Alberto Franzin, Fabio Palese

Sistemi Intelligenti

January 15th, 2013

Introduction

Introduction

Bayesian networks
Definition
Naive Bayes
Naive Bayes for spam classification

SpamBayes Implementation

Tests

Results and Conclusions

Bayes rule:

Introduction

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

▶ P(A|B) is the *a posteriori* probability of event *A*, knowing the event *B* has already occurred.

Bayes rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

- ▶ P(A|B) is the *a posteriori* probability of event *A*, knowing the event *B* has already occurred.
- ▶ P(B|A) is the *likelihood*

Bayes rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

- ▶ P(A|B) is the *a posteriori* probability of event *A*, knowing the event *B* has already occurred.
- ightharpoonup P(B|A) is the *likelihood*
- ▶ P(B|A)P(A) is the *prior* probability

Bayes rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

- ightharpoonup P(A|B) is the a posteriori probability of event A, knowing the event *B* has already occurred.
- ightharpoonup P(B|A) is the *likelihood*
- ightharpoonup P(B|A)P(A) is the prior probability
- ► $P(B) = \sum_{a \in A} P(B|A = a)P(A = a)$ is the *total* probability

Bayes rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

► In other words, we can estimate the probability of an hypothesis, given that we know the consequences.

Bayes rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

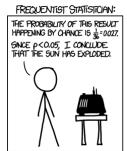
- ► In other words, we can estimate the probability of an hypothesis, given that we know the consequences.
- ▶ This has led to two different interpretations of the theorem.

Frequentists vs. Bayesians



from http://xkcd.com/1132, see also http://en.wikipedia.org/wiki/Sunrise_problem

Frequentists vs. Bayesians



from http://xkcd.com/1132, see also http://en.wikipedia.org/wiki/Sunrise_problem

The frequentist relies on the theoretical probability of the events.

Frequentists vs. Bayesians



from http://xkcd.com/1132, see also http://en.wikipedia.org/wiki/Sunrise_problem

The bayesian observes the past events occurred, and adapts the probability accordingly.

BAYESIAN NETWORKS

A Bayesian network is a way to describe causal relationships between events (J. Pearl, 1985).

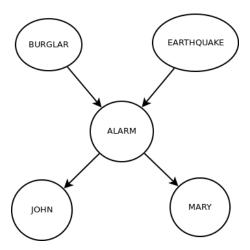
- ► Nodes = events
- ► (Directed) Edges = causal relationship

BAYESIAN NETWORKS

A Bayesian network is a way to describe causal relationships between events (J. Pearl, 1985).

- ► Nodes = events
- ► (Directed) Edges = causal relationship
- ► Two nodes are connected by an edge: the child of an arc is influenced by its ancestor in a probabilistic way

AN EXAMPLE



NAIVE BAYES

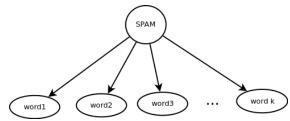
► Computing all the probabilities in a Bayesian network requires exponential time. We introduce the assumption of independence among variables.

NAIVE BAYES

- ► Computing all the probabilities in a Bayesian network requires exponential time. We introduce the assumption of independence among variables.
- ► It is called *naive*, since it's often unrealistic, but it yields good results.

NAIVE BAYES

- ► Computing all the probabilities in a Bayesian network requires exponential time. We introduce the assumption of independence among variables.
- ► It is called *naive*, since it's often unrealistic, but it yields good results.
- ► In spam classification:



NAIVE BAYES FOR SPAM CLASSIFICATION

We want to build a classifier that distinguishes good mails from undesired mails:

- ▶ good mails: ham
- ▶ undesired mails: *spam*

NAIVE BAYES FOR SPAM CLASSIFICATION

Formulas:

► For each word:

$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails}}{\text{\# total occurrences of word}}$$

$$P_{spam|word} = \frac{P_{word|spam}P_{spam}}{P_{word}}$$

NAIVE BAYES FOR SPAM CLASSIFICATION

Formulas:

