Section 8.1: Point and Interval Estimates of Population Parameters

8.1 Health care

a) This study will estimate the population proportion who have health insurance and the mean dollar amount spent on health insurance by the population.
b) The sample proportion and the sample mean can be used to estimate these parameters.

8.2 Video on demand

a) This study will estimate the proportion of U.S. adults watching content time-shifted and the mean amount of hours spent watching over the Internet.
b) The proportion in the sample watching content time-shifted and the mean number of hours watching over the Internet of those sampled can be used to estimate these parameters.
c) The sample mean number of hours watched online is the unbiased estimator of the mean number of hours watched online for the entire population of U.S. adults. The sampling distribution of the mean number of hours watched online is centered at the true mean number of hours watched online in the population.

8.3 Projecting winning candidate

a) The point estimate is 54.8%, or 0.548.
b) The interval estimate is given by the point estimate plus or minus the margin of error: 0.548 – 0.03 = 0.518 to 0.548 + 0.03 = 0.578.
c) A point estimate is one specific number, such as a proportion. An interval estimate is a range of numbers.

8.4 Youth professional football

The point estimate is 313/1009 = 0.31 or 31%.

8.5 Government spying

a) The point estimate is the proportion of the sample: 900/1000 = 0.90, or 90%.
b) With a probability of 95%, the point estimate of 0.9 falls within a distance of 0.045 of the actual proportion of German citizens who find it unacceptable.

8.6 Game apps

a) The sample point estimate is the mean of these responses, (1.09 + 4.99 + 1.99 + 1.99 + 2.99)/5 = $2.61.
b) With a probability of 95%, the point estimate of $2.61 falls within a distance of $1.85 of the actual mean fee charged for paid games in the app store.

8.7 Nutrient effect on growth rate

a) The point estimate is the mean of the heights of the six tomato plants, 62.2.
b) The interval would be: 62.2 – 4.9 = 57.3 mm to 62.2 + 4.9 = 67.1 mm. It includes all heights within one margin of error on either side of the mean.
c) A point estimate alone may be highly inaccurate, especially with a small sample. An interval estimate gives us a sense of the accuracy of the point estimate.

8.8 More youth professional football

a) The margin of an error for a 95% confidence interval would be 1.96 times the standard deviation of 0.015, which is 0.03. It is very likely that the population proportion is no more than 0.03 lower or 0.03 higher than the reported sample proportion.
b) The 95% confidence interval includes all points within the margin of error of the mean. Lower endpoint: 0.31 – 0.03 = 0.28; upper endpoint: 0.31 + 0.03 = 0.34. The confidence interval goes from 0.28 to 0.34. This is the interval containing the most believable values for the parameter.

8.9 Feel lonely often?

a) This solution uses the 1972–2010 Cumulative data file.

Response	Percentage
0	54.0
1	13.9
2	8.6
3	6.6
4	4.1
5	3.0
6	1.4
7	8.4

The mean is 1.5 and the standard deviation is 2.21. Most respondents said that they were never lonely, but on the average subjects were lonely 1.5 days a week.

b) The standard deviation of the sample mean, 0.06, refers to the standard deviation of the sampling distribution for samples of size 1450.

8.10 CI for loneliness

The confidence interval would range from 1.5 – 0.12 = 1.38 to 1.5 + 0.12 = 1.62. This is the range that includes the most believable values for the population mean.

8.11 Barack Obama as president

a) The population parameter is "Obama job approval proportion", the value of the sample for the period from 5 April to 7 April 2016 is 0.51, the point estimate is 0.51, and the margin of error is 0.03.

b) It is very likely that the population proportion is no more than 0.03 lower or 0.03 higher than the reported sample proportion.

Section 8.2: Constructing a Confidence Interval to Estimate a Population Proportion

8.12 Putin

For a 95% confidence interval, multiply 1.96, the *z*-score for a 95% confidence interval, by the standard error. The margin of error is $1.96\sqrt{\hat{p}(1-\hat{p})/n} = 1.96\sqrt{0.83(1-0.83)/2000} = 0.016$, or 1.6%.

8.13 Flu shot

a) The point estimate of the proportion of the population who were victims would be 24/3900 = 0.00615.

b) The standard error would be $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.00615(1-0.00615)/3900} = 0.00125$.

c) The margin of error would be (1.96)(0.00125) = 0.00245.

d) The 95% confidence interval would include all proportions within one margin of error of the mean proportion of 0.00615, namely 0.00615 – 0.00245 = 0.0037 to 0.00615 + 0.00245 = 0.0086, or (0.0037, 0.0086). We are 95% confident that the proportion of people receiving the flu shot but still developing the flu is between 0.37% and 0.86%.

e) Yes. The upper limit of the confidence interval is 0.86%, which is less than 1%.

8.14 Renewable energy usage in India

a) The point estimate is 1479/1700 = 0.87.

b) The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.87(1-0.87)/1700} = 0.008$. The margin of error is (1.96)(0.008) = 0.016.

c) 0.87 – 0.016 = 0.854
0.87 + 0.016 = 0.886.
The numbers represent the most believable values for the population proportion.

d) We must assume that the data were obtained randomly, and that a large enough sample size is used so that the number of successes and the number of failures both are greater than 15. Both seem to hold true in this case.

8.15 Make industry help environment?

a) The data must be obtained randomly, which the text assures us we can assume for the GSS. We also must assume that the number of successes and the number of failures both are greater than 15, also true in this case.

b) The point estimate is 1403/1497 = 0.937.
The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.9372(1-0.9372)/1497} = 0.006$.
0.937 – (1.96)(0.006) = 0.925
0.937 + (1.96)(0.006) = 0.949
We can be 95% confident that the proportion of the population who believe it should be the government's responsibility to impose strict environmental laws is between 0.937 – 0.925 = 0.925 and 0.937 + 0.949 = 0.949. Since the lower limit is above 50%, we can conclude that a majority of the population would answer yes.

8.16 Favor death penalty

a) We can obtain the value reported under "Sample p" by dividing the number of those in favor by the total number of respondents, 1183/1824.

b) We can be 95% confident that the proportion of the population who are in favor of the death penalty is between 0.626665 and 0.670484, or rounding, (0.627, 0.670).

c) 95% confidence refers to a probability that applies to the confidence interval method. If we use this method over and over for numerous samples, in the long run we make correct inferences (that is, the confidence interval contains the parameter) 95% of the time.

d) We can conclude that more than half of all American adults were in favor because all the values in the confidence interval are above 0.50.

8.17 Oppose death penalty

If 0.649 are in favor, then 1 – 0.649 = 0.351 are opposed. We can figure the margin of error by subtracting the proportion who favor the death penalty from one of the limits of the confidence interval. The margin of error, 0.649 – 0.627 = 0.022, can then be added to and subtracted from the proportion who are opposed to get a confidence interval of 0.351 – 0.022 = 0.329 to 0.351 + 0.021 = 0.373. These are also 1 minus the endpoints of the interval in the previous exercise.

8.18 Stem cell research

a) The "Sample p", 1521/2113 = 0.720, is the proportion of all respondents who believe that stem cell research has merit. The "95% CI" is the 95% confidence interval. We can be 95% confident that the population proportion falls between 0.701 and 0.739.

b) The margin of error is (0.7390 – 0.7007)/2 = 0.019.

8.19 *z*-score and confidence level

a) 1.645

b) 2.33

c) 3.29

8.20 Trusting CNN news?

We can be 95% confident that the population proportion of Americans likely voters, who appoint CNN as the TV news or commentary source they trust the most, is between 0.113 and 0.155.

8.21 Budget impact on opportunities for young Canadians

The inference applies to the population of Canadian adults. We can be 95% confident that the population proportion of Canadian adults who believe that the first federal budget delivered by Finance Minister Bill Morneau would have a positive impact on the opportunities for young Canadians is between 0.3 and 0.36.

8.22 Operations growth in Luxembourg

a) The data must be obtained randomly. We also must assume that the number of successes and the number of failures both are greater than 15.

