Section 5.1 How Probability Quantifies Randomness

5.1 Probability

The long run relative frequency definition of probability refers to the probability of a particular outcome as the proportion of times that the outcome would occur in a long run of observations.

5.2 Minesweeper

a) With a relatively short run, such as 10 attempts of completing the game, the cumulative proportion of successful attempts can fluctuate a lot.

b) We would have to attempt the game many times. In the long run, the cumulative proportion approaches the actual probability of an outcome.

5.3 Counselor availability

No. In the short run, the proportion of a given outcome can fluctuate a lot. Only in the long run does a given proportion approach the actual probability of an outcome.

5.4 Airline accident deaths

a) This would be considered a "long run" of trials. A long run refers to many, many trials. 825 million passengers per year certainly would qualify as many trials.

b) The probability of dying on a particular flight is $265/825{,}000{,}000 = 0.00000032$, or about 1 in 3 million.

c) We generally assume that crashes are independent of each other. Therefore, the chance of dying stays constant throughout the entire year.

5.5 World Cup 2014

a) 35.9% is the subjective probability that Brazil will win the World Cup. It is not based on a frequency of events.

b) Any event with nonzero probability could occur. Even Algeria could have won the World Cup.

5.6 Pick the incorrect statement

Part (b) is not correct because in the short run probabilities of each digit being generated can fluctuate a lot.

5.7 Sample size and sampling accuracy

With a biased sampling design, having a large sample does not remove problems from the sample not being selected to represent the entire population.

5.8 Heart transplant

In the absence of pre-existing data, Dr. Barnard relied on the subjective definition of probability. He used his own judgment rather than objective information such as data.

5.9 Nuclear war

We would be relying on our own judgment rather than objective information such as data, and so would be relying on the subjective definition of probability.

5.10 Simulate coin flips

a) The ten outcomes will be different each time this exercise is completed. The outcomes will likely show a good deal of variation.

b) The ten outcomes will be different each time this exercise is completed.

c) The sample proportions will tend to vary less than the proportions based on $n = 10$ or $n = 100$.

d) As the number of trials increases from 1 to 10 to 1000, the variability of the proportion decreases. The law of large numbers indicates that as the number of independent trials increases, the cumulative proportion approaches the actual probability of a given outcome. Each of the two outcomes will occur closer to 0.50 of the time as the number of trials increases.

5.11 Unannounced pop quiz

a) The results will be different each time this exercise is conducted.

b) We would expect to get about 50 questions correct simply by guessing.

c) The results will depend on your answer to (a).

d) 42% of the answers were "true." We would expect this percentage to be 50%. They are not necessarily identical, because observed percentages of a given outcome can fluctuate in the short run.

5.11 (continued)

e) There are some groups of answers that appear nonrandom. For example, there are strings of five "trues" and eight "falses", but this can happen by random variation. Typically, the longest strings of trues or falses that students will have will be much shorter than these.

5.12 Stock market randomness

a) The students will probably see runs of consecutive Hs or Ts.

b) It will probably not be unusual to see a run of about 5 Hs in a row.

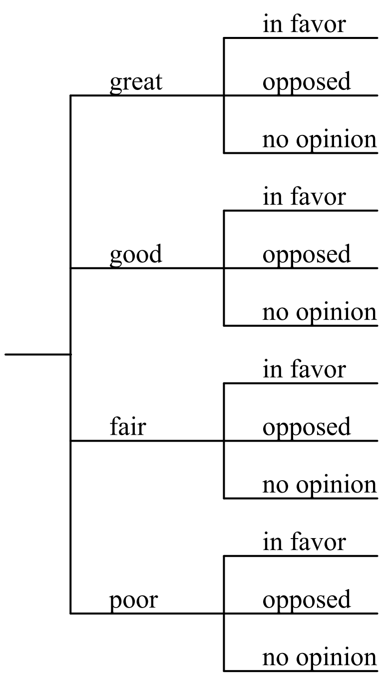
c) If you are a serious investor, you should not get too excited if you see a sequence of increases in the stock market over several days. This would not be a surprising outcome if the market's direction (rising or falling) were randomly generated.

Section 5.2 Finding Probabilities

5.13 Student union poll

a) There are $4 \times 3 = 12$ possible responses: great/in favor, great/opposed, great/no opinion, good/in favor, good/opposed, good/no opinion, fair/in favor, fair/opposed, fair/no opinion, poor/in favor, poor/opposed, poor/no opinion.

b)



5.14 Songs

a) The sample space is the set of all possible outcomes; in this case, it is all possible tracks: 1, 2, 3, 4, 5, 6, 7, 8, …, and 100.

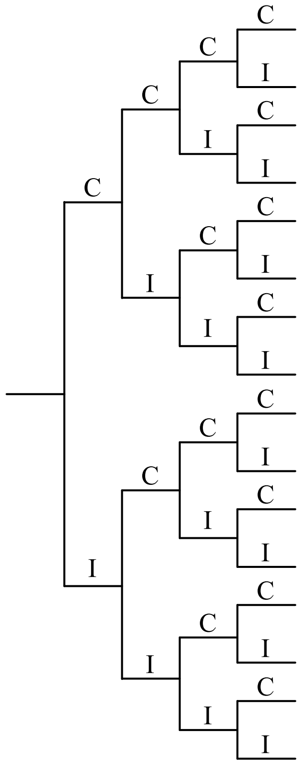
b) The probability for each possible outcome is 0.01 because there are 100 possible outcomes, and each has an equal chance of being selected.

c) The probability of that the track randomly chosen from the playlist is of your favorite artist is 15/100 = 0.15.

d) The probability of that the track randomly chosen from the playlist is not of your favorite artist is 85/100 = 0.85

5.15 Pop quiz

a)



b) There are $2 \times 2 \times 2 \times 2 = 16$ possible outcomes; therefore, the probability of each possible individual outcome is 1/16 = 0.0625.

c) Looking at the tree diagram, there are five outcomes in which the student would pass: CCCC, CCCI, CCIC, CICC, and ICCC. The probability of each of these outcomes is 0.0625. Thus, the probability that the student would pass is 0.0625 + 0.0625 + 0.0625+ 0.0625+ 0.0625 = 0.3125 (rounds to 0.313).

5.16 More true-false questions

a) For each question, you could give one of two answers: "true" or "false," and there are 10 questions; thus, the number of possible outcomes is $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10} = 1024$.

b) The complement of the event of getting at least one of the questions wrong is the event of getting none of the questions wrong.

c) Only one of these outcomes is entirely correct; therefore, the possibility of getting them entirely correct with random guessing is 1/1024 = 0.00098. The possibility of getting at least one wrong is 1.0 – 0.00098 = 0.999.

5.17 Curling

You should disagree. Not all events from the sample space are equally likely. Specifically, there are four ways of getting a sum of 7 points (2 in the first shot and 5 in the second, 3 and 4, 4 and 3, 5 and 2) whereas there is only one way of getting a 0 (0 in both shots).

5.18 On-time arrival probabilities

a) The sample space includes OO, OL, LO, and LL; where O stands for on-time arrival and L for late arrival.

b) The probability that both flights are on time is P(O and O) = P(O) × P(O) = 0.82 × 0.82 = 0.6724.

c) The probability that both flights are late is P(L and L) = P(L) × P(L) = 0.18 × 0.18 = 0.0324.

5.19 Three children

a) The sample space for the possible genders of three children is BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG.

b) There are 8 possible outcomes, each equally likely, so the probability of any particular outcome is 1/8.

5.19 (continued)

c) There are 3 outcomes with two girls and one boy so the probability is 3/8 = 0.375.
d) There are only 2 outcomes, BBB and GGG, that do not have children of both genders so that the probability is 1– 2/8 = 6/8 = 3/4.

5.20 Pick the incorrect statement

Part (a) is not correct. Outcomes 0 through 4 are not equally likely. There are 4 possible outcomes for the event "you get answered once by a female employee" FMMM, MFMM, MMFM, MMMF; where F stands for female and M for Male. The probability of each of these outcomes is $(0.5)^3 \times 0.5$. Thus, the overall probability of this event is $4 \times (0.5)^3 \times 0.5 = 0.25 \neq 1/5$.

5.21 Insurance

20 heads has probability $(1/2)^{20}$, which is $1/1{,}048{,}576 = 0.000001$. The risk of a one in a million death is $1/1{,}000{,}000 = 0.000001$.

5.22 Pick the incorrect statement

Part (a) is not correct. Based on your last night's experience you could more likely select the same restaurant you visited yesterday for tonight's dinner. However, if your choice is made randomly each time then the probability of choosing any restaurant will remain the same.

