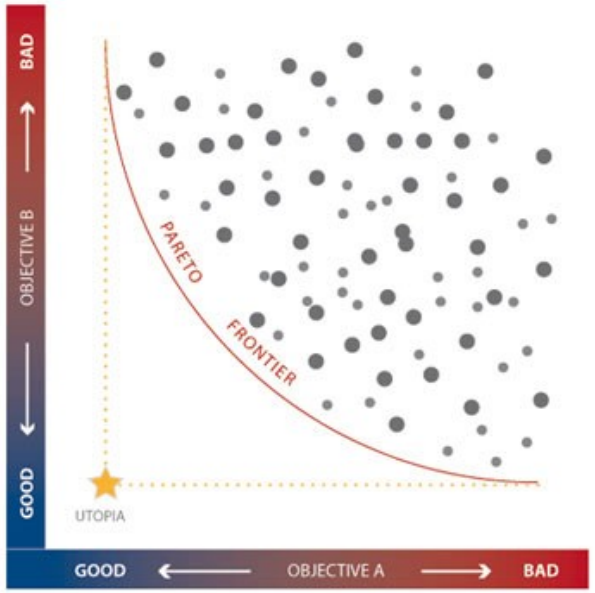
Multi-Objective Optimization

Alberto TONDA, Senior Researcher (DR)

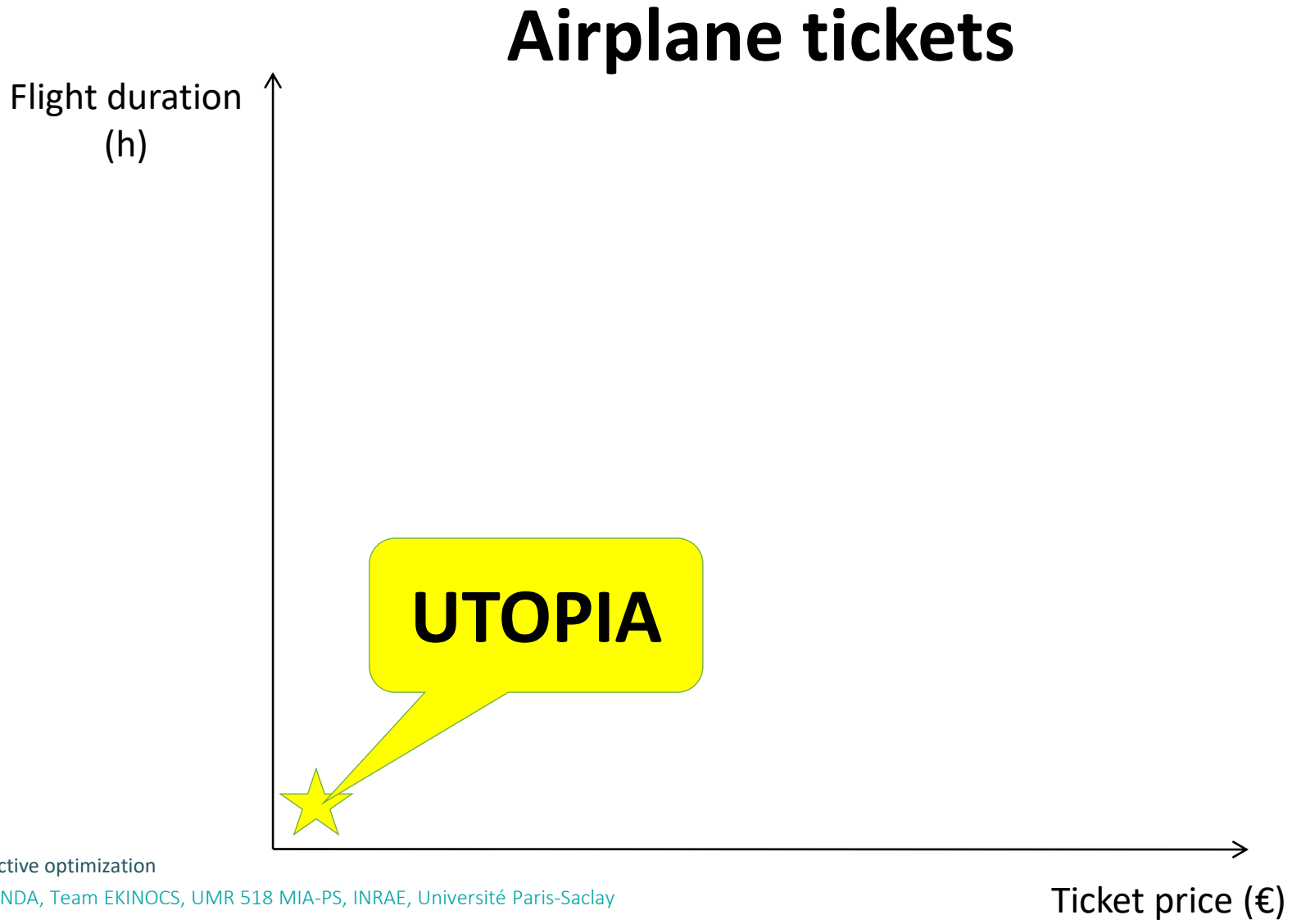
UMR 518 MIA-PS (Applied Mathematics and Computer Science)
INRAE, AgroParisTech, Université Paris-Saclay
Institut des Systèmes Complexes, Paris-Ile-de-France