8.22 (continued)

b) The mean is 0.46 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.46(1-0.46)/5000} = 0.007$. The confidence interval is 0.46 – (1.96)(0.007) = 0.446 ; 0.46 + (1.96)(0.007) = 0.474. We can be 95% confident that the proportion of the population who intend to grow their operations in Luxembourg over the next two years is between 0.446 and 0.474. We can conclude that a minority of the population would intend to grow their operations in Luxembourg over the next two years.

8.23 Chicken breast

a) The sample proportion is 207/316 = 0.655. The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} =$ $\sqrt{0.655(1-0.655)/316} = 0.0267$. The 99% confidence interval is $\hat{p} \pm z_{.005}(se) = 0.655 \pm 2.58(0.0267)$, or (0.586, 0.724). At the 99% confidence level, a range of plausible values for the population proportion of chicken breast that contain E.coli is 0.586 to 0.724. We can conclude that the population proportion exceeds 50% because 50% is below the lowest believable value of the confidence interval.

b) The 95% confidence interval would be narrower because the margin of error will decrease. The *z*-score will be 1.96 compared to 2.58 for the 99% confidence interval.

8.24 Dispute over unlocking iPhone

The 95% confidence interval is (0.48, 0.54). No, the proportion of Americans supporting Justice Department than Apple in the dispute over unlocking iPhone could be just below 50%.

8.25 Exit poll predictions

a) The sample proportion is 660/1400 = 0.471.
The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.471(1-0.471)/1400} = 0.013$.
The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.471 \pm 1.96(0.013)$, or (0.446, 0.496).
We could predict the winner because 0.50 falls outside of the confidence interval. It does not appear that the Democrat received more than half of the votes.

b) The 99% confidence interval $\hat{p} \pm z_{.005}(se) = 0.471 \pm 2.58(0.013)$, or (0.437, 0.506). We now cannot predict a winner because it is plausible that the Democrat received more than half of the votes. The more confident we are, the wider the confidence interval.

8.26 Exit poll with smaller sample

a) The sample mean proportion is the same, 0.471.
The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.471(1-0.471)/140} = 0.042$.
The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.471 \pm 1.96(0.042)$, or (0.389, 0.553).
We cannot predict the winner because 0.50 falls within this interval. It is possible that the Democrat received more than half of the votes.

b) Larger sample sizes increase the denominator, making the standard error smaller. A smaller standard error leads to smaller margins of error and narrower confidence intervals.

8.27 Simulating confidence intervals

The results of the simulation will be different each time it is conducted. The percentage we'd expect would be 95% and 99%, but the actual values may differ a bit because of sampling variability.

8.28 Simulating confidence intervals with poor coverage

a) The results of the simulation will be different each time it is conducted, but around 15% would fail to contain the true value.

b) We would expect 5% not to contain the true value given that we're using a 95% confidence interval. This suggests at least one requirement for calculating the 95% confidence interval has not been met.

c) The results of the simulation will be different each time it is conducted, but around 15% would still fail to contain the true value.

d) The results of the simulation will be different each time it is conducted, but the distribution will be right-skewed, suggesting the central limit theorem does not apply.

Section 8.3: Constructing a Confidence Interval to Estimate a Population Mean

8.29 Average temperature in Florida

a) The point estimate of the population mean is 64.264°F.

b) The standard error of the sample mean is $s/\sqrt{n} = 3.109/\sqrt{122} = 0.281$.

c) We were 95% confident that the population mean falls between 63.707 and 64.821.

d) It is not plausible that the population mean is 63°F because it falls outside of the confidence interval.

8.30 Average temperature in the United States

a) The point estimate of the population mean is 41.699°F and the standard error is $s/\sqrt{n} = 2.948/\sqrt{122} = 0.267$.

b) We can be 95% confident that the population mean falls between 41.170 and 42.227. If random samples of size 122 were repeatedly drawn under the same conditions and 95% confidence intervals were constructed for each sample, the proportion of these intervals that would contain the population mean would be about 0.95.

c) Yes. The sample means are very different with roughly the same standard deviations, and the two intervals do not overlap at all. The means for Florida and the United States are very different.

8.31 Using *t* table

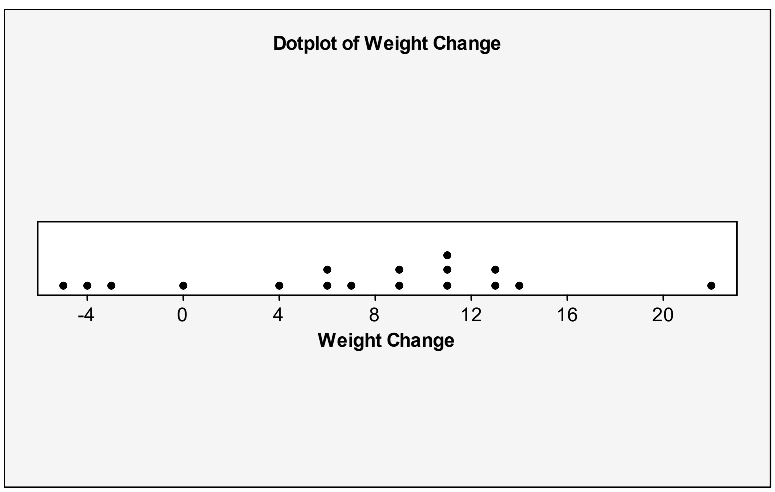
a) 2.776

b) 2.145

c) 2.977

8.32 Anorexia in teenage girls

a)



b) The mean and standard deviation can be verified on MINITAB, or other technology.

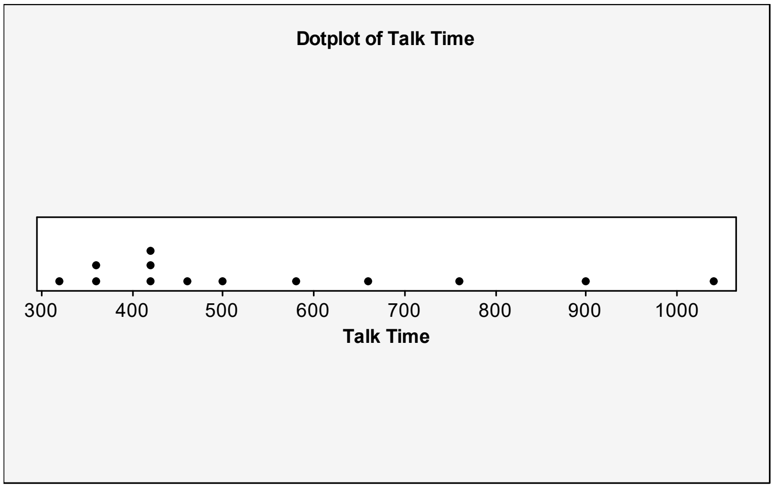
c) The standard error can be verified on MINITAB.

d) $df = n - 1 = 17 - 1 = 16$; a 95% confidence interval uses the *t*-score equal to 2.120 because that is the *t*-score for 16 degrees of freedom and a confidence interval of 95% (.025 beyond the *t*-score on either side).

e) The margin of error would be (2.120)(1.74) = 3.7; therefore, the confidence interval would be 7.29 – 3.7 = 3.59 to 7.29 + 3.6 = 10.89. The true mean weight change is likely positive because no negative scores fall in the confidence interval. The true mean weight change also could be very small, because the lower end point of 3.6 pounds is near 0.

8.33 Talk time on smartphones

a)



The shape of the distribution is right-skewed. The assumptions are a random sample (fulfilled) and an approximately normal distribution (questionable because of right-skew, but the *t*-interval is robust to deviation from normal). The outlier at 1050 might make validity of results questionable.

b) (i) IQR = 650 – 420 = 230; 1.5 × IQR = (1.5)(230) = 345
Q1 – IQR = 420 – 345 = 75 and Q3 + IQR = 650 + 345 = 995
There is one potential outlier according to this criterion: 1050.