5.23 Seat belt use and auto accidents

a) The sample space of possible outcomes is YS; YD; NS; and ND.
b) P(D) = 2111/577,006 = 0.004; P(N) = 164,128/577,006 = 0.284
c) The probability that an individual did not wear a seat belt and died is 1601/577,006 = 0.003. This is the probability that an individual will fall in both of these groups – those who did not wear seat belts and those who died.
d) If the events N and D were independent, the answer would have been P(N and D) = P(N) × P(D) = (0.004)(0.284) = 0.001. In the context of these data, this means that more people died than one would expect if these two events were independent since 0.001 is not equal to 0.003. This indicates that the chance of death depends on seat belt use.

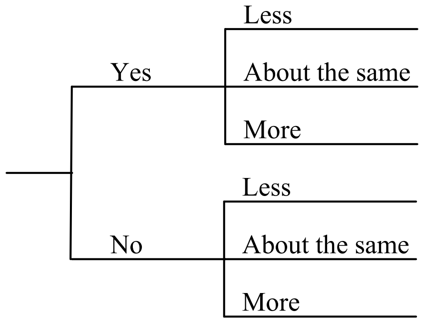
5.24 Protecting the environment

	GRNPRICE			
GRNGROUP	**Yes**	**No**	**Not Sure**	**Total**
Yes	293	71	66	430
No	2211	1184	1386	4781
Total	**2504**	**1255**	**1452**	**5211**

a) (i) P(GRNGROUP) = 430/5211 = 0.083
(ii) P(GRNPRICE) = 2504/5211 = 0.481
b) P(GRNGROUP and GRNPRICE) = 239/5211 = 0.056
c) If the variables were independent, P(GRNGROUP and GRNPRICE)= P(GRNGROUP) × P(GRNPRICE) = (430/5211) × (2504/5211) = 0.040. This is smaller than the actual probability which was computed in (b). If a respondent is a member of an environmental group it is more likely that they are also willing to pay higher prices to protect the environment.
d) (i) P(GRNGROUP or GRNPRICE) = (239 + 71 + 66 + 2111)/5211 = 2641/5211 = 0.507
(ii) P(GRNGROUP or GRNPRICE) = P(GRNGROUP)+ P(GRNPRICE) – P(GRNGROUP and GRNPRICE) = 430/5211+ 2504/5211 – 239/5211 = 2641/5211 = 0.508

5.25 Global warming and trees

a)



b) If A and B were independent events, P(A and B) = P(A) $\times$ P(B). Since P(A and B) > P(A) $\times$ P(B), A and B are not independent. Thus, whether or not a person plans to use less fuel in the future depends on whether they believe that global warming is happening. The probability of responding “yes” on global warming and “less” on future fuel use is higher than what is predicted by independence.

5.26 Newspaper sales

a)

	Weekday		
Weekend	**Yes**	**No**	**Total**
Yes	0.25	0.20	0.45
No	0.05	0.50	0.55
Total	**0.30**	**0.70**	**1.00**

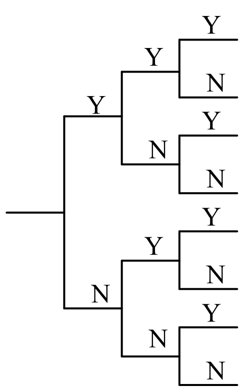
b) P(W) = 0.25 + 0.10 = 0.30; P(S) = 0.25 + 0.20 = 0.45

c) The event “W and S” means that they bought the paper on the weekday and weekend. P(W and S) = 0.25.

d) If W and S were independent, we’d expect that P(W and S) = P(W) $\times$ P(S) = (0.30)(0.45) = 0.135, not 0.25. These are not the same, so the events are dependent. It makes sense that customers who buy on the weekend are more likely to buy on the weekday, because you already know that they like reading the newspaper.

5.27 Arts and crafts sales

a) There are eight possible outcomes as seen in the tree diagram.



b) The probability of at least one sale to three customers is 0.488. Of the 8 outcomes, only one includes no sale to any customer. The probability that this would occur = (0.8)(0.8)(0.8) = 0.512. Thus, 1 – 0.512 = 0.488, the probability that at least one customer would buy.

c) The calculations in (b) assumed that each event was independent. That outcome would be unrealistic if the customers are friends or members of the same family, and encourage each other to buy or not to buy.

Section 5.3 Conditional Probability

5.28 Recidivism rates

Using R for reincarcerated, B for blacks, and W for whites, the conditional probabilities are: P(R | B) = 0.81; P(R | W) = 0.73.

5.29 Smoke alarms statistics

38% = $P(D/P^C)$ and 21% = P[D | (P and F)].

5.30 Audit and low income

a) P(Audited | Income < \$200,000) = P(Audited and Income < \$200,000)/P(Income < \$200,000) = 0.0085/(0.0085 + 0.9556) = 0.0088

b) P(Income < \$200,000 | Audited) = P(Income < \$200,000 and Audited)/P(Audited) = 0.0085/(0.0085 + 0.0009 + 0.0003) = 0.8763

5.31 Religious affiliation

a) The probability that a randomly selected individual is identified as Christian is (57,199 + 36,148 + 16,834 + 11,366 + 51,855)/228,182 = 0.7599.

b) P(Catholic | Christian) = P(Catholic and Christian)/P(Christian) = (57,199/228,182)/0.7599 = 0.3299

c) P(No Religion | Answered) = P(No Religion and Answered)/P(Answered) = (34,169/228,182)/[(228,182 – 11,815)/(228,182)] = 0.1497/0.9482 = 0.1579

5.32 Labor force

a) There are three possible events among employees which are government employees. If a government employee is G, a full-time employee is F, a part-time employee is P, and a retired employee is R, then the three possible events among government employees are GF, GP, and GR. The latter three probabilities (of employees who are government employees, 7.5% are part-time employees, 60% are full-time employees, and 32.5% are retired) are conditional probabilities.

b) P(G and F) = P(F | G) × P(G) = (0.60)(0.205) = 0.123.

5.33 Revisiting seat belts and auto accidents

a) P(D) = 2111/577,006 = 0.004

b) P(D | wore seat belt) = 510/412,878 = 0.001
P(D | didn't wear seat belt) = 1601/164,128 = 0.010

c) Neither P(D | wore seat belt) nor P(D | didn't wear seat belt) equals P(D); specifically, 0.001 and 0.010 are different from 0.004. Thus, the events are not independent.

5.34 Go Celtics!

a)

	Free Throw Success		
	2nd free throw made	**2nd free throw missed**	**Total**
1st free throw made	251	34	285
1st free throw missed	48	5	53
Total	**299**	**39**	**338**

b) (i) P(made first) = 285/338 = 0.84
(ii) P(made second) = 299/338 = 0.88

c) P(made second | made first) = 251/285 = 0.88; it seems as if his success on the second shot depends hardly at all on whether he made the first. P(made second | made first) = P(made second)

5.35 Identifying spam

a)

	Identified as Spam by ASG	
Spam	**Yes**	**No**
Yes	7005	835
No	48	

b) 7005/(7005+835) = 0.8935

c) 7005/(7005+48) = 0.9932

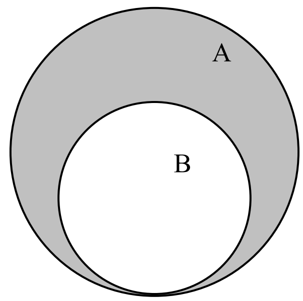
5.36 Homeland security

a)

	Detected by Device	
Radioactive Material	**Yes**	**No**
Yes	a	b
No	c	d

The cell corresponding to the false alarms the NYPD fears are given by the cell containing “c”.

b)



c) Since the event A contains the event B, if B is known to have occurred, it must be that A has also occurred. Thus, P(A | B) = 1. However, knowing that A has occurred does not guarantee the occurrence of B since B is a subset of A. Thus, P(B | A) < 1.

5.37 Down syndrome again

a) P(D | NEG) = 6/3927 = 0.0015

b) P(NEG | D) = 6/54 = 0.111; these probabilities are not equal because they are built on different premises. One asks us to determine what proportion of fetuses with negative tests actually has Down syndrome. This is a small number based on a very large pool of fetuses who had negative tests (3927). The other asks us to calculate what proportion of fetuses with Down syndrome had a negative test. This is a larger number because even though the number of false negatives is small, it’s based on a small pool of fetuses (just 54).

