Ejercicios de Recurrencia

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Lógica y Métodos Discretos
1º Grado Ingeniería Informática

RECURRENCIAS - ALBERTO LLAMAS GONZÁLEZ

Ejercicio 1.7. Obtén una recurrencia lineal homogénea para cada una de las sucesiones siguientes definidas $\forall n \geq 0$:

① $x_n = 4n+1$

$$4n + 1 = 4n \cdot 1^n + 1 \cdot 1^n \quad \left\{ \begin{array}{l} 4n \text{ es un polinomio de grado } 1 \\ 1 \text{ es un polinomio de grado } 0 \end{array} \right.$$

$$p(x) = (x-1)^2 \cdot (x-1) = (x-1)^3 = x^3 - 3x^2 + 3x - 1$$

Polinomio característico

Luego la recurrencia Lineal Homogénea es:

$$\underline{x_n - 3x_{n-1} + 3x_{n-2} - x_{n-3} = 0}$$

② $Y_n = 2^n + n$

$$2^n + n = 1 \cdot 2^n + 1^n \cdot n \quad \left\{ \begin{array}{l} 1 \text{ es un polinomio de grado } 0 \\ n \text{ es un polinomio de grado } 1 \end{array} \right.$$

$$p(x) = (x-2)^1 \cdot (x-1)^2 = (x-2)(x^2 - 2x + 1) = x^3 - 2x^2 + x - 2x^2 + 4x - 2 =$$

$$= x^3 - 4x^2 + 5x - 2$$

Recurrencia Lineal Homogénea:

$$\underline{Y_n - 4y_{n-1} + 5y_{n-2} + 2y_{n-3}}$$

③ $z_n = 2^n + 3^n(n+1)$

$2^n + 3^n(n+1) = 2^n \cdot 1 + 3^n(n+1)$ $\Big\{$ 1 polinomio de grado 0; n+1 polinomio de grado 1

$p(x) = (x-2)(x-3)^2 = (x-2)(x^2-6x+9) = x^3 - 6x^2 + 9x - 2x^2 + 12x - 18 =$

$= x^3 - 8x^2 + 21x - 18$

$z_n - 8z_{n-1} + 21z_{n-2} - 18z_{n-3} = 0$

Escaneado con CamScanner

(1.8) Sucesión de los números de Fibonacci

$F_0 = 0 \quad F_1 = 1 \quad y \quad F_n = F_{n-1} + F_{n-2}$ para $n \geq 2$

Demostrar:

1. $F_{n+2} > 2F_n \quad \forall n \geq 2$

$F_{n+2} = F_{n+1} + F_n = F_n + F_{n-1} + F_n = 2F_n + F_{n-1} > 2F_n$

2. $\sum_{i=0}^{n} (F_i)^2 = F_n \cdot F_{n+1} \quad \forall n \geq 0$ Demostramos por inducción

Para $n=0$

$\sum_{i=0}^{0} (F_i)^2 = F_0^2 = 0 = F_0 \cdot F_1 = 0 \cdot 1 = 0$

Suponemos cierto para n.

Veamos $n+1$

$\sum_{i=0}^{n+1} (F_i)^2 = \sum_{i=0}^{n} (F_i)^2 + (F_{n+1})^2 = F_n \cdot F_{n+1} + (F_{n+1})^2 = F_{n+1}(F_n + F_{n+1}) = F_{n+1} \cdot F_{n+2}$

3. 5 divide a F_{5n} $\forall n \geq 0$ Demostramos por inducción $F_{5n} = 5k$

Para $n=0$

$F_{5 \cdot 0} = F_0 = 0 \quad , \quad 5 \mid 0$

Suponemos cierto para n. Vemos para $n+1$

$F_{5n+5} = F_{5n+4} + F_{5n+3} = F_{5n+3} + F_{5n+2} + F_{5n+2} + F_{5n+1} =$

$= 5F_{5n+3} + 2F_{5n+2} + F_{5n+1} = F_{5n+2} + F_{5n+1} + 2F_{5n+1} + 2F_{5n} + F_{5n+1} =$

$= F_{5n+1} + F_{5n} + F_{5n+1} + 2F_{5n+1} + 2F_{5n} + F_{5n+1} = 5F_{5n+1} + 3F_{5n} \underset{H.I}{=}$

