

Direct Comparative Analysis of Nature-inspired Optimization Algorithms on Community Detection Problem in Social Networks

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Abstract. Nature-inspired optimization Algorithms (NIOAs) are nowadays a popular choice for community detection in social networks. Community detection problem in social network is treated as optimization problem, where the objective is to either maximize the connection within the community or minimize connections between the communities. To apply NIOAs, either of the two, or both objectives are explored. Since NIOAs mostly exploit randomness in their strategies, it is necessary to analyze their performance for specific applications. In this paper, NIOAs are analyzed on the community detection problem. A direct comparison approach is followed to perform pairwise comparison of NIOAs. The performance is measured in terms of five scores designed based on prasatul matrix and also with average isolability. Three widely used real-world social networks and four NIOAs are considered for analyzing the quality of communities generated by NIOAs.

Keywords: Nature Inspired Optimization Algorithms, Community Detection, Fitness Function, Direct Comparison

1 Introduction

In today's world, majority of the problems are complex in nature and requires optimization of diverse objectives such as minimization of costs, energy consumption and/or maximization of efficiency, sustainability and performance. Specifically, optimization problems are often subject to a set of complex, non-linear constraints. To solve optimization problems in an effective and time efficient manner, numerous Nature-inspired Optimization Algorithms (NIOAs) are developed [1–3]. NIOAs are typically based on randomization concept and are used for both continuous and discrete optimization problems. An extensive comparative study of several NIOAs algorithms for continuous and discrete optimization has been performed in [4, 5]. In another work [6], a comparative analysis of NIOAs on ten continuous and discrete optimization problems has been carried out. In addition to this, numerous methods have been introduced which developed the discrete version of a continuous optimization problem [7, 8]. An

example of a discrete optimization problem is community detection. It is discrete in the sense that each of the solution element in a solution vector with N-dimensions can take only discrete values. Several NIOAs algorithms on community detection have been proposed [9]. Comparative study of few NIOA based community detection has also been carried out [10, 11].

The general principle to solve the community detection problem is to maximize intra-community connectivity (vertices/ entities of the same community are strongly connected) and minimize inter-community connectivity (vertices/ entities belonging to different communities are loosely connected). However, the measure of cohesiveness may vary depending on the type of network (unweighted, weighted, directed, undirected, multiple edges, dynamic etc.). In this paper, we have considered only undirected and unweighted networks for carrying out our experiments and analyze the performance of NIOAs algorithms on community detection. The contributions of this paper are listed as follows:

- A considerable variety of NIOAs algorithms such as Grey Wolf Optimizer (GWO), Moth-Flame Optimization (MFO), Sine-Cosine Algorithm (SCA) and Whale Optimization Algorithm (WOA) have been used to detect communities in a network.
- A comparative performance analysis based on Average Isolability (AVI) has been carried to determine the quality of communities identified by the corresponding baselines.
- Communities obtained from the respective baseline algorithms are directly compared with each other based on D-scores (direct comparison) and K-scores (overall comparison).

The organization of the rest of the paper is as follows: Section 2 emphasizes on the baseline NIOAs algorithms, Section 3 briefs about the community detection problem, Section 4 discusses about the direct comparative analysis measure, Section 5 is dedicated to experimental analysis and Section 6 concludes the paper.

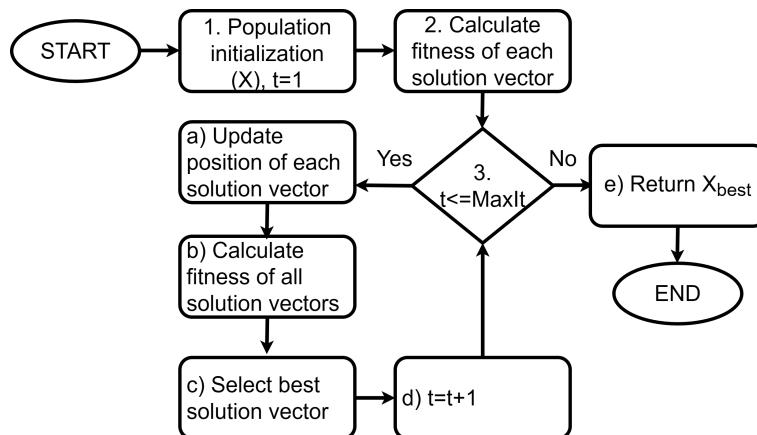


Fig. 1: Generic Flow diagram of NIOAs.

