Università degli Studi di Napoli “Federico II”

SCUOLA POLITECNICA E DELLE SCIENZE DI BASE
DIPARTIMENTO DI INGEGNERIA ELETTRICA E TECNOLOGIE DELL’INFORMAZIONE

FIELD AND SERVICE ROBOTICS (FSR)
HOMEWORK n. 1

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CAPITOLO 1. ATLAS ROBOT

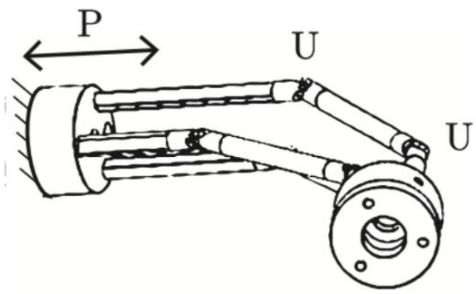
a) **“While standing, ATLAS is fully actuated ” TRUE**
Assimilating the ATLAS robot to the case of a human being, we can consider this statement to be true. When the robot is standing and in contact with the ground, it is fully actuated because, in the state x=(q,qdot), the dynamics function f is suriective: i.e. any desired instantaneous acceleration can be achieved at that precise moment, since (under the assumption that actuators can generate un-limited torques) there are no constraints limiting the possibility of control. The contact with the ground provides the necessary reactions to balance the system and generate external forces/torques.

b) **“While doing backflip, ATLAS is fully actuated.” FALSE**
This statement is false. At the moment ATLAS is in flight, its centre of mass follows a parabolic trajectory determined by the initial conditions of velocity and position at the moment of detachment from the ground. Since it is unable to generate force or torque with respect to the environment (unless we consider interactions with the air, which we neglect), the robot cannot modify the global motion of its centre of mass. It can control the relative configuration of its parts by means of internal actuators, adjusting its body attitude, but it cannot change its trajectory. Similarly, a jumping human being can move his limbs to change orientation, but cannot change his overall motion without external interaction. This makes ATLAS underactuated in flight, since not all of its degrees of freedom are instantly controllable.

CAPITOLO 2. NUMBER OF THE DEGREES OF FREEDOM

7 DOFs

$$\text{DoFs} = m(N-1-J) + \sum_{i=1}^{J} f_i$$



Where:

- m=6 (3D)
- n: number of links (including the ground link) =1(ground/base)+1(end_effector)+6links =8
- J=6 universal joints (U) + 3 prismatic joint = 9
- Sum(fi)= 1x3 prismatico + 2x6 universale = 15

Grübler's formula provides a lower bound, DoFs=6*(8-1-9) +15=3

Comparing the result with my intuition:

The resulting formula gives a value of 3 degrees of freedom, but this only represents a lower bound and does not take into account any redundant constraints in the system, which could lead to a different result.

Let us consider a single arm that has a prismatic joint and two universal joints. If we analyse this arm independently, the position of its end effector can be determined by 5 degrees of freedom.

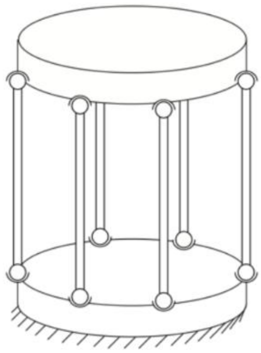
Now let us assume that we freeze the end effector and consider the other 2 arms. They will have the basic link and the end effector constrained, we only need to analyse the internal movements of the arm structure. The arm will only be free to move its own prismatic joint without violating the constraints. In this way we can calculate the degrees of freedom of the system and its topology

3 prismatic joints $I^1 x I^1 x I^1 = I^3$

2 universal joints $T^2 \; x \; T^2$

Topology: $T^2 \; x \; T^2 \; x \; I^3$

9 DOFs



Consider a system made up of:

- 12 spherical joint: provides 3 degrees of freedom (DoF) of rotation

$$\text{DoFs} = m(N - 1 - J) + \sum_{i=1}^{J} f_i$$

Where:

- m=6 (3D)
- n: number of links (including the ground link) =1(ground/base)+1(plate)+6links =8
- J=12 spherical joint = 12
- Sum(fi)= 3x12 spherical joint = 32

Grübler's formula provides a lower bound, DoFs=6(8-1-12)+36=6

Comparing the result with my intuition:

Initially, I expected the system to have 3 degrees of freedom (DoF) related to the position of the upper plate. However, after applying Grübler's formula, which provides a lower bound for the degrees of freedom of an articulated mechanism, I obtained a result of 6. This value prompted me to reflect more deeply on the system's behavior.

I then considered the possibility of modifying the orientation of each of the six supporting links independently, without necessarily moving the other links. By analyzing this configuration, I realized that Grübler's formula indeed provides a lower bound, but it does not always coincide with the actual number of degrees of freedom of the system.