Outline

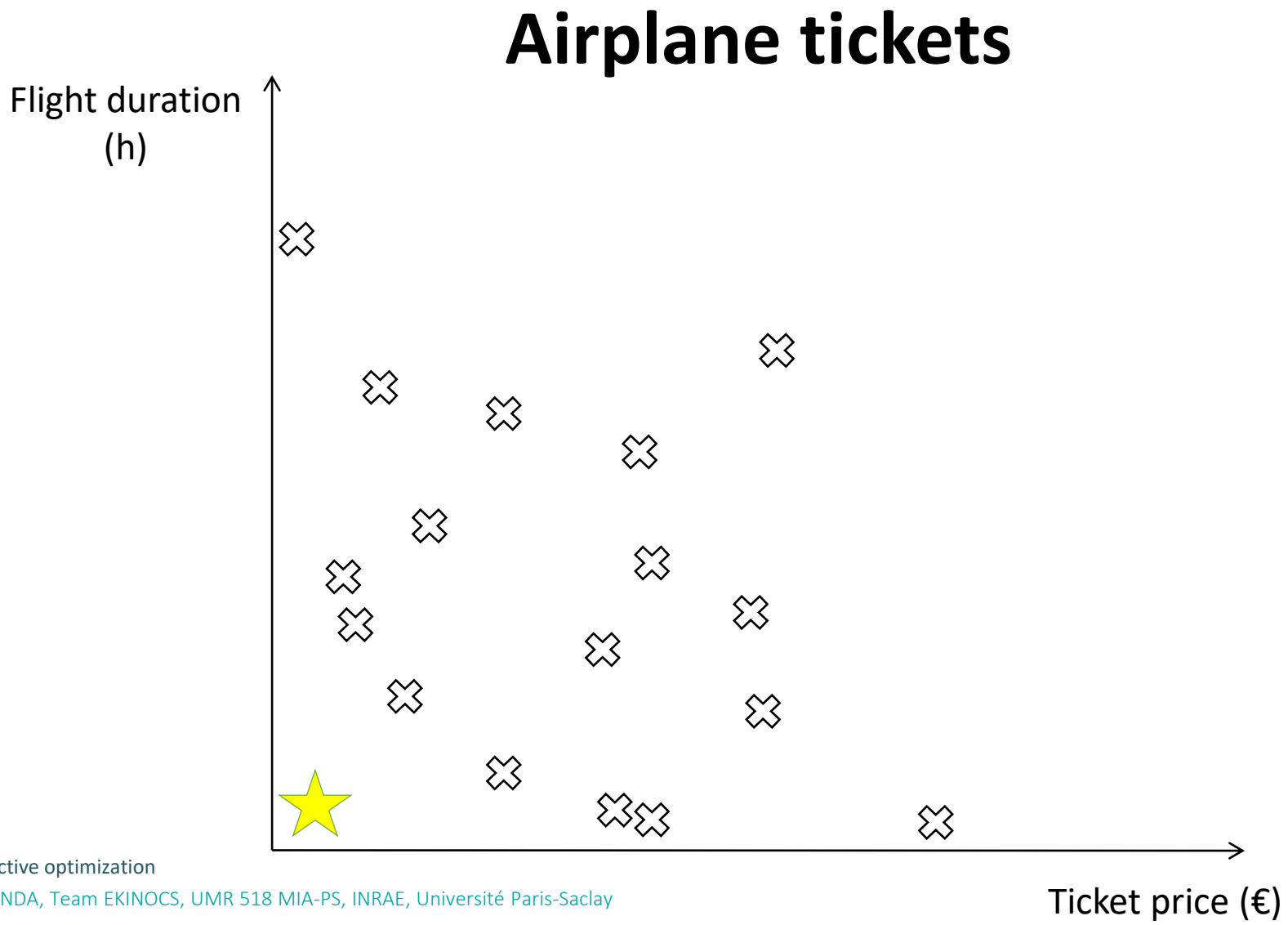
- Multi-objective problems
- Multi-objective optimization
- Real-world examples
- NSGA-II
- Other approaches
- **Many**-objective optimization…?



