

# A Posterior Distribution for Anti-spam Bayesian Statistical Model

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**Abstract**—This paper deals with Bayesian models applied to anti-spam. In our previous work involved in mathematically modeling the behavior of spam, we worked on defining the priors distributions that summarize the behavior of spam. In this paper, taking into account certain priors distributions and we define the posterior distributions that summarize the probability that a received message is spam. These laws a priori and a posteriori certainly help improve the spam detection and increase the reliability decision.

**Keywords** - Beta distribution, Normal distribution, Conditional density, prior law, posterior law, Spam, Ham, Bayesian statistical model, transfer theorem, Classification.

## I. INTRODUCTION

This article aims to contribute to the study of anti-spam methods by providing a statistical Bayesian model built on two laws: prior and posterior. The negative effects of spam on information systems were widely described in [1]. Several research works and efforts are being made to reduce their scale and impact. Though not satisfied with this phenomenon, the community has nevertheless been taking into account its existence for designing network architectures and security implementation. Solutions have been developed but are still not sufficient for a total eradication. These solutions are based on several methods of treatment, and can be found in [2]. Many of these methods require a specific network infrastructure or cooperation between network elements (e.g. mail server directories) based on particular organizations that require specific costs. Anti-spam Bayesian methods operate as "standalone" and are based on statistical and probabilistic mathematical models. These methods benefit from an important research in the field of language processing. Indeed, fundamental results in this field have been obtained [3] and make it possible to apply it to anti-spam field. Paul Graham [4] was probably a pioneer in suggesting a complete solution to filter messages based on Bayesian statistical methods [5,6]. Several classification-based solutions also exist for which a summary can be found in [2]. The author proposes three main classes :

- Filtering the content amounts to distinguish the spam from the non spam (called ham). As mentioned in [2], this type of process is prone to errors, and subject to false negatives [1] and false positives [2]. The goal is to reduce the number of false positives. For this purpose

there are several filtering methods: heuristic, adaptive or Bayesian filtering, collaborative and honey pot.

- Identifying one or more fields of the message. Classical cryptographic functions may be used (digital signature and authentication). This method is effective against phishing [3] or any approach of spoofing.
- Establishing a cost for message transmission. This method reduces a significant amount of resources at spammer side. Several approaches based on costs exist.

Because of their limitations when they are used separately, administrators and users sometimes combine several of these methods.

In [7], since  $P(C = spam)$  and  $P(C = ham)$  are estimated by dividing the number of training messages of category (spam or ham) by the total number of training messages, authors consider the probability having a spam  $P(C = spam)$  as a random variable. Based on Jeffrey's law and Fisher's information they determine its law which is a  $Beta(0.5, 0.5)$ . Our work in this paper extends the result obtained in [4] focusing on the posterior law.

In this paper, we consider the probability  $p(\vec{x}/s)$  as a random variable. Using the transfer theorem, we give its law from that of  $p$ , depending on the parameter  $c$ . Graham [4] and Robinson [8] worked on messages content by assuming the probability of a message being a spam to be equal to that of a ham ( $P(spam) = P(ham) = 0.5$ ). By considering a prior law on the probability of a message being a spam, we obtained the corresponding law after observing the parameter  $c$  in a message; that is the posterior law.

In [10], some Naive Bayes (NB) classifiers are proposed : multi-variate Bernoulli NB, multinomial NB, Boolean attributes, multi-variate Gauss NB.