2 Nature Inspired Optimization Algorithms

NIOAs share a set of steps that is portrayed by the generic workflow of the algorithm in Figure 1. In the first step, the algorithm generates a set of candidate solutions. This candidate solution generation is called population initialization (X) which requires setting of three parameters such as population size, number of dimensions and setting the range of value of solution element. The second step deals with evaluation of the goodness of each of the candidate solution using fitness function. Following this, the termination criteria or the maximum number of iterations (MaxIt) is assigned in third step. Until the termination criteria is satisfied, a set of procedures are repeated as enumerated in the given figure by a), b), c) and d). Firstly, position of each solution vector is updated. Next, the fitness of the updated position vector is computed and compared with the previous fitness. Subsequently, the best solution vector is selected and current iteration counter is incremented by one. Then, after the termination condition is satisfied, the algorithm returns the best solution vector. In this section, we have discussed about some of the best performing algorithms in NIOAs realm which are as follows.

2.1 Grey Wolf Optimizer (GWO)

This is a population based optimization algorithm inspired by the hunting mechanism of grey wolves found in nature [12]. The wolves are categorized in descending order of leadership hierarchy as α , β , δ and ω such that α, ω lies at the top and bottom of hierarchy respectively. GWO algorithm starts with population initialization followed by computation of fitness of wolves where the best three wolves are designated as α , β and δ . Next, the distance between each wolf and prey is computed by,

$$\vec{D} = |\vec{C} \cdot \vec{X}_p(t) - \vec{X}(t)| \quad (1)$$

where t represents number of iterations, \vec{C} indicates coefficient vector, \vec{X}_p , \vec{X} is location vector of prey and grey wolf respectively. Thereafter, position of grey wolf is updated using the following formula,

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D} \quad (2)$$

where A is a vector coefficient in $[0,2]$. Then, position of prey is updated according to the following formula,

$$\vec{X}_p(t+1) = \frac{(\vec{X}_1 + \vec{X}_2 + \vec{X}_3)}{3}, \quad (3)$$

where $\vec{X}_1, \vec{X}_2, \vec{X}_3$ represents position vector of α, β, δ wolves respectively. These set of steps are repeated until termination criteria is satisfied. Ultimately, GWO algorithm returns the best position vector for α which indicates the best solution of the problem under consideration.

2.2 Sine-Cosine Algorithm (SCA)

It is also a population based optimization algorithm where the search for optimal solution is inspired by the sine and cosine trigonometric functions [13]. Initially, SCA algorithm starts with population initialization where each individual is represented by $X_i = (x_{i1}, \dots, x_{ij}, \dots, x_{iD})$ in the D-dimensional search space. Next, the optimal solution is obtained using sine and cosine functions depicted by the following formula,

$$X_i^{t+1} = X_i^t + r_1 \times \sin(r_2) \times |r_3 X_{best}^t - X_i^t|, \quad r_4 < 0.5 \quad (4)$$

$$X_i^{t+1} = X_i^t + r_1 \times \cos(r_2) \times |r_3 X_{best}^t - X_i^t|, \quad r_4 \geq 0.5, \quad (5)$$

where X_i^t indicates the position of search space at t^{th} iteration, X_{best}^t refers to the best position in t^{th} iteration. Equation 4 and 5 indicates that SCA comprises of four key parameters such as r_1 , r_2 , r_3 and r_4 where r_1 represents the search region. This region lies either between the search agent and target or outside, r_2 refers to the extent the movement is done towards or outside the target, r_3 is used to emphasize ($r_3 > 1$) or de-emphasize ($r_3 < 1$) the current optimal solution in order to compute the distance to be covered by search agents and r_4 is used to explore the search space deterministically by switching between sine and cosine functions.

2.3 Moth-Flame Optimization (MFO)

It is a population based optimization algorithm inspired by the transverse orientation of moths around light sources [14]. Moths travel long distances in a straight line by maintaining a fixed angle with the moon. MFO algorithm basically comprises of three primary steps. The first step is population initialization of moths using a matrix $M(t)$ in a D-dimensional search space. Next, fitness of individual moths are stored in an array.

