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# ➤ Constrained Optimization and Linear Programming

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# ➤ Outline

- Constrained optimization
- Linear programming
- Other approaches



# ➤ Constrained optimization

- Optimizing objective function is not enough
  - We also have to respect additional **constraints** on variables
  - In general, different from **boundaries** (although, terminology...)
  - Boundaries can turn into constraints ( $x \in [0,2] \rightarrow x \geq 0, x \leq 2$ )
  - Constraints are usually **equalities** and **inequalities**

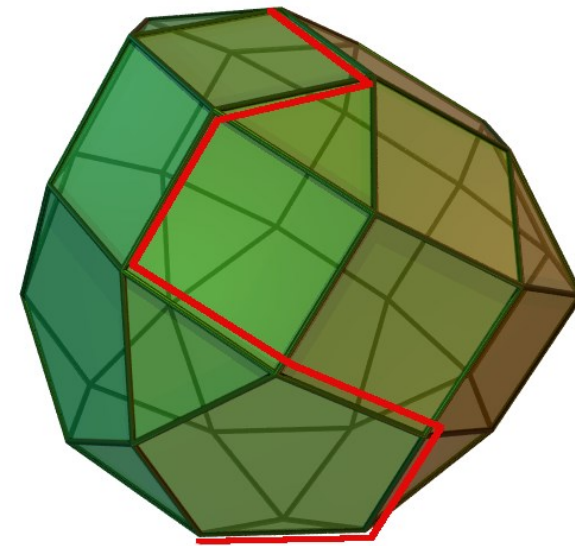
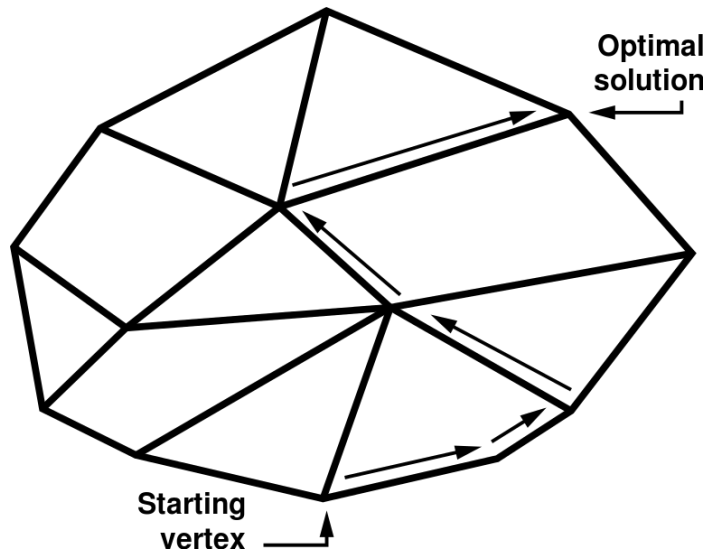
$$\begin{aligned}
 &\underset{\mathbf{x}}{\text{minimize}} && f(\mathbf{x}) \\
 &\text{subject to} && h_i(\mathbf{x}) = 0 \text{ for all } i \text{ in } \{1, \dots, \ell\} \\
 & && g_j(\mathbf{x}) \leq 0 \text{ for all } j \text{ in } \{1, \dots, m\}
 \end{aligned}$$

## ➤ Constrained optimization

- Example: optimize the diet of soldiers
  - Minimize cost of meals
  - Attain at least a certain amount of each nutrient group
  - Original problem had 9 nutrient groups and 77 food items