The classifiers are built on Bayes' theorem: if a message is represented by a vector  $\vec{x} = (x_1, \dots, x_n)$ , then the probability that the message belongs to the class  $c$  (spam or ham) is given by the formula

$$P(c/\vec{x}) = \frac{P(c).P(\vec{x}/c)}{P(\vec{x})} \quad (1)$$

As there are two classes (spam and ham) the denominator is written

$$P(\vec{x}) = P(\vec{x}/s).P(s) + P(\vec{x}/h).P(h) \quad (2)$$

In this paper, we work on Formulas (1) and (2) which contain 4 parameters :  $P(s)$ ,  $P(h) = 1 - P(s)$ ,  $P(\vec{x}/s)$  and  $P(\vec{x}/h)$ . Given the relationship between them we reduce that number to 2. We also take into account the results obtained in [4], i.e.  $P(s)$  is a random variable

The criterion for classifying a message as spam is given by

$$\frac{P(s) \cdot P(\vec{x}/s)}{P(\vec{x}/s) \cdot P(s) + P(\vec{x}/h) \cdot P(h)} > T \quad (3)$$

where  $T$  is a fixed threshold.

The criterion for classifying is adapted for each classifier:

- 1) For the multivariate Bernoulli NB distribution, the criterion becomes

$$\frac{P(s) \cdot \prod_{i=1}^m P(t_i/s)^{x_i} \cdot (1 - P(t_i/s))^{(1-x_i)}}{\sum_{c \in \{s,h\}} P(c) \cdot \prod_{i=1}^m P(t_i/c)^{x_i} \cdot (1 - P(t_i/c))^{(1-x_i)}} > T$$

where

- $x_i = 1$  if the token  $t_i$  occurs in the message, otherwise  $x_i = 0$ .
- $p(t/c) = \frac{1+M_{t,c}}{2+M_c}$  where  $M_{t,c}$  is the number of training messages of category  $c$  that contain token  $t$ , while  $M_c$  is the total number of training messages of category  $c$ .

- 2) For the multinomial NB distribution, (3) becomes

$$\frac{P(s) \cdot \prod_{i=1}^m P(t_i/s)^{x_i}}{\sum_{c \in \{s,h\}} P(c) \cdot \prod_{i=1}^m P(t_i/c)^{x_i}} > T$$

- 3) For the multivariate Gaussian NB distribution, (3) gives

$$\frac{P(s) \cdot \prod_{i=1}^m g(x_i; \mu_{i,s}, \sigma_{i,s})}{\sum_{c \in \{s,h\}} P(c) \cdot \prod_{i=1}^m g(x_i; \mu_{i,c}, \sigma_{i,c})} > T$$

where

$$g(x_i; \mu_{i,s}, \sigma_{i,s}) = \frac{1}{\sigma_{i,s} \sqrt{2\pi}} e^{-\frac{(x_i - \mu_{i,s})^2}{2\sigma_{i,s}^2}}$$

and the mean ( $\mu_{i,s}$ ) and standard deviation ( $\sigma_{i,s}$ ) of each distribution are estimated from the training data.

In those classifiers,  $P(s)$  and  $P(h)$  are typically estimated by dividing the number of training messages of category ( $s$  or  $h$ ) by the total number of training messages.

In [11], the criterion for classifying a message as a spam is

$$\frac{P(C = spam/\vec{X} = \vec{x})}{P(C = ham/\vec{X} = \vec{x})} > \lambda \quad (4)$$

where

$$P(C = c/\vec{X} = \vec{x}) = \frac{P(C = c) \prod_{i=1}^n P(X_i = x_i/C = c)}{\sum_{k \in \{spam, ham\}} P(C = k) \prod_{i=1}^n P(X_i = x_i/C = k)}$$

where  $n$  is the size of the vector  $\vec{x}$ .

For this classifier,  $P(C = spam)$  and  $P(C = ham)$  are

also estimated by dividing the number of training messages of category (spam or ham) by the total number of training messages.

In [12], authors use a similar formulae to (1) and (2) as follows

$$\begin{aligned} p(s/\vec{x}) &= \frac{P(s) \cdot p(\vec{x}/s)}{P(\vec{x}/s) \cdot P(s) + P(\vec{x}/h) \cdot P(h)} \\ &= \frac{\frac{P(\vec{x}/s)}{P(\vec{x}/h)} P(s)}{\frac{P(\vec{x}/s)}{P(\vec{x}/h)} P(s) + P(h)} \end{aligned} \quad (5)$$

In this paper, we consider only the ratio  $\frac{P(\vec{x}/s)}{P(\vec{x}/h)}$ .  $P(C = spam)$  and  $P(C = ham)$  are also estimated by dividing the number of training messages of category (spam or ham) by the total number of training messages.