(ii) $\bar{x} - 3s = 553 - 3(227) = -128$ and $\bar{x} - 3s = 553 + 3(227) = 1234$, so there are no potential outliers according to this criterion.

c) The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 553 \pm 2.179\left(227/\sqrt{13}\right)$, or (416, 690). We can be 95% confident that the population mean talk time is between 416 and 690 minutes.

d) With 1050 removed: $\bar{x} = 512$, $s = 178$, $df = 11$.

The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 512 \pm 2.201\left(178/\sqrt{12}\right)$, or (399, 625). The new confidence interval is narrower than the one using all of the data and centered at a lower value (512 instead of 553).

8.34 Birth weights of elephants

a) According to StatCrunch:
95% confidence interval results:
μ: Mean of variable

Variable	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
Elephant weight	116	6.4420494	4	98.114004	133.886

b) We could increase the sample size or decrease the confidence level for the confidence interval.

c) According to StatCrunch:
99% confidence interval results:
μ : Mean of variable

Variable	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
Elephant weight	116	6.4420494	4	86.340182	145.65982

It is wider because we have more assurance of a correct inference by using a higher confidence level, which gives a larger *t*-value for the margin of error.

d) The assumptions in (a) are that the data are produced randomly and the population distribution is approximately normal. The former is more important than the latter. This method is robust in terms of the normal population assumption.

8.35 Buy it now

a) The assumptions are a random sample (fulfilled) and approximately normal distribution of the Buy It Now price. The dotplot and box plot suggest a bell-shaped distribution except for an outlier indicated by the box plot that might make results questionable. However, its z-score is $\frac{675-630.8}{20.47} = 2.2$, so it is within three standard deviations of the mean.

b) The standard error is $s/\sqrt{n} = 20.47/\sqrt{9} = 6.82$.

c) We are 95% confident that the population mean But It Now price of the iPhone 5s on eBay is between \$615 and \$647.

d) Yes, all plausible values for the mean Buy It Now price are larger than the plausible values for mean closing price of auctions.

e) No, the interval computed without \$675 still lies above (569, 599).

8.36 Time spent on e-mail

a) The margin of error is $t_{.025}\left(s/\sqrt{n}\right) = 1.96\left(13.05/\sqrt{1050}\right) = 0.79$. The margin of error can also be found using the confidence interval given in the exercise; (7.680 – 6.100)/2 = 0.79.

b) We have 95% confidence that the mean number of hours spent on e-mail per week is between 6.6 and 7.1 hours.

c) There is no concern because the sample size is large (n = 1050). The central limit theorem applies, and the sampling distribution of the sample mean is approximately normal.

8.37 Grandmas using e-mail

a) The mean is 3.07, the standard deviation is 3.38, and the standard error is $s/\sqrt{n} = 3.38/\sqrt{14} = 0.90$.

b) The 90% confidence interval is $\bar{x} \pm t_{.05}(se) = 3.07 \pm 1.771(0.90)$, or (1.47, 4.67). We are 90% confident that the population mean number of hours per week spent sending and answering e-mail for women of at least age 80 is between 1.5 and 4.7 hours.

c) Many women of age 80 or older will spend 0 or 1 hour per week on e-mail, with only a few spending more time. This leads to a right-skewed distribution. However, the t-interval is robust against departures from the normal distribution assumption. Also, there are no outliers in the preceding sample, so the inference should still be valid.

8.38 Wage discrimination?

a) The confidence interval refers to means, not individual scores.

b) This should say, "if random samples of 9 women were repeatedly selected, then 95% of the time, *the confidence interval would contain the population mean*."

c) We know that $\bar{x}$ falls in the confidence interval; this is the sample mean on which the confidence interval is based.

d) If we sample the entire population and take the mean, we're going to get the exact mean of the population.

8.39 How often read a newspaper?

a) It is not plausible that $\mu = 7$. This value is well outside the confidence interval.

b) The sample mean is 4.1, the standard error is $s/\sqrt{n} = 3.0/\sqrt{240} = 0.194$.

The confidence interval is $\bar{x} \pm t_{.025}(se) = 4.1 \pm 1.97(0.194)$, or (3.7, 4.5).

If the sample size increased to 240, the margin of error would decrease.

c) The standard deviation is fairly large relative to the mean, an indication that the population distribution might be skewed. (The lowest possible value of 0 is only (0 – 4.1)/3.0 = –1.37, or 1.37 standard deviations below the mean.) This would not affect the validity of this analysis because the sample size is bigger than 30. With a large random sample size, the sampling distribution approaches a bell shape.

d) The term "robust" means that even if the normality assumption is not completely met, this analysis is still likely to produce valid results.

8.40 Works hours per week

a) To construct the confidence interval, we would first calculate the margin of error. The margin of error equals the standard error, 1.963, multiplied by the t-score for 39 degrees of freedom (n – 1) and a 95% confidence interval (2.02). To calculate the confidence interval, we would then subtract the margin of error from the mean, 38.7, to get the lower limit of the confidence interval and add it to the mean to get the higher limit of the confidence interval.

b) We can conclude that the population mean is not larger than 43 because 43 falls above the upper endpoint of the confidence interval.

c) (i) A 99% confidence interval would be wider than a 95% confidence interval.

(ii) A larger sample size would lead to narrower confidence interval than would a small sample size.

8.41 Length of hospital stay for child birth

That indicates that the sample mean is expected to fall within 1.0 of the true mean about 95% of the time. In other terms, the confidence interval extends from 0.029 below the mean to 0.029 above the mean: 2.343 to 2.401.

$$s = \text{Margin of error} \times \frac{\sqrt{n}}{t_{0.025}} = 0.029 \times \frac{\sqrt{2962}}{1.96} = 0.805.$$

8.42 Effect of *n*

a) The standard error is $s/\sqrt{n} = 100/\sqrt{25} = 20$ and the margin of error is (2.0639)(20) = 41.3.

b) The standard error is $s/\sqrt{n} = 100/\sqrt{100} = 10$ and the margin of error is (1.9842)(10) = 19.8.

As the sample size increases, the margin of error decreases.

8.43 Effect of confidence level

i) The standard error is $s/\sqrt{n} = 100/\sqrt{25} = 20$ and the margin of error is (2.0639)(20) = 41.3.

ii) The margin of error is (2.7969)(20) = 55.9. The margin of error increases as the chosen confidence level increases.

8.44 Catalog mail-order sales

a) It is not plausible that the population distribution is normal because a large proportion are at the single value of 0. Because we are dealing with a sampling distribution of a sample greater than size 30, this is not likely to affect the validity of a confidence interval for the mean. Large random samples lead to sampling distributions of the sample mean that are approximately normal.

b) The sample mean is 10, the standard error is $s/\sqrt{n} = 10/\sqrt{100} = 1$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 10 \pm 1.984(1)$, or (8.0, 12.0). It does seem that the sales per catalog declined with this issue. $15 is not in the confidence interval, and, therefore, is not a plausible population mean for the population from which this sample came.

8.45 Number of children

a) The standard error is $s/\sqrt{n} = 1.67/\sqrt{1971} = 0.04$.

b) The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 1.89 \pm 1.96(0.04)$, or (1.81, 1.97). We can conclude that the population mean is less than 2.0 because the entire confidence interval is below 2.0.

8.46 Simulating the confidence interval

a) The results will differ each time this exercise is conducted.

b) We would expect 5% of the intervals not to contain the true value.

c) Close to 95% of the intervals contain the population mean even though the population distribution is quite skewed. This is so because with a large random sample size, the sampling distribution is approximately normal even when the population distribution is not. The assumption of a normal population distribution becomes less important as n gets larger.

Section 8.4: Choosing the Sample Size for a Study

8.47 Unemployment percentage

$$n = \frac{\hat{p}(1-\hat{p})z^2}{m^2} = \frac{0.5(1-0.5)1.96^2}{0.05^2} = 384.16.$$

385 families are needed to estimate the population proportion of families having at least one unemployed member within 0.05 at a 95% confidence level.

