# Tutorial 10

## Question 1

Show that 3COL is in PSPACE.

3COL is in PSAPCE because there exists a non-deterministic decider M which decided 3COL using polynomial space. M is defined as follows:

M = on input G:

* Non-deterministically select an assignment v to the nodes of the graph
* If no adjacent nodes have equal value then accept. Otherwise, reject

It is trivial to see that M is a polynomial decider as the space complexity is O(m) where m is the number of nodes.

## Question 2

## Question 3

Show that if a decision problem X has complexity 3n, then X is in EXPTIME.

In order to prove that 3n is in EXPTIME, I am going to convert 3n into 2p for some p.

* 3n = 2p => p= nlog23
* Thus, 3n = 2nlog3

Clearly, 2nlog3= O(2n2).

**Proof** 2nlog3 = O(2n2)

For c = 1 and for all n ≥ 1, 2nlog3 < 2n2

Thus, X is a problem contained in EXPTTIME.

## Question 4

Show that every PSPACE-hard problem is NP-hard.

Let L be a problem in the PSPACE-HARD class.

Consequently, we can infer the following:

All languages in PSAPCE reduce to L in deterministic polynomial time.

We have already proved that NP ⊆ PSPACE. Thus, all languages in NP can be reduced to L in deterministic polynomial time.

In conclusion, L is NP-hard which means that every problem in the class PSPACE-HARD is also in the class NP-HARD.

So, if a deterministic polynomial time is found for a PSPACE-Hard problem then P==NP.