

# Quantum Information and Computing

Assignment 6 (due in two weeks)

November 19, 2024

1. **Density Matrices** Consider a quantum system composed by  $N$  subsystems (spins, atoms, particles etc..) each described by a wave function  $\psi_i \in \mathcal{H}^D$  where  $\mathcal{H}^D$  is a  $D$ -dimensional Hilbert space. How do you write the total wave function of the system  $\Psi(\psi_1, \psi_2, \dots, \psi_N)$  ?
  - (a) Write a code (Fortran or Python) to describe the composite system in the case of  $N$ -body non interacting, separable pure state;
  - (b) and in the case of a general  $N$ -body pure wave function  $\Psi \in \mathcal{H}^{DN}$ ;
  - (c) Comment and compare their efficiency;
  - (d) Given  $N=2$ , write the density matrix of a general pure state  $\Psi$ ,  $\rho = |\Psi\rangle\langle\Psi|$ ;
  - (e) Given a generic density matrix of dimension  $D^N \times D^N$  compute the reduced density matrix of either the left or the right system, e.g.  $\rho_1 = \text{Tr}_2 \rho$ .
  - (f) Test the functions described before (and all others needed) on two-spin one-half (qubits) with different states.