8.48 Binge drinkers

$$n = \frac{\hat{p}(1-\hat{p})z^2}{m^2} = \frac{0.44(1-0.44)1.96^2}{0.05^2} = 379$$

8.49 Abstainers

a) $n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.5(1-0.5)1.96^2}{0.05^2} = 385$

b) $n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.19(1-0.19)1.96^2}{0.05^2} = 237$

c) Strategy (a) is inefficient if we are quite sure we'll get a sample proportion that is far from 0.50 because it overestimates the sample size by quite a bit. The first sample size would be more costly than needed.

8.50 How many businesses fail?

a) $n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.5(1-0.5)1.96^2}{0.10^2} = 97$

b) $n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.5(1-0.5)1.96^2}{0.05^2} = 385$

c) $n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.5(1-0.5)2.575^2}{0.05^2} = 664$

d) As the margin of error decreases, we need a larger sample size to guarantee estimating the population proportion correct within the given margin of error and at a given confidence level. As the confidence level increases, we also need a larger sample size to get the desired results.

8.51 Employment percentage in the United States

Using the prior estimation for the population proportion we have from 2015, we obtain

$$n = \frac{\hat{p}(1-\hat{p})z^2}{m^2} = \frac{0.803(1-0.803)1.96^2}{0.03^2} = 675.23.$$

676 families are needed to estimate the population proportion of families having at least one unemployed member in the U.S. within 0.03 at a 95% level of confidence.

8.52 Farm size

a) $n = \dfrac{\sigma^2 z^2}{m^2} = \dfrac{(200)^2(1.96)^2}{25^2} = 246$

b) We can use the same formula as in (a):

$$246 = \frac{(300)^2(1.96)^2}{m^2} \Rightarrow m^2 = \frac{(300)^2(1.96)^2}{246} \Rightarrow m = \sqrt{\frac{(300)^2(1.96)^2}{246}} = 37.5$$

8.53 Income of Native Americans

We guess that the range of a bell-shaped distribution is about 6*s*. One sixth of the range, 120,000, is 20,000, a good guess for the standard deviation.

$$n = \frac{\sigma^2 z^2}{m^2} = \frac{(20,000)^2(2.58)^2}{1000^2} = 2663$$

8.54 Population variability

For a very diverse population, we'd have a wider range of observed values, and hence, a larger standard deviation. Larger standard deviations result in larger standard errors and wider confidence intervals. For a homogeneous population, we would have a smaller standard deviation, and would not need the large sample size for the denominator of the standard error formula. When estimating the mean income for all medical doctors in the U.S., we'd have a fairly wide income: from the lower range of incomes of rural family doctors to the extremely high range of incomes of specialists at major teaching hospitals. A sample from this population would likely have a large standard deviation. For a population of McDonald's entry-level employees in the U.S., however, we'd have a much smaller range. They'd all likely be making minimum wage or slightly higher. The standard deviation of a sample from this population would be relatively small.

8.55 Web survey to get large *n*

They are better off with the random sample of 100 responses than with the website. The website does not produce the data randomly, one of the assumptions to make the inferences we've been discussing.

8.56 Do students like statistics?

a) The sample proportion is 10/10 = 1.0.

b) The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{1(1-1)/10} = 0$. The usual interpretation of standard error does not make sense. The sampling distribution is likely to have some variability.

c) The margin of error is (1.96)(0) = 0, and thus, the confidence interval is 1.0 to 1.0. It is not reasonable to conclude that all students at the college like statistics. The ordinary large-sample approach works poorly in this case because we do not have more than 15 successes and 15 failures.

d) It is more appropriate to add two to the successes and the failures and then repeat the process. This would give twelve who said they liked statistics, and two who said they did not like, for a total of 14. The new sample proportion would be 12/14 = 0.857.
The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.857(1-0.857)/14} = 0.094$.
The margin of error is (1.96)(0.094) = 0.184.
The confidence interval is 0.673 to 1.041 (or 1.0 because p cannot exceed 1).
We can be 95% confident that the proportion of students who like statistics is within this interval.

8.57 Movie recommendation

a) We first add two to the numbers of successes and failures. We then have 10 successes and 6 failures, for a total of 16. The sample proportion would be 10/16 = 0.625 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.625(1-0.625)/16} = 0.121$.
The margin of error is (1.96)(0.121) = 0.237 and the confidence interval is 0.388 to 0.862.

b) It is plausible that it is successful for only half the population. 0.50 is within the confidence interval.

8.58 Google Glass

a) The sample proportion is 0/500 = 0 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0(1-0)/500} = 0$.

b) The margin of error is (1.96)(0) = 0, so the confidence interval is 0 – 0 = 0 to 0 + 0 = 0, or (0, 0). It is not sensible, the true proportion may be very small, but it is larger than zero. (Google sold some glasses.)

c) With this small a sample, we expect 500(0) = 0 successes, which is well below the 15 required for the large sample method to work. Adding two to the number of successes and failures, we have 2 successes and 502 failures, for a total of 504. The sample proportion is now 2/504 = 0.004 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.004(1-0.004)/504} = 0.0028$. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.004 \pm 1.96(0.0028)$, or (0, 0.0095) since the lower proportion cannot be negative.

d) Yes it is plausible, since the entire interval lies below 0.01.

Section 8.5: Using Computers to Make New Estimation Methods Possible

8.59 Why bootstrap?

One purpose of using the bootstrap method is to determine confidence intervals for a given point estimate when we do not have a standard error or confidence interval formula that works well.

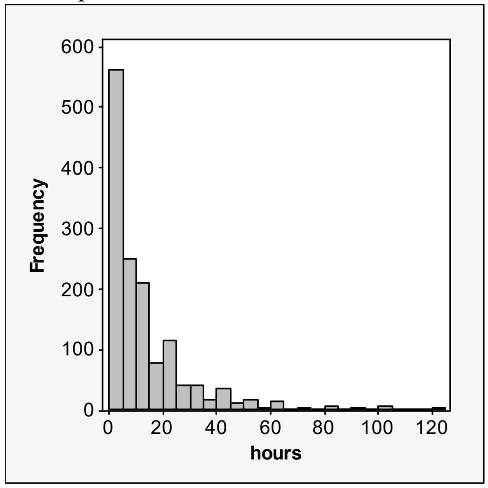
8.60 Bootstrap technique for estimation

a) We would sample with replacement from the 10 closing speed values, taking 10 observations and finding the standard deviation. We would do this many, many times–10,000 times, perhaps. The 95% confidence interval would be the values in the middle 95% of the standard deviation values (from 2.5% to 97.5%).

b) Results are to be obtained using the book's web app or a software.

8.61 Bootstrap interval for the mean

a) The sample mean is 11.6, the sample median is 6.



b) The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 11.6 \pm 1.96\left(15/\sqrt{1399}\right)$, or (10.8, 12.4).

c) Sample, with replacement, the1399 values from the sample (where each value has a 1 in 1399 chance of being selected). Compute the mean for each such resample. Do this 10,000 times. The 2.5th and 97.5th percentile of these 10,000 values are the lower and upper bounds for a confidence interval for the mean.

d) Results will vary for each simulation. One simulation, using 10,000 resamples, resulted in a 95% confidence interval of (10.8, 12.4).

e) The bootstrap distribution of $\bar{x}$ looks approximately normal. This is not surprising, as the central limit theorem predicts that for large sample sizes (here 1399), the sampling distribution of will have this shape, regardless of the shape of the population distribution.

8.62 Bootstrap interval for the proportion

a) Results will vary for each simulation. One simulation resulted in a sample proportion of 0.014.

b) Results will vary for each simulation. One simulation resulted in a confidence interval of (0.04, 0.18).

c) Judging by the histogram, the sampling distribution is right-skewed and not bell shaped. Hence, the interval extends more to the right than to the left relative to the sample proportion.