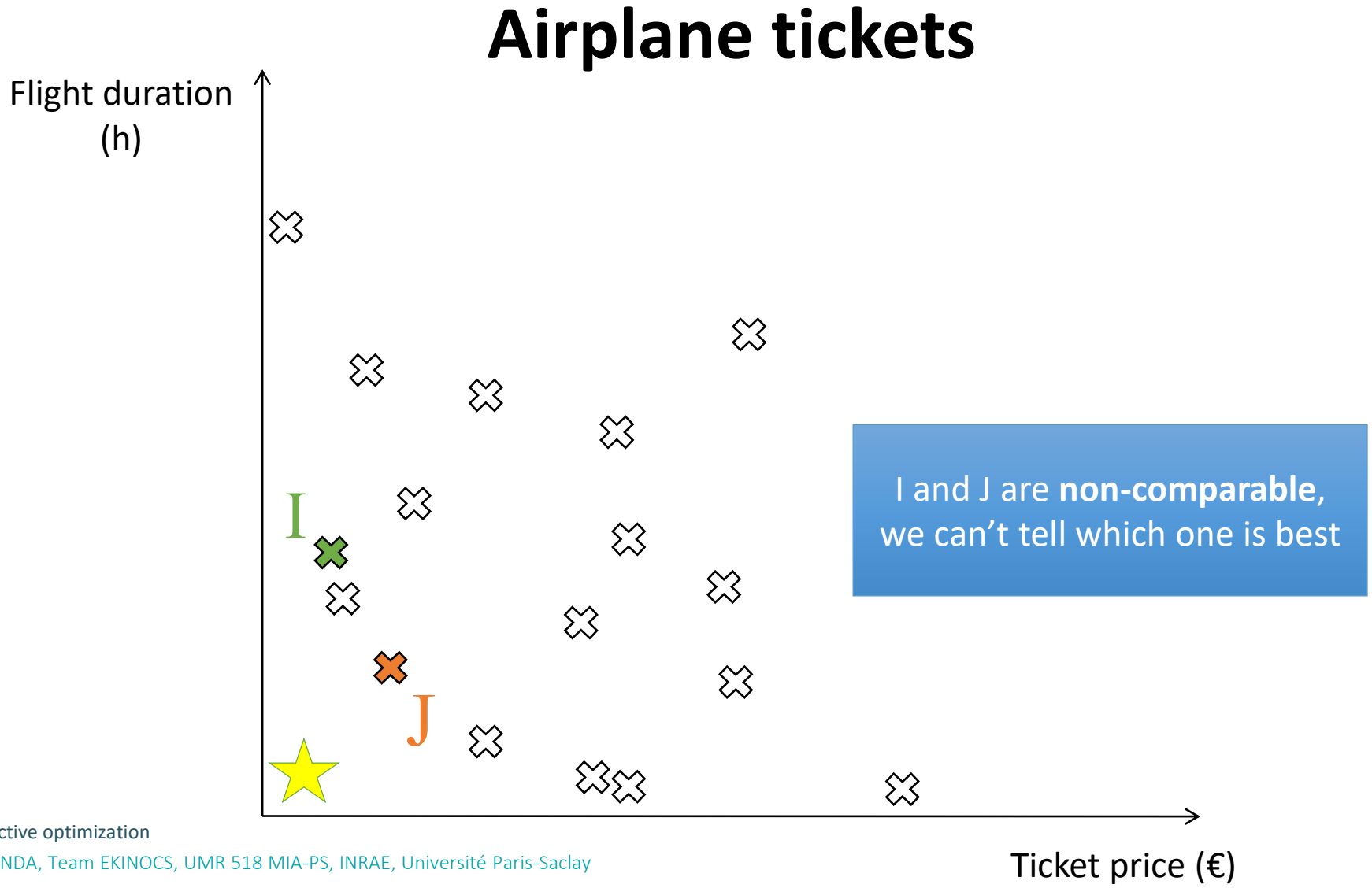
Multi-objective problems



Multi-objective problems

