Structural Equation Modeling

Course material by Dirk Heerwegh, revised by Ahu Alanya

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Lab Session 3

Objectives

- Multiple Group Analysis
  - MIMIC
  - MGCFA
  - MGSEM
- “Groups” can be. . .
  - Countries
  - Schools
  - Males / Females
  - Time points

Multiple Groups Research

- Substantive research questions
  - Focus on factor means, factor (co-)variances, regression coefficients
  - We hope to find some interesting differences
- But before we can compare:
  - Is the measurement instrument performing in the same way in each of the groups?
  - We need to establish “measurement equivalence” before we can make meaningful comparisons:
  - Same number of factors?
  - Same factor loadings?
  - Same intercepts / thresholds?
  - Same residual variances of the factor indicators?
  - We hope to find no differences

Multiple Groups Research - Levels of Measurement Equivalence

- Configural
- Metric
- Scalar

Multiple Groups Research - Approaches

- MGCFA
  - Simultaneously testing a CFA model in multiple groups
  - Possible to test equality of each parameter
  - Requires a larger sample size than the MIMIC approach
  - Requires more programming
  - Not very elegant for comparing many groups
- MIMIC
  - Group membership captured in dummy-coded variable(s)
  - Not possible to test equality of each parameter
  - Smaller sample sizes
  - Easier to program
  - Good approach when dealing with many groups

Sequence of tests

Testing equivalencies across groups: sets of parameters are tested in a logically ordered and increasingly restrictive manner:
- Factor loadings (metric equivalence)
- Intercepts (scalar equivalence)

And, in some cases (depending on the purpose of the research):
- Factor variances / covariances
- Structural regression paths
- Error variances / covariances
- Disturbance terms

Lavaan syntax

```
setwd("~/Documents/KUL/SEM/2015")
ds<-read.table("session3")
library(lavaan)
model<-'f =~ x1 + x2 + x3 + x4'
# configural equivalence
fit1<-cfa(model,data=ds,group="group")

# metric equivalence: set the factor loadings equal across groups
fit2<-cfa(model,data=ds,group="group",group.equal=c("loadings"))

# scalar equivalence: set the factor loadings and the intercepts equal across groups
fit3<-cfa(model,data=ds,group="group",group.equal=c("loadings","intercepts"))
```

Lavaan alternative syntax

```
# configural equivalence
model.conf.eq<-'f =~ x1 + x2 + x3 + x4'
```

```r
fit.conf.eq<-cfa(model.conf.eq,data=ds,group="group")

# metric equivalence: set the factor loadings equal across groups
model.metr.eq<-'f =~ c(l1,l1)*x1 + c(l2,l2)*x2 + c(l3,l3)*x3 + c(l4,l4)*x4'
fit.metr.eq<-cfa(model.metr.eq,data=ds,group="group")

# scalar equivalence: set the factor loadings and the intercepts equal across groups
model.scal.eq<-'f =~ c(l1,l1)*x1 + c(l2,l2)*x2 + c(l3,l3)*x3 + c(l4,l4)*x4
x1 ~ c(t1,t1)*1
x2 ~ c(t2,t2)*1
x3 ~ c(t3,t3)*1
x4 ~ c(t4,t4)*1
f ~ c(a1,a2)*1
a1 == 0'
fit.scal.eq<-cfa(model.scal.eq,data=ds,group="group")
```

Model fit

```r
anova(fit1,fit2)
```

```
## Chi Square Difference Test
##
##      Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit1  4 21729 21863  4.3422
## fit2  7 21788 21906 69.9430     65.601       3  3.731e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(fit1,fit3)
```

```
## Chi Square Difference Test
##
##      Df   AIC   BIC    Chisq Chisq diff Df diff Pr(>Chisq)
## fit1  4 21729 21863   4.3422
## fit3 10 22066 22167 353.7448      349.4       6  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Lavaan syntax - partial equivalence

- Results suggest that setting all factor loadings equal across groups is too restrictive
- Use model modification indexes to determine which factor loadings should be freed
- Specify which part of the model should not be constrained equal across groups using the group.partial parameter in the cfa function

```r
fit2b<-cfa(model,data=ds,group="group",group.equal=c("loadings"),
           group.partial=c("f =~ x2"))
anova(fit1,fit2b)
```

```
## Chi Square Difference Test
##
##       Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit1   4 21729 21863 4.3422
## fit2b  6 21728 21851 7.2372      2.895       2     0.2352
```

Lavaan syntax - partial equivalence

- Based on "model 2b", check whether intercepts are equal across groups