Chapter Problems: Practicing the Basics

8.63 Unemployed college grads

a) These are point estimates.

b) The information here is not sufficient to construct confidence intervals. We also need to know sample sizes.

8.64 Approval rating for president

We could tell someone who had not taken a statistics course that we do not know the exact percentage of the population who approve of the job Barack Obama is doing as president, but we are quite sure that it is within 3% of 42%, that is, between 39% and 45%.

8.65 British monarchy

The first sample proportion is 0.86. It has a standard error $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.86(1-0.86)/1667} = 0.0085$, and the margin of error is $z_{0.025}(se) =$ (1.96)(0.0085) = 0.017.

The second sample proportion is 0.73. It has a standard error $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.73(1-0.73)/1667}$ 0.0109, and the margin of error is $z_{0.025}(se) =$ (1.96)(0.0109) = 0.021.

8.66 Technology and productivity

a) 46% is a point estimation of the proportion of all working online adults who consider that technology helps them to become more productive. A point estimation involves the use of sample data to calculate a single value to serve as a "best guess" or "best estimate" of an unknown population parameter.
b) This is only an estimate based on one sample of 535 working online adults. The true population percentage may not be exactly the same as the sample percentage.

8.67 Life after death

The 95% confidence interval is 0.81 – 0.018 = 0.792 to 0.81 + 0.018 = 0.828, or (0.792, 0.828). We can say that we are 95% confident that the population percentage of people who believe in life after death falls between 79.2% and 82.8%.

8.68 Female belief in life after death

X, 822, is the number of females who said that they believe in life after death out of N, 977, the total number of females in the sample, regardless of their response. "Sample p" is the proportion, 0.841, of females who said that they believe in life after death. "95.0% CI" is the 95% confidence interval; we can be 95% confident that the proportion of the population of females who believe in life after death is between 81.8% and 86.4%.

8.69 Work agreement for nannies

a) We must assume that the data were obtained randomly, the number of successes and number of failures are more than 15.
b) The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.75(1-0.75)/5000} = 0.006124$.

   The confidence interval is $\hat{p} \pm z(se)$. Thus the lower limit is 0.75 – 2.58(0.006124) = 0.07342 and the upper limit is 0.75 + 2.58(0.006124) = 0.07658.
   This is an illustration of how a very large sample size contributes to a small standard error by providing a very large denominator for the standard error calculation.
c) Because 70% falls below the lowest believable value in the confidence interval. We can conclude that more than 70% of all families in New York City who employ nannies, do not enter into a written work agreement with them.

8.70 Alternative therapies

This is not correct. A 95% confidence interval refers to an interval, not to a point estimate.

8.71 Population data

It doesn't make sense to construct a confidence interval because we have data for the entire population. We can actually know the proportion of vetoed bills; we don't have to estimate it.

8.72 Wife supporting husband

a) 122 said they strongly agree, and 359 said that they agree, for a total of 481. The sample size in 2008 was 1308.
b) From (b) the sample proportion is 481/1308 = 0.368 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} =$ $\sqrt{0.368(1-0.368)/1308} = 0.0133$.
c) The 99% confidence interval for the population proportion who would agree is $\hat{p} \pm z_{.005}(se) =$ $0.368 \pm 2.58(0.0133)$, or (0.33, 0.40). We can be 99% confident that the population proportion of women who agree with this statement falls within this interval.

8.73 Legalize marijuana?

a) 496 said "legal." 751 said "not legal." The sample proportions are 0.398 and 0.602, respectively.

b) The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.398(1-0.398)/1247} = 0.0139$. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.398 \pm 1.96(0.0139)$, or (0.37, 0.43). We can conclude that a minority of the population supports legalization because 0.50 is above the upper endpoint of the 95% confidence interval.

c) It appears that the proportion favoring legalization is increasing over time.

8.74 Insomnia in France

a) The data were obtained randomly, the number of successes and number of failures are more than 15.

b) The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.73(1-0.73)/12778} = 0.004$.

The confidence interval is $\hat{p} \pm z(se)$.

Thus the lower limit is 0.73 – 3.1(0.004) = 0.07176 and the upper limit is 0.73 + 3.1(0.004) = 0.07424. When the sample size is extremely large, the standard error is extremely small. This is because the standard error is $\sqrt{\hat{p}(1-\hat{p})/n}$ so that as n increases, the standard error decreases and will be quite small for very large values of n. Since the confidence interval is computed by taking the sample mean and adding and subtracting the appropriate multiple of the standard error, when n is extremely large the confidence interval will be quite narrow even for large confidence levels.

8.75 Streaming

a) The sample proportion is 0.43, the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.43(1-0.43)/2300} = 0.0103$. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.43 \pm 1.96(0.0103)$, or (0.41, 0.45). We are 95% confident that the percentage of U.S. adults regularly watching television shows via streaming is between 41% and 45%.

b) A 99% confidence interval would be wider. To have more confidence, we need a wider interval. Technically, the z-score changes from 1.96 to 2.58, resulting in a wider interval.

c) The confidence interval would be narrower. With 5000 adults, we have more information, resulting in a smaller standard error for the sample proportion. A smaller standard error leads to a narrower margin of error and thus narrower interval.

8.76 Edward Snowden

a) True, this is the definition of a confidence interval.

b) False, a confidence interval is about a population parameter, not about the result of a sample.

8.77 More NSA spying

a) False, the true percentage may be as large as 52%.

b) True, because 76% is no longer a plausible value and is outside the confidence interval

8.78 Grandpas using e-mail

a) The first result is a 90% confidence interval for the mean hours spent per week sending and answering e-mail for males of at least age 80. The sample mean, $\bar{x}$, is listed as 3.000 hours. Thus, the estimated mean hours spent per week sending and answering e-mail for males of at least age 80 is 3.000 hours. The sample standard deviation is 4.093. This quantity estimates the population standard deviation which tells us how far we can expect a typical observation to vary from the mean. These estimates are based on a sample of size 9.

b) The sample mean is 3.000, the standard error is $s/\sqrt{n} = 4.093/\sqrt{9} = 1.3643$. The 90% confidence interval is $\bar{x} \pm t_{.05}(se) = 3.000 \pm 1.8595(1.3643)$, or (0.463, 5.537). We can be 90% confident that the population mean number of hours spent per week sending and answering email for males of at least age 80 is between 0.5 and 5.5 hours.

c) With only 9 observations, hard to determine shape, but distribution may be bell shaped or skewed right. There are no outliers in the sample. By robustness of t-interval, results should be valid.

8.79 Travel to work

a) With a very large sample, the standard error is very small (because the denominator of the standard error formula is so large). With a small standard error, the margin of error also is small.

b) We would need to know a standard deviation for the mean travel time.

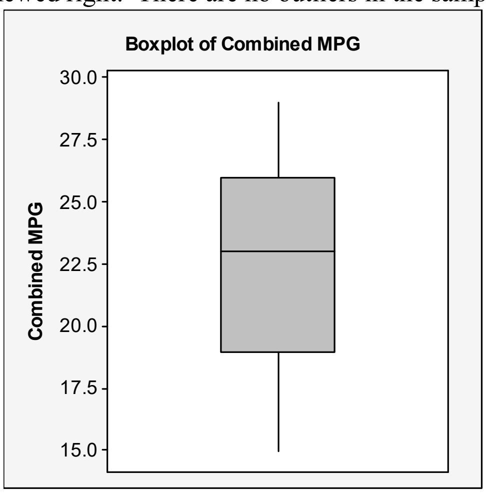
8.80 *t*-scores

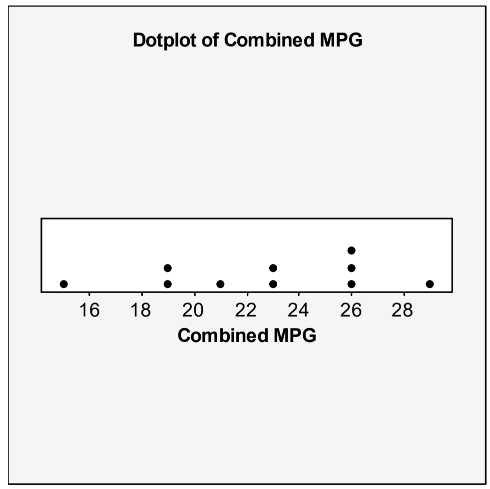
a) The *t*-score for a 95% confidence interval with a sample size of 10 is 2.262. For a sample size of 20 the *t*-score is 2.093. For a sample size of 30 the *t*-score is 2.045. For a sample size of infinity the *t*-score is 1.96 (the *z*-score for a 95% confidence interval).

b) The answer in (a) suggests that the *t* distribution approaches the standard normal distribution as the sample size gets larger.

