0.0252	0.0253	0.0253	0.0253
0.0134	0.0134	0.0134	0.0134
0.0099	0.0099	0.0099	0.0101
0.0086	0.0086	0.0086	0.0086

Identified Modes

x_1	x_2	x_3	x_4
0.1668	0.4013	0.3405	0.0976
0.2613	0.1847	-0.2768	-0.1685
0.2420	-0.3169	-0.1154	0.1908
0.1174	-0.3306	0.3766	-0.1561

Reconstructed FRFs through Modal Approach

First of all it is possible to build the mass, damping and stiffness matrices in modal coordinates from the parameters just evaluated through the residual minimization technique, remembering that it was imposed $m_{qkk} = 1\ \forall k$.

$$[M_q] = \begin{bmatrix} m_{q11} & 0 & 0 & 0 \\ 0 & m_{q22} & 0 & 0 \\ 0 & 0 & m_{q33} & 0 \\ 0 & 0 & 0 & m_{q44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[C_q] = \begin{bmatrix} c_{q11} & 0 & 0 & 0 \\ 0 & c_{q22} & 0 & 0 \\ 0 & 0 & c_{q33} & 0 \\ 0 & 0 & 0 & c_{q44} \end{bmatrix}$$

$$[K_q] = \begin{bmatrix} k_{q11} & 0 & 0 & 0 \\ 0 & k_{q22} & 0 & 0 \\ 0 & 0 & k_{q33} & 0 \\ 0 & 0 & 0 & k_{q44} \end{bmatrix}$$

The modal frequency response matrix is evaluated as:

$$[H_q] = \left[-\Omega^2 [M_q] + i\Omega [C_q] + [K_q]\right]^{-1}$$

Now we define the modal matrix as the combination of the mode shapes we found:

$$[\phi] = \left[\underline{X}^{(1)}\ \underline{X}^{(2)}\ \underline{X}^{(3)}\ \underline{X}^{(4)}\right]$$

Finally the frequency response function with respect to the measurement points is:

$$[H] = [\phi] [H_q] [\phi]^T$$

The elements of the first row of the matrix are the reconstructed FRFs. Below are the individual modal FRFs and a comparison with the experimental ones.