```
fit3b<-cfa(model,data=ds,group="group",group.equal=c("loadings","intercepts"),
           group.partial=c("f =~ x2"))
anova(fit1,fit3b)
```

```
## Chi Square Difference Test
##
##       Df   AIC   BIC    Chisq Chisq diff Df diff Pr(>Chisq)
## fit1   4 21729 21863   4.3422
## fit3b  9 22003 22109 288.3441        284       5  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
fit3c<-cfa(model,data=ds,group="group",group.equal=c("loadings","intercepts"),
           group.partial=c("f =~ x2","x3 ~ 1"))
anova(fit1,fit3c)
```

```
## Chi Square Difference Test
##
##       Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit1   4 21729 21863  4.3422
## fit3c  8 21727 21839 10.2978     5.9557       4     0.2025
```

Lavaan alternative syntax - partial equivalence

- Loadings and intercepts kept equal, except for loading of x2 and intercept of x3 ("fit3c")

```
model.partial.scal.eq<-'f =~ c(l1,l1)*x1 + c(l21,l22)*x2 + c(l3,l3)*x3 + c(l4,l4)*x4
x1 ~ c(t1,t1)*1
x2 ~ c(t2,t2)*1
x3 ~ c(t31,t32)*1
x4 ~ c(t4,t4)*1
f ~ c(a1,a2)*1
a1 == 0'
fit.partial.scal.eq<-cfa(model.partial.scal.eq,data=ds,group="group")
```

Comparing means

- Once (partial) measurement equivalence is demonstrated, the latent means can be compared
- Use the summary function for object fit3c to check difference in latent means

- The latent mean of the first group is set to 0; the latent mean of the second group is free to deviate.
- The part you need to look at is called "Intercepts", and you want to look at the intercept of f
- The latent mean is different in Group 2 than in Group 1 as the intercept of f in Group 2 equals 0.169 (p=2.44e-07)
- If we had not corrected for partial invariance, we would have estimated the difference in latent means as 0.044 (p=0.21)

Ex. 1.

- Use file "exS3_1" to perform a two-group CFA (2 factors with 4 indicators each)
- The variable names are x1-x8, with x1-x4 loading on factor 1 and x5-x8 loading on factor 2
- No cross-loadings are assumed

1.1 Statistically test for measurement equivalence and compare the latent means of both factors across the 2 groups; are they different?
1.2 Statistically test whether the variance of each factor is equal across groups (do this in the model that results from Ex. 1.1 so that any measurement non-equivalence is accounted for)
1.3 Statistically test whether the covariance between the 2 factors is equal across groups (based on the model from 1.2)
1.4 Build a MIMIC model in which the group variable is used as independent variable and test whether you reach the same conclusions as with the MGCFA approach regarding measurement equivalence and difference in means

Ex. 1 - Solution 1.1

```
ds<-read.table("exS3_1")
model<-"f1 =~ x1 + x2 + x3 + x4
f2 =~ x5 + x6 + x7 + x8"
fit1<-cfa(model,data=ds,group="group")
fit2<-cfa(model,data=ds,group="group",group.equal=c("loadings"))
fit3<-cfa(model,data=ds,group="group",group.equal=c("loadings","intercepts"))
anova(fit1,fit2)
```

```
## Chi Square Difference Test
##
##      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit1 38 33725 33993 48.982
## fit2 44 33723 33959 58.844     9.8627       6     0.1306
```

```
anova(fit1,fit3)
```

```
## Chi Square Difference Test
##
##      Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit1 38 33725 33993  48.982
## fit3 50 33814 34018 161.703     112.72      12  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Ex. 1 - Solution 1.1

```
# Use summary function to detect largest MIs - x3 and x6 appear to need different
# intercepts across groups
fit3b<-cfa(model,data=ds,group="group",group.equal=c("loadings","intercepts"),
           group.partial=c("x3 ~ 1","x6 ~ 1"))
anova(fit1,fit3b)
```

```
## Chi Square Difference Test
##
##       Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit1  38 33725 33993 48.982
## fit3b 48 33716 33931 60.033     11.051      10     0.3536
```

```
# Check overall model fit of specification 3b
c(inspect(fit3b,"fit")["chisq"],inspect(fit3b,"fit")["df"],inspect(fit3b,"fit")["pvalue"])
```

```
##      chisq         df     pvalue
## 60.0326006 48.0000000  0.1140915
```