In this specific case, the total number of degrees of freedom is 9, distributed as follows:

- 6 degrees of freedom to describe the orientation of the six supporting links around their respective axes; $S^1 x\, S^1 x\, S^1 x\, S^1 x\, S^1 x\, S^1 = T^6$
- 2 degrees of freedom to describe the movement of the centre of mass of the upper plate on a spherical cap. S^2
- 1 degree of freedom to describe the orientation of the top plate around the z-axis S^1

Topology: $T^6 x\, S^2 x\, S^1$

<u>CAPITOLO 3.</u> SENTENCES REGARDING UNDERACTUATION

a) **A car with inputs the steering angle and the throttle is underactuated. TRUE**
A normal car in a plane has at least three DoFs given by the planar rigid body chassis, but usually only has 2 inputs (steering and accelerator). This means that it is not possible to control the acceleration of generalised coordinates independently. I cannot achieve an instant movement of the machine in a lateral direction.

b) **The KUKA youBot system on the slides is fully actuated.TRUE**
The KUKA youBot is a mobile manipulator robot with an omnidirectional base (equipped with 4 Mecanum wheels, each with its own motor) and a robotic arm with actuators on each joint. The base, being omnidirectional, can control all planar degrees of freedom (two translations and one rotation in plane), while the arm is also fully actuated. Overall, each degree of freedom in the system is associated with an actuator, obviously neglecting the degrees of freedom of the wheels.

c) **The hexarotor system with co-planar propellers is fully actuated.FALSE**
A hexarotor with all co-planar propellers cannot generate lateral thrust without tilting, nor can it independently control translations and rotations over all 6 degrees of freedom (3 translations + 3 rotations). It must tilt to move horizontally, and this constrains translational and rotational motion. In other words, it is underactuated

d) **The KUKA iiwa 7-DOF robot is redundant and it cannot be underactuated because we know that all redundant systems are not. FALSE**
It is true that the 7-joint KUKA iiwa is a redundant manipulator, and is also fully actuated (each joint has its own motor). However, the argument ‘if it is redundant, it cannot be under-actuated’ is not universally correct: in general, a robot can be redundant and still be under-actuated, such as a drone with 8 parallel-axis propellers

CAPITOLO 4. DISTRIBUTIONS AND ANNIHILATOR

a. $\Delta_1 = \left\{ \begin{bmatrix} -3x_2 \\ 1 \\ -1 \end{bmatrix} \right\}, \boldsymbol{U} \in \mathbb{R}^3$

$\Delta 1$ is generated by a single vector field, therefore, it is a 1-dimensional distribution.
A 1-dimensional distribution is always involutive.
Annihilator
$\dim(\Delta^{\perp}) = 3 - 1 = 2$

$$(w_1 w_2 w_3) * \begin{pmatrix} -3x_2 \\ 1 \\ -1 \end{pmatrix} = 0 => \{-w_1 3x_2 + w_2 = w_3$$

By choosing $w_1 = 1 \quad w_2 = 0 \quad and\ w_1 = 0 \quad w_2 = 1$

$$\Delta^{\perp} = span\{(1,0,-3x_2),(0,1,1)\}$$

b. $\Delta_2 = \left\{ \begin{bmatrix} -1 \\ 0 \\ x_3 \end{bmatrix}, \begin{bmatrix} x_2 \\ -\alpha \\ x_1 \end{bmatrix} \right\}, \boldsymbol{U} \in \mathbb{R}^3$ with α the last digit of your matriculation number.

my matriculation number is P38000261, so $\Delta_2 = span\{ \begin{pmatrix} -1 \\ 0 \\ x_3 \end{pmatrix}, \begin{pmatrix} x_2 \\ -1 \\ x_1 \end{pmatrix} \}$

$$\text{Det}\begin{bmatrix} -1 & x_2 \\ 0 & -1 \end{bmatrix} = 1 \;=> \text{Dim}\ (\Delta) = 2\ \forall\ x \in U$$

$$\left[\begin{pmatrix} -1 \\ 0 \\ x_3 \end{pmatrix}, \begin{pmatrix} x_2 \\ -1 \\ x_1 \end{pmatrix} \right] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ x_3 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ -1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 - x_1 \end{bmatrix}$$

$$\text{Det}\begin{bmatrix} -1 & x_2 & 0 \\ 0 & -1 & 0 \\ x_3 & x_1 & -1 - x_1 \end{bmatrix} = -1 - x_1 \;\neq 0 \;=> \; is\ not\ involutive\ unless\ x_1 = -1$$

Annihilator
$\dim(\Delta^{\perp}) = 3 - 2 = 1$

$$(w_1 w_2 w_3) * \begin{pmatrix} -1 & x_2 \\ 0 & -1 \\ x_3 & x_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} w_1 = x_3 w_3 \\ w_2 = w_3 x_1 + x_3 w_3 x_2 \end{cases}$$