In [13], the following criterion is used :

Given an input document  $d$ , its target class (spam, ham) can be found by choosing the one with the high posterior probability.

$$H(d) = \arg \max_{c_j \in C} \sum_{w \in F} \frac{P(w/c_j) P(c_j)}{\sum_{c' \in C} P(w/c') P(c')} P(w/d') CHI(w, c_j)$$

where  $F$  is a text feature vector;  $P(w/c_j)$  is the ratio of frequency of word  $w$  in  $d$  among the total number of words,  $CHI(w, c_j)$  is a  $\chi^2$  statistic of word  $w$  and class  $c_j$  which measures the dependence between word  $w$  and  $c_j$ .

$P(c_j)$  is the number of documents with class label  $c_j$  divided by the total number of documents.

In [14], a simple linear regression model is considered. This model  $Y_M = a + bR_M + \varepsilon_M$  relates the class label to the rank of the message, where  $Y_M$  is a dichotomous outcome of the  $M^{th}$  message (spam or ham),  $R_M$  is the rank of the  $M^{th}$  message and  $\varepsilon_M$  is a random error. Assuming that  $E(\varepsilon_M) = 0$ , i.e. no random error on the average, then  $E(Y_M) = a + bR_M$ . Let  $p(Y_M = spam) = \Pi_M$  be the probability that the  $M^{th}$  message is a spam. They took

$$p(Y_M = spam) = \frac{e^{(a+bR_M)}}{1 + e^{(a+bR_M)}}.$$

In [15], the maximum a posterior (MAP) class is obtained by calculating

$$e_{MAP} = \arg \max_{e_i \in E} P(e_i) \prod p(k_j/e_i)$$

where  $k_j$  is the word found in the  $j^{th}$  position in the unseen query.  $e_i$  is the tutorial example/page.

$$P(e_i) = \frac{\text{Number of queries in } Q \text{ with } e_i}{\text{Number of queries in } Q}$$

where  $Q$  is the queries set.

In [16,17,18], the degree of confidence of  $\vec{x}$  being a spam is introduced:

$$\begin{aligned}
W_S^{NB}(\vec{x}) &:= P(\text{spam}/\vec{x}) \\
&= \frac{p(\text{spam}) \cdot \prod_{i=1}^m P(x_i/\text{spam})}{\sum_{k \in \{\text{spam}, \text{ham}\}} P(k) \cdot \prod_{i=1}^m P(x_i/k)}
\end{aligned}$$

and  $P(C = \text{spam})$  and  $P(C = \text{ham})$  are also estimated by dividing the number of training messages of category (spam or legitime) by the total number of training messages. In [19], authors deliver a discussion about the implementation of Binomial and Poisson distribution in Bayesian spam filter, to calculate the probability of a mail being a spam, containing words that are not already stored in a database (*i.e.*, encountered by the filter for the first time). They use :

- A Binomial random variable  $R$  with parameters  $n$  and  $p$ , then the probability function of  $R$  is given as follows :

$$f(r) = P(R = r) = \binom{n}{r} p^r q^{n-r}, \quad r = 0, 1, 2, 3, \dots, n$$

where  $p + q = 1$ .

- A Poisson random variable  $R$ , with probability function

$$f(r) = P(R = r) = e^{-m} m^r / r!, \quad r = 0, 1, 2, 3, \dots, n$$

where  $m = np$ .

$p$  is the probability to have a spam word, which is assumed to be constant from trial to trial (0.4 for the Binomial distribution and 0.04 for the Poisson distribution).

