COMP 2210 Theory of Computing
Tutorial 1

1. Design a DFA over the alphabet $\{a, b\}$ that accepts precisely the:

   a) strings with an even number of 'a's.

   b) strings with an odd number of 'a's.

   c) strings with number of 'a's divisible by 3.

   d) strings with number of 'a's divisible by n, for some $n \in \mathbb{N}$.

2. a) Given a DFA M, how can we obtain a DFA M' such that M' accepts exactly those strings that are rejected by M? (Hint: think about the solutions to a) and b) in the previous question)

   b) Prove that the class of regular languages is closed under *complement*. That is, if L is a regular language then so is $\sim L = \{x \in \Sigma^* \mid x \notin L\}$.

3. Prove the following:

   a) $\varnothing$ and Σ^* are regular languages.

   b) if L_1 and L_2 are regular then $L_1 \cup L_2$ is regular.

   c) If L_1 and L_2 are regular then $L_1 \cap L_2$ is regular.

   d) if L_1 and L_2 are regular then $L_1 L_2 = \{xy \mid x \in L_1, y \in L_2\}$ is regular.

   e) If L is regular then $L^* = \{x_1 \dots x_k \mid k \in \mathbb{N}, x_i \in L\}$ is regular.