Ex. 1 - Solution 1.1

```
# Equal means? Check output you get from summary(fit3b) or use below statements
int <- inspect(fit3b,"est")[10]$alpha["f1","intercept"] # gives intercept of f1
z <- inspect(fit3b,"est")[10]$alpha["f1","intercept"] /
  inspect(fit3b,"se")[10]$alpha["f1","intercept"] # gives z value
p<-(1-pnorm(abs(z)))*2 # gives 2 sided p-value
paste("Factor 1: alpha=",round(int,3),", z=",round(z,3),", p=",round(p,3), sep="")
```

```
## [1] "Factor 1: alpha=0.053, z=1.508, p=0.132"
```

```
int <- inspect(fit3b,"est")[10]$alpha["f2","intercept"] # gives intercept of f2
z <- inspect(fit3b,"est")[10]$alpha["f2","intercept"] /
  inspect(fit3b,"se")[10]$alpha["f2","intercept"] # gives z value
p <- (1-pnorm(abs(z)))*2 # gives 2 sided p-value
paste("Factor 2: alpha=",round(int,3),", z=",round(z,3),", p=",round(p,3), sep="")
```

```
## [1] "Factor 2: alpha=0.209, z=6.512, p=0"
```

Ex. 1 - Solution 1.2

```
model3 <- "f1 =~ c(l1,l1)*x1+ c(l2,l2)*x2 + c(l3,l3)*x3 + c(l4,l4)*x4
f2 =~ c(l5,l5)*x5 + c(l6,l6)*x6 + c(l7,l7)*x7 + c(l8,l8)*x8
x1 ~ c(t1,t1)*1
x2 ~ c(t2,t2)*1
x3 ~ c(t31,t32)*1      # free intercept for x3 across groups
```

```
x4 ~ c(t4,t4)*1
x5 ~ c(t5,t5)*1
x6 ~ c(t61,t62)*1       # free intercept for x6 across groups
x7 ~ c(t7,t7)*1
x8 ~ c(t8,t8)*1
f1 ~ c(a11,a12)*1
f2 ~ c(a21,a22)*1
a11 == 0                # set intercept of f1 to 0 in first group
a21 == 0                # set intercept of f2 to 0 in first group
f1 ~~ c(psi11,psi11)*f1    # equal variance of factor 1 across groups
f2 ~~ c(psi22,psi22)*f2    # equal variance of factor 2 across groups"
fit4<-cfa(model3,data=ds,group="group")
```

Ex. 1 - Solution 1.2

```
anova(fit3b,fit4)
```

```
## Chi Square Difference Test
##
##       Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit3b 48 33716 33931 60.033
## fit4  50 33713 33917 60.799    0.76638       2     0.6817
```

- The non-significant p-value indicates that we can constrain the factor variances to be equal across groups (Note that since we are testing 2 parameters at once, it is possible that this finding does not remain when testing one at the time)

Ex. 1 - Solution 1.3

```
model2<-"f1 =~ c(l1,l1)*x1+ c(l2,l2)*x2 + c(l3,l3)*x3 + c(l4,l4)*x4
f2 =~ c(l5,l5)*x5 + c(l6,l6)*x6 + c(l7,l7)*x7 + c(l8,l8)*x8
x1 ~ c(t1,t1)*1
x2 ~ c(t2,t2)*1
x3 ~ c(t31,t32)*1       # free intercept for x3 across groups
x4 ~ c(t4,t4)*1
x5 ~ c(t5,t5)*1
x6 ~ c(t61,t62)*1       # free intercept for x6 across groups
x7 ~ c(t7,t7)*1
x8 ~ c(t8,t8)*1
f1 ~ c(a11,a12)*1
f2 ~ c(a21,a22)*1
a11 == 0                # set intercept of f1 to 0 in first group
a21 == 0                # set intercept of f2 to 0 in first group
f1 ~~ c(psi11,psi11)*f1     # equal variance of factor 1 across groups
f2 ~~ c(psi22,psi22)*f2     # equal variance of factor 2 across groups
f1 ~~ c(psi1,psi1)*f2   # constrain the factor covariance to be equal"

fit5<-cfa(model2,data=ds,group="group")
```

Ex. 1 - Solution 1.3

```
anova(fit4,fit5)
```

```
## Chi Square Difference Test
##
##      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit4 50 33713 33917 60.799
## fit5 51 33711 33910 60.962    0.16322       1     0.6862
```