► For each word:

$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails}}{\text{\# total occurrences of word}}$$

$$P_{spam|word} = \frac{P_{word|spam}P_{spam}}{P_{word}}$$

► Final for spam is:

$$P_{spam} = \prod_{words \in mail} P_{spam|word}$$

NAIVE BAYES FOR SPAM CLASSIFICATION

Formulas:

► For each word:

$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails}}{\text{\# total occurrences of word}}$$

$$P_{spam|word} = \frac{P_{word|spam}P_{spam}}{P_{word}}$$

► Final for spam is:

$$P_{spam} = \prod_{words \in mail} P_{spam|word}$$

▶ same for ham

NAIVE BAYES FOR SPAM CLASSIFICATION

Formulas:

► For each word:

$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails}}{\text{\# total occurrences of word}}$$

$$P_{spam|word} = \frac{P_{word|spam}P_{spam}}{P_{word}}$$

► Final for spam is:

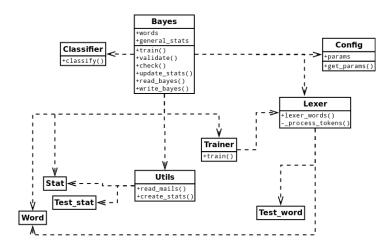
$$P_{spam} = \prod_{words \in mail} P_{spam|word}$$

- same for ham
- ► Outcome is the class that maximizes the probability of belonging to that class.

SPAMBAYES

- ► SpamBayes: applying Naive Bayes to spam classification
- ► Python with Ply and BeautifulSoup
- ► dataset: SpamAssassin archive
- ► Code and documentation available at http: //code.google.com/p/sist-int-2012project/

SPAMBAYES



► Smoothing:

$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails} + k}{\text{\# total occurrences of the word} + |C| \times k}$$

SpamBayes

NOTES ON IMPLEMENTATION

► Smoothing:

$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails} + k}{\text{\# total occurrences of the word} + |C| \times k}$$

► Calculations can be simplified: some words bring little contribution to the mail status

NOTES ON IMPLEMENTATION

► Smoothing:

$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails} + k}{\text{\# total occurrences of the word} + |C| \times k}$$

- ► Calculations can be simplified: some words bring little contribution to the mail status
- ► Several mail features detected

NOTES ON IMPLEMENTATION

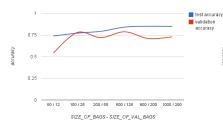
► Smoothing:

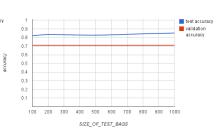
$$P_{word|spam} = \frac{\text{\# occurrences of word in spam mails} + k}{\text{\# total occurrences of the word} + |C| \times k}$$

- ► Calculations can be simplified: some words bring little contribution to the mail status
- ► Several mail features detected
- ► Several parameters to be tuned: we describe the more relevant ones

Introduction

Size of training/validation/test sets:

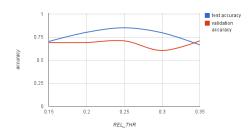




Tests

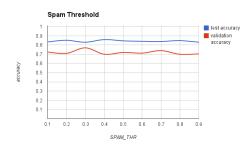
PARAMETERS

Relevance threshold:



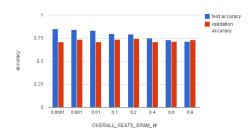
PARAMETERS

"Spamicity" threshold:



PARAMETERS

Feature statistic threshold:



► About the dataset:

- ► About the dataset:
 - ► General features

- ► About the dataset:
 - ► General features
 - ► Single words

- ► About the dataset:
 - ► General features
 - ► Single words
- ► About the classification:

- ► About the dataset:
 - ► General features
 - ► Single words
- ► About the classification:
 - ► Accuracy

- ► About the dataset:
 - ▶ General features
 - ► Single words
- ► About the classification:
 - ► Accuracy
 - ► How to improve?