$= 5F_{5n+1} + 3\,5k = 5(F_{5n+1} + 3k)$

4. $F_{n-1} \cdot F_{n+1} = (F_n)^2 + (-1)^n \quad \forall n \geq 1$

Para $n=1$

$$F_0 \cdot F_1 = F_1 + (-1) = 0 \cdot 1 = 1 - 1 = 0$$

Suponemos cierto para n y comprobamos $n+1$

$$F_{n-1} \cdot F_{n+1} = (F_n)^2 + (-1)^n \overset{\text{Despejamos}}{\Rightarrow} (F_n)^2 = F_{n-1} \cdot F_{n+1} - (-1)^{n+1} \, *$$

$$F_n \cdot F_{n+2} = F_n(F_n + F_{n+1}) = F_n^2 + F_n F_{n+1} \overset{H.I}{=} F_{n-1} F_{n+1} + F_n F_{n+1} + (-1)^{n+1} =$$

$$= F_{n+1}(F_n + F_{n-1}) + (-1)^{n+1} = \underline{F_{n+1}^2 + (-1)^{n+1}}$$

1.8

5. $mcd(F_n, F_{n+1}) = 1 \quad \forall n \geq 0$

$F_0 = 0 \quad F_1 = 1$

$F_n = F_{n-1} + F_{n-2} \quad n \geq 2$

Para $n = 0$

$mcd(F_0, F_1) = mcd(0, 1) = 1$ cierto para $n = 0$

Supongo que es cierto para n

Para $n+1$ $\quad mcd(F_{n+1}, F_{n+2}) \overset{\text{definición recursividad}}{=} mcd(F_{n+1}, F_{n+1} + F_n)$

Algoritmo de las restas sucesivas de Euclides

$$mcd(F_{n+1}, F_n) \underset{\text{Hipótesis de inducción}}{=} 1$$

1.9 Resuelve las ecuaciones en recurrencia siguientes:

1. $x_0 = 1$, $x_1 = 1$ $x_n = 2x_{n-1} - x_{n-2}$ para $n \geq 2$

Polinomio característico:

$$p(x) = x^2 - 2x + 1 = (x-1)^2$$

$$x = \frac{2 \pm \sqrt{4-4}}{2} = 1$$

$$x_n = C_1 + C_2 n \Rightarrow \begin{cases} x_0 = 1 \Rightarrow C_1 = 1 \\ x_1 = 1 \Rightarrow 1 = 1 + C_2 \Rightarrow C_2 = 0 \end{cases}$$

$$\boxed{x_n = 1}$$

2. $x_0 = 1$ $x_1 = 2$ $x_n = 5x_{n-1} - 6x_{n-2}$ para $n \geq 2$

$$x_n - 5x_{n-1} + 6x_{n-2} = 0$$

$$p(x) = x^2 - 5x + 6$$

$$x = \frac{5 \pm \sqrt{25-24}}{2} \begin{cases} x = 3 \\ x = 2 \end{cases}$$

$$p(x) = (x-3)(x-2)$$

$$x_n = C_1 \cdot 3^n + C_2 \cdot 2^n \rightarrow \left\{ \begin{array}{l} x_0 = 1 \rightarrow 1 = C_1 + C_2 \\ x_1 = 2 \rightarrow 2 = 3C_1 + 2C_2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \begin{array}{l} -3 = -\cancel{3C_1} - 3C_2 \\ 2 = \cancel{3C_1} + 2C_2 \\ \hline +1 = +C_2 \quad , \quad C_1 = 0 \end{array}$$

$$\boxed{x_n = 2^n}$$

Escaneado con CamScanner

3. $x_0 = 1$, $x_1 = 1$ $\quad x_n = 3x_{n-1} + 4x_{n-2}$ para $n \geq 2$

$x_n - 3x_{n-1} - 4_{n-2} = 0$, $p(x) = x^2 - 3x - 4.$

$$x = \frac{3 \pm \sqrt{9+16}}{2} \quad \begin{cases} x = 4 \\ x = -1 \end{cases}$$