This is followed by storing the flames which are the best positions obtained by moths when searching the search space and is similarly represented in matrix $F(t)$ and it's corresponding fitness values are stored in array $OF(t)$. Next, as the moths come across flames/ artificial light, they try to maintain a similar fixed angle with the flames resulting into a deadly spiral path towards the flames. Therefore, the second step is associated with updating the position of moths using the following formula,

$$M_i(t) = Dis_i(t) \times e^{bk} \times \cos(2\pi k) + F_j(t), \quad (6)$$

$$Dis_i(t) = |F_j(t) - M_i(t)|, \quad (7)$$

where $M_i(t)$ refers to the moth's position in i^{th} iteration, $Dis_i(t)$ represents the distance moth $M_i(t)$ and corresponding flame $F_j(t)$, k is a random number that lies in the range [-1,1], b depicts shape of logarithmic spiral.

2.4 Whale Optimization Algorithm (WOA)

It is a population based optimization algorithm inspired by the hunting mechanism of humpback whales [15]. Firstly, population of search agents is initialized and fitness of individual search agents is computed. Considering the fitness values, the current best search agent is assumed to be the target prey. Secondly, the position of other search agents are updated near the target prey based on parameters p and A . These parameters controls position updating by incorporation of these parameters into three different rules such as encircling prey where $p < 0.5$ and $|A| < 1$, search for prey where $p < 0.5$ and $|A| \geq 1$ and spiral updating position where $p \geq 0.5$. The position of search agent $\vec{X}(t+1)$ using is updated by encircling prey at iteration $t+1$ using Equation 8 and Equation 9.

$$\vec{D} = |\vec{C} \cdot \vec{X}^* - \vec{X}(t)| \quad (8)$$

$$\vec{X}(t+1) = |\vec{X}^*(t) - \vec{A} \cdot \vec{D}|, \quad (9)$$

$$\vec{A} = 2\vec{a} \cdot \vec{r} - \vec{a} \quad (10)$$

$$\vec{C} = 2 \cdot \vec{r} \quad (11)$$

\vec{X}^* represents the best search agent in the current iteration t , $\vec{X}(t)$ represents position of a search agent at iteration t , the value of a decreases from 2 to 0 over the iterations, r is a random number in range $[0,1]$. Next, searching for prey is similar to encircling prey. However, the only difference is that \vec{X}^* is replaced with a randomly selected search agent \vec{X}_{rand} . In spiral position update, the positions of individual search agents are updated using the following equation,

$$\vec{X}(t+1) = \vec{D} \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t), \quad (12)$$

where $\vec{D} = |\vec{X}^* - \vec{X}(t)|$ which indicates the difference of the distance between the target prey and the search agent at the current iteration, b is constant, $l \in [-1, 1]$. The position of search agents are updated until the termination criteria and finally WOA algorithm returns the best search agent.

3 Community Detection Problem

The problem of community detection in networks belong to the class of graph partitioning problem, and it is thus a NP-hard problem [16]. Therefore, it has received a lot of attention in recent years and several community detection methods have been introduced for identifying communities in networks. A network comprises of a set of entities and relationships/ connections shared by the entities. Networks are represented in the form of a graph indicated by $G(V, E)$ comprising of nodes (V) referring to entities and edges (E) specifying connections. The problem is to divide the network into several

communities $C = \{C_1, C_2, C_3, \dots, C_k\}$ where each community say C_i , $\forall i = 1, 2, \dots, k$ consists of a set of nodes belonging to V such that the number of connections within C_i should be maximized and number of connections between C_i and other communities should be minimized. These maximization or minimization requires the use of fitness function in order to obtain the best solution.

Suppose, $G(V, E)$ is divided into l feasible partitions $P = \{P_1, P_2, P_3, \dots, P_l\}$. Then, community detection problem is formulated as an optimization problem using the following equation,

$$f(P^{best}) = \max f(P), \quad (13)$$

where P^{best} is the desired partition of the network obtained by incorporating a fitness function f which evaluates the goodness of the network.

Fitness function: It is required to find the best solution in an optimization problem. Here, as we are considering community detection as an optimization problem, so for fitness computation, community evaluation metrics such as are modularity, Normalized Mutual Information (NMI), purity, Adjusted Random Index (ARI) etc. are used [17, 18]. Modularity is used to measure the quality of community, whereas NMI, purity, ARI is used to measure accuracy of community. Depending on the cardinality of fitness function used, community detection problem is classified as single-objective optimization problem and multi-objective optimization problem [19].

