Constrained Optimization and Linear Programming

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Outline

- Constrained optimization
- Linear programming
- Other approaches

Constrained optimization

- Optimizing objective function is not enough
  - We also have to respect additional **constraints** on variables
  - In general, different from **boundaries** (although, terminology…)
  - Boundaries can turn into constraints ($x \in [0,2] \rightarrow x \geq 0, x \leq 2$)
  - Constraints are usually **equalities** and **inequalities**

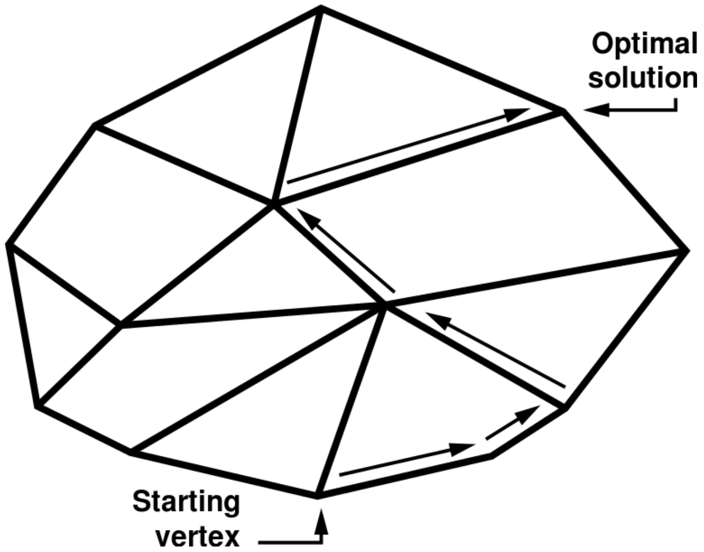
$$\begin{array}{lll} \underset{\mathbf{x}}{\text{minimize}} & f(\mathbf{x}) & \\ \text{subject to} & h_i(\mathbf{x}) = 0 & \text{for all } i \text{ in } \{1, \ldots, \ell\} \\ & g_j(\mathbf{x}) \leq 0 & \text{for all } j \text{ in } \{1, \ldots, m\} \end{array}$$

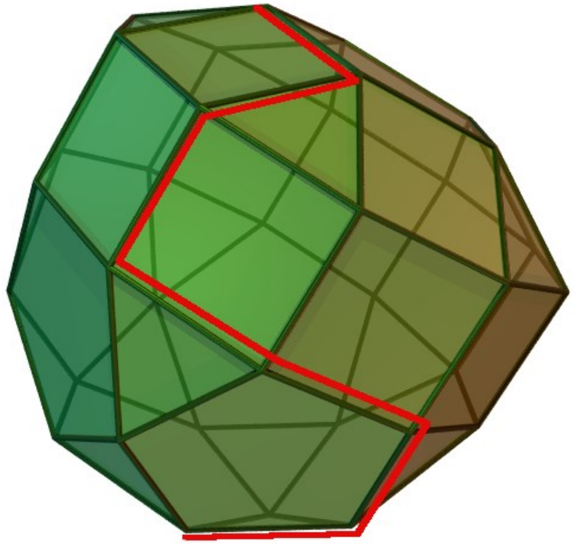
Constrained optimization

- Example: optimize the diet of soldiers
  - Minimize cost of meals
  - Attain at least a certain amount of each nutrient group
  - Original problem had 9 nutrient groups and 77 food items

Linear programming

- Linear objective function, linear constraints
  - Stating the problem properly ensures **finding global optimum**
  - **Simplex**: explore vertices of a polytope (hyper-polygon)
  - Also evaluate **feasibility** of the problem (e.g. no possible solutions)