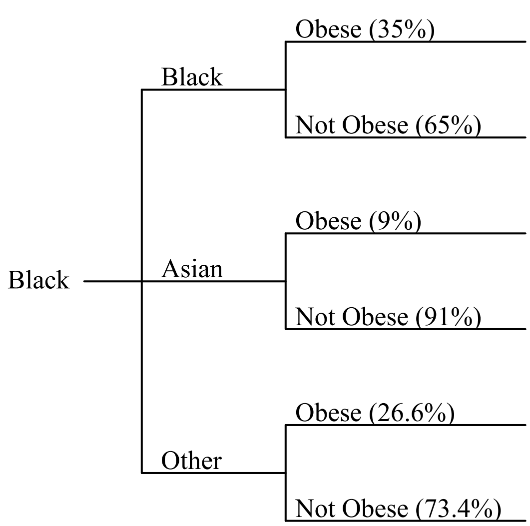
5.38 Obesity in America

a) 35% and 9% are conditional. They are the chance of being obese given your race.

b)

	Black	**Asian**	**Other**
Obese	8446	518	64,570
Not Obese	15,685	5234	177,894
Total	**24,131**	**5752**	**242,464**

c)



5.39 Happiness in relationship

a) P(very happy) = 147/317 = 0.46
b) (i) P(very happy | male) = 69/146 = 0.47
(ii) P(very happy | female) = 78/171 = 0.46
c) From (a), P(very happy) = 0.46 and from (b), P(very happy | male) = 0.47. Since these values are very close, we can say the events of being happy and being male are independent.

5.40 Petra Kvitova serves

a) P(1^{st} serve good) = 28/41 = 0.68
b) P(double fault | first serve is a fault) = P(double fault)/P(first serve is a fault) = (3/41)/((41 – 28)/41) = 3/13 = 0.23
c) 3/41 = 0.07; She double faults on 7% of her service points.

5.41 Answering homework questions

a) P(answering correctly the first and the second parts) = P(answering correctly the second | answered correctly the first) × P(answering correctly the first) = (0.60)(0.75) = 0.45
b) (i) P(misses second and answering correctly the first) = P(misses second | answers correctly the first) × P(answering correctly the first) = (1 – 0.60)(0.75) = (0.40)(0.75) = 0.30 and P(answering correctly the second and miss the first) = P(answered correctly the second | miss first) × P(misses first) = (0.40)(1 – 0.75) = (0.40)(0.25) = 0.10. Thus, the probability of answering correctly one of the two parts is 0.30 + 0.10 = 0.40.
(ii) As shown in part (a), the probability of answering both parts correctly is 0.45. P(misses second and misses first) = P(misses second | misses first) × P(misses first) = (1 – 0.40)(1 – 0.75) = (0.60)(0.25) = 0.15. Thus, the probability that you answer correctly both or neither is 0.45 + 0.15 = 0.6 and the probability that you answer correctly only one part is 1 – 0.60 = 0.40.
c) The results of the two parts are not independent because the probability that your answer correctly the second part depends on whether your answered correctly the first part or not.

5.42 Discussion with students

a) False. Once one student of a business program has been selected, there are only 19 out of 29, which is not two-thirds. If two students of a business program have been selected, there are now 18 out of 28, which also is not two-thirds. The correct probability is P(first student of a business program) × P(second student of a business program | first student of a business program) × P(third student of a business program | first and second students of a business program) = (20/30)(19/29)(18/28) = 0.281
b) A and B are not independent because your chances of getting a student of a business program on the second draw are lower if you got a student of a business program on the first draw. On the first draw, there was a 20/30 probability of drawing a student of a business program, but on the second draw, there is now a 19/29 probability.
c) redo (a) and (b).
   a) True because at each draw, two-thirds of the students are of business programs.
   b) Yes because the students are replaced, the first student drawn has no effect on the second student drawn.

5.43 Drawing more cards

P(winning) = P(two diamonds) = P(first card is a diamond)×P(second card is a diamond | first card is a diamond) = (10/47)(9/46) = 0.0416

5.44 Big loser in Lotto

P(0 winning numbers) = P(have none) × P(have none | had none on first) × P(have none | had none on first and second) × P(have none | had none on first, second or third) × P(have none | had none on first through fourth) × P(have none | have none on first through fifth) = (43/49)(42/48)(41/47)(40/46)(39/45)(38/44) = 0.436

5.45 Online sections

a) P(C | A) = P(C and A)/P(A) = P(C)/P(A) = (0.5 × 0.5)/(0.5) = 0.5; P(C | B) = P(C and B)/P(B) = P(C)/P(B) = (1/4)/(3/4) = 0.333 (P(B) = 1 – P(both sections are not online-based) = 1 – 0.5 × 0.5 = 0.75).
b) These are not independent evens because P(C | A), 0.5, is not equal to P(C), 0.25.

5.45 (continued)

c) P(C | A) is the probability of both sections being online-based given that the first is online-based. This is the same as the probability that the second course is online-based, 0.5. P(C | B) is the probability that both sections are online-based given that one of the sections is online-based. There are 3 possibilities where one of the sections is online-based: OO, OO^C, O^CO. For only one of these are both sections online-based so this probability is smaller, namely 0.333.

5.46 Checking independence

a) P(A) = 0.50; P(B) = 0.50; P(C) = (0.5)(0.5) = 0.25; P(D) = (0.5)(0.5)(0.5) = 0.125.

b) A and B are independent. The other sets would not be independent because if D (three visits are TS1) occurs, we know the other three have occurred; thus, DC, DB, and DA are not independent. If C occurs, we know that A and B have occurred; thus, CB and CA are not independent.

Section 5.4 Applying the Probability Rules

5.47 Heart disease

It is easiest to find the probability of none or one only of the 50 selected persons has the disease and then subtract from one.

P(none of the 50 selected persons has the disease) = $(999/1000)^{50}$ = 0.9512;

P(only one of the 50 selected persons has the disease) = 50(1/1000) $(999/1000)^{49}$ = 0.0476. Thus, the probability to have at least two persons with this disease among 50 people is 1 – (0.9512 + 0.0476) = 0.0012. This is highly coincidental.

5.48 Matching your birthday

a) Because we're noting a specific date rather than students with the same birthday on any date, the probability would be lower.

b) There's a 0.57 chance that two students out of 25 will share a birthday. However, if we are interested in finding the probability that one of the remaining 24 students also has a January first birthday, the probability is one minus the probability no one shares your birthday. Thus, it is 1 – $(364/365)^{24}$ = 0.06.

5.49 Lots of pairs

Each student can be matched with 24 other students, for a total of (25)(24) = 600 pairs. But this considers each pair twice (e.g., student 1 with student 2, and student 2 with student 1), so the answer is (25)(24)/2 = 300.

5.50 Holes in one at Masters

a) P(no holes in one during a round of golf) = (1 – 0.0005)(1 – 0.0015)(1 – 0.0005)(1 – 0.0025) = 0.995

b) P(no holes in one during the next 20 rounds of golf) = $(0.995)^{20}$ = 0.905

c) P(at least one hole in one during the next 20 rounds of golf) = 1 – P(no holes in one during the next 20 rounds of golf) = 1 – 0.905 = 0.095

5.51 Failure and repair of photocopiers

a) The probability that all the 5 copiers fail and require reparation is 0.1 × 0.1 × 0.08 × 0.08 × 0.05 = 0.0000032.

b) The probability that none of the 5 copiers fails and requires reparation is (1 – 0.1) × (1 – 0.1) × (1 – 0.08) × (1 – 0.08) × (1 – 0.05) = 0.6513.

c) The probability that one of the 5 copiers fails and requires reparation is 2(0.1 × 0.9 × 0.92 × 0.92 × 0.95) + 2(0.9 × 0.9 × 0.08 × 0.92 × 0.95) + (0.9 × 0.9 × 0.92 × 0.92 × 0.05) = 0.2923

d) The probability that one of the small copiers fails and requires reparation 2(0.1 × 0.9) = 0.18.

e) The probability that at least one of the 5 copiers fails and requires reparation is 1 – P(none of the 5 copiers fails and requires reparation) = 1 – 0.6513 = 0.3487.

5.52 Horrible 11 on 9/11

Because of the huge number of possible occurrences, combined with our tendency as humans to look for patterns, we are going to see coincidences fairly frequently. Coincidences, however, are not amazing when we consider them in the context of all the possible random occurrences at all times. The percentage that constitute coincidences is very small; we just pay more attention to coincidental than to non-coincidental occurrences.