$p(x) = (x-4)(x+1)$

$x_n = A \cdot 4^n + B(-1)^n$

$$\left.\begin{array}{l} x_0 = 1 \Rightarrow \quad 1 = A + B \\ x_1 = 1 \Rightarrow \quad 1 = 4A - B \end{array}\right\} \Rightarrow \begin{array}{l} B = 1 - A \Rightarrow \underline{B = 3/5} \\ 4A - 1 + A = 1 \,;\ \underline{A = 2/5} \end{array}$$

$$\boxed{x_n = \tfrac{2}{5} 4^n + 3/5\,(-1)^n}$$

4. $x_0 = 1$ $\quad x_1 = 2$ $\quad x_n = -x_{n-1} + 6x_{n-2}$ para $n \geq 2$

$x_n + x_{n-1} - 6x_{n-2} = 0$

$p(x) = x^2 + x - 6$

$$x = \frac{-1 \pm \sqrt{25}}{2} \quad \begin{cases} x = 2 \\ x = -3 \end{cases}$$

$p(x) = (x-2)(x+3)$

$x_n = A \cdot 2^n + B \cdot (-3)^n$

$$\left.\begin{array}{l} A + B = 1 \\ 2 = 2A - 3B \end{array}\right\} \Rightarrow \begin{array}{l} A = 1 - B \,;\ \underline{A = 1} \\ 2 = 2 - 2B - 3B \,;\ \underline{B = 0} \end{array}$$

$$\boxed{x_n = 2^n}$$

5. $x_0 = 0 \quad x_1 = 1 \quad x_n = 2x_{n-1} - 2x_{n-2}$ para $n \geq 2$

$$x_n - 2x_{n-1} + 2x_{n-2} = 0$$

$$p(x) = x^2 - 2x + 2$$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} \begin{cases} 1+i \\ 1-i \end{cases}$$

$$p(x) = (x - (1+i))(x - (1-i))$$

$$x_n = A(1+i)^n + B(1-i)^n$$

$$0 = A + B \quad ; \quad A = -B$$

$$1 = A + Ai + B - Bi \Rightarrow 1 = A(1+i) + B(1-i) \Big\} \Rightarrow -B(1+i) + B(1-i) = 1$$

$$-Bi - Bi = 1$$

$$\underline{B = -\frac{1}{2i}} \quad ; \quad \underline{A = \frac{1}{2i}}$$

$$\boxed{x_n = \frac{1}{2i}(1+i)^n - \frac{1}{2i}(1-i)^n}$$

6. $x_0 = 5 \quad x_1 = 12 \quad x_n = 6x_{n-1} - 9x_{n-2}$ para $n \geq 2$

$$x_n - 6x_{n-1} + 9x_{n-2} = 0 \qquad p(x) = x^2 - 6x + 9$$

$$x = \frac{6 \pm \sqrt{0}}{2} = 3$$

$$p(x) = (x-3)^2$$

$$x_n = (An + B)3^n$$

$$x_0 = 5 \; ; \; \underline{5 = B}$$

$$x_1 = 12 \; ; \; 12 = (A + 5)3 \; ; \; \frac{12 - 15}{3} = A \; ; \; \underline{A = -1}$$

$$\boxed{x_n = (-n + 5)3^n}$$

7. $x_0 = 1 \quad x_1 = 1 \quad x_2 = 2 \qquad x_n = 5x_{n-1} - 8x_{n-2} + 4x_{n-3}$ para $n \geq 3$

$x_n - 5x_{n-1} + 8x_{n-2} - 4x_{n-3} = 0$

$p(x) = x^3 - 5x^2 + 8x - 4$

	1	-5	8	-4
1		1	-4	4
	1	-4	4	$\boxed{0}$
2		2	-4	
	1	-2	$\boxed{0}$	
2		2		
	1	$\boxed{0}$		

$p(x) = (x-1)(x-2)^2$

$x_n = A \cdot 1^n + (Bn + C)2^n$

$$\left.\begin{aligned} 1 &= A + C \\ 1 &= A + (B+C)2 \\ 2 &= A + (2B+C)4 \end{aligned}\right\} \left.\begin{aligned} 1 &= A + C \\ 1 &= A + 2B + 2C \\ 2 &= A + 8B + 4C \end{aligned}\right\}$$