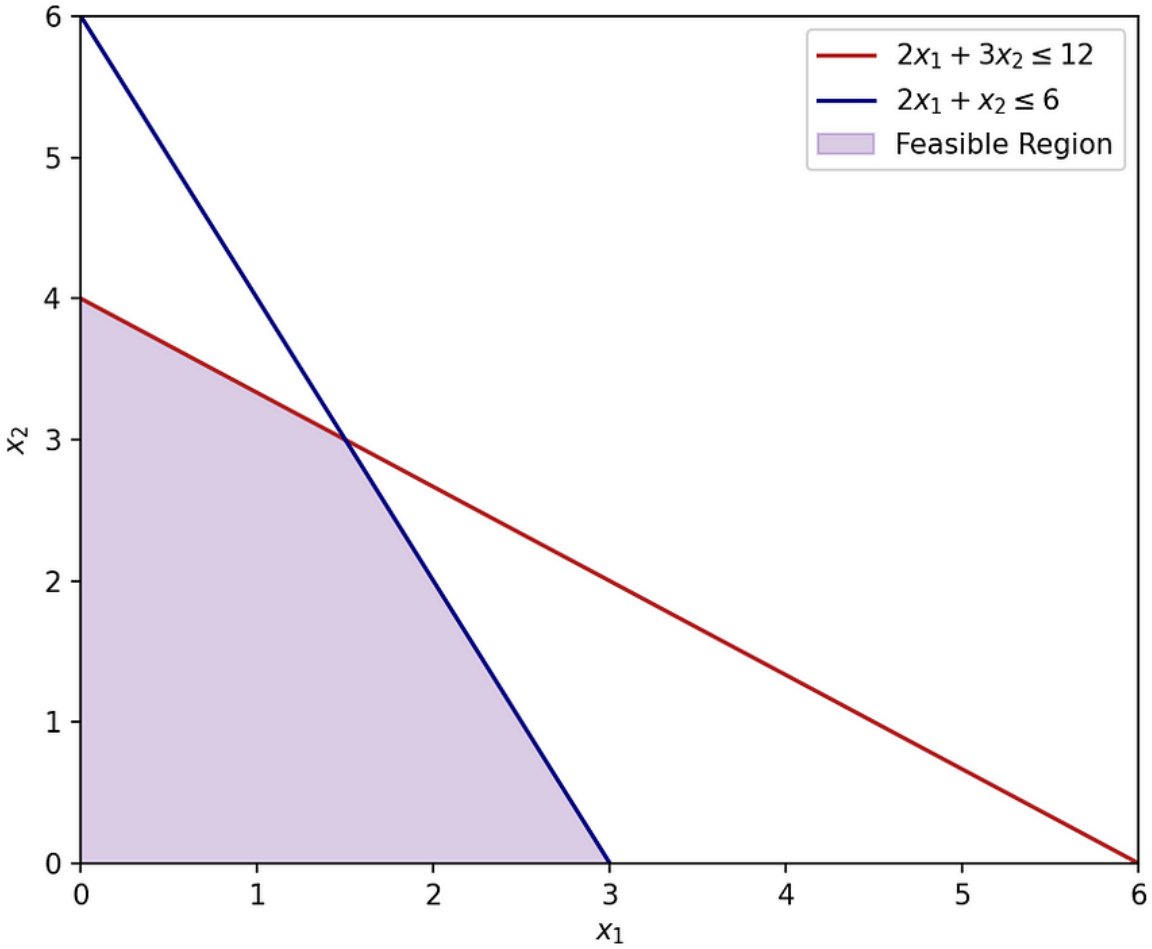




Linear programming

- Example for 2 variables

$$\text{maximize } f(x_1, x_2) = x_1 + x_2$$
$$x_1 \in [0,6]; x_2 \in [0,6]$$
$$2x_1 + 3x_2 \leq 12$$
$$2x_1 + x_2 \leq 6$$



Other approaches

- If constraints and/or objective function are not linear
- Variations of Linear Programming
  - Quadratic Programming
  - Non-linear Programming
- Or convert problem into unconstrained using **penalties**

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \\ & \mathbf{h}(\mathbf{x}) = \mathbf{0} \end{aligned} \quad \Longrightarrow \quad \begin{gathered} \underset{\mathbf{x}}{\text{minimize}}\, f(\mathbf{x}) + \rho \cdot p_{\text{count}}(\mathbf{x}) \\ p_{\text{count}}(\mathbf{x}) = \sum_i (g_i(\mathbf{x}) > 0) + \sum_j (h_j(\mathbf{x}) \neq 0) \end{gathered}$$

Questions?

Bibliography

- Kochenderfer & Wheeler, *Algorithms for Optimization*, MIT Press, 2019
- Vanderbei, *Linear Programming: Foundations and Extensions*, 2014