5.53 Coincidence in your life

The response will be different for each student. The explanation, however, will discuss the context of the huge number of the possible random occurrences that happen in one's life, and the likelihood that at least some will happen (and appear coincidental) just by chance.

5.54 Monkeys typing Shakespeare

If we assume that each time the monkey hits a key, it is independent of the other times he/she hits a key, we can use the multiplicative rule. (1/50)(1/50)(1/50)(1/50)(1/50)(1/50)(1/50) = 0.0000000000013

5.55 A *true* coincidence of emergency

a) The probability that the first person will call multiplied by the probability that the second person will call and so on for all 2 million, that is (1.37/1000) taken to the 2 million power, which is zero to a huge number of decimal places.

b) This solution assumes that each person decides independently of all others. This is not realistic because when such a big number of people decide to call together that is a case of a common big emergency in the region.

5.56 Rosencrantz and Guildenstern

a) The sample space would be all possible combinations of coin flips. Thus, one could get 91 heads, or 90 heads in a row and then one tail, or 89 heads in a row then a tail then another head, and so on for 2 to the 91st power, which is 2.48×10^{27} combinations!

b) Guildenstern's outcome for this sample space is 91 heads:
HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH
HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH.
If the second flip were a tail, the outcome would be:
HTHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH.
HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH.

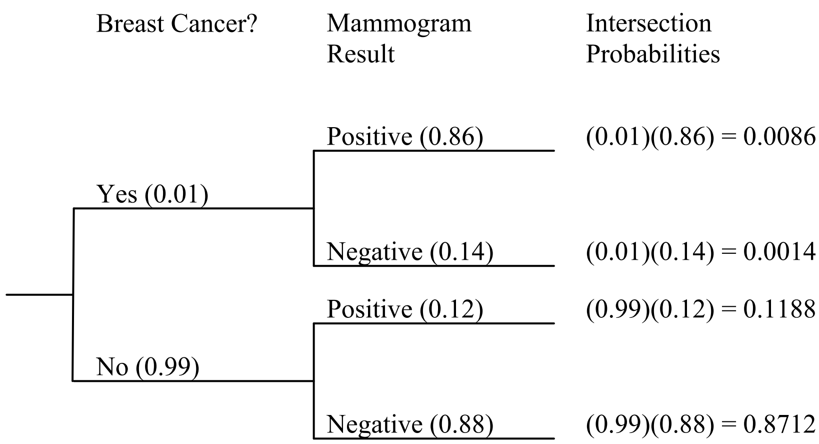
c) The probability of the event of getting a head 91 times in a row is the probability of getting no tails, $(1./2)^{91}$, essentially 0.

d) The probability of at least one tail is 1.0 minus the probability of no tails which is essentially 1.

e) Our probability model assumes that each coin flip is independent, and that each has the same probability of success.

5.57 Mammogram diagnostics

a)



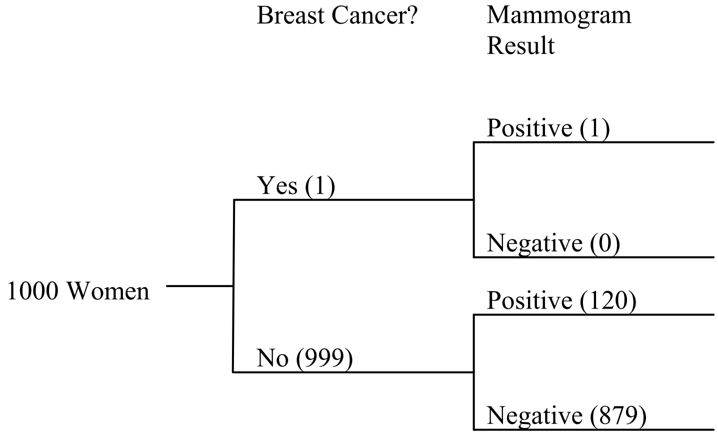
b) P(POS) = P(S and POS) + P(S^c and POS) = 0.0086+0.1188 = 0.1274

c) P(S | POS) = P(POS and S)/P(POS) = (0.0086/0.1274) = 0.068

d) The frequencies on the branches are calculated by multiplying the proportion for each branch by the total number. For example, (0.01)(100) = 1, the frequency on the "yes" branch. Similarly, we can multiply that 1 by 0.86 to get 0.86, which rounds up to 1, the number on the "pos" branch. There are 13 positive tests in this example, only one of which indicates breast cancer. Thus, the proportion of positive tests with breast cancer is 1/(1 + 12) = 0.077 (rounds to 0.08).

5.58 More screening for breast cancer

a) The four intersection probabilities are now 0.0009, 0.0001, 0.1200, and 0.8791.
P(POS) = P(S and POS) + P(S^c and POS) = 0.0009+0.1200 = 0.121
P(S | POS) = P(POS and S)/P(POS) = (0.0009/0.121) = 0.007



b) The tree diagram shows that only 1 out of 121 positive tests indicated actual breast cancer. 1/121 = 0.008.

c) As can be seen from the two tree diagrams, 12 out of 13 were false positives for older women, whereas 120 out of 121 were false positives among younger women. Since it is less likely for a younger woman to have breast cancer, yet the specificity is the same for the two groups, the proportion of false positives will be greater for younger women.

5.59 Was OJ actually guilty?

a) 45 is entered based on the proportion of 100,000 women who are murdered annually; it is the sum of women who are murdered by their partners and women who are murdered by someone other than their partners. 5 is entered because it is the number, out of 100,000 who are murdered by someone other than their partners.

b) The blanks in the tree diagram will be 40 for those women who are murdered by their partners, and 99,955 for women who are not murdered.

c) Of the women who suffer partner abuse and are murdered, 40/45, or 89%, are murdered by their partners. On the other hand, the probability that a woman is murdered by her partner given that she has suffered partner abuse is 40/100000 or 0.0004. O.J.'s wife had been abused by him and murdered, so it is the 89% statistic that is relevant in this case. These statistics differ drastically because most women who suffer partner abuse are not murdered. However, those who are murdered are typically murdered by their partner.

5.60 Convicted by mistake

a) We are given the following information: P(Convicted | Guilty) = 0.95, P(Acquitted | Innocent) = 0.95, P(Guilty) = 0.90. We are asked to find P(Innocent | Convicted). Based on these values we can fill in the table as follows:

	Guilty	**Innocent**	**Total**
Convicted	(0.95)(0.9) = 0.855	0.10 – 0.095 = 0.005	0.855 + 0.005 = 0.86
Acquitted	0.90 – 0.855 = 0.045	(0.95)(0.10) = 0.095	0.045 + 0.095 = 0.14
Total	**0.90**	**1 – 0.90 = 0.10**	

P(Innocent | Convicted) = P(Innocent and Convicted)/P(Convicted) = 0.005/0.86 = 0.0058

b) Making the change P(Guilty) = 0.50 we have the following:

	Guilty	**Innocent**	**Total**
Convicted	(0.95)(0.50) = 0.475	0.50 – 0.475 = 0.025	0.475 + 0.025 = 0.50
Acquitted	0.50 – 0.475 = 0.025	(0.95)(0.50) = 0.475	0.025 +0.475 = 0.50
Total	**0.50**	**1 – 0.50 = 0.50**	

P(Innocent | Convicted) = P(Innocent and Convicted)/P(Convicted) = 0.025/0.50 = 0.05

5.60 (continued)

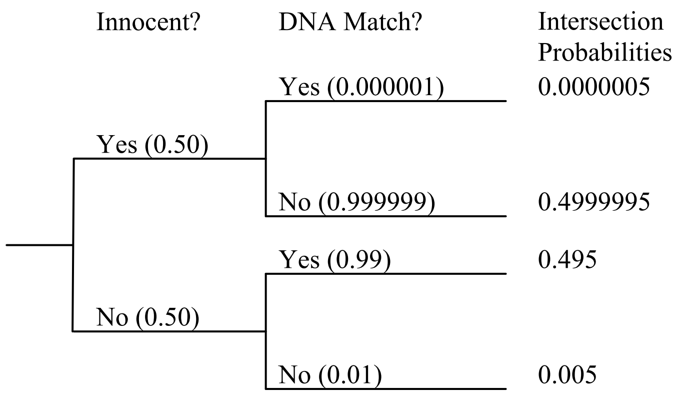
c) We are given the following information: P(Convicted | Guilty) = 0.99, P(Acquitted | Innocence) = 0.75 and P(Guilty) = 0.90. We are asked to find P(Innocent | Convicted). Based on these values we can fill in the table as follows:

	Guilty	Innocent	Total
Convicted	(0.99)(0.9) = 0.891	0.10 – 0.075 = 0.025	0.891 + 0.025 = 0.916
Acquitted	0.90 – 0.891 = 0.009	(0.75)(0.10) = 0.075	0.009 + 0.075 = 0.084
Total	**0.90**	**1 – 0.90 = 0.10**	

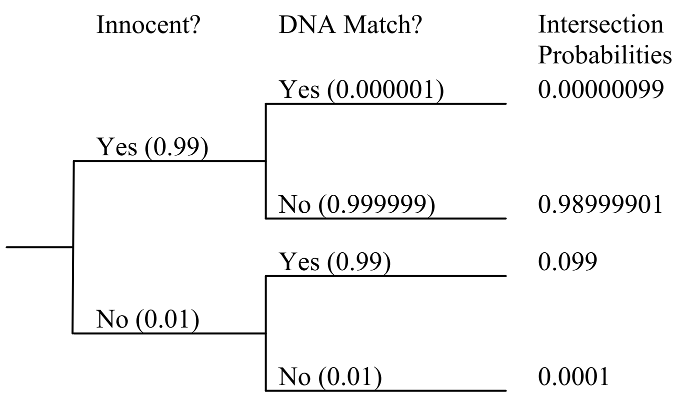
P(Innocent | Convicted) = P(Innocent and Convicted)/P(Convicted) = 0.025/0.916 = 0.0273.

5.61 DNA evidence compelling?

a) P(Innocent | Match) = P(Innocent and Match)/P(Match) = 0.0000005/0.4950005 = 0.000001



b) P(Innocent | Match) = P(Innocent and Match)/P(Match) = 0.00000099/0.00990099 = 0.0001
When the probability of being innocent is higher, there's a bigger probability of being innocent given a match.



c) P(Innocent | Match) can be much different from P(Match | Innocent).

5.62 Triple Blood Test

a) The prevalence is 54/5282 = 0.01.

b) (i) The sensitivity is 48/54 = 0.89.
(ii) The specificity is 3921/5228 = 0.75.

c) (i) The positive predictive value is 48/1355 = 0.035.
(ii) The negative predictive value is 3921/3927 = 0.998.

d) There are four ways of describing the probability that a diagnostic test makes a correct decision. First, in (b-i), we see the probability that the test would be positive if one had Down syndrome. Second, in (b-ii), we see the probability that the test would be negative given that one does not have Down syndrome. In (c-i), we see the probability that one would have Down syndrome if the test is positive. Finally, in (c-ii), we see the probability that one would not have Down syndrome given a negative test.

5.63 Simulating arrivals to local holiday center

a) The results of this exercise will be different each time it is conducted. One would make the assumption of independence with respect to visitors.

b) We would multiply the chances of the first person not being married (0.464) by the chances of the second person not being married (1 – 0.536 = 0.464) by the chances of the third person not being married (0.464), etc. This results in multiplying (0.464) × (0.464) × (0.464)× ⋯ × (0.464) – a grand total of twenty 0.464's. This product of these probabilities is 2.14×10^{-7}.

5.64 Probability of winning

a) The simulation in the example only consisted of 20 repetitions. When using the sample proportion to estimate a probability, the estimates will be better approximations when the number of trials is larger.

b) Continuing where the example left off in the random number table, the next 180 repetitions result in a much closer to the actual probability of winning.

c) As the number of repetitions in the simulation increases, the difference between the simulated probability and the actual probability will decrease.

5.65 Probability of winning

a) Multiply the table results in Exercise 16 by 4 seconds to get the answers.

b) Answers will vary depending on the simulation.

c) Answers will vary depending on the simulation, but rapid succession is likely faster.

Chapter Problems: Practicing the Basics

5.66 Peyton Manning completions

a) No. What it means is that in 100 passes we expect to see about 65 completions, but the actual number may vary somewhat.

b) If Manning is still at his typical playing level, it would be quite surprising if his completion percentage over a large number of passes differed significantly from 0.65. The more passes he throws, the closer the observed percentage should be to 0.65.

5.67 Due for a boy?

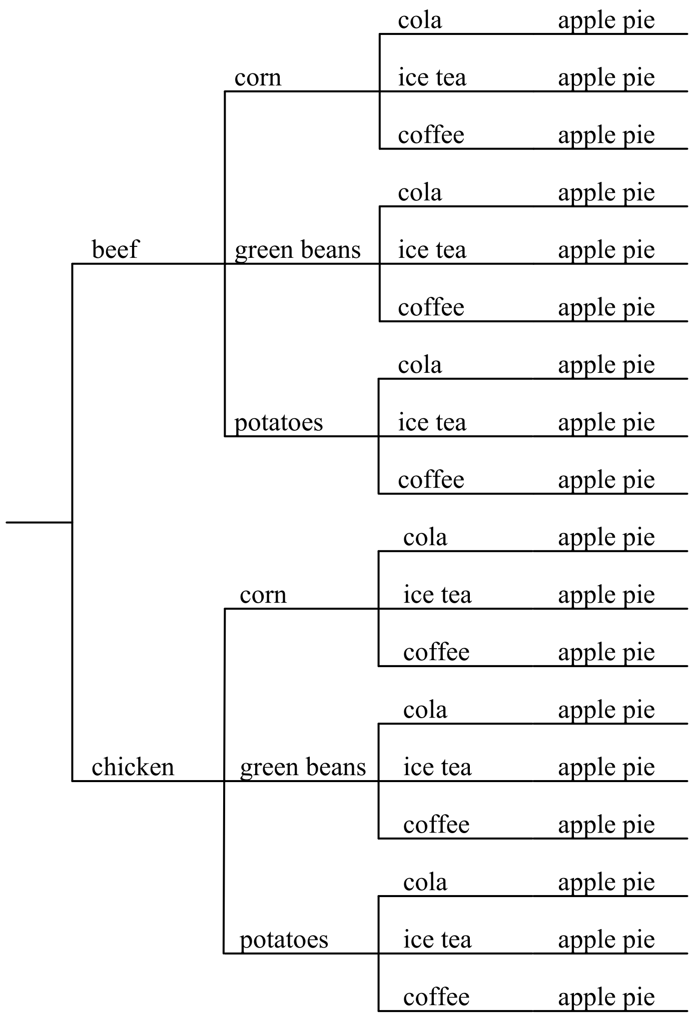
The gender of each child is independent of the genders of the previous children. Thus, the chance that this child is a boy is still 1/2.

5.68 P(life after death)

The relative frequency refers to the probability of an outcome as a long-run proportion; that is, we observe how many times a given event occurs out of a certain number of trials. Subjective definitions of probability refer to our degree of belief that a given outcome will occur. Because we cannot observe whether people have life after death, we can only give a subjective definition of (a) the probability of life after death. On the other hand, we could observe (b) how often we remember at least one dream that we had the previous night. Over the long-run, we could calculate the relative frequency of remembering a dream, and estimate the probability that this will occur.

5.69 Choices for lunch

a) Given that all customers select one dish from each category, there are $2 \times 3 \times 3 \times 1 = 18$ possible meals.



b) In practice, it would not be sensible to treat all the outcomes in the sample space as equally likely for the customer selections we'd observe. Typically, some menu options are more popular than are others.

5.70 Caught doctoring the books

a) If we were to randomly pick one of the digits between 1 and 9 using a random numbers table, the probability for each digit would be $1/9 = 0.111$.

b) The probability of a 5 or a 6 as the first digit using (i) Benford's Law would be $0.08 + 0.07 = 0.15$, and using (ii) random selection would be $0.111 + 0.111 = 0.222$.

5.71 Life after death

a) The estimated probability that a randomly selected adult in the U.S. believes in life after death is $1455/1787 = 0.8142$.

b) The probability that both subjects believe in life after death is $0.8142 \times 0.8142 = 0.6629$.

c) The assumption used in the answer to (b) is that the responses of the two subjects are independent. This is probably unrealistic because married couples share many of the same beliefs.

5.72 Death penalty jury

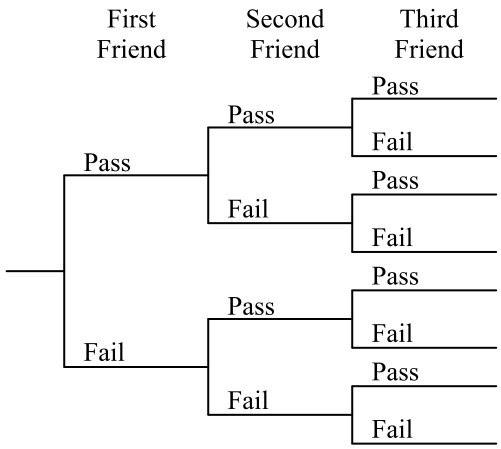
a) The probability of all 12 jurors being white is the probability that the first is white (0.90) multiplied by the probability that the second is white (0.90), and so on for all 12 jurors. $0.90 \times 0.90 \times 0.9 \times \cdots \times 0.90 = 0.28$.