$A = 1 - C$

$2B + 2C + 1 - C = 1 \;;\; 2B + C = 0$

$2B = -C$

$\underline{A = 2} \quad \underline{C = -1}$

$\underline{B = 1/2}$

$x_n = 2 + 2^n(\frac{1}{2}n - 1)$

8. $x_0 = 1 \quad x_1 = 1 \quad x_2 = 2, \quad x_n = x_{n-1} + x_{n-2} - x_{n-3}$ para $n \geq 3$

$x_n - x_{n-1} - x_{n-2} + x_{n-3} = 0$

$p(x) = x^3 - x^2 - x + 1$

	1	-1	-1	1
1		1	0	-1
	1	0	-1	$\boxed{0}$
-1		-1	1	
	1	-1	$\boxed{0}$	
+1		+1		
	1	$\boxed{0}$		

$p(x) = (x-1)^2(x+1)$

$x_n = (An + B) + (-1)^n C$

$$\left.\begin{aligned} 1 &= B + C \\ 1 &= (A+B) - C \\ 2 &= 2A + B + C \end{aligned}\right\}$$

$B = 1 - C$

$1 = A + 1 - C - C$

$2 = 2A + 1 - \cancel{C} + \cancel{C}$

$\underline{A = 1/2}$

$+1/2 = +2C \;;\; \underline{C = 1/4} \quad \underline{B = 3/4}$

$$\boxed{x_n = (\tfrac{1}{2}n + \tfrac{3}{4}) + \tfrac{1}{4}(-1)^n}$$

9. $x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_n = -2x_{n-1} + x_{n-2} + 2x_{n-3} \quad n \geq 3$

$$x_n + 2x_{n-1} - x_{n-2} - 2x_{n-3} = 0$$

$$p(x) = x^3 + 2x^2 - x - 2 = (x+2)(x+1)(x-1)$$

$$\begin{array}{r|rrrr}
 & 1 & 2 & -1 & -2 \\
-2 & & -2 & 0 & 2 \\
\hline
 & 1 & 0 & -1 & \boxed{0} \\
-1 & & -1 & 1 & \\
\hline
 & 1 & -1 & \boxed{0} & \\
1 & & 1 & & \\
\hline
 & 1 & \boxed{0} & &
\end{array}$$

$$x_n = A(-2)^n + B(-1)^n + C$$

$$\left.\begin{array}{l}
0 = A + B + C \\
1 = -2A - B + C \\
3 = +4A + B + C
\end{array}\right\}
\begin{array}{l}
3A = 3 \Rightarrow \underline{A = 1} \\
-B + C = 3 \\
B + C = -1
\end{array}
\Big\}
\begin{array}{l}
\underline{B = -2} \\
\underline{C = 1}
\end{array}$$

$$\boxed{x_n = (-2)^n - 2(-1)^n + 1}$$

10. $x_0 = 1 \quad x_1 = 1 \quad x_2 = 3 \quad x_n = 4x_{n-1} - 5x_{n-2} + 2x_{n-3}$ para $n \geq 3$

$$x_n - 4x_{n-1} + 5x_{n-2} - 2x_{n-3} = 0$$

$$p(x) = x^3 - 4x^2 + 5x - 2 = (x-1)^2(x-2)$$

$$\begin{array}{r|rrrr}
 & 1 & -4 & 5 & -2 \\
1 & & 1 & -3 & 2 \\
\hline
 & 1 & -3 & 2 & \boxed{0} \\
1 & & 1 & -2 & \\
\hline
 & 1 & -2 & \boxed{0} & \\
2 & & 2 & & \\
\hline
 & 1 & 0 & &
\end{array}$$

$$x_n = (An + B) + C(2)^n$$

$$\left.\begin{array}{l}
1 = B + C \\
1 = A + B + 2C \\
3 = 2A + B + 4C
\end{array}\right\}
\begin{array}{l}
A + C = 0 \\
B + C = 1 \\
B + 2C = 3
\end{array}
\quad
\begin{array}{l}
\Rightarrow A = -C \\
\underline{C = 2} \\
\underline{A = -2} \\
\underline{B = -1}
\end{array}$$

$$\boxed{x_n = -2n - 1 + 2^{n+1}}$$

11. $x_0 = 1 \quad x_1 = 3 \quad x_2 = 7 \qquad x_n = 3x_{n-1} - 3x_{n-2} + x_{n-3}$ para $n \geq 3$

$$x_n - 3x_{n-1} + 3x_{n-2} - x_{n-3} = 0$$

$$p(x) = x^3 - 3x^2 + 3x - 1 = (x-1)^3$$

	1	-3	3	-1
1		1	-2	1
	1	-2	1	$\boxed{0}$
1		1	-1	
	1	-1	$\boxed{0}$	
1		1		
	1	$\boxed{0}$		