8.81 Fuel efficiency

a) With only 10 observations, it is hard to determine shape, but the distribution may be bell shaped or skewed right. There are no outliers in the sample.





b) From technology, the sample mean is 22.7 and the standard deviation is 4.244. The standard error is $s/\sqrt{n} = 4.244/\sqrt{10} = 1.3421$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 22.7 \pm 2.2622(1.3421)$, or (19.7, 25.7). We have 95% confidence that the mean combined mpg for SUVs manufactured from 2012 to 2015 is between 19.7 and 25.7 mpg.

8.82 Time spent on emails per week

a) The standard error is $s/\sqrt{n} = 9.543/\sqrt{7446} = 0.1105$. The confidence interval is $\bar{x} \pm t(se)$ has bounds 5.234 – (1.96)(0.1105) = 5.0174 and 5.234 + (1.96)(0.1105) = 5.451. We can be 95% confident that the population mean of email usage per week is between 5.0174 and 5.451hours.

b) The assumption of a normal population distribution is not required in this case because of the very large sample size. However, the distribution of the emailing hours per week is normal, thanks to the Central Limit Theorem.

c) If the population distribution is not normal, even with a small *n*, the results are not necessarily invalidated. The method is robust in terms of the normal distribution assumption. However, if there were some extreme outliers, this might not hold true.

8.83 More time on emails per week

The first column, "Variable" refers to the variable in which we are interested, emailing hours per week. The second column tells us that the mean time spent on emails per week for these 7446 Americans was 5.234. The third column, "Std. Err," tells us the standard error, the standard deviation of the sampling distribution for samples of size 7446. The fourth column "DF" is for the degree of freedom which is $n - 1$. Finally, the two last columns tell us the 95% confidence interval for emailing hours per week. We can be 95% confident that the population mean emailing hours per week is between 5.0174 and 4.51.

8.84 How long lived in town?

a) The population distribution is not likely normal because the standard deviation is almost as large as the mean. In fact, the lowest possible value of 0 is only (0 – 20.3)/18.2 = –1.1, or 1.1 standard deviations below the mean. Moreover, the mean is quite a bit larger than the median. Both of these are indicators of skew to the right.

b) We can construct a 95% confidence interval, however, because the normal population assumption is much less important with such a large sample size.
The sample mean is 20.3, the standard error is $s/\sqrt{n} = 18.2/\sqrt{1415} = 0.484$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 20.3 \pm 1.96(0.484)$, or (19.4, 21.2). We can be 95% confident that the population mean number of years lived in a given city, town, or community is between 19.4 and 21.2.

8.85 How often do women feel sad?

a) The sample mean is 1.81 and the standard error is $s/\sqrt{n} = 1.98/\sqrt{816} = 0.069$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 1.81 \pm 1.963(0.069)$, or (1.67, 1.95). We can be 95% confident that the population mean number of days women have felt sad over the past seven days is between 1.67 and 1.95.

b) This variable is not likely normally distributed given that the means for both women and men are much larger than their respective medians, and both standard deviations are larger than their respective means. In fact, the lowest possible score of 0 is (0 – 1.81)/1.98 = –0.9, or 0.9 standard deviations below the mean for women. Because the sample size is so large, however, there is no problem with the confidence interval method used in (a).

8.86 How often feel sad?

From technology, the sample mean is 1.4 and the standard deviation is 2.221. The standard error is $s/\sqrt{n} = 2.221/\sqrt{10} = 0.702$. The 90% confidence interval is $\bar{x} \pm t_{.05}(se) = 1.4 \pm 1.833(0.702)$, or (0.1, 2.7). We can be 90% confident that the mean for the population of Wisconsin students is between 0.1 and 2.7. For this inference to apply to the population of all University of Wisconsin students, we must assume that the data are randomly produced and that we have an approximately normal population distribution.

8.87 Happy often?

a) We can verify these numbers using the GSS. The sample size is 1451.

b) The standard error for the sample mean is $s/\sqrt{n} = 2.05/\sqrt{1451} = 0.054$, 0.05.

c) We have to assume that the data are produced randomly (true for the GSS, although not actually a simple random sample), and we have to assume that the population distribution is approximately normal. Given our large sample size, the second assumption is met. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 5.27 \pm 1.962(0.05)$, or (5.2, 5.4). Since the confidence level is completely above 5, we can conclude that the population mean is at least 5.0.

8.88 Revisiting mountain bikes

a) The sample mean is \$628.30 and the standard error is $s/\sqrt{n} = 341.4/\sqrt{12} = \$98.55$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 628.3 \pm 2.2010(98.55)$, or (411.4, 845.3). We can be 95% confident that the population mean for mountain bike price falls between \$411 and \$845.

b) To form the interval, we need to assume that the data are produced randomly, and that the population distribution is approximately normal. The population does not seem to be distributed normally. The data seem to cluster on the low and high ends, with fewer in the middle. Unless there are extreme outliers, however, this probably does not have much of an effect on this inference. This method is fairly robust with respect to the normal distribution assumption.

8.89 eBay selling prices

a) We could estimate the parameter μ, which represents the mean Buy It Now selling price for the population of Samsung S5 16GB smartphones.

b) From technology, the point estimate of μ is the sample mean of \$576.90.

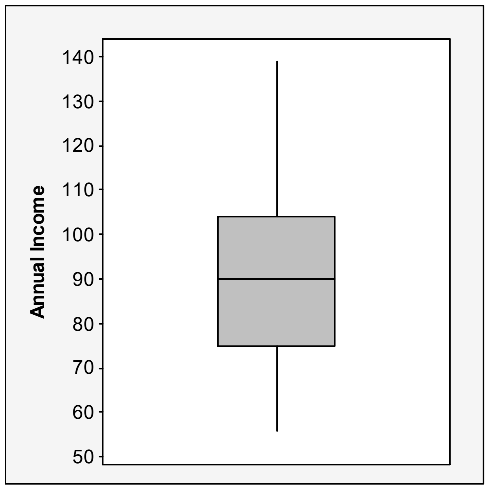
8.89 (continued)

c) The standard deviation is \$36.70, and the standard error is $s/\sqrt{n} = 36.7/\sqrt{12} = \$10.60$. The average deviation of prices from the sample mean of \$576.90 is 36.70. The average deviation of the sample mean, $\bar{x}$ to the population mean μ is \$10.6.

d) The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 576.9 \pm 2.201(10.6)$, or (554, 600). We can be 95% confident that the population mean Buy It Now selling price for the population of Samsung S5 16GB smartphones is between \$554 and \$600.

8.90 Income for families in public housing

a)



From the box plot, we can predict that the population distribution is skewed to the right. There does not seem to be an extreme outlier, so this should not affect the population inferences. This method is fairly robust with respect to the normal distribution assumption.

b) From technology, the mean is 90.2 and the standard deviation is 20.99.

c) The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 90.2 \pm 2.0484\left(20.99/\sqrt{29}\right)$, or (82.26, 98.27). We can be 95% confident that the mean income for the population of families living in public housing in Chicago is between \$8,226 and \$9,827.