5.72 (continued)

b) This problem would be solved in the same manner as (a), substituting 0.50 for 0.90. The probability is now 0.0002.

5.73 Driver's exam

a) There are $2 \times 2 \times 2 = 8$ possible outcomes.



b) If the eight outcomes are equally likely, the probability that all three pass the exam is 1/8 = 0.125. This could also be calculated by multiplying the probability that the first would pass (0.5), by the probability that the second would pass (0.5), by the probability that the third would pass (0.5). $0.5 \times 0.5 \times 0.5 = 0.125$.

c) If the three friends were a random sample of their age group, the probability that all three would pass is $0.7 \times 0.7 \times 0.7 = 0.343$.

d) The probabilities that apply to a random sample are not likely to be valid for a sample of three friends because the three friends are likely to be similar on many characteristics that might affect performance on such a test (e.g., IQ). In addition, it is possible that they studied together.

5.74 Independent on coffee?

a) These events are dependent since if the student has visited Europe in the past 12 months, it is likely that they flew there.

b) One could explain that independent means that the first event has no bearing on the second event – that one could not predict the second event from the first.

c) Dependent. Individuals who visit Europe are likely to visit more than one country, particularly ones that are close together.

d) Independent. These two events seem to be unrelated.

e) The pairs of events in (a) are most dependent, those in (d) the least.

5.75 Health insurance

a) The probability that a patient does not have health insurance is 0.16. Thus, the probability that a patient has health insurance is 1 – 0.16 = 0.84.

b) P(private | health insurance) = P(private and health insurance)/P(health insurance) = 0.59/0.84 = 0.70

5.76 Teens and drugs

The second probability that is given, 26%, refers to a conditional probability. The event is conditional on the teen reporting that they go to clubs for music or dancing at least once a month. The probability refers to the event that a teen says that drugs were usually available at the club events.

5.77 Teens and parents

a) The last two percentages, 31% and 1%, are conditional probabilities. The 31% is conditioned on the event that the teen says that parents are never present during the parties they attend. The 1% is conditioned on the event that the teen says that parents are present at the parties they attend. For both percentages, the event to which the probability refers is a teen reporting that marijuana is available at the parties they attend.

5.77 (continued)

b)

	Parents Present	
Marijuana available	**Yes**	**No**
Yes	9	133
No	860	295

c) The probability that parents are present given that marijuana is not available at the party is P(parents are present and marijuana is not available)/P(marijuana is not available) = 860/(860 + 295) = 0.74.

5.78 Laundry detergent

a) The probability that a randomly chosen consumer would have seen advertising for the new product and tried the product is 0.10.

b) The probability that the person has tried the product given that the person has seen the product advertised is P(tried the product and seen product advertised)/P(seen product advertised) = 0.10/0.35 = 0.29.

c) from (a): P(A and B), from (b): P(A | B)

d) P(A) = 0.15; P(A | B) = 0.29. Because P(A) does not equal P(A | B), A and B are not independent.

5.79 Car and Phone

a) The sample space of all possible outcomes for the pairs of labels is as follows:
(1, 1); (1, 2); (1, 3); (1, 4); (1, 5); (1, 6); (2, 1); (2, 2); (2, 3); (2, 4); (2, 5); (2, 6); (3, 1); (3, 2); (3, 3); (3, 4); (3, 5); (3, 6); (4, 1); (4, 2); (4, 3); (4, 4); (4, 5); (4, 6).

b) The outcomes in *A* are (1, 1); (2, 2); (3, 3) and (4, 4); the probability of this is 4/24 = 0.167.

c) The outcomes in *B* are (1, 6); (2, 5); (3, 4) and (4, 3); the probability of this is 4/24 = 0.167.

d) (i) There are no outcomes that include both *A* and *B*; that is, none of the similarly labeled pairs add up to seven. Thus, the probability of *A* and *B* is 0.

(ii) The probability of *A* or *B* = (4/24) + (4/24) = 0.333.

(iii) The probability of *B* given *A* is 0. If you select a pair with the same label, it cannot add up to seven.

e) *A* and *B* are disjoint. There cannot be a combination where the car label and phone label have the same value and these sum of these values is 7.

5.80 Passing scores

a) *P*(*B* and *D*) = 0.3 which also is *P*(*B*); when an event *B* is contained within an event *D*, *P*(*B* and *D*) = *P*(*B*) because all *B*'s are also *D*'s. The requirement that the event also be *D* does not constrain *B* anymore than it already is constrained.

b) *P*(*B* or *D*) = 0.70 which also is *P*(*D*); when an event *B* is contained within an event *D*, *P*(*B* or *D*) = *P*(*D*) because *B* does not add anything to *D*. It already is part of it.

5.81 Conference dinner

P(Dinner | Breakfast) = P(Dinner and Breakfast)/P(Breakfast) = 0.40/0.50 = 0.80

5.82 Waste dump sites

a) Let A = violation at the first project and B = violation at the second project. Then, if the two projects are disjoint, P(A or B) = P(A) + P(B) = 0.30 + 0.25 = 0.55.

b) $P(B \mid A^c) = P(B \text{ and } A^c)/P(A^c) = 0.25/0.7 = 0.357$. Note that if the projects A and B are disjoint, the probability of (B and A^c) is the same as the probability of B.

c) (a) If independent, P(A or B) = P(A) + P(B) – P(A and B) = 0.30 + 0.25 – 0.075 = 0.475. (Note: If independent, P(A and B) = P(A) × P(B) = 0.3 × 0.25 = 0.075.)

(b) If independent, $P(B \mid A^c) = P(B \text{ and } A^c)/P(A^c) = (0.175)/(0.70) = 0.25$. (Note: If independent, $P(B \text{ and } A^c) = P(A^c) \times P(B) = 0.7 \times 0.25 = 0.175$.)

5.83 A dice game

There are 36 possible combinations of dice. Of these, eight add up to seven or eleven [1,6; 2,5; 3,4; 4,3; 5,2; 6,1; 5,6; and 6,5], and four add up to two, three, or twelve [1,1; 1,2; 2,1; and 6,6]. Twenty-four add up to other sums. If you roll another sum, it doesn't affect whether you win or lose. Given that you roll a winning or losing combination, there is an 8 in 12 chance of winning. Thus, there is a 0.67 chance of winning. This game would not be played in a casino because the game favors the player to win in the long run, not the house.

5.84 No coincidences

a) The probability that we would never have a coincidence on these 100 topics is the probability that we would not have a coincidence on the first multiplied by the probability that we would not have a coincidence on the second multiplied by the probability that we would not have a coincidence on the third, and so on through 100 topics. P(disagree on first) $\times$ P(disagree on second) $\times$ P(disagree on third) $\times \cdots \times$ P(disagree on 100th) = $(0.98)^{100}$ = 0.13.

b) The probability that we would have a coincidence on at least one topic is 1 – 0.13 = 0.87.

5.85 Amazing roulette run?

a) This strategy is a poor one as the roulette wheel has no memory. The chance of an even slot or of an odd slot is the same on each spin of the wheel.

b) $(18/38)^{26}$, which is essentially 0.

c) It would not be surprising if sometime over the past 100 years one of these wheels had 18 evens in a row. Events that seem highly coincidental are often not so unusual when viewed in the context of *all* the possible random occurrences at all times.

5.86 Death penalty and false positives

a) One error would be a false positive, where an individual who is innocent is convicted. The other error would be a false negative where a guilty defendant acquitted.

b) (i) The probability that all were truly guilty is the probability of the first being guilty multiplied by the probability of the second being guilty, and so on through the 1234th person. For 1234 people this equals $(0.99)^{1234}$. The probability that all were truly guilty, therefore, is 0.00000411 or close to 0.

(ii) The probability that at least one was actually innocent is 1 – 0.00000411 = 0.99999589, or close to 1.

c) The answers in (b) become (i) essentially 0 and (ii) essentially 1.

5.87 Screening smokers for lung cancer

False negatives would be when the helical computed tomography diagnostic test indicates that an adult smoker does not have lung cancer when he or she does have lung cancer. Conversely, a false positive would occur when this test indicates the presence of lung cancer when there is none.