$x_n = (An^2 + Bn + C)$

$1 = C$

$3 = A + B + C$

$7 = 4A + 2B + C$

$\left.\begin{array}{l} 2B = A + B \\ 6 = 4A + 2B \end{array}\right\} \begin{array}{l} A = 1 \\ B = 1 \end{array}$

$$\boxed{x_n = n^2 + n + 1}$$

12 $x_0 = 0 \quad x_n = 2x_{n-1} + 1 \qquad n \geq 1$ (Torres de Hanoi)

$x_n - 2x_{n-1} = 1 \qquad 1 = b^n p(n) \qquad b = 1 \quad p(n) = 1$

Polinomio característico de la recurrencia homogénea que contiene las soluciones de nuestra ecuación

$p(x) = (x-2)(x-1) \qquad x_n = A \cdot 2^n + B$ (Solución general)

~~p~~polinomio homogénea — polinomio no homogénea

Extendemos las condiciones iniciales

$x_1 = 1 \quad x_0 = 0$

$\left.\begin{array}{l} 0 = A + B \\ 1 = 2A + B \end{array}\right\} \begin{array}{l} A = 1 \\ B = -1 \end{array} \Rightarrow x_n = 2^n - 1$ (Solución particular)

Extendemos las condiciones para una solución ~~[illegible]~~ arbitraria

$x_0 = a \qquad x_1 = 2a + 1$

Solución general de la no homogénea

$\left.\begin{array}{l} a = A + B \\ 2a + 1 = 2A + B \end{array}\right\} \left.\begin{array}{l} 2a + 1 = A + a \\ A = a + 1 \\ B = -1 \end{array}\right\} \boxed{x_n = (a+1)2^n - 1}$

13 $x_0 = 1 \quad x_n = x_{n-1} + n \qquad n \geq 1$ (regiones plano)

$x_n - x_{n-1} = n \qquad n = b^n p(n) \quad b = 1 \quad p(n) = n \quad gr(p) = 1$

Polinomio característico: $p(x) = (x-1)^3$

Solución generalísima : $x_n = An^2 + Bn + C$

Extendemos las condiciones iniciales

$x_0 = 1 \quad x_1 = 2 \quad x_2 = 4$

$$\left.\begin{array}{l} 1 = C \\ 2 = A+B+C \\ 4 = 4A+2B+C \end{array}\right\} \left.\begin{array}{l} A+B = 1 \\ 4A+2B = 3 \end{array}\right\} \left.\begin{array}{l} A = 1/2 \\ B = 1/2 \end{array}\right\} \boxed{x_n = \frac{1}{2}n^2 + \frac{n}{2} + 1 \text{ (Solución particular)}}$$

Extendemos las condiciones para una solución arbitraria

$x_0 = a \quad x_1 = a+1 \quad x_2 = a+3$

Solución general de la no homogénea

$$\left.\begin{array}{l} a = C \\ a+1 = A+B+C \\ a+3 = 4A+2B+C \end{array}\right\} \left.\begin{array}{l} A+B = 1 \\ 4A+2B = 3 \end{array}\right\} \left.\begin{array}{l} A = 1/2 \\ B = 1/2 \end{array}\right. \Rightarrow x_n = \frac{n^2}{2} + \frac{n}{2} + a$$