By choosing $w_3 = 1$

$$\Delta^{\perp} = spam\{(x_3, x_1 + x_3 x_2, 1)\}$$

c. $\Delta_3 = \left\{ \begin{bmatrix} 2x_3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2x_2 \\ x_1 \\ -1 \end{bmatrix} \right\}, U \in \mathbb{R}^3$

$$\text{Det}\begin{bmatrix} 1 & x_1 \\ 0 & -1 \end{bmatrix} = -1 \;=>\text{Dim}\,(\Delta) = 2\;\forall\, x \in U$$

$$\left[\begin{pmatrix} 2x_3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2x_2 \\ x_1 \\ -1 \end{pmatrix}\right] = \begin{bmatrix} 0 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2x_3 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2x_2 \\ x_1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2+2 \\ 2x_3 \\ 0 \end{bmatrix}$$

$$\text{Det}\begin{bmatrix} 2x_3 & -2x_2 & 0 \\ 1 & x_1 & 2x_3 \\ 0 & -1 & 0 \end{bmatrix} = \;4{x_3}^2 \neq 0 \;=>\; is\ not\ involutive\ unless\ x_3 = 0$$

Annihilator
$\dim(\Delta^{\perp}) = 3 - 2 = 1$

$$(w_1 w_2 w_3) * \begin{pmatrix} 2x_3 & -2x_2 \\ 1 & x_1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} w_2 = -2x_3 w_1 \\ w_3 = -2x_3 w_1 x_1 - \;2\, w_1 x_2 \end{cases}$$

By choosing $w_1 = 1$

$$\Delta^{\perp} = span\{(1, -2x_3, -2x_3 x_1 - \;2x_2)\}$$

<u>CAPITOLO 5.</u> PFAFFIAN CONSTANT CONSTRAINTS

Suppose to have a mechanical system subject to kinematic constraints in a Pfaffian form A$\dot{q} = 0$ with A$\in R^{kxn}$ with A constant and k > 0

$\dot{q} = G(q)u$ where G$\in R^{nxm}$ is the null of the matrix A

If the matrix A is constant, it means that its coefficients do not depend on q and therefore the imposed linear relationships between the variables also remain fixed. This also implies that the kernel ker (A)=G does not depend on q.
The holonomy or nonholonomy can be discriminated through $dim(\Delta_A(q))$

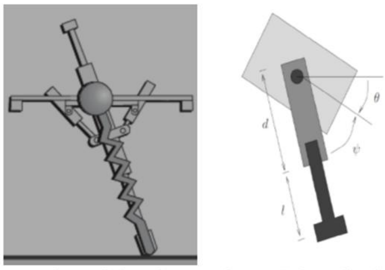
$$\Delta_A(q) = span\{(g1, g2, \dots, gm, [g1, g2], \dots, [gm-1, gm])\}$$

Lie derivatives are null since g_i does not depend on q (all jacobian are null) this implies that:

$$dim(\Delta_A(q)) = dim(G) = m = v$$

The system is completely holonomic and has holonomic constraints only

CAPITOLO 6. RAIBERT'S HOOPER ROBOT



Consider the Raibert's hooper robot of the picture above. It has the following kinematic constraint in the Pfaffian form $(I + m(l + d)^2)\dot{\theta} + m(l + d)^2\dot{\psi} = 0$, with $q = [\theta \quad \psi \quad l]^T$, with I the moment of inertia of the body and m the leg mass concentrated at the foot. Compute a kinematic model of such a robot and show whether this system is holonomic or not. *[Hint: Use Matlab symbolic toolbox and the null command to ease your work. Do not care about the physical meaning of the kinematic inputs.]*

Steps to follow

1) Write the matrix A
2) Find the null of the matrix A (G)
3) The holonomy or nonholonomy can be discriminated through $dim(\Delta_A(q))$

Command Window

```
matrix A
[I + m*(d + l)^2, m*(d + l)^2, 0]

null space of A   G(q):
[-(m*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I), 0]
[                                           1, 0]
[                                           0, 1]

input u:
u1
u2

qdot=G(q)*u
-(m*u1*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I)
                                             u1
                                             u2

F=span(g1,g2,[g1,g2])
[-(m*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I), 0, -(m*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I)^2]
[                                           1, 0,                                              0]
[                                           0, 1,                                              0]

rank(F)=dim(deltaA(q))=v=
     3

qdot=G(q)u is controllable
The system is subject to nonholonomic constraints only
dim(deltaA(q))=3=v=n
A(q)*qdot=0 is not integrable, and it is a set of nonholonomic constraints in the Pfaffian form
```

The F matrix always has full rank if we analyse the system taking into account the physical meaning of variables and parameters in the denominator.