In [20], author defines a local probability. For example:

$$P_{\text{local-spam}} = 0.5 + \frac{(N_{\text{spam}} - N_{\text{nonspam}})}{C_1 \times (N_{\text{spam}} + N_{\text{nonspam}} + C_2)}.$$

Each word feature generates one such local probability. These local probabilities are then used in a Bayesian chain rule to calculate an overall probability that an incoming text is a spam.

The Bayesian chain rule is

$$P(\text{in class}/\text{feature}) =$$

$$\frac{P(\text{feat}/\text{in class}) \times P(\text{in class})}{P(\text{feat}/\text{in class}) \times P(\text{in class}) + P(\text{feat}/\text{not in class}) \times P(\text{not in class})}$$

which is applied iteratively to each local chain probability feature into an overall probability for the text. Here  $p(\text{in class})$  is also estimated by dividing the number of training messages of class (spam or ham) by the total number of training messages.

In [21], they use the formula

$$\begin{aligned}
y &= \log \left[ \frac{P(c_0/\vec{x})}{P(c_1/\vec{x})} \right] \\
&= \log[P(c_0)] - \log[P(c_1)] + \sum_{i=1}^n w_i
\end{aligned}$$

where  $w_i = \log \left[ \frac{P(x_i/c_0)}{P(x_i/c_1)} \right]$  and calculate

$$P(c_0/\vec{x}) = \frac{1}{1 + 2^{-y}}.$$

$P(C = \text{spam})$  and  $P(C = \text{ham})$  are also estimated by dividing the number of training messages of category (spam or ham) by the total number of training messages.

In [22], they introduce a score notion as follows. If an e-mail is represented by  $\vec{x} = (x_1, \dots, x_n)$  then

$$\begin{aligned}
\text{score}(\vec{x}) &= \log[P(\text{spam})] + \sum_k \log[P(x_k/\text{spam})] \\
&\quad - \log[P(\text{leg})] - \sum_k \log[P(x_k/\text{leg})].
\end{aligned}$$

Therefore, if  $\text{score}(\vec{x}) > 0$ , the e-mail will be a spam, and legitime otherwise.

In this work, we complete the work done in [4] by proposing a posterior law for anti-spam using Formula (1). So, we give the posterior law which completes the statistical model started in [21] modeling the evolution of spam.

This work differs from the previous works listed above; it does not assume that the probability of having a spam is equal to 0.5. It's the following of work done in [23] where they consider that spam law varies following beta law, in the light of this consideration it gives the nature of the posterior law.

We introduce a parameter  $c$  which is the ratio  $\frac{P(\vec{x}/s)}{P(\vec{x}/h)}$ , and we control the probability  $P(c/\vec{x})$  depending on the parameter  $c$ .

To ensure the accuracy of this work, we repeat the same work with the normal law as *a priori*, find the posterior law and at the end, we compare the results between them and we give some conclusions.

## II. TRANSFER THEOREM[24]

In this section we present the transfer theorem which will be used in our method.

**THEOREM:**

Let  $X$  be a random variable taking its values in  $R^d$ ,  $f$  its probability density, and let  $\varphi$  a Borelian function from  $R^d$  to  $R$  (*i.e.* measurable function).

Then if one of the terms of the equality  $E[\varphi(X)] = \int_{R^d} \varphi(u) f(u) du$  has a meaning, then the other also does, and the equality occurs.

Conversely, if the above equality takes place for any bounded Borelian function  $\varphi$  then  $f$  is a probability density of  $X$ .

**EXAMPLE:**

If  $X$  is a random variable with standard normal distribution, *i.e.* its density is  $f_X = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$  for all real  $x$ .

Using the transfer theorem we can find the law of the random variable  $X^2$ .

