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Algorithms Lab

Exercise – *Attack of the Clones*

The battle on Geonosis is raging. The Separatists, led by Count Dooku, have surrounded senator Padme and n Jedi. Capturing Padme would be a disastrous hit for the Empire, and she must be protected at all costs!

To model the battle, assume that Padme is standing at the center of a circle C such that no separatist is located within the disk bounded by C. The circle C is split into m segments of equal length, numbered from 1 to m in clockwise order. Jedi are positioned around Padme, and for every Jedi you know which segments of the circle (s)he protects. However, if two Jedi protect the same segment of C and they both participate in the battle, then their lightsabers clash, resulting in a big explosion. It is of paramount importance for the safety of Padme to avoid such an explosion. Therefore, some of the Jedi may have to sit back and stay out of the battle. As the commander of the unit protecting Padme, you are asked to calculate the largest number of Jedi that can participate in the fight.

More formally, Jedi are numbered from 1 to n. For every Jedi you are given two numbers $a_i, b_i \in \{1, \ldots, m\}$, denoting that the i-th Jedi protects segments in the set $\mathcal{I}(a_i, b_i)$, defined as

$$\mathcal{I}(a_i, b_i) = \begin{cases} \{a_i, a_i + 1, \ldots, b_i\}, & \text{if } a_i \leq b_i \\ \{a_i, a_i + 1, \ldots, m, 1, \ldots, b_i\}, & \text{if } a_i > b_i. \end{cases}$$

You are required to calculate the maximum size of a subset $S \subseteq \{1, \ldots, n\}$ such that $\mathcal{I}(a_i, b_i) \cap \mathcal{I}(a_j, b_j) = \varnothing$, for every two distinct $i, j \in S$.

Input The first line of the input contains the number $t \leq 35$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains two integers `n m`, separated by a space and such that $1 \leq n \leq 5 \cdot 10^4$, $1 \leq m \leq 10^9$. Here n denotes the number of Jedi, and m denotes the number of segments on the circle C.
- The following n lines define the segments of C protected by each Jedi. The i-th such line contains two integers `a b`, separated by a space and such that $1 \leq a, b \leq m$, denoting the set of segments $\mathcal{I}(a, b)$ protected by the i-th Jedi.

It is guaranteed that in each test case there exists a segment $s \in \{1, \ldots, m\}$ which belongs to at most 10 subsets $\mathcal{I}(a_i, b_i)$, that is, at most 10 Jedi are protecting segment s.

Output For each test case output the size of the largest subset S of Jedi that can fight at the same time. Formally, output the size of the largest set S such that $\mathcal{I}(a_i, b_i) \cap \mathcal{I}(a_j, b_j) = \varnothing$ for every two distinct $i, j \in S$.

Points There are three groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 40 points, you may assume $n, m \leq 10^3$ and there exists a segment $s \in \{1, \ldots, m\}$ such that $s \notin \bigcup_{i=1}^{n} \mathcal{I}(a_i, b_i)$, that is, no Jedi protects the segment s.
2. For the second group of test sets, worth 30 points, you may assume there exists a segment $s \in \{1, \ldots, m\}$ such that no Jedi protects s.
3. For the third group of test sets, worth 30 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`i`.in/out`, for $i \in \{1, 2, 3\}$.

Sample Input

```
3
5 10
1 3
4 7
8 10
9 2
3 8
5 10
9 2
3 4
5 6
7 8
1 4
5 15
13 2
4 9
5 8
11 1
10 11
```

Sample Output

```
3
4
3
```