# Exercise 3

Show that the class P is closed under union, concatenation and complement.

## Closed under union

Let A be a language in P. Let B a language in P. Thus there exists a polynomial time deterministic Turing machine M such that L(M) = A and a polynomial time deterministic Turing machine M’ such that L(M’) = B.

Let N be the following deterministic Total Turing machine.

* On input string x:
  + Run M with input string x
  + Run M’ with input string x
  + If either M or M’ accept then accept, otherwise reject

As we can easily see, L(N) = A ∪ B. Furthermore, N runs in deterministic polynomial time because both M and M’ run in deterministic polynomial time. Therefore, (A ∪ B) is in P as well.

## Closed under intersection

Let A be a language in P. Let B a language in P. Thus there exists a polynomial time deterministic Turing machine M such that L(M) = A and a polynomial time deterministic Turing machine M’ such that L(M’) = B.

Let N be the following deterministic Total Turing machine.

* On input string x:
  + Run M with input string x
  + Run M’ with input string x
  + If both M and M’ accept then accept, otherwise reject

As we can easily see, L(N) = A ∩ B. Furthermore, N runs in deterministic polynomial time because both M and M’ run in deterministic polynomial time. Therefore, (A ∩ B) is in P as well.

## Closed under complement

Let A be a language in P. Thus there exists a polynomial time deterministic Turing machine M such that L(M) = A.

Let N be the following deterministic Total Turing machine.

* On input string x:
  + Run M with input string x
  + If M accepts then reject otherwise accept

As we can easily see, L(N) = ~A. Furthermore, N runs in deterministic polynomial time because M runs in deterministic polynomial time. Therefore, (A ∩ B) is in P as well.