Let  $\varphi$  a bounded and continuous function. Then, by a change

of variable

$$\begin{aligned} E[\varphi(X^2)] &= \int_{-\infty}^{+\infty} \varphi(x^2) \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= \int_0^{+\infty} \varphi(u) \left[ \frac{1}{\sqrt{2\pi}} u^{-\frac{1}{2}} e^{-\frac{u}{2}} \right] du. \end{aligned}$$

Thus, the density of  $X^2$  is :

$$\begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{\sqrt{2\pi}} u^{-\frac{1}{2}} e^{-\frac{u}{2}} & \text{if } x \geq 0 \end{cases}$$

#### A. Calculation of the posterior law for beta law

Using Formulae (1) (with  $c = s$ ) and (2) we get

$$\begin{aligned} P(s/\vec{x}) &= \frac{1}{1 + \frac{P(\vec{x}/h)}{P(\vec{x}/s)} \times \frac{P(h)}{P(s)}} \\ &= \frac{1}{1 + \frac{P(\vec{x}/h)}{P(\vec{x}/s)} \times \left( \frac{1}{P(s)} - 1 \right)} \end{aligned}$$

by  $P(h) = 1 - P(s)$ .

From [4],  $P(s)$  is a random variable with beta(0.5;0.5) law and noted  $X$ .

The ratio  $\frac{P(\vec{x}/s)}{P(\vec{x}/h)}$  is noted  $c$ .

Thus, the aim is to seek the law of probability of the random variable

$$Y = \frac{1}{1 + \frac{1}{c} \times \left( \frac{1}{X} - 1 \right)} = \frac{X}{\left( 1 - \frac{1}{c} \right) X + \frac{1}{c}} \quad (6)$$

where  $X$  is a beta-distributed random variable with density  $f_X$  and  $c$  is a real parameter.

In the following, we use the transfer theorem. Let  $\varphi$  a measurable function, then

$$\begin{aligned} E[\varphi(Y)] &= \int_{-\infty}^{+\infty} \varphi \left[ \frac{x}{\left( 1 - \frac{1}{c} \right) x + \frac{1}{c}} \right] f_X(x) dx \\ &= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 \left[ \frac{x}{\left( 1 - \frac{1}{c} \right) x + \frac{1}{c}} \right] x^{\alpha-1} (1-x)^{\beta-1} dx \end{aligned} \quad (7)$$

Using the change of variable

$$y = \frac{x}{x \left( 1 - \frac{1}{c} \right) + \frac{1}{c}}, \quad (8)$$

we get

$$E[\varphi(Y)] = \int_{-\infty}^{+\infty} \varphi(y) f_Y(y) dy$$

where

$$f_Y(y) = \frac{1}{c} \times \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\left( \frac{1}{c} y \right)^{\alpha-1} (1-y)^{\beta-1}}{\left[ 1 - y \left( 1 - \frac{1}{c} \right) \right]^{\alpha+\beta}} \cdot 1_{[0,1]}(y)$$

is the probability density function of  $Y$ , according to the transfer theorem.

For  $\alpha = \beta = 0.5$ , we have  $\Gamma(\alpha) = \Gamma(\beta) = \sqrt{\pi}$  and

$$\Gamma(\alpha + \beta) = \Gamma(1) = 1.$$

The posterior density of  $Y$  is then

$$f_Y(y) = -\frac{\sqrt{c}}{\pi} \cdot \frac{1}{\sqrt{y(1-y)} \cdot [(c-1)y - c]} \cdot 1_{[0,1]}(y)$$

and its distribution function is

$$\begin{aligned} F_Y(y) &= \int_{-\infty}^y -\frac{\sqrt{c}}{\pi} \cdot \frac{1}{\sqrt{t(1-t)} \cdot [(c-1)t - c]} \cdot 1_{[0,1]}(t) dt \\ &= \begin{cases} 0 & \text{if } y \leq 0, \\ \int_0^y -\frac{\sqrt{c}}{\pi} \cdot \frac{1}{\sqrt{t(1-t)} \cdot [(c-1)t - c]} dt & \text{if } 0 < y < 1, \\ 1 & \text{if } y \geq 1. \end{cases} \end{aligned}$$