- The non-significant p-value indicates that we can constrain the factor covariance to be equal across groups

Ex. 1 - Solution 1.4

```
# Dummy-code group for ease of interpretation
ds$grpB <- ifelse(ds$group=="B",1,0)
# Test for measurement non-equivalence (only intercepts of indicators are checked)
model<-"f1 =~ x1 + x2 + x3 + x4
f2 =~ x5 + x6 + x7 + x8
f1 ~ grpB
f2 ~ grpB
x1  ~  0*grpB
x2  ~  0*grpB
x3  ~  0*grpB
x4  ~  0*grpB
x5  ~  0*grpB
x6  ~  0*grpB
x7  ~  0*grpB
x8 ~ 0*grpB"
fit.mimic<-cfa(model,data=ds)
mi<-inspect(fit.mimic,"mi")  # You should review all modification indexes!
mi<-subset(mi,substr(lhs,1,1)=="x" & op=="~" & rhs=="grpB") # But here we will
                  # restrict to the direct effects of grpB on the x-variables
mi.sorted<-mi[order(-mi$mi),]  # sort from high to low
mi.sorted[1:3,]  # only display some large MI values
```

```
##   lhs op  rhs     mi    epc sepc.lv sepc.all sepc.nox
## 1  x3  ~ grpB 67.375  0.391   0.391    0.192    0.385
## 2  x6  ~ grpB 32.116 -0.303  -0.303   -0.156   -0.311
## 3  x8  ~ grpB  8.955  0.175   0.175    0.088    0.175
```

Ex. 1 - Solution 1.4

```
# Allow intercept of x3 and x6 to vary by group
model2<-"f1 =~ x1 + x2 + x3 + x4
f2 =~ x5 + x6 + x7 + x8
f1 ~ grpB
```

```
f2 ~ grpB
x1 ~ 0*grpB
x2 ~ 0*grpB
x3 ~ grpB
x4 ~ 0*grpB
x5 ~ 0*grpB
x6 ~ grpB
x7 ~ 0*grpB
x8 ~ 0*grpB"
fit2.mimic<-cfa(model2,data=ds)
```

- Review the model output and verify you obtain similar substantive results as with MGCFA

Ex. 2. Major Depression - Gender analysis

- On page 272 of Brown's book "Confirmatory Factor Analysis for Applied Research" (2006), correlation matrices and mean and standard deviation vectors are given for 375 Females and 375 Males
- Nine variables (mdd1-mdd9) are supposed to measure a single latent variable "Major Depression". One error covariance between mdd1 and mdd2 is assumed for substantive reasons
- Read in the summary data and fit the 8 models that Brown presents on page 273
- Review the fit of each of the models and compare to Brown's results

Ex. 2 - Solution: reading in the data (Females)

```
# Females, n=375
lower1<-'1
0.616   1
0.315   0.313   1
0.349   0.332   0.261   1
0.314   0.25    0.27    0.327   1
0.418   0.416   0.298   0.328   0.317   1
0.322   0.313   0.096   0.117   0.13    0.14    1
0.409   0.415   0.189   0.314   0.303   0.281   0.233   1
0.318   0.222   0.051   0.115   0.14    0.15    0.217   0.222   1'
sds1<-c(1.717,2.015,2.096,2.212,2.132,2.005,2.062,2.156,1.791)
mean1<-c(4.184,3.725,1.952,3.589,2.256,3.955,3.869,3.595,1.205)

cov1<-getCov(lower1,names=paste("mdd",1:9,sep=''),sds=sds1)
```

Ex. 2 - Solution: reading in the data (Males)

```
# Males, n=375
lower2<-'1
0.689   1
0.204   0.218   1
0.335   0.284   0.315   1
0.274   0.32    0.153   0.265   1
0.333   0.333   0.221   0.364   0.268   1
```

```
0.258   0.211   0.114   0.139   0.185   0.132   1
0.319   0.346   0.176   0.207   0.231   0.279   0.146   1
0.316   0.269   0.111   0.14    0.117   0.131   0.263   0.163   1'
sds2<-c(1.598,2.018,2.094,2.232,2.108,2.113,2.286,2.174,1.788)
mean2<-c(4.171,3.685,1.739,3.357,2.235,3.661,3.421,3.517,1.259)

cov2<-getCov(lower2,names=paste("mdd",1:9,sep=''),sds=sds2)
```