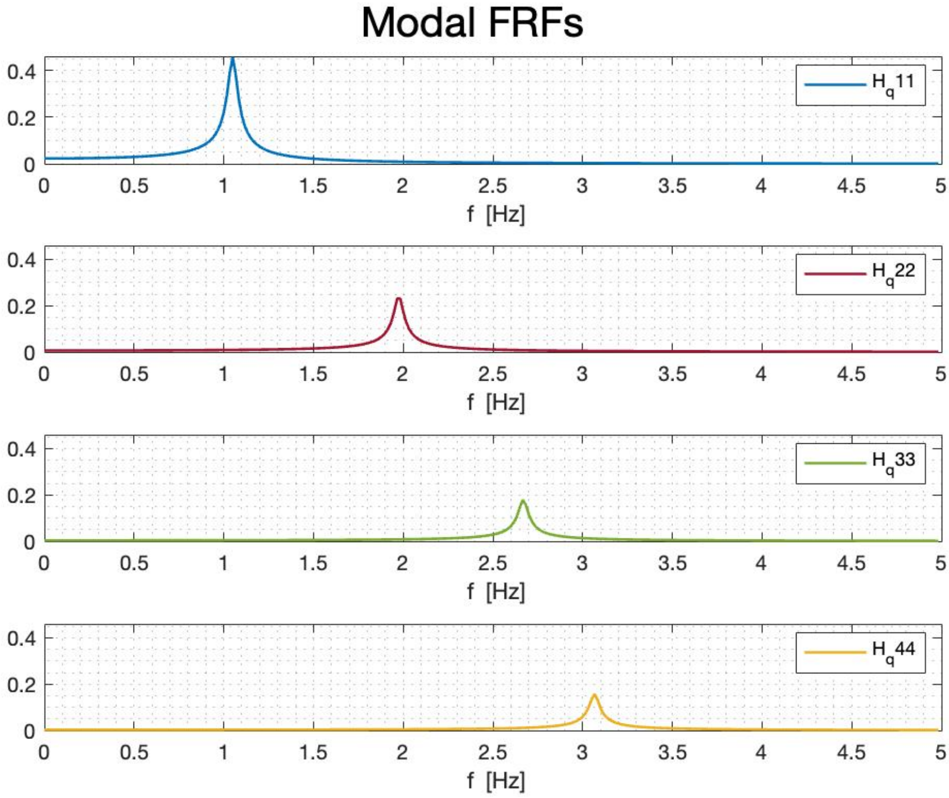

Figure 6: Individual Modal FRFs

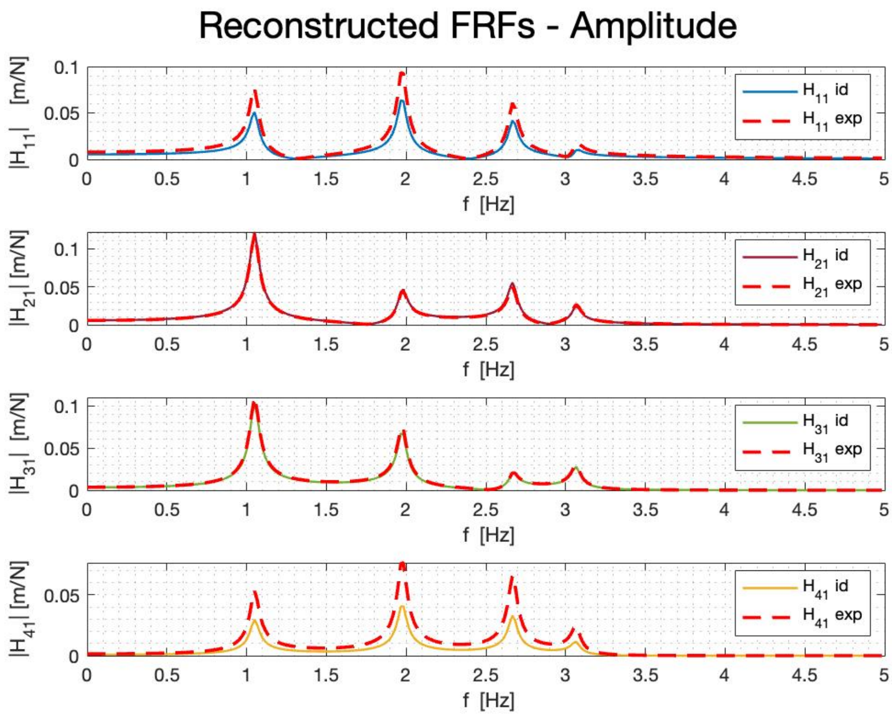

Figure 7: Amplitude comparison between original and reconstructed FRFs

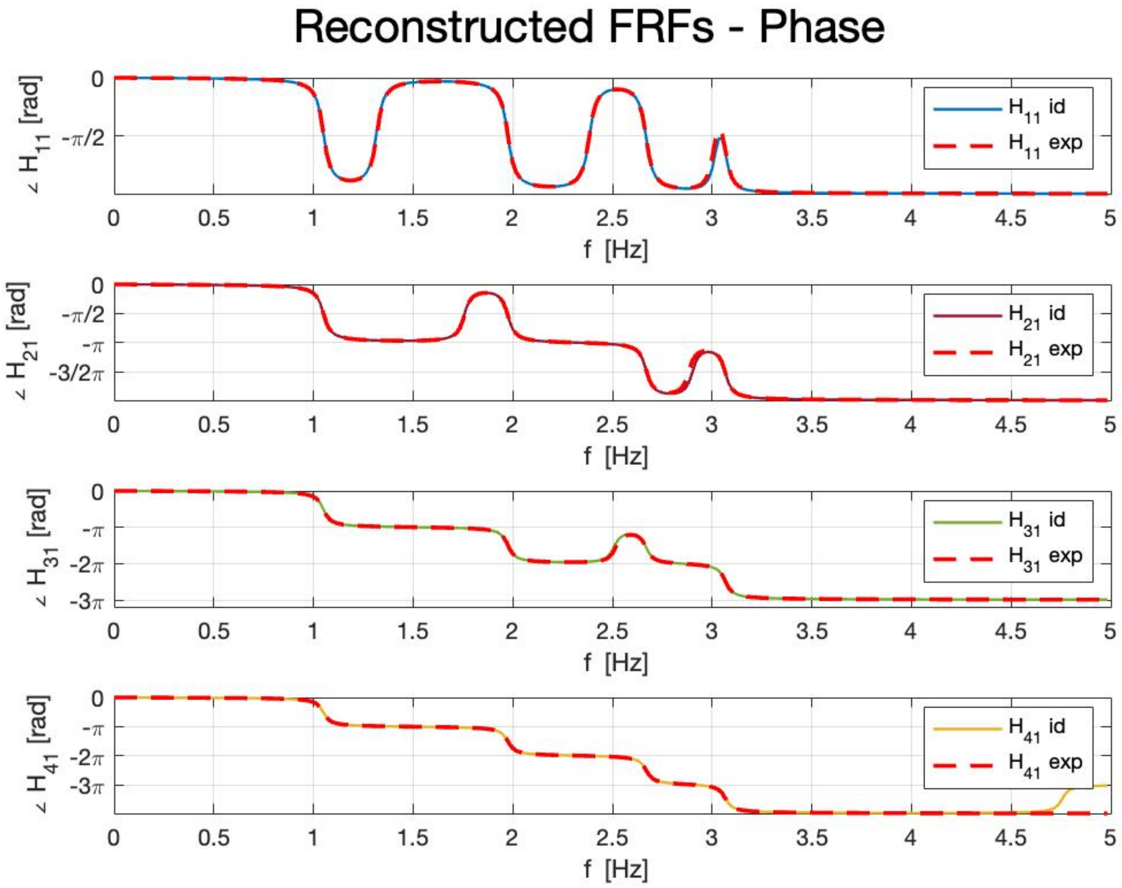

Figure 8: Phase comparison between original and reconstructed FRFs

Co-Located FRF of the Second Point of Measurement

The co-located FRF of x_2 is simply the element of the frequency response matrix $[H]$ at the second row and second column: H_{22}.