4 Direct comparative analysis

The rapid growth of NIOAs have necessitated the performance evaluation of the respective algorithms. Though several statistical measures such as mean, standard deviation and median are used for performance comparison purpose, but these measures do not directly compare the solutions given by two separate algorithms say primary algorithm (A_p) and alternative algorithms (A_q), where A_p refers to those algorithms whose performance is to be evaluated and A_q refers to the set of algorithms with which A_p is to be compared. In this paper, we have used D-scores and K-scores for direct comparison and overall comparison respectively to evaluate the quality of communities [20].

Direct Optimality (DO:) A_p is compared with A_q in terms of optimality by combining the comparative performance considering best performance, average performance and worst performance of A_p with respect to A_q denoted by O_1, O_2, O_3 respectively and is defined by,

$$DO = O_1 + 0.5 * O_2 - O_3 \quad (14)$$

Direct Comparability (DC): A_p is compared with algorithm A_q in terms of three levels of abstractions such as win, tie and loose denoted by C_1, C_2 and C_3 respectively and is defined by,

$$DC = C_1 + 0.5 * C_2 - C_3 \quad (15)$$

Table 1: Dataset Statistics. First column contains dataset details, # Nodes refers to number of nodes, # Edges refers to number of edges, Avg. degree indicates average degree of the graph.

Dataset	# Nodes	# Edges	Avg. degree
Karate [21]	34	78	4.58
Dolphin [22]	62	159	5.12
Football [23]	115	613	10.66

Overall Optimality (KO): The overall optimality of A_p is computed based on three levels of abstraction such as best, average and worst irrespective of win or loose indicated by K_1^0 , K_2^0 and K_3^0 respectively and is defined by,

$$KO = K_1^0 + 0.5 * K_2^0 - K_3^0 \quad (16)$$

Overall Comparability (KC): A_p is compared with A_q by considering overall comparability in all three levels of abstraction such as win, tie and loose indicated by K_1^c , K_2^c and K_3^c respectively and is defined by,

$$KC = K_1^c + 0.5 * K_2^c - K_3^c \quad (17)$$

Overall Together (KT): It is used to interpret that A_p performs better than A_q considering that abstraction levels such as best & average and win & tie are overlapping and is defined by,

$$KT = \frac{a + b + d + e}{n} \quad (18)$$

where a,b, c and d represents the overlapping abstraction levels, n indicates total number of possible combinations of abstraction levels.

5 Experimental Analysis

In this work, experiments are conducted on several widely used real-world datasets such as karate network [21], dolphin network [22] and football network [23] summarized in Table 1. Several state-of-the-art NIOAs algorithms such as GWO, MFO, SCA and WOA have been used on community detection to perform a comparative analysis of these algorithms using average isolability and five different performance measures based on optimality and comparability [24]. Also, the performance of NIOAs algorithms for community detection is highly dependent on parameter settings. Therefore, in this section, we discuss about algorithm parameter settings, average isolability and result analysis.

5.1 Algorithm parameter settings

There are two types of parameters in NIOAs algorithms namely, common parameters and algorithm specific parameters. Parameters that are common in all NIOAs algorithms

are called common parameters and parameters specific to a particular NIOAs algorithm are the algorithm specific parameters. There are particularly three common parameters namely population size, number of dimensions and number of iterations which are described below.

Number of dimensions: In community detection context, number of dimensions is equal to the total number of nodes present in a network. The size of candidate solution is equal to the number of dimensions. Total number of such candidate solutions indicates population size.

Population size: The population size needs to be carefully initialized because the best solution might be dependent on population size. Setting a high population size improves the search capability but leads to increase in time complexity of the algorithm. In our experiments, we have set the population size as 30.

Number of iterations: It is also a key parameter to find the optimal solution. Initially, current iteration is set to 1. For specification of number of iterations, two aspects are to be considered. Firstly, if the number of iterations is small, then the optimal solution might not be found. Whereas, large number of iterations increases time complexity of optimization algorithms and may lead to redundancy i.e. iterations may continue even after attaining the best solution. Therefore, number of iterations must be carefully set.