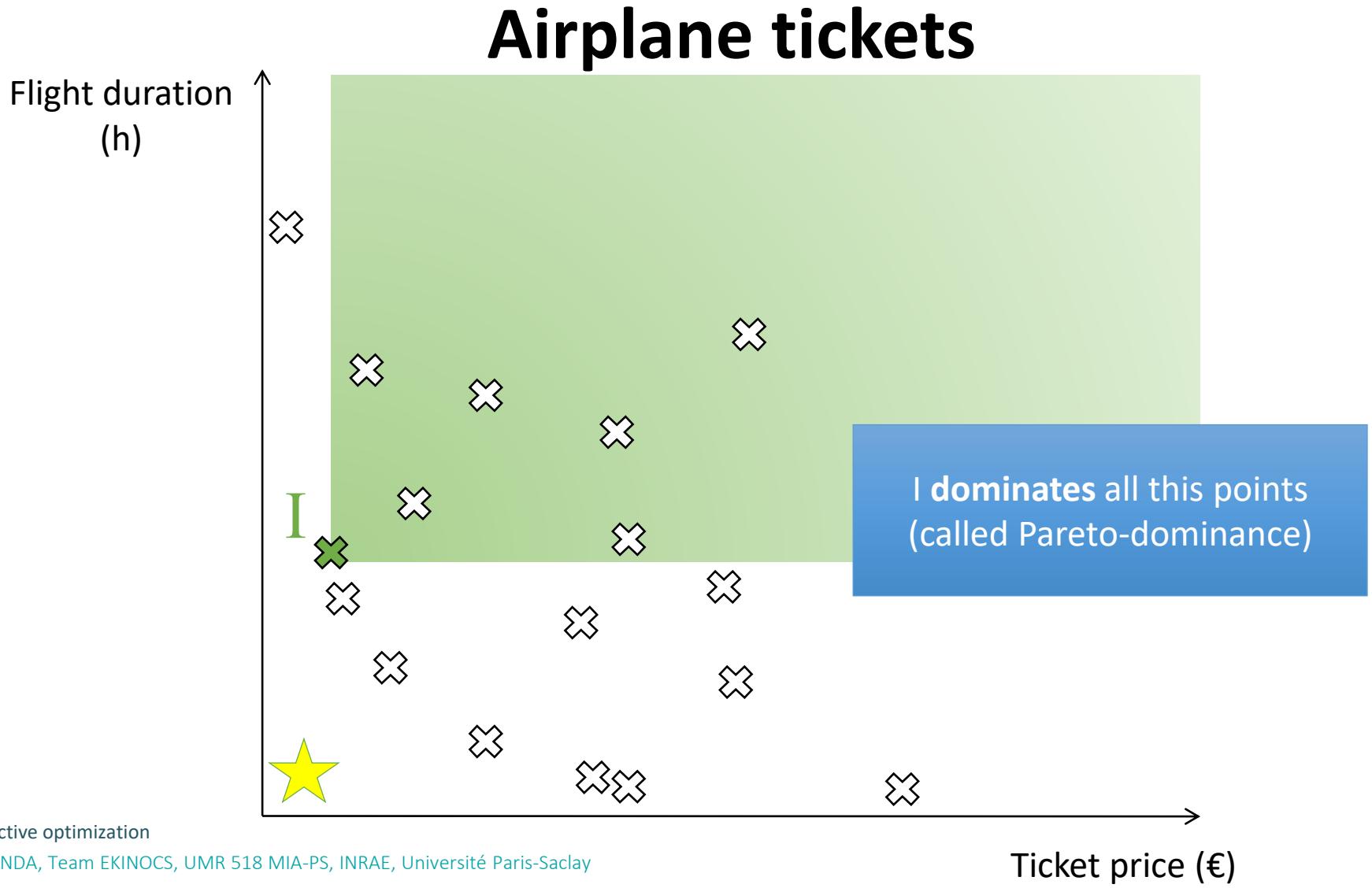


Multi-objective problems