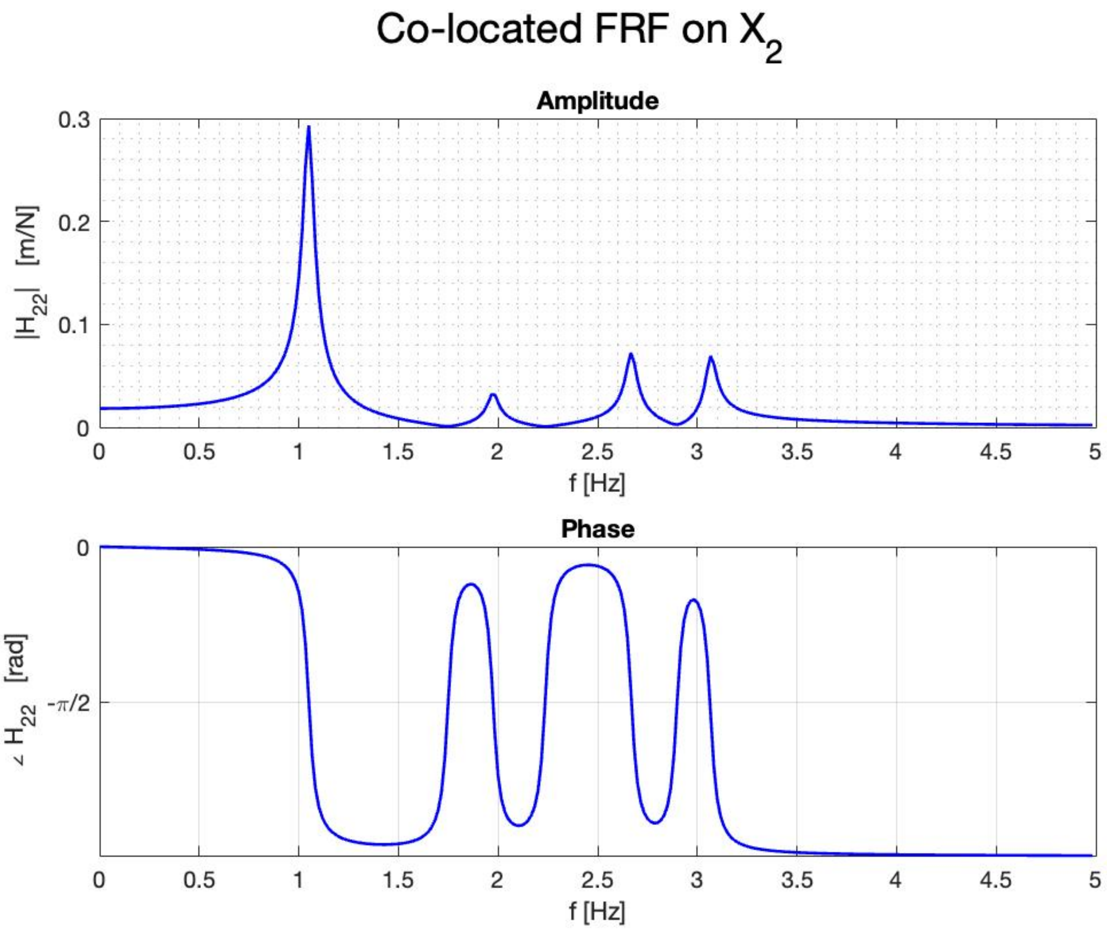

Figure 9: Co-Located FRF

Mass, Damping an Stiffness matrices

The mass, damping and stiffness matrices with respect to the point of measurements can be evaluated through the modal matrices as follows:

$$[M] = \left[[\phi]^T\right]^{-1} [M_q] \, [\phi]^{-1}$$

$$[C] = \left[[\phi]^T\right]^{-1} [C_q] \, [\phi]^{-1}$$

$$[K] = \left[[\phi]^T\right]^{-1} [K_q] \, [\phi]^{-1}$$

The following figure shows the obtained results.

Mass matrix

x_1	x_2	x_3	x_4
3.9584	-0.5267	2.2389	-0.6591
-0.5267	6.2021	-1.1392	2.5579
2.2638	-1.1392	6.4744	-1.7666
-0.6591	2.5579	-1.7666	4.7802

Damping matrix

x_1	x_2	x_3	x_4
1.3222	-0.1744	0.7419	-0.2123
-0.1744	2.0657	-0.3712	0.8446
0.7419	-0.3712	2.1528	-0.5838
-0.2123	0.8446	-0.5838	1.5936

Stiffness matrix

1.0e+03*

x_1	x_2	x_3	x_4
0.8455	-0.7256	0.5660	-0.3873
-0.7256	1.4064	-1.1035	0.7532
0.5660	-1.1035	1.5863	-1.0866
-0.3873	0.7532	-1.0866	1.3709