Assignment 7

Quantum Ising Model

December 3rd 2024

QUANTUM ISING MODEL

Consider N spin-1/2 particles on a one-dimensional lattice, described by the Hamiltonian:

$$\hat{H} = \lambda \sum_{i}^{N} \sigma_i^z + \sum_{i}^{N-1} \sigma_i^x \sigma_{i+1}^x$$

where σ_x and σ_z are the Pauli matrices and λ is the strength of the external field.

1) Write a program to compute the $N \times N$ matrix representation of the Hamiltonian $\hat{H}$ for different size N.

2) Diagonalize $\hat{H}$ for different $N = 1,\ldots,N_{max}$ and $\lambda \in [0,-3]$. What is the largest is N_{max} you can reach

3) Plot the first k levels as a function of λ for different N and comment on the spectrum.

you can compute the ground state analitically
and the first excited: you obtain it when one
spin flip (VEDI FOTO)

PLOT RENORMALIZED ENERGY VALUES (vedi foto)

you can choose the coundary conditions. OPEN or CLOSE
Let us consider open

Quantum Ising model is one of the simplest non trivial many-body quantum systems. Let's consider a linear chain of interacting spin-1/2 in the presence of an external field of intensity λ.

The two pieces of the Hamiltonian do not commute:

$$[\mathbb{I}_1 \otimes \sigma_2^z, \sigma_1^x \otimes \sigma_2^x] \neq 0$$

Remember that:

$$\sigma_i^z = \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes \ldots \otimes \sigma_i^z \otimes \mathbb{I}_{i+1} \ldots \otimes \mathbb{I}_N$$

$$\sigma_i^x \otimes \sigma_i^x = \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes \ldots \otimes \sigma_i^x \otimes \sigma_{i+1}^x \otimes \mathbb{I}_{i+2} \ldots \otimes \mathbb{I}_N$$

So the overall size is? Hamiltonian with size 2^N

Two regimes:
i- lambda = 0
ii- lambda -> infinity

Depending on the sign of J they will be alligned or anti-alligned
J>1 --> anti-ferro magnetic

Between those two: quantum phase transition from an ordered to a disordered phase

VEDI FOTO

CODE DEVELOPMENT

Matrix representation of the operator $A \otimes B$, with $A = (M_r, M_c) \quad B = (N_r, N_c)$, labeled as:

$$A \otimes B = (M_r \times N_r, M_c \times N_c)$$

$$((ii-1)N_r + kk, (jj-1)N_c + mm) \text{ with } \begin{array}{r} ii \in [1, M_r] \\ jj \in [1, M_c] \\ kk \in [1, N_r] \\ mm \in [1, N_c] \end{array}$$

$$A \otimes B = \begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1M_c}B \\ A_{21}B & A_{22}B & \dots & A_{2M_c}B \\ \vdots & \vdots & \dots & \vdots \\ A_{M_r1}B & A_{M_r2}B & \dots & A_{M_rM_c}B \end{bmatrix}$$

Optional: to observe the quantum phase transition

Look at observable, example at the magnetization

Compute entanglement (von neumann entropy)

.....Take a bipartition....

magnetization close to 1 when lambda infinite and 0 when lambda is zero

GIOVANNI:
Look at what happens for delta varying lambda, where delta is a difference between ...

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