1.9.14

$x_0 = 1 \quad \cancel{\ldots} \; x_n - 2x_{n-1} = n \quad n \geq 1$

$n = b^n p(n) \Rightarrow b = 1 \quad p(n) = n \quad gr(p) = 1$

$p_H(x) = x - 2$

$p(x) = (x-2)(x-1)^2 = (x-2)(x^2 - 2x + 1) = x^3 - 2x^2 + x - 2x^2 + 4x - 2 =$

$= x^3 - 4x^2 + 5x - 2$

$g_n = A\,2^n + (Bn + C)\,1^n = \underline{A\,2^n + Bn + C}$

Solución general. Sacamos la particular

$x_0 = 1 \quad x_1 = 2x_0 + 1 = 3 \quad x_2 = 2x_1 + 1 = 8$

$1 = A + C \Rightarrow C = 1 - A$

$3 = 2A + B + C \qquad 2 = A + B \Rightarrow B = 2 - A = -1 \qquad C = -2$

$8 = 4A + 2B + C \qquad 7 = 3A + 2B \qquad 7 = 3A + 4 - 2A$

$x_n = 3 \cdot 2^n + (-n + 2) = \boxed{3 \cdot 2^n - n - 2} \qquad \underline{3 = A}$

15. $x_0 = 0 \quad x_n - 2x_{n-1} = 3^n \qquad n \geq 1$

$3^n = \cancel{3} b^n p(n) \qquad b = 3 \quad p(n) = 1$

$p(x) = (x-2)(x-3)$ (Polinomio característico)

$x_n = A \cdot 2^n + B\, 3^n$ (Solución generalísima)

Extendemos las condiciones iniciales

$x_0 = 0$

$x_1 = 3$

$$\left.\begin{array}{l} 0 = A + B \\ 3 = 2A + 3B \end{array}\right\} \left.\begin{array}{l} A = -B = \underline{-3} \\ \underline{B = 3} \end{array}\right\}$$ Solución particular: $\boxed{x_n = -3(2^n) + 3^{n+1}}$

Solución arbitraria

$x_0 = a \quad x_1 = 2a + 3$

$$\left.\begin{array}{l} A + B = a \\ 2A + 3B = 2a + 3 \end{array}\right\} \left.\begin{array}{l} A = a - B = \underline{a-3} \\ 2(a - B) + 3B = \cancel{2a} + 3 \;;\; \underline{B = 3} \end{array}\right\} \Rightarrow$$ Solución general de la no homogénea $\boxed{x_n = (a-3)2^n + 3^{n+1}}$

16. $x_0 = 0 \quad x_n - 2x_{n-1} = (n+1)\, 3^n$ para $n \geq 2$

$(n+1)\, 3^n = b^n p(n) \qquad b = 3 \quad p(n) = n+1 \quad gr(p) = 1$

Polinomio característico : $p(x) = (x-2)(x-3)^2$

Solución generalísma : $x_n = A\, 2^n + (Bn + C)\, 3^n$

Extendemos :

$x_0 = 0 \quad x_1 = 6 \quad x_2 = 39$

$$\left.\begin{array}{l} A + C = 0 \\ 2A + 3B + 3C = 6 \\ 4A + 28B + 9C = 39 \end{array}\right\} \left.\begin{array}{l} 12A + 18B + 18C = 36 \\ 4A + 18B + 9C = 39 \end{array}\right\} \left.\begin{array}{l} 8A + 9C = -3 \\ A + C = 0 \end{array}\right\} \begin{array}{l} C = -3 \\ A = 3 \\ \underline{B = 3} \end{array}$$

Solución particular : $x_n = 3 \cdot 2^n + (3n - 3)\, 3^n = 3\, 2^n + 3^{n+1}(\cancel{3}n - 1)$

Escaneado con CamScanner

17. $x_0 = 1/2 \quad x_1 = 3 \quad x_n = 2x_{n-1} + x_{n-2} + 3$ para $n \geq 2$

$x_n - 2x_{n-1} - x_{n-2} = 3$

$3 = b^n p(n) \quad b = 1 \quad p(n) = 3 \quad gr(p) = 0$

$p_H(x) = x^2 - 2x - 1$

$x = \dfrac{2 \pm \sqrt{8}}{2} \begin{cases} 1+\sqrt{2} \\ 1-\sqrt{2} \end{cases}$