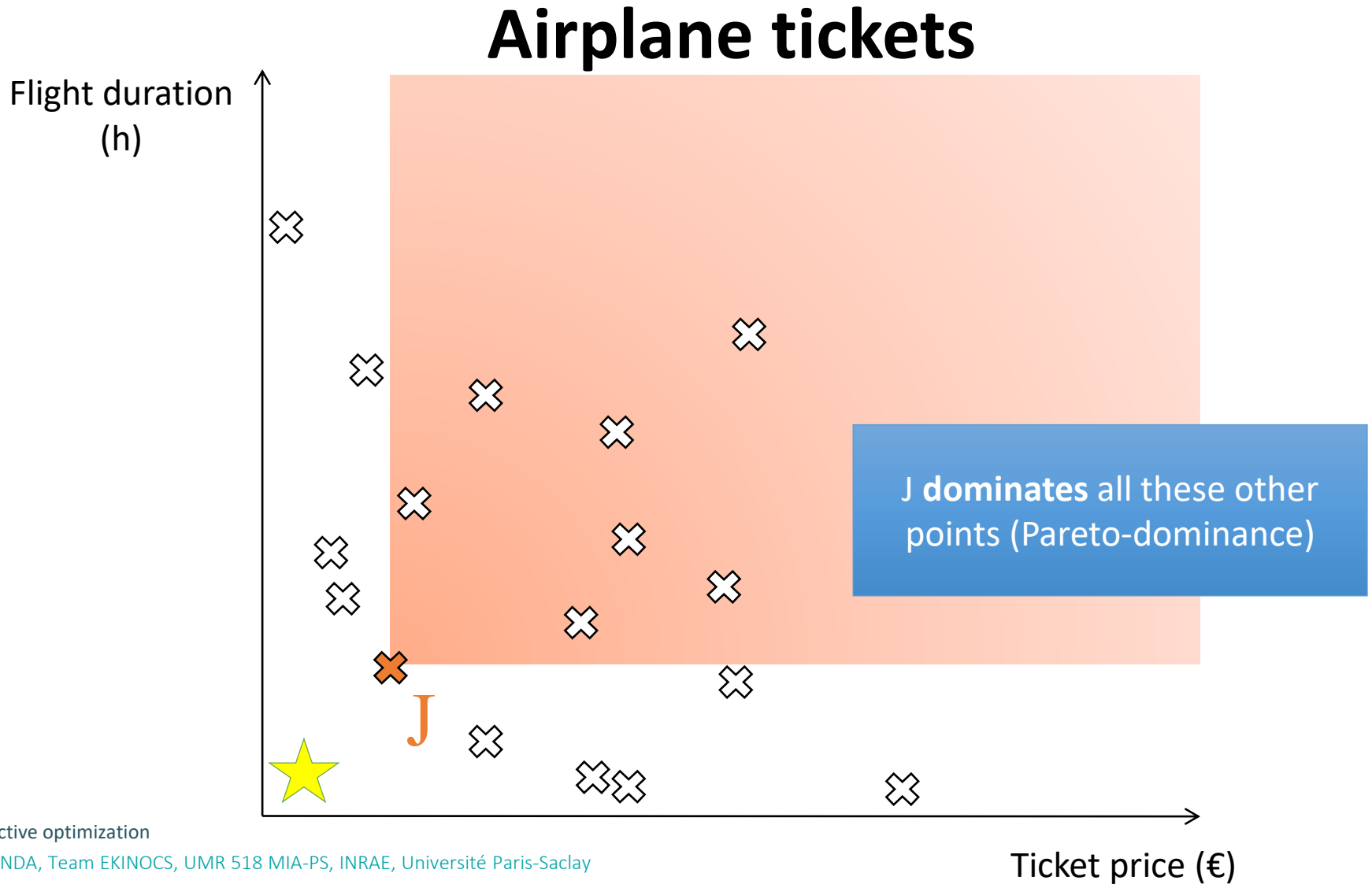


I and J are **non-comparable**, we can't tell