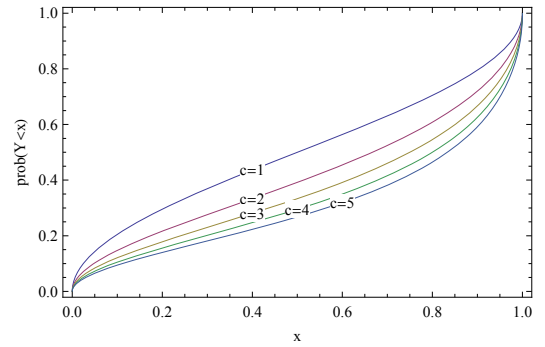


Fig. 1. Distribution functions for posterior law

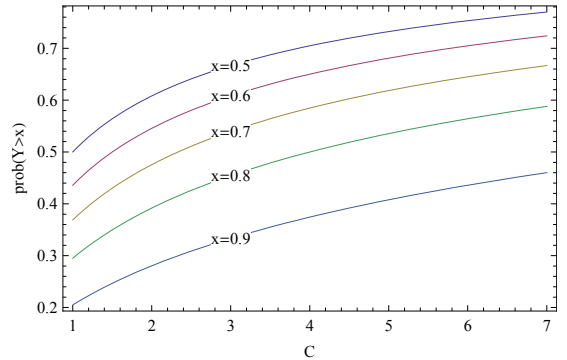


Fig. 2. Probability that  $Y$  is greater or equal than  $y$  as a function of  $c$ , for Beta distribution

#### B. Calculation of the posterior law for uniforme law

We consider the case where the random variable  $X$  follows uniforme on  $[0;1]$  distribution. The density function is then:

$$f_X(x) = 1_{[0,1]}(x)$$

In order to get the law of the random variable  $Y$  defined by (6) we use the same steps as in (7) and (8) and we get

$$E[\varphi(Y)] = \int_{-\infty}^{+\infty} \varphi(y) \cdot f_Y(y) dy$$

where

$$f_Y(y) = \frac{1}{c \left[ y \left( 1 - \frac{1}{c} \right) - 1 \right]^2} \cdot 1_{[0,1]}(y)$$

is the density of  $Y$ , by the transfer theorem.  
and its distribution function is

$$F_Y(y) = \int_{-\infty}^y \frac{1}{c \left[ t \left( 1 - \frac{1}{c} \right) - 1 \right]^2} \cdot 1_{[0,1]}(t) dt$$

$$= \begin{cases} 0 & \text{if } y \leq 0, \\ \frac{-y}{c((c-1)y-c)} & \text{if } 0 < y < 1, \\ 1 & \text{if } y \geq 1. \end{cases}$$

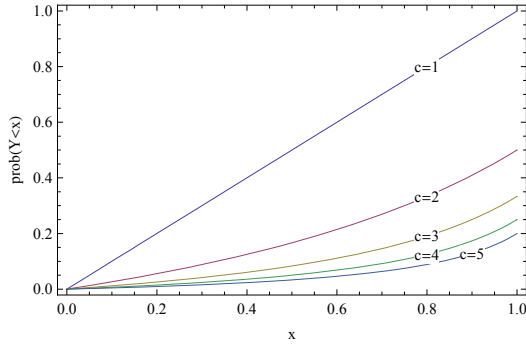


Fig. 3. Distribution functions for posterior law

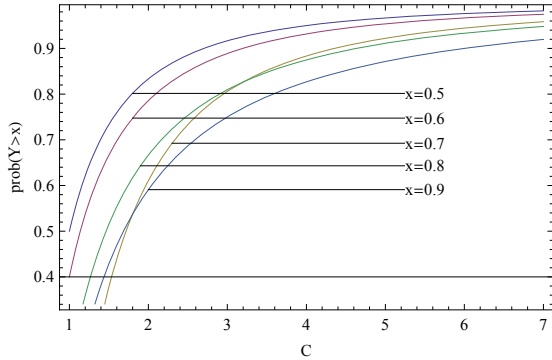


Fig. 4. Probability that  $Y$  is greater or equal than  $x$  as a function of  $c$ , for uniform distribution

### III. RESULTS FOR THE POSTERIOR LAW OF BETA LAW

In this section, we provide the values of the probability  $P(Y \geq y)$  for some values of  $c$ .