Ex. 2 - Solution: fitting the model in both groups separately

```
model1<-'f =~ mdd1 + mdd2 + mdd3 + mdd4 + mdd5 + mdd6 + mdd7 + mdd8 + mdd9
mdd1 ~~ mdd2'

# Single group analysis - Females
fit1.females<-cfa(model1,sample.cov=cov1,sample.mean=mean1,sample.nobs=375)

# Single group analysis - Males
fit1.males<-cfa(model1,sample.cov=cov2,sample.mean=mean2,sample.nobs=375)
```

Ex. 2 - Solution: Measurement Equivalence

```
# Configural Equivalence
fit2<-cfa(model1,sample.cov=list(cov1,cov2),sample.mean=list(mean1,mean2),
          sample.nobs=list(375,375))

# Metric Equivalence (equal factor loadings)
fit3<-cfa(model1,sample.cov=list(cov1,cov2),sample.mean=list(mean1,mean2),
          sample.nobs=list(375,375),group.equal=c("loadings"))

# Scalar Equivalence (equal factor loadings + intercepts)
fit4<-cfa(model1,sample.cov=list(cov1,cov2),sample.mean=list(mean1,mean2),
          sample.nobs=list(375,375),group.equal=c("loadings","intercepts"))

# Equal indicator error variances
fit5<-cfa(model1,sample.cov=list(cov1,cov2),sample.mean=list(mean1,mean2),
          sample.nobs=list(375,375),group.equal=c("loadings","intercepts","residuals"))
```

Ex. 2 - Solution: Population heterogeneity

```
# Equal Factor variance
fit6<-cfa(model1,sample.cov=list(cov1,cov2),sample.mean=list(mean1,mean2),
          sample.nobs=list(375,375),
          group.equal=c("loadings","intercepts","residuals","lv.variances"))

# Equal Latent Mean
fit7<-cfa(model1,sample.cov=list(cov1,cov2),sample.mean=list(mean1,mean2),
          sample.nobs=list(375,375),
          group.equal=c("loadings","intercepts","residuals","lv.variances","means"))
```

Ex. 2 - Solution: summary (1)

Single Group Solutions	Chi-squared	df	RMSEA	CFI	TLI
Women (n=375)	53.14	26	0.053	0.96	0.94
Men (n=375)	46.03	26	0.045	0.97	0.95

Ex. 2 - Solution: summary (2)

Measurement Invariance	Chi-squared	df	RMSEA	CFI	TLI
Equal form	99.17	52	0.049	0.96	0.95
Equal factor loadings	103.12	60	0.044	0.97	0.96
Equal ind. intercepts	115.63	68	0.043	0.96	0.96
Equal ind. error var.	125.33	77	0.041	0.96	0.96

Ex. 2 - Solution: summary (3)

Population heterogeneity	Chi-squared	df	RMSEA	CFI	TLI
Equal factor variance	126.12	78	0.041	0.96	0.97
Equal latent mean	128.04	79	0.041	0.96	0.96

Ex. 3. Holzinger-Swineford data

This exercise uses the HolzingerSwineford1939 dataset that comes with lavaan.

3.1. Fit the 3-factor model in each school separately (Grant-White and Pasteur). x1-x3 and x9 should load on the Spatial factor; x4-x6 on the Verbal factor; and x7-x9 on the Speed factor.
3.2. Fit the model in both groups simultaneously and verify that the sum of the Chi-squared values from 3.1. equals the Chi-squared value in 3.2. (the same should also hold for the df)
3.3. Test for metric equivalence; if necessary, build a model with partial metric equivalence.
3.4. Test for scalar equivalence; if necessary, build a model with partial scalar equivalence.
3.5a. Compare the latent means across the two groups: which are statistically different and which are not?
3.5b. If measurement non-equivalence is observed: what if measurement non-equivalence had not been accounted for? Would the same conclusions with regard to latent means have been attained?
3.6. Are all 3 factor variances equal across groups?

Ex. 3 - Solution 3.1.

```
model<-"spatial  =~ x1 + x2 + x3 + x9
verbal  =~ x4 + x5 + x6
speed =~ x7 + x8 + x9"
fit1.GW  <- cfa(model,data=subset(HolzingerSwineford1939,school=="Grant-White"))
fit1.P   <- cfa(model,data=subset(HolzingerSwineford1939,school=="Pasteur"))
```

- Request the summary for both model outputs and review them
- Note that the model fits well in het Grant-White school, but not well inthe Pasteur school.