# ➤ Linear programming

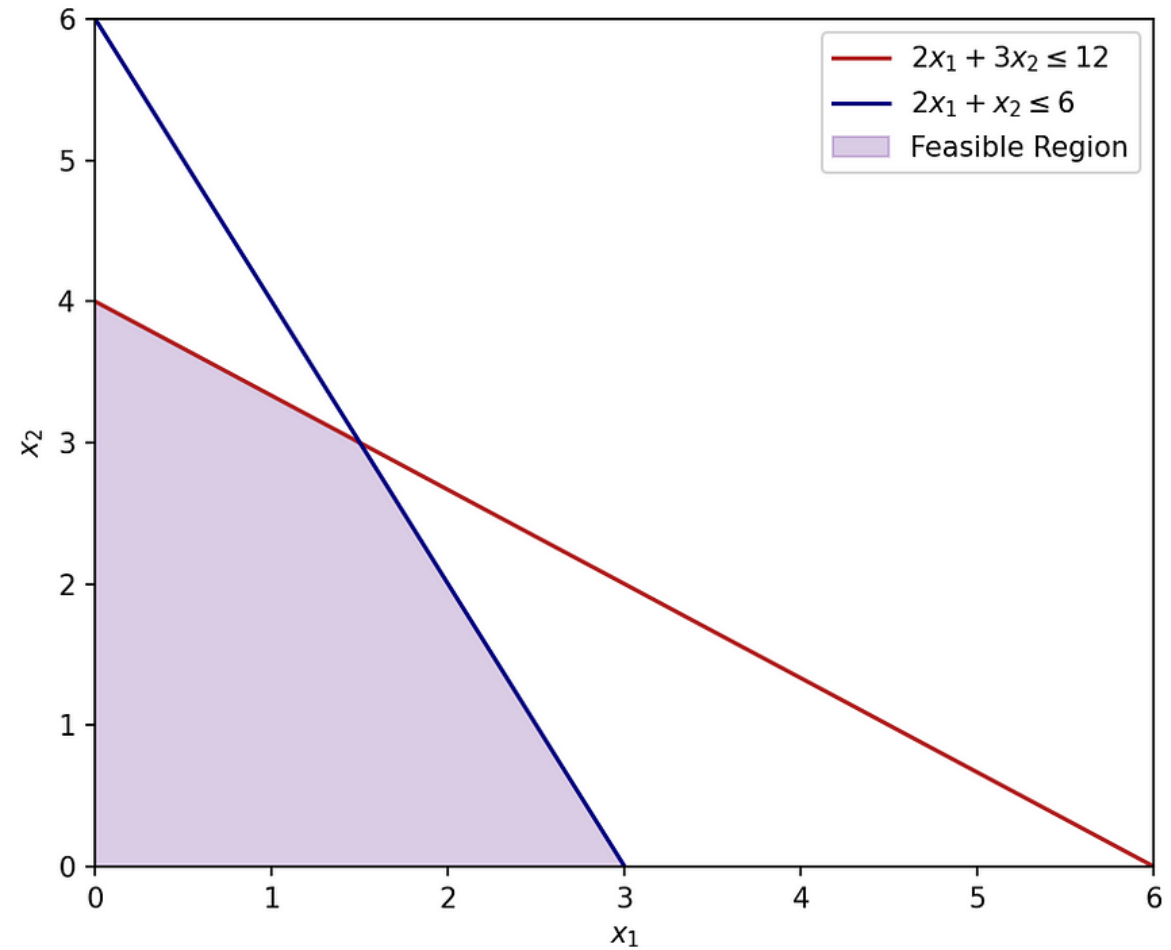
- Linear objective function, linear constraints
  - Stating the problem properly ensures **finding global optimum**
  - **Simplex**: explore vertices of a polytope (hyper-polygon)
  - Also evaluate **feasibility** of the problem (e.g. no possible solutions)



# ➤ Linear programming

- Example for 2 variables

$$\begin{aligned} \text{maximize } f(x_1, x_2) &= x_1 + x_2 \\ x_1 &\in [0, 6]; x_2 \in [0, 6] \\ 2x_1 + 3x_2 &\leq 12 \\ 2x_1 + x_2 &\leq 6 \end{aligned}$$



## ➤ Other approaches

- If constraints and/or objective function are not linear
- Variations of Linear Programming
  - Quadratic Programming
  - Non-linear Programming
- Or convert problem into unconstrained using **penalties**

$$\begin{array}{ll}
 \underset{\mathbf{x}}{\text{minimize}} & f(\mathbf{x}) \\
 \text{subject to} & \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \\
 & \mathbf{h}(\mathbf{x}) = \mathbf{0}
 \end{array}
 \quad \Rightarrow \quad
 \begin{array}{l}
 \underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x}) + \rho \cdot p_{\text{count}}(\mathbf{x}) \\
 p_{\text{count}}(\mathbf{x}) = \sum_i (g_i(\mathbf{x}) > 0) + \sum_j (h_j(\mathbf{x}) \neq 0)
 \end{array}$$

The logo for INRAE, featuring the word "INRAE" in a bold, teal, sans-serif font.The logo for université PARIS-SACLAY, featuring the word "université" in a purple, sans-serif font above the words "PARIS-SACLAY" in a bold, purple, sans-serif font.

## ➤ Questions?

### Bibliography

- Kochenderfer & Wheeler, *Algorithms for Optimization*, MIT Press, 2019
- Vanderbei, *Linear Programming: Foundations and Extensions*, 2014

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