8.91 Females watching TV

a) We would not expect that TV watching has a normal distribution, but rather that it is right-skewed. One indication is that the standard deviation is almost as large as the mean. The other is that the lowest possible value is zero which is only (0 – 3.08)/2.7 = –1.14, or 1.14 standard deviations below the mean.

b) The confidence interval is based on the assumptions that the data were produced randomly and that the population distribution is approximately normal. Although the distribution is not normal, the confidence interval is still valid since the method used to calculate the interval is robust to violations of the normality assumption.

c) The interval refers to the mean number of hours of TV watched in a typical day for adult females. It does not refer to the range of possible hours of TV watched by a typical female 95% of the time. In repeated samples of adult females of size 698, we would expect 95% of the confidence intervals calculated to contain the population mean, the number of hours of TV watched by adult females in a typical day.

8.92 Males watching TV

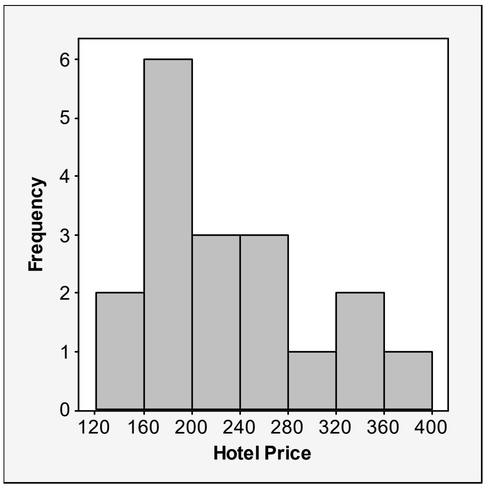
We can be 95% confident that the mean number of hours spent watching TV for the population of males falls between 2.67 and 3.08.

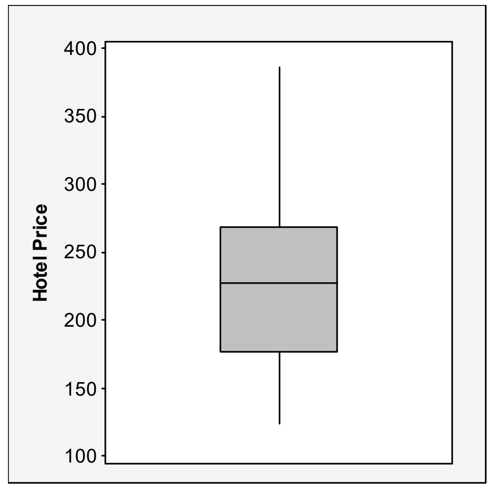
8.93 Working mother

a) This choice of scoring assumes that strongly agree and agree are the same distance apart as are strongly disagree and disagree. It assumes, however, that there is a larger difference between agree and disagree, almost as if there's another category in the middle (e.g., neutral).
b) Based on this scoring, we would interpret this sample mean as close to this middle "neutral" area, but slightly toward disagree.
c) We could make inferences about proportions for these data by looking at the proportion who responded in each way. For example, the proportion who responded "strongly agree" is 104/1308 = 0.0795.

8.94 Miami spring break

a)





b) The sample mean is \$230.80 and the standard error is $s/\sqrt{n} = 70.8/\sqrt{18} = \$16.69$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 230.8 \pm 2.1098(16.69)$, or (196, 266). With 95% confidence, the mean price for a double room in Miami over spring break is between \$196 and \$266.
c) You should not be too concerned, the distribution is not too far from normal and the *t*-interval is robust with respect to the normal distribution assumption. The potential outlier of \$386 is not too influential. Using technology, the interval without the potential outlier is (190, 253).

8.95 Sex partners in previous year

a) The standard error is $s/\sqrt{n} = 1.22/\sqrt{1766} = 0.029$.
b) The distribution was probably highly skewed to the right because the standard deviation is larger than the mean. In fact, the lowest possible value of 0 is only $\frac{0-1.11}{1.22} = -0.91$, or 0.91 standard deviation below the mean.
c) The skew need not cause a problem with constructing a confidence interval for the population mean, unless there are extreme outliers, because this method is robust with respect to the normal distribution assumption.

8.96 Brexit

a) The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.29(1-0.29)/700} = 0.01715$
b) $n = \frac{\hat{p}(1-\hat{p})z^2}{m^2} = \frac{0.5(1-0.5)1.96^2}{0.01^2} = 9604$

8.97 Driving after drinking

a) $n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.2(1-0.2)1.96^2}{0.04^2} = 385$

b) $n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.5(1-0.5)1.96^2}{0.04^2} = 601$

This is larger than the answer in (a). If we can make an educated guess about what to expect for the proportion, we can use a smaller sample size, saving possibly unnecessary time and money.

8.98 Adaptability to meet consumer demands

$n = \dfrac{\hat{p}(1-\hat{p})z^2}{m^2} = \dfrac{0.36(1-0.36)1.96^2}{0.01^2} = 8851.046,$ which rounds up to 8852. If we assume a 95% confidence interval and 1% margin of error, we can estimate the sample size to have been about 8852.

8.99 Mean property tax

a) $n = \dfrac{\sigma^2 z^2}{m^2} = \dfrac{(1000)^2(1.96)^2}{100^2} = 385$

The solution makes the assumption that the standard deviation will be similar now.

b) The margin of error would be more than \$100 because the standard error will be larger than predicted.

c) With a larger margin of error, the 95% confidence interval is wider; thus, the probability that the sample mean is within \$100 (which is less than the margin of error from part b) of the population mean is less than 0.95.

8.100 Accept a credit card?

Since the number of successes is less than 15, add two to the successes and failures. The sample proportion is now 2/104 = 0.019.

The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.019(1-0.019)/104} = 0.013$. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.019 \pm 1.96(0.013)$, or (–0.006, 0.044), which should be reported as (0, 0.044), because 0 is the lowest possible proportion.

They can conclude that fewer than 10% of their population would take the credit card.

8.101 Kicking accuracy

a) With the small sample size, we'd have to add two to each outcome. We'd then have 2 failures and 12 successes, for a sample proportion of 12/14 = 0.857 successes. We can now use the large sample method to find the confidence interval.

The standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.857(1-0.857)/14} = 0.094$. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.857 \pm 1.96(0.094)$, or (0.67, 1.04), which should be reported as (0.67, 1.00), because 1 is the highest possible proportion.

b) The lowest value that is plausible for that probability is 0.67.

c) The random sample assumption might not be met. For example, under this anxiety-producing situation of trying out, the player might react well to a first success, increasing his or her chances of future successes. Alternately, he or she might miss the first one, increasing anxiety and diminishing the chances of making later kicks.

Chapter Problems: Concepts and Investigations

8.102 Religious beliefs

Each student's one-page report will be different, but will explain the logic behind random sampling and the effect of sample size on margin of error.

8.103 TV watching and race

We can compare black and white subjects by creating a confidence interval for each group. The assumptions on which the confidence intervals are based are that the data were randomly produced and that the population distributions are approximately normal. As calculated below, the 95% confidence interval for white subjects is (2.84, 3.12) and for black subjects it is (3.87, 4.89). It seems that blacks watch more TV than do whites, on the average.

White subjects: The sample mean is 2.98, the standard error is $s/\sqrt{n} = 2.66/\sqrt{1324} = 0.073$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 2.98 \pm (1.96)(0.073)$, or (2.84, 3.12).

Black subjects: The sample mean is 4.38 and the standard error is $s/\sqrt{n} = 3.58/\sqrt{188} = 0.261$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 4.38 \pm (1.97)(0.261)$, or (3.87, 4.89).

8.104 Housework and gender

We can compare men and women by creating a confidence interval for each gender. The assumptions on which the confidence interval is based are that the data were randomly produced and that the population distributions are approximately normal. As calculated below, the 95% confidence interval for men is (17.7, 18.5) and for women is (32.2, 33.0). It seems that women do more housework than do men, on the average.