Ex. 3 - Solution 3.2.

```
fit2 <- cfa(model,data=HolzingerSwineford1939,group="school")
fit2
```

```
## lavaan (0.5-18) converged normally after  56 iterations
##
##   Number of observations per group
##   Pasteur                                            156
##   Grant-White                                        145
##
##   Estimator                                           ML
##   Minimum Function Test Statistic                 81.547
##   Degrees of freedom                                  46
##   P-value (Chi-square)                             0.001
##
## Chi-square for each group:
##
##   Pasteur                                         53.253
##   Grant-White                                     28.293
```

Ex. 3 - Solution 3.3.

```
fit3 <- cfa(model,data=HolzingerSwineford1939,group="school",
            group.equal=c("loadings"))
anova(fit2,fit3)
```

```
## Chi Square Difference Test
##
##      Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit2 46 7454.1 7683.9 81.547
## fit3 53 7447.8 7651.7 89.266     7.7196       7      0.358
```

- Metric equivalence seems to hold

Ex. 3 - Solution 3.4.

```
fit4 <- cfa(model,data=HolzingerSwineford1939,group="school",
            group.equal=c("loadings","intercepts"))
anova(fit3,fit4)
```

```
## Chi Square Difference Test
##
##      Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit3 53 7447.8 7651.7  89.266
## fit4 59 7477.4 7659.0 130.828     41.562       6  2.244e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Scalar equivalence does not hold, review modification indexes to pin point the problematic item(s)

```
mi<-inspect(fit4,"mi")  # You should review all modification indexes!
mi<-subset(mi,substr(lhs,1,1)=="x" & op=="~1" & rhs=="") # But here we will
                   # restrict to the direct effects of grpB on the x-variables
mi.sorted<-mi[order(-mi$mi),]  # sort from high to low
```

Ex. 3 - Solution 3.4.

```
mi.sorted[1:5,]  # only display some large MI values
```

```
##   lhs op rhs group     mi    epc sepc.lv sepc.all sepc.nox
## 1  x3 ~1          1 10.118  0.267   0.267    0.224    0.224
## 2  x3 ~1          2  7.862 -0.207  -0.207   -0.196   -0.196
## 3  x7 ~1          1  7.669  0.220   0.220    0.200    0.200
## 4  x7 ~1          2  5.937 -0.170  -0.170   -0.161   -0.161
## 5  x2 ~1          1  3.238 -0.168  -0.168   -0.135   -0.135
```

- Relax equality constraint on the intercept of x3

```
fit4b <- cfa(model,data=HolzingerSwineford1939,group="school",
             group.equal=c("loadings","intercepts"),
             group.partial=c("x3~1"))
anova(fit3,fit4b)
```

```
## Chi Square Difference Test
##
##       Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit3  53 7447.8 7651.7  89.266
## fit4b 58 7458.1 7643.4 109.528     20.262       5   0.001116 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Ex. 3 - Solution 3.4.

```
mi<-inspect(fit4b,"mi")  # You should review all modification indexes!
mi<-subset(mi,substr(lhs,1,1)=="x" & op=="~1" & rhs=="") # But here we will
                   # restrict to the direct effects of grpB on the x-variables
mi.sorted<-mi[order(-mi$mi),]  # sort from high to low
mi.sorted[1:5,]  # only display some large MI values
```

```
##   lhs op rhs group    mi    epc sepc.lv sepc.all sepc.nox
## 1  x7 ~1          1 6.289  0.198   0.198    0.181    0.181
## 2  x7 ~1          2 4.893 -0.154  -0.154   -0.145   -0.145
## 3  x8 ~1          1 3.781 -0.135  -0.135   -0.138   -0.138
## 4  x8 ~1          2 3.436  0.123   0.123    0.119    0.119
## 5  x2 ~1          1 1.231 -0.102  -0.102   -0.083   -0.083
```

Ex. 3 - Solution 3.4.