Polinomio Característico: $p(x) = (x - (1+\sqrt{2}))(x + (1-\sqrt{2}))(x-1)$

Solución generalísima: $x_n = A(1+\sqrt{2})^n + B(1-\sqrt{2})^n + C$

$x_0 = 1/2 \quad x_1 = 3 \quad x_2 = 2x_1 + x_0 + 3 = 19/2$

$$\left.\begin{array}{l} A+B+C = 1/2 \\ (1+\sqrt{2})A + (1-\sqrt{2})B + C = 3 \\ (1+\sqrt{2})^2 A + (1-\sqrt{2})^2 B + C = 19/2 \end{array}\right\} \quad A = \frac{8+5\sqrt{2}}{8} \quad B = \frac{8-5\sqrt{2}}{8} \quad C = -3/2$$

$$\boxed{x_n = \frac{8+5\sqrt{2}}{8}(1+\sqrt{2})^n + \frac{8-5\sqrt{2}}{8}(1-\sqrt{2})^n - 3/2}$$

18. $x_0 = 0 \quad x_1 = 1 \quad x_n = 3x_{n-1} - 2x_{n-2} + 2^n \quad n \geq 2$

$x_n - 3x_{n-1} + 2x_{n-2} = 2^n \qquad 2^n = b^n p(n) \quad b = 2 \quad p(n) = 1$

$p_H(x) = x^2 - 3x + 2$

$x = \dfrac{3 \pm \sqrt{9-4\cdot 2}}{2} \begin{cases} 2 \\ 1 \end{cases}$

$p(x) = (x-2)^2(x-1)$ Solución generalísima: $x_n = A + (Bn + C)2^n$

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 7$

$$\left.\begin{array}{l} A+C = 0 \\ A+2B+2C = 1 \\ A+8B+4C = 7 \end{array}\right\} \begin{array}{l} A = -C \\ 2B+C = 1 \\ 8B+3C = 7 \end{array} \left.\right\} \begin{array}{l} 8B+4C = 4 \\ 8B+3C = 7 \end{array} \left.\right\} \begin{array}{l} C = -3 \\ A = 3 \\ B = 2 \end{array}$$

Solución particular: $\boxed{x_n = 3 - 3\cdot 2^n + 2n \cdot 2^n}$

19. $x_0=0 \quad x_n - 2x_{n-1} = n + 2^n \quad n\geq 2$

$$n = b^n p(n) \qquad b=1 \quad p(n)=n \qquad gr(p)=1$$

$$2^n = c^n g(n) \qquad c=2 \quad g(n)=1 \qquad gr(g)=0$$

$p(x) = (x-2)^2 (x-1)^2$

Solución generalísima : $x_n = An + B + (Cn+D)2^n$

$x_0 = 0 \quad x_1 = 3 \quad x_2 = 12 \quad x_3 = 35$

$$\left.\begin{array}{l} B+D=0 \\ A+B+2C+2D=1 \\ 2A+B+8C+4D=12 \\ 3A+B+24C+8D=35 \end{array}\right\} \quad \begin{array}{l} \underline{A=-1} \\ \underline{B=-2} \\ \underline{C=1} \\ \underline{D=2} \end{array}$$

Solución ~~general~~ particular: $\boxed{x_n = -2-n + 2^n(2+n)}$

20. $x_0=0, \; x_1=1 \quad x_n = 3x_{n-1} - 2x_{n-2} + 2^n + 2n$ para $n\geq 2$

$x_n - 3x_{n-1} + 2x_{n-2} = 2^n + 2n$

$$2^n = b^n p(n) \quad b=2 \quad p(n)=1$$

$$2n = c^n g(n) \quad c=1 \quad g(n)=2n$$

$x^2 - 3x + 2 = (x-2)(x-1)$

$p(x) = (x-1)^3 (x-2)^2$

Solución general : $x_n = An^2 + Bn + C + (Dn+E)2^n$

Solución particular

$x_0=0 \quad x_1=1 \quad x_2 = 3x_1 - 2x_0 + 2^2 + 2\cdot 2 = 11 \quad x_3 = 45 \quad x_4 = 137$

$$\left.\begin{array}{l} C+E=0 \\ A+B+C+2D+2E=1 \\ 4A+2B+C+8D+4E=11 \\ 9A+3B+C+24D+8E=45 \\ 16A+4B+C+64D+16E=137 \end{array}\right\} \quad \begin{array}{l} \underline{A=-1} \\ \underline{B=-5} \\ \underline{C=-3} \\ \underline{D=2} \\ \underline{E=3} \end{array}$$

Solución particular:

$$\boxed{x_n = -3-5n-n^2 + 2^n(3+2n)}$$