Figure 2 shows the curves of  $P(Y \geq y)$ , which is the probability of the spam's acceptance area, as a function of parameter  $c$ .

In the following we try to be more specific in giving the value of the probability that  $Y$  belongs to an interval  $[a, b]$ , the probability  $P(a \leq Y \leq b)$ , for some values of  $c$ .

$c$	$P(Y \geq 0.5)$	$P(Y \geq 0.6)$	$P(Y \geq 0.7)$	$P(Y \geq 0.8)$
0.5	0.391827	0.333333	0.275999	0.216347
1	0.5	0.435906	0.36901	0.295167
1.5	0.564094	0.5	0.430245	0.349802
2	0.608173	0.545629	0.47549	0.391827
2.5	0.640983	0.580431	0.510978	0.425876
3	0.666667	0.608173	0.539893	0.454371

TABLE I  
PROBABILITY THAT  $Y$  IS GREATER THAN  $y$

$c$	[0.5-0.6]	[0.6-0.7]	[0.7-0.8]	[0.8-0.9]
0.5	0.0625445	0.0701385	0.0836639	0.111389
1	0.0640942	0.0668957	0.0738429	0.0903345
1.5	0.0615717	0.0618285	0.0657539	0.0775833
2	0.0584932	0.0573345	0.0596519	0.0689838
2.5	0.0555534	0.0535583	0.0596519	0.0627079
3	0.0528955	0.0503843	0.0511411	0.1594

TABLE II  
PROBABILITY THAT  $Y$  IS IN  $[a, b]$  FOR BETA DISTRIBUTION

Table II shows the probability of the acceptance area of the spam in interval  $[0.5 - 0.6]$ ,  $[0.6 - 0.7]$ ,  $[0.7 - 0.8]$  and  $[0.8 - 0.9]$ .

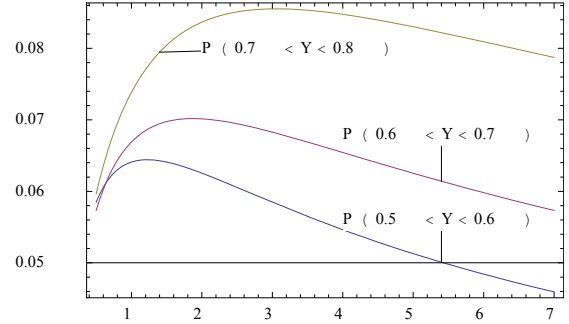


Fig. 5. Probability that  $Y$  is in  $[a, b]$  as function of  $c$ , for Beta distribution

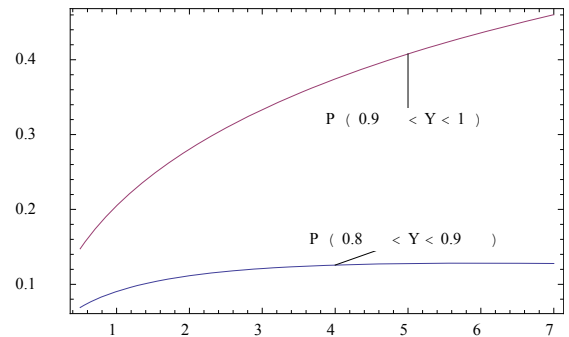


Fig. 6. Probability that  $Y$  is in  $[0.8, 0.9]$  or  $[0.9, 1]$  for Beta distribution

### IV. CONCLUSION AND FUTURE WORK

In this paper, we have shown using the transfer theorem that the law which summarize the initial information (beta

law) built on the Fisher information is well suited to define a statistical model containing a prior law and posterior law which gives better results than the calculations based only on static case.

For future work, we intend to study a new method to detect spam considering the occurrence number of some words appearing in a message. Other prior distributions will be tested against the beta law.

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