5.2 Average Isolability

It is required to compare and improve the candidate solutions to obtain a near optimal solution. In our experiment, we have considered individual cluster specific fitness function namely, AVerage Isolability (AVI) [24] where the objective is to examine the ability of a cluster to isolate itself from rest of the network by examining the nodes based on the strength of connections. Therefore, to find the optimal solution, we have maximized AVI. For an undirected graph, Isolability of a cluster C_i is defined by,

$$\text{Isolability}(C_i) = \frac{\{(u, v) \mid u \in_{C_i} v\}}{\{\{(u, v); (u, w)\} \mid u \in_{C_i} v \& w \notin C_i\}}, \quad (19)$$

where, the numerator term indicates connections within the community C_i and denominator is the total number of connections. Next, AVI is defined by,

$$Q_{AVI}(G, C) = \frac{1}{K} \sum \text{Isolability}(C_i), \quad (20)$$

where k indicates total number of clusters in $G(V, E)$.

5.3 Result Analysis

Quality of communities given by GWO, MFO, SCA and WOA have been analyzed on three widely used real-world datasets. The analysis has been carried based on the emphasizing on the quality of the community given by each baseline algorithm and performing comparative evaluation. AVI value is used for quality evaluation. In addition to this, performance analysis based on one-to-one comparison (D-scores) and one-to-many comparison (K-scores) is performed.

Table 2: Comparative performance of MFO algorithm with alternative algorithms based on D-scores and K-scores.

Dataset	GWO					SCA					WOA				
	DO	DC	KO	KC	KT	DO	DC	KO	KC	KT	DO	DC	KO	KC	KT
Karate	0.98	0.98	1.00	1.00	1.00	0.98	0.98	1.00	1.00	1.00	0.06	0.75	0.53	1.0	1.0
Dolphin	1.44	0.76	1.00	0.78	1.00	1.44	0.76	1.00	0.78	1.00	0.17	0.17	0.31	0.30	0.64
Football	1.44	0.75	1.00	0.76	1.00	1.44	0.75	1.00	0.76	1.00	1.44	0.75	1.00	0.76	1.00

5.3.1 Result analysis with Average Isolability: The AVI scores of the communities given by GWO, MFO, SCA and WOA on real-world datasets are shown in Figure 2. Let us try to analyze the performance of these algorithms with the help of this figure. Here, the X-axis represent real-world datasets namely karate, dolphin and football; Y-axis represents AVI score. The performance of GWO, MFO, SCA and WOA is shown using teal, lime, yellow and green colored bars respectively. The values corresponding to each bar indicates AVI score of the respective algorithms on a given dataset. Higher AVI score indicates good performance of corresponding algorithm and the performance deteriorates with decrease of AVI score. Therefore, the results shown in Figure 2 indicates that MFO algorithm gives the best performance on all the datasets and WOA algorithm shows the worst performance on karate and dolphin dataset. Whereas, GWO algorithm shows the worst performance on football dataset.

5.3.2 Result analysis based on D-scores and K-scores: D-scores and K-scores are used to evaluate the performance of all possible combinations of the baseline algorithms in terms of the quality of communities given by the respective algorithms. All such combinations of baseline algorithms indicated by (A_p, A_q) is considered as a comparable algorithm pair. The results of comparable algorithm pairs such as $(A_p = MFO, A_q =$

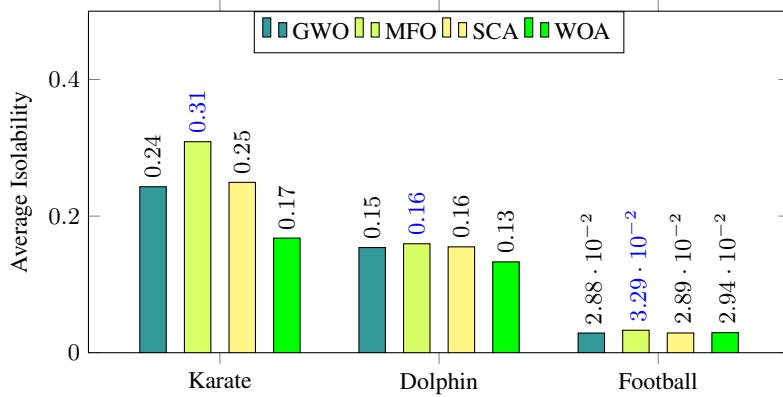


Fig. 2: Comparative analysis of GWO, MFO, SCA and WOA based on Average Isolability.