Men: The sample mean is 18.1, the standard error is $s/\sqrt{n} = 12.9/\sqrt{4252} = 0.198$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 18.1 \pm (1.96)(0.198)$, or (17.7, 18.5).

Women: The sample mean is 32.6, the standard error is $s/\sqrt{n} = 18.2/\sqrt{6764} = 0.221$. The 95% confidence interval is $\bar{x} \pm t_{.025}(se) = 32.6 \pm (1.96)(0.221)$, or (32.2, 33.0).

8.105 Women's role opinions

Running the house:

The sample proportion is 275/1831 = 0.150 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.15(1-0.15)/1831}$ = 0.008. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.150 \pm 1.96(0.008)$, or (0.134, 0.166).

Because 0.50 does not fall in this range, we can conclude that fewer than half of the population agrees with the statement that women should take care of running their homes and leave running the country up to the men.

Man as the achiever:

The sample proportion is 627/1835 = 0.342 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} = \sqrt{0.342(1-0.342)/1835}$ = 0.011. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) =$ $0.342 \pm 1.96(0.011)$, or (0.320, 0.364).

Because 0.50 does not fall in this range, we can conclude that fewer than half of the population agrees with the statement that it is better for everyone involved if the man is the achiever outside the home and the woman takes care of the home and the family.

Preschool child:

The sample proportion is 776/1830 = 0.424 and the standard error is $\sqrt{\hat{p}(1-\hat{p})/n} =$ $\sqrt{0.424(1-0.424)/1830}$ = 0.012. The 95% confidence interval is $\hat{p} \pm z_{.025}(se) = 0.424 \pm 1.96(0.012)$, or (0.400, 0.448).

Because 0.50 does not fall in this range, we can conclude that fewer than half of the population agrees with the statement that a preschool child is likely to suffer if her mother works.

The full one-page report will differ for each student, but should include the above findings, along with interpretations and descriptions, and information about assumptions.

8.106 Types of estimates

If we know the confidence interval of (4.0, 5.6), we know the mean falls in the middle because the confidence interval is calculated by adding and subtracting the same number from the mean. In this case, the mean equals 4.8. On the other hand, if we only knew the mean of 4.8, we could not know the confidence interval, and would have much less of an idea of how accurate this point estimate is likely to be.

8.107 Width of a confidence interval

When we use larger confidence levels, we want to be able to be even more accurate, and thus, we have to have a wider interval. For example, if we try to guess someone's age, we're more likely to be accurate if we guess a wider range of ages. Mathematically, a higher confidence level gives us a higher *t*- or *z*-score. This score is what we multiply by the standard error to get the confidence interval; if it's bigger, we have a bigger confidence interval. When we use a larger sample size, on the other hand, we end up with a narrower interval. This makes sense if we think about the likelihood of a larger sample being more accurate. Also, mathematically, the larger sample size in the denominator of the standard error calculation gives us a smaller standard error. Because standard error is part of the calculation of margin of error, a smaller standard error gives us a smaller margin of error, and hence, a smaller confidence interval.

8.108 99.9999% confidence

An extremely large confidence level makes the confidence interval so wide as to have little use.

8.109 Need 15 successes and failures

a) (0.5)(30) = 15
b) (0.3)(50) = 15
c) (0.1)(150) = 15

In all cases, if we have a smaller sample size, this number of successes mathematically <u>cannot</u> be 15.

8.110 Outliers and CI

When the fifth observation is changed to 875, the 95% confidence interval becomes (588, 718), which is much wider than original interval. An outlier can dramatically affect the standard deviation (21 for original data, 84 for modified data) and hence the margin of error.

8.111 What affects *n*?

a) An increase in the confidence level, leads to a higher *z*-score, which leads to a higher *n* because it increases the numerator. We would need a larger sample size to be more confident.
b) A decrease in the margin of error, *m*, would lead to a higher *n* because it decreases the denominator. We would need a larger sample size to have less error.

8.112 Multiple choice: CI property

The best answer is (a).

8.113 Multiple choice: CI property 2

The best answer is (b).

8.114 Multiple choice: Number of close friends

Both (b) and (e) are correct.

8.115 Multiple choice: Why *z*?

The best answer is (a).

8.116 Mean age at marriage

a) The confidence interval refers to the population, not the sample, mean.
b) The confidence interval is an interval containing possible means, not possible individual scores.
c) $\bar{x}$ is the sample mean; we know exactly what it is.
d) If we sampled the entire population even once, we would know the population mean exactly.

8.117 Interpret CI

If we repeatedly took samples of 50 records from the population, approximately 95% of those intervals would contain the population mean age.

8.118 True or false

False, it should be the population proportion.

8.119 True or false

False, it is the sampling distribution that must be approximately normal, not the population distribution.

8.120 True or false

False, a volunteer sample is not a random sample, thus violating one of the necessary assumptions.

8.121 True or false

True, since the denominator of the margin of error is $\sqrt{n}$, quadrupling *n* doubles the value of the denominator, thereby halving the margin of error.

8.122 Women's satisfaction with appearance

False, margin of error depends on standard error. Standard error uses the sample proportion, which would differ for each of these responses, in its calculation.

♦♦8.123 Opinions over time about the death penalty

a) When we say we have 95% confidence in an interval for a particular year, we mean that in the long-run (that is, if we took many random samples of this size from this population), the intervals based on these samples would capture the true population proportion 95% of the time.

b) The probability of all intervals containing the population mean is 0.264.

$$P(26) = \frac{26!}{26!(26-26)!}(0.95)^{26}(1-0.95)^{26-26} = 0.264$$

c) The mean of the probability distribution of *X* is $np = (26)(0.95) = 24.7$.

d) To make it more likely that all 26 inferences are correct, we could increase the confidence level, to 99% for example.

♦♦8.124 Why called "degrees of freedom"?

The mean is the sum of all observations divided by the sample size. If we don't know one observation, but we do know the mean, we can solve algebraically to find the observation. For example, if we know that two of three observations are 1 and 2, and we know the mean is 2, we can solve as follows:

$$2 = \frac{1+2+x}{3} \Rightarrow 6 = 1+2+x \Rightarrow 6 = 3+x \Rightarrow x = 3$$

♦♦8.125 An alternative interval for the population proportion

a) $|\hat{p} - p| = 1.96\sqrt{p(1-p)/n} \Rightarrow \left|\frac{20}{20} - p\right| = 1.96\sqrt{p(1-p)/10}$

If we substitute $p = 1$, we get 0 on both sides of the equation.
If we substitute $p = 0.83887$, we get 0.16113 on both sides of the equation.

b) The confidence interval formed using the method in this exercise seems more believable because it forms an actual interval and contains more than just the value of 1. It seems implausible that all of the students have iPods.

♦♦8.126 *m* and *n*

The *z*-score of 1.96 is approximately 2.0. So, the numerator of the formula for *n* is approximately 0.50(1 – 0.50)(2.0)(2.0), which is 1.

♦♦8.127 Median as point estimate

If the population is normal, the standard error of the median is 1.25 times the standard error of the mean. A larger standard error means a larger margin of error, and therefore, a wider confidence interval. The sample mean tends to be a better estimate than the sample median because it is more precise.

Chapter Problems: Student Activities

♦♦8.128 Randomized response

a) If a head is obtained on the first flip (probability = 0.5), P(HH) = 1/2 $\times$ 1/2 = 1/4 and P(HT) = 1/2 $\times$ 1/2 = 1/4. If a tail is obtained on the first flip (probability = 1/2), the student answers H with probability p for P(TH) = 1/2 $\times$ p = p/2 and the student answers T with probability of 1 – p for P(TT) = (1 – p)/2.

b) The proportion who report head for their second response is the sum of P(HH) and P(TH) = 0.25 + p/2. Thus we can estimate p as $\hat{p} = 2(\hat{q} - 0.25) = 2\hat{q} - 0.5$.

c) Answers will be different for each class.

💻8.129 GSS project

The results will be different each time this exercise is conducted.