- Relax equality constraint on the intercept of x3 and x7

```
fit4c <- cfa(model,data=HolzingerSwineford1939,group="school",
             group.equal=c("loadings","intercepts"),
             group.partial=c("x3~1","x7~1"))
anova(fit3,fit4c)
```

```
## Chi Square Difference Test
##
##       Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit3  53 7447.8 7651.7 89.266
## fit4c 57 7445.6 7634.7 95.064     5.7981       4     0.2147
```

Ex. 3 - Solution 3.5a / 3.5b

- Taking into account measurement non-equivalence (partial measurement equivalence):

Factor	Mean in Group 2	p-value
Spatial	0.032	0.8
Verbal	0.576	0
Speed	-0.079	0.404

- Not taking into account measurement non-equivalence:

Factor	Mean in Group 2	p-value
Spatial	-0.119	0.32
Verbal	0.576	0
Speed	-0.2	0.042

Ex. 3 - Solution 3.6

```
fit5 <- cfa(model,data=HolzingerSwineford1939,group="school",
             group.equal=c("loadings","intercepts","lv.variances"),
             group.partial=c("x3~1","x7~1"))
anova(fit4c,fit5)
```

```
## Chi Square Difference Test
##
##       Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit4c 57 7445.6 7634.7 95.064
## fit5  60 7442.2 7620.2 97.704     2.6392       3     0.4507
```

- The non-significant p-value reflects that we can set all 3 error variances equal across groups (Note: testing each factor variance at the time may give different results)

Ex. 4. MGSEM

This exercise builds on ex. 3
We might hypothesize that students' abilities increase as they move to the next grade. So we would expect to see a positive effect of "grade" on each of the three latent variables. But is this positive effect observed in both schools? And is the effect equal in both schools? Or would students in one school progress less quickly than students in the other school?

4.1. Extend the model from ex. 3 that takes into account any measurement non-equivalence by including the variable "grade". Regress the three latent variables on this covariate. Allow the regression parameters to be different across both schools.
4.2. Test whether it is possible to constrain each of the three regression parameters to be equal across the two schools. If this is not possible, develop a model which holds as many regression parameters equal across groups as possible.
4.3. To verify the robustness of findings from 4.2, test for measurement equivalence (loadings and intercepts of the indicator variables) of the measurement instrument by grade

Ex. 4 - Solution 4.1

```
model1.sem<-"spatial  =~ x1 + x2 + x3 + x9
verbal  =~ x4 + x5 + x6
speed =~ x7 + x8 + x9
spatial  ~ grade
verbal  ~ grade
speed ~ grade"
fit1.sem <- sem(model1.sem,data=HolzingerSwineford1939,group="school",
             group.equal=c("loadings","intercepts"),
             group.partial=c("x3~1","x7~1"))
```

- Review the output using the summary() function

Ex. 4 - Solution 4.2

```
fit2.sem <- sem(model1.sem,data=HolzingerSwineford1939,group="school",
             group.equal=c("loadings","intercepts","regressions"),
             group.partial=c("x3~1","x7~1"))

anova(fit1.sem,fit2.sem)
```

```
## Chi Square Difference Test
##
##          Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit1.sem 69 7818.2 8029.3 114.32
## fit2.sem 72 7812.7 8012.7 114.88    0.55911       3     0.9057
```

- The non-signifcant p-value suggests that all regression coefficients can be held equal across the two schools. (Note: this is an omnibus test. Testing each regression separately may give different results)

Ex. 4 - Solution 4.3

```
model<-"spatial  =~ x1 + x2 + x3 + x9
verbal  =~ x4 + x5 + x6
speed =~ x7 + x8 + x9"

# Subset the data, removing any missing grades
hs<-subset(HolzingerSwineford1939,grade != "NA")

# Measurement equivalence - Configural Equivalence
fit3.ce <- cfa(model,data=hs,group="grade")

# Measurement equivalence - Metric Equivalence
fit3.me <- cfa(model,data=hs,group="grade",
            group.equal=c("loadings"))

# Measurement equivalence - Scalar Equivalence
fit3.se <- cfa(model,data=hs,group="grade",
            group.equal=c("loadings","intercepts"))
```

Ex. 4 - Solution 4.3

```
anova(fit3.ce,fit3.me)
```

```
## Chi Square Difference Test
##
##         Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit3.ce 46 7469.2 7698.8 74.445
## fit3.me 53 7458.6 7662.3 77.908     3.4623       7     0.8392
```

```
anova(fit3.ce,fit3.se)
```

```
## Chi Square Difference Test
##
##         Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit3.ce 46 7469.2 7698.8 74.445
## fit3.se 59 7455.4 7636.9 86.702     12.256      13     0.5068
```

- Non-significant p-values suggest measurement equivalence, and so results from 4.2. should not be affected by measurement non-equivalence.