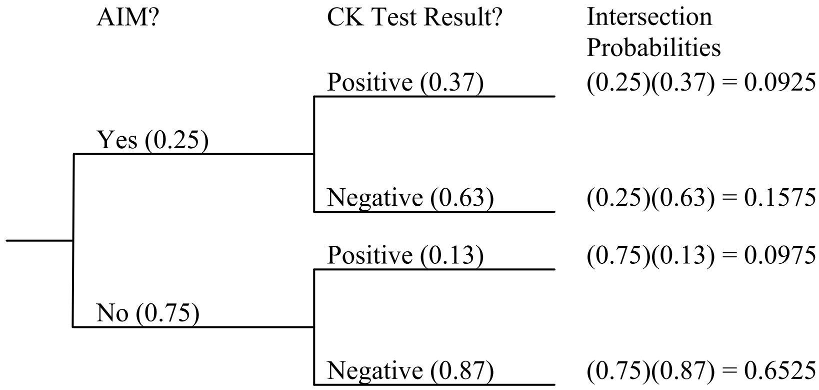
5.88 Screening for heart attacks

a) The sensitivity is the probability of a positive test given that someone actually has the condition. In this case, given that someone has AMI, there is a 37% chance that they have a positive CK test.

b) The specificity is the probability of a negative test given that someone does not have the condition. In this case, given that the individual does not have AMI, there is an 87% chance that they have a negative CK test.

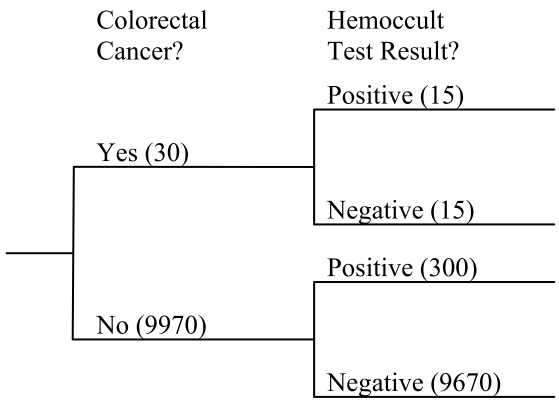
5.88 (continued)

c)



5.89 Screening for colorectal cancer

a)



b) 15/315 = 0.048 of those who have a positive hemoccult test actually have colorectal cancer. Because so few people have this cancer, most of the positive tests will be false positives. There are so many people without this cancer that even a low false positive rate will result in many false positives.

5.90 Color blindness

a) Given that one is a man, one has a 0.05 probability of being color blind. Given that one is a woman, one has a 0.0025 probability of being color blind.

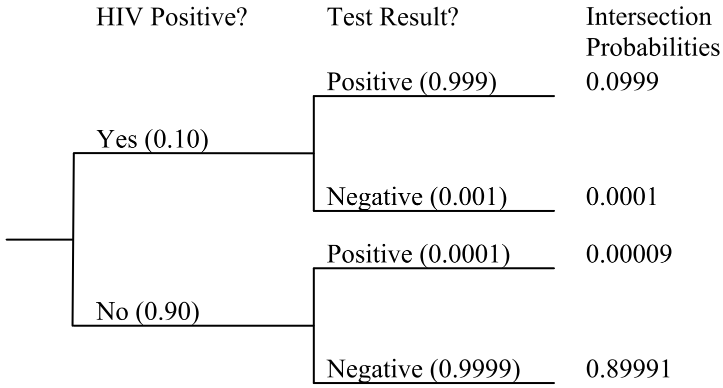
b) If the population is half male and half female, the proportion of the population that is color blind is 525/20,000 = 0.026.

	Color Blind	**Not Color Blind**	**Total**
Male	500	9500	10,000
Female	25	9975	10,000
Total	**525**	**19,475**	**20,000**

c) Given that a randomly chosen person is color blind, the probability that the person is female is 25/525 = 0.048.

5.91 HIV testing

a)



b)

	Positive	Negative	Total
HIV	0.0999	0.0001	0.1
No HIV	0.00009	0.89991	0.9
Total	**0.09999**	**0.90001**	**1.0**

c) Given that someone has a positive test result, the probability that this person is truly HIV positive is 0.0999/0.09999 = 0.999.

d) A positive result is more likely to be in error when the prevalence is lower as relatively more of the positive results are for people who do not have the condition. With fewer people with HIV, the chances of a false positive are higher. The contingency tables below demonstrate that with a prevalence rate of 10%, there is likely to be one false positive out of 1000 positive tests, whereas with a prevalence rate of 1%, there is likely to be 1 false positive out of only 101 positive tests.

10% Rate	**Positive**	**Negative**	**Total**
HIV	999	1	1000
No HIV	1	8999	9000
Total	**1,000**	**9000**	**10,000**

1% Rate	**Positive**	**Negative**	**Total**
HIV	100	0	100
No HIV	1	9899	9900
Total	**101**	**9899**	**10,000**

5.92 Prostate cancer

a) If 10% of those who took the PSA test truly had prostate cancer, the probability that a man truly had prostate cancer, given that he had a positive test, is 0.086/0.689 = 0.125.

	Positive	Negative	Total
Prostate cancer	0.086	0.014	0.10
No prostate cancer	0.603	0.297	0.90
Total	**0.689**	**0.311**	**1.00**

b)

	Positive	Negative	Total
Prostate cancer	86	14	100
No prostate cancer	603	297	900
Total	**689**	**311**	**1000**

5.92 (continued)

c) If the cases increase for which a test is positive, the sensitivity will go up because more people will have positive tests – this group will include more of the people who actually have prostate cancer. On the other hand, because more people are testing positive for prostate cancer, the probability of a false positive is increasing as well. This means that the specificity, the probability that someone has a negative test given that they don't have prostate cancer, will go down. An increase in false positives means a decrease in correct negatives.

5.93 U Win

Answers will vary. To set up the simulation, one can assign the digit 0 to the letter U, assign the digits 1–3 to the letter W, assign the digits 4–6 to the letter I and the digits 7–9 to the letter N. Choose a row of the random number table to start and read off sets of 5 digits at a time. In this simulation, it is possible to receive the same letter more than once, so duplicate digits should not be discarded.

5.94 Win again

Answers will vary. To set up the simulation, one can make the letter assignments as in Exercise 5.93 and then select random digits until one of each letter has been chosen. Record the number of random digits that were required. Repeat this process 20 times. To estimate the expected number of combo meals one would need to purchase in order to win the free shake, sum the required number of digits from the 20 repetitions and divide by 20.

Chapter Problems: Concepts and Investigations

5.95 Simulate law of large numbers

a) The cumulative proportions for (i) through (iv) will differ for each student who conducts this exercise. Students will notice, however, that the cumulative proportion of heads approaches 0.50 with larger numbers of flips. This illustrates the law of large numbers and the long-run relative frequency definition of probability in that as the number of trials increases, the proportion of occurrences of any given outcome (in this case, of heads) approaches the actual proportion in the population "in the long run."

b) The outcome will be similar to that in (a), with the cumulative proportion of heads approaching one third with larger numbers of flips.

5.96 Illustrate probability terms with scenarios

a) The sample space is the set of possible outcomes for a random phenomenon.

(i) Examples will differ for each student. One example for a designed experiment might involve participants being randomly assigned to either listen to music or sit in silence while attempting to solve a puzzle, with the possibilities of success or failure. The sample space would consist of music and success, music and failure, silence and success, and silence and failure.

(ii) Examples will differ for each student. One example for an observational study is what students choose for a drink and a snack from a vending machine. If students must choose one of each, and the drink choices are regular and diet soda, and the snack choices are cookies and chips, then the sample space includes regular soda and cookies, regular soda and chips, diet soda and cookies, and diet soda and chips.

b) Disjoint events are events that do not share any outcomes in common. Examples will differ for each student. One example of two events that are disjoint are having your first-born child be a girl, and having your first-born child be a boy.

c) A conditional probability occurs when one assesses the probability of one event occurring given that another event already has occurred.

(i) Examples will differ for each student. One possible example of independent events from everyday life is the car in front of you on the highway. The car in front of you on your commute home is likely independent of the car in front of you on your commute to school.

(ii) Examples will differ for each student. One possible example of dependent events from everyday life is meals. What you eat for dinner likely depends to some degree on what you ate for lunch.

5.97 Short term versus long run

a)
   (i) The cumulative proportion of heads would be 59/110 = 0.536.
   (ii) The cumulative proportion of heads is now 504/1010 = 0.499.
   (iii) The cumulative proportion of heads is now 4950/10,010 = 0.4945.

b) As n increases, the cumulative proportion tends toward 0.494. Short-term aberrations do not affect the long run.