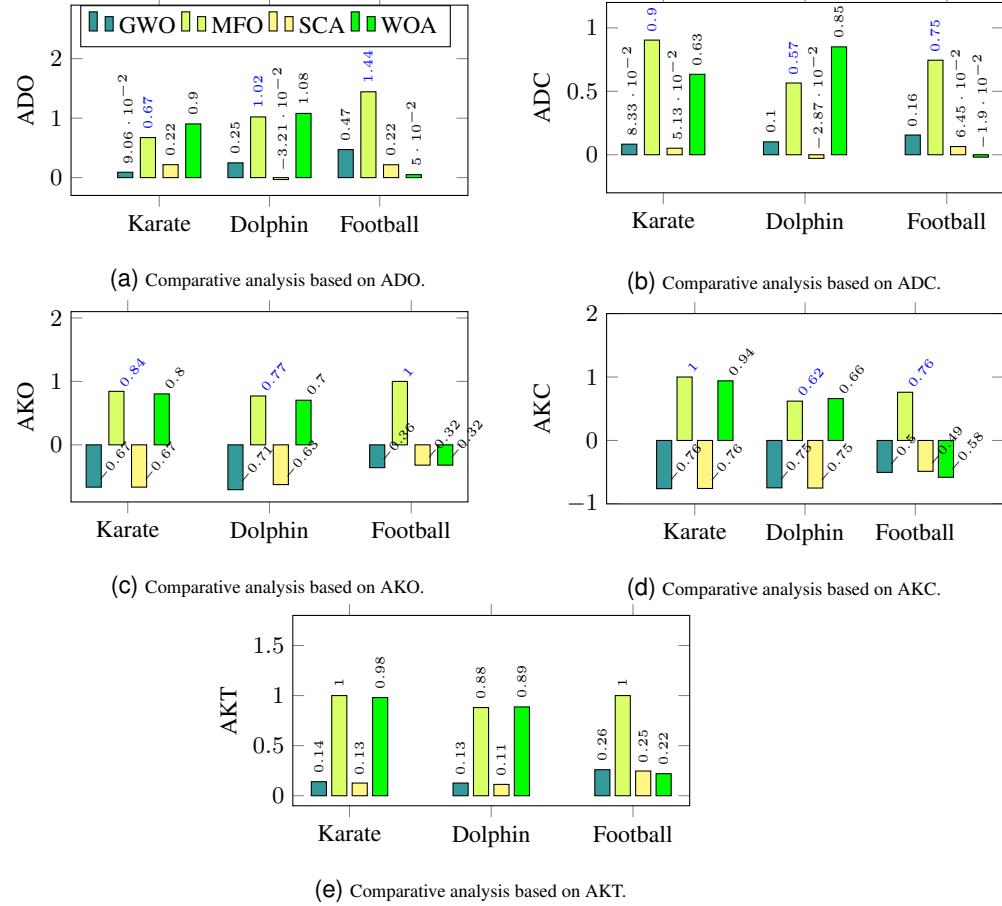


Fig.3: Comparative analysis based on average D-score and K-scores i.e. ADO, ADC, AKO, AKC and AKT values for the of communities identified with GWO, MFO, SCA and WOA on real-world datasets.

GWO), ($A_p = MFO, A_q = SCA$) and ($A_p = MFO, A_q = WOA$) in terms of D-scores and K-scores on karate, dolphin and football dataset are summarized in Table 2. Then, average DO (ADO), average DC (ADC) score, average KO (AKO), average KC (AKC) and average KT (AKT) is obtained by summation of corresponding DO, DC, KO, KC and KT scores of all comparable algorithm pairs with $A_p = MFO$ divided by the total number of such pairs and the results are shown in Figure 3. High ADO, ADC, AKO, AKC, AKT scores indicate that A_p performs better than A_q in terms of optimality and comparability. For each dataset and corresponding performance measure, highest positive score obtained by the respective algorithm is ranked as 1, second highest is ranked as 2 and so on. Following this ranking procedure, MFO algorithm is ranked as 1

and hence, it is the best performing algorithm in terms of D-scores, K-scores. Following this strategy, SCA gives the worst performance.

6 Conclusion

A quality measure based on connection strength associated with a cluster called average isolability and a direct comparison approach based on five scores designed based on prasatul matrix is used to evaluate the quality of communities considering optimality and comparability. Four NIOAs and three widely used real-world datasets are used to perform comparative analysis. Results based on average isolability indicate that the MFO algorithm gives the best performance on all datasets. Whereas, WOA algorithm has the worst performance on karate and dolphin datasets, GWO algorithm has the worst performance on football datasets. Following this, the performance analysis based on the five scores derived from prasatul matrix suggests that the MFO algorithm achieves the best performance and the SCA algorithm gives the worst performance.

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