which one is best

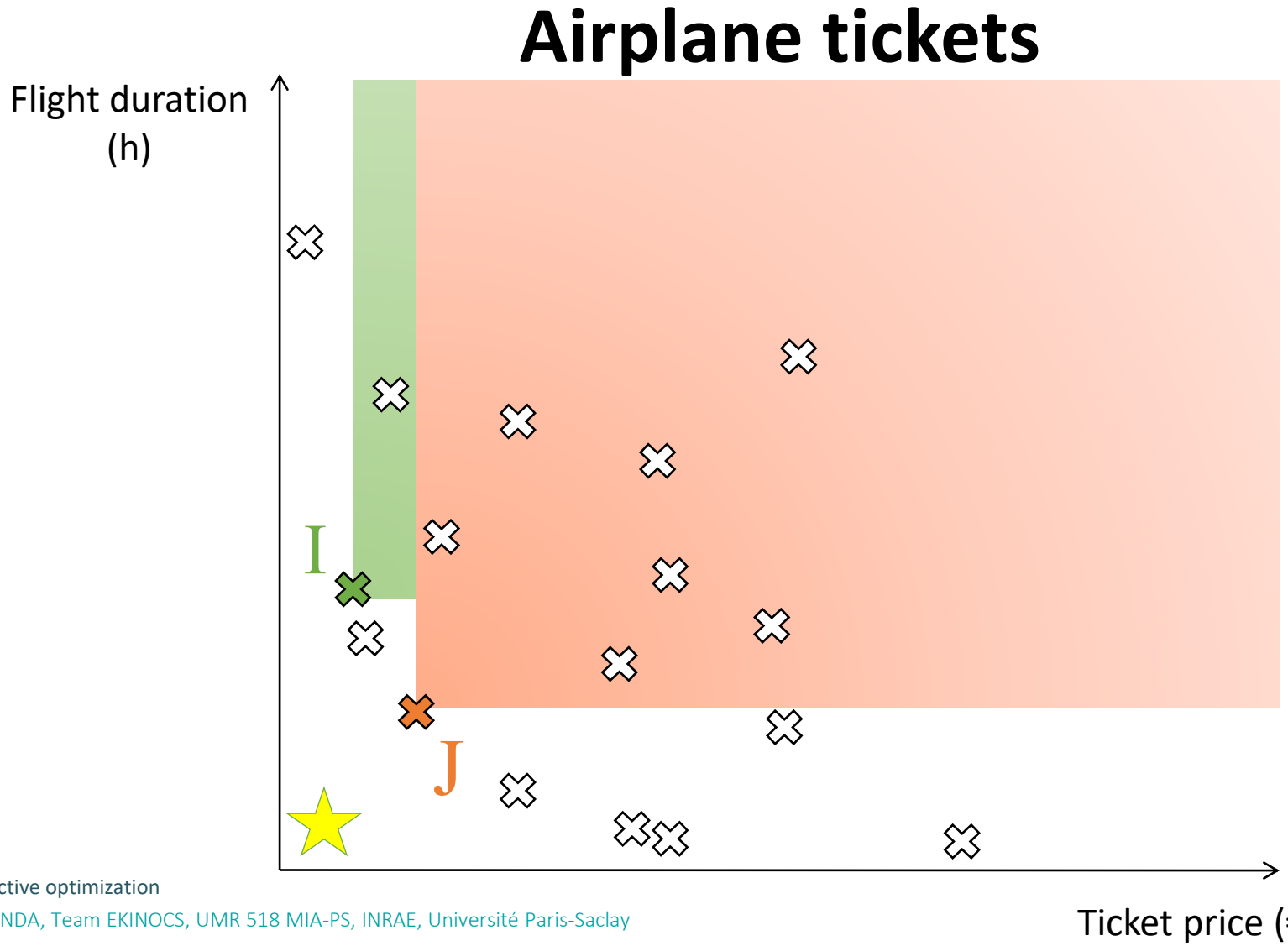
Multi-objective problems



