

COMP1201 Assignment 3

(due 22nd May, 4pm)

This assignment is worth 5% of the module marks. Submission is via the Handin system.

The assignment contains three algorithmic problem solving questions. Please justify your answers with detailed explanations. We will not accept answers without explanations, even if they are correct.

Q1 Prove that in any Facebook community, there must exist two people who have the same number of friends. Mathematically, prove that any **simple** graph G with at least two vertices must contain two vertices of the same degree. [4 marks]

Q2 (a) Consider a minimum spanning tree (MST) of a connected, weighted graph. If we remove an edge (u, v) of the MST, then we get two separate trees. Are these two trees the MSTs on their respective sets of nodes? Is the edge (u, v) a least-weight edge crossing between those two sets of nodes? [3 marks]

(b) Consider the following algorithm for finding an MST on a graph: split the nodes of the graph arbitrarily into two nearly equal-sized sets, and find an MST on each of those sets. Then connect the two MSTs with the least-cost edge between them. Would this algorithm always return an MST of the original graph? [3 marks]

If your answer to these questions is Yes, prove them; if your answer is No, disprove them by constructing a counterexample.

Q3 (a) An automotive company has three factories, each associated with different costs of producing vehicles (the cost of raw materials, the cost of labour, and the environmental footprint per vehicle produced):

	Materials	Labour	CO_2 emissions
Factory 1	6	18	10
Factory 2	5	14	17
Factory 3	7	11	20

Each month the company can afford to pay for 4000 hours of labour and 4000 units of raw materials. Additionally, labour politics require at least 100 cars to be produced at Factory 1 and environmental regulations allow the company to emit at most 3000 units of CO_2 .

What is the optimal allocation of manufacturing capacity between the factories (i.e. how many vehicles need to be produced by each factory to ensure that the company produces the maximum possible number of vehicles each month)? (Hint: use Linear Programming) [4 marks]

(b) Suppose that labour politics changes and the company is required to produce at least 75 vehicles at each factory (at least 225 vehicles in total), other requirements remaining in place as before. What would be the effect of such a development?

[1 mark]