5.98 Risk of space shuttle

a) We often form our beliefs about probability based on subjective information rather than solely on objective information such as data. We assess the probability of an outcome by taking into account all available information. In this case, the available information is 20 safe missions, which leads NASA workers to trust the safety of the missions more than after just one mission.

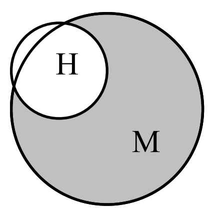
b) The problem with this belief is that each particular mission has no memory of the previous mission. A given mission has pretty much the same risk of disaster as another mission. Of course, missions are not entirely independent of other missions, so the risks might differ somewhat. With a coin flip, each event truly is independent of the previous one. Having 20 heads in a row gives us no information about the outcome of the next flip. Similarly, having 20 safe missions in a row doesn't necessarily predict the outcome of the next mission.

5.99 Mrs. Test

(1) 99% accurate could refer to specificity, meaning that if you are pregnant, you'll have a positive test 99% of the time. (2) It could refer to sensitivity, meaning that if you're not pregnant, you'll have a negative test 99% of the time. (3) 99% chance that you are pregnant given a positive test. (4) 99% chance that you are not pregnant given a negative test.

5.100 Marijuana leads to heroin?

The H set is very small compared to the M set and the H set is almost completely contained within the M set.



5.101 FIFA World cup

a) Use the extension of the multiplication rule with conditional probabilities. P(groups stage) × P(round of 16 | groups stage) × P(quarterfinals | groups stage and round of 16) × P(semifinals | groups stage, round of 16, and quarterfinals) × P(final match | groups stage, round of 16, quarterfinals and semifinals) = (32/32)(16/32)(8/16)(4/8)(2/4) = 0.0625 = (2/32).

b) Of those who played round of 16, 50% get into quarter finals; of these, 50% get into semifinals; of these, 50% get into the final match (0.50)(0.50)(0.5) = 0.125.

5.102 How good is a probability estimate?

a) $1/\sqrt{n} = 1/\sqrt{5282} = 0.014$; Thus, the margin of error would lead to a predicted range of 0.257 – 0.014 = 0.243 to 0.257 + 0.014 = 0.271.

b) The margin of error gets smaller and smaller (approaches zero) as n gets larger. The implication of this is that the estimate of probability is more accurate with a larger n.

5.103 Protective bomb

The fallacy of his logic is that the event of a person bringing a bomb is independent of the event of any other person bringing a bomb. Thus, if the probability of one person bringing a bomb on the plane is one in a million, that is true whether or not this person has a bomb on the plane. The probability of another person bringing a bomb given that this person has a bomb is the same as the probability of another person bringing a bomb given that this person does not have a bomb.

5.104 Streak shooter

a) Over the course of a season, there will be many runs. Randomness produces runs. For example, if you flip a coin six times, it's not unlikely that in the long run, you'll have streaks of six heads. Using Coselli's logic, this would only have a $(0.5)^6 = 0.016$ chance, but over the long run, such streaks will occur just randomly. In fact, if you flip a balanced coin 2000 times, the longest run of heads you can expect during those flips is about 10! In the current example, you should think about whether this is like seeing six baskets in a row on the next six shots, or more like seeing six baskets in a row in a long series of shots (over a season, for example).

b) This example demonstrates how things that look highly coincidental may not be so when viewed in a wider context. If you choose only the "streaks" and ignore the rest of the season, the sets of baskets in a row look like streaks. If you look at the context of the whole season, the "streaks" look like the kinds of runs that would occur if each basket were truly independent of the others.

5.105 Multiple choice

Both (c) and (d) are correct.

5.106 Multiple choice

The best answer is (d).

5.107 Multiple choice: Coin flip

The best answer is (e).

5.108 Multiple choice: Dream come true

The best answer is (b).

5.109 Multiple choice: Comparable risks

The best answer is (b).

5.110 True or false

a) False, any given sequence is equally likely to happen as any other possible sequence.

b) True, there is only one sequence of ten flips resulting in ten heads but there are many combinations resulting in five heads.

5.111 Obesity in Australia

Statement (a) is false because the sample space is a set of four non-equally likely outcomes. The probability that both the two selected Australian adults are overweight or obese is 0.634^2, while the probability that none of them is overweight or obese is $(1 - 0.634)^2$.

Statement (b) is false because the event that only one of them is overweight or obese can occur in two different ways. Therefore, the probability of this event should be two times the suggested probability in (b).

5.112 Driving versus flying

The reasoning behind this statement is that driving has a higher probability of death than does flying. Thus, the more people who switch to driving, the higher the rate of death.

5.113 Prosecutor's fallacy

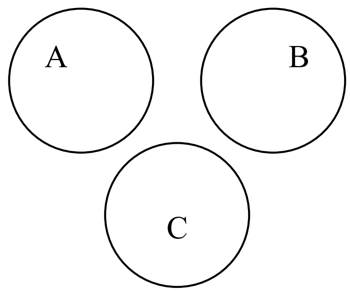
Being not guilty is a separate event from the event of matching all the characteristics listed. It might be easiest to illustrate with a contingency table. Suppose there are 100,000 people in the population. We are given that the probability of a match is 0.001. So out of 100,000 people, 100 people would match all the characteristics. Now the question becomes, of those 100 people, what is the probability that a person would not be guilty of the crime? We don't know that probability. However, let's SUPPOSE in the population of 100,000, 5% of the people could be guilty of such a crime, 95% are not guilty. Here's a contingency table that illustrates this concept.

5.113 (continued)

	Guilty	Not Guilty	Total
Match		???	100
No Match			99,900
Total	**5000**	**95,000**	**100,000**

Thus, the P(not guilty | match) = ???/ 100.

♦♦5.114 Generalizing the addition rule



As we can see from the Venn diagram, disjoint events don't overlap. Thus, the probability of any one of them occurring is the sum of the probability of each one occurring.

♦♦5.115 Generalizing the multiplication rule

When two events are not independent, P(A and B) = P(A) $\times$ P(B | A); if we think about (A and B) as one event, we can see that P[C and (A and B)] = P(A and B) $\times$ P(C | A and B). If we replace P(A and B) with its equivalent, P(A) $\times$ P(B | A), we see that: P(A and B and C) = P(A) $\times$ P(B | A) $\times$ P(C | A and B).

♦♦5.116 Bayes's rule

a) P(A | B) = P(A and B)/P(B) and P(B | A) = P(A and B)/P(A)
If we rearrange the second of the above formulas, we get P(A and B) = P(A)P(B | A).
Now, we can replace P(A and B) in the first formula in the first line above with the rearrangement of the second formula. We now get: P(A | B) = [P(A)P(B | A)]/P(B).

b) The probability that B occurs is the sum of the probability that B occurs and A occurs, and the probability that B occurs and A does not occur. Thus, we're adding the probabilities that B occurs in both the presence and the absence of A.

c) If we replace P(B and A) with the rearrangement of the formula above, P(A)P(B | A), and P(B and A^c) with a similar rearrangement, P(A^c)P(B | A^c), we can rearrange P(B) = P(B and A) + P(B and A^c) to get the formula in the exercise: P(B) = P(A) P(B | A) + P(A^c)/P(B | A^c)

d) Based on the information in (a) through (c), we can replace parts of the following formula: P(A | B) = P(A and B)/P(B). The numerator P(A and B) can be replaced with the rearrangement from (a). The denominator P(B) can be replaced with the rearrangement from (c).
P(A | B) = P(A)P(B | A) / [P(A) P(B | A) + P(A^c)P(B | A^c)]

Chapter Problems: Student Activities

5.117 Simulating matching birthdays

a) The results will be different each time this exercise is conducted.

b) The simulated probability should be close to 1.

5.118 Simulate table tennis

a) The probability of winning a point will be different each time this exercise is conducted. Theoretically, the probability of winning a game is the probability that you get 11 points while your opponent still gets at less than 10 points. That is, $\sum_{k=10}^{9}\binom{10+k}{10}(0.45)^{11}(0.55)^{k}$ = 0.25 (using Excel).

b) The results will be different each time this exercise is conducted.

5.119 Which tennis strategy is better?

a) The results will be different each time this exercise is conducted.
b) The results will be different each time this exercise is conducted.

5.120 Saving a business

a) The results will be different each time this exercise is conducted.
b) The results will be different each time this exercise is conducted.
c) The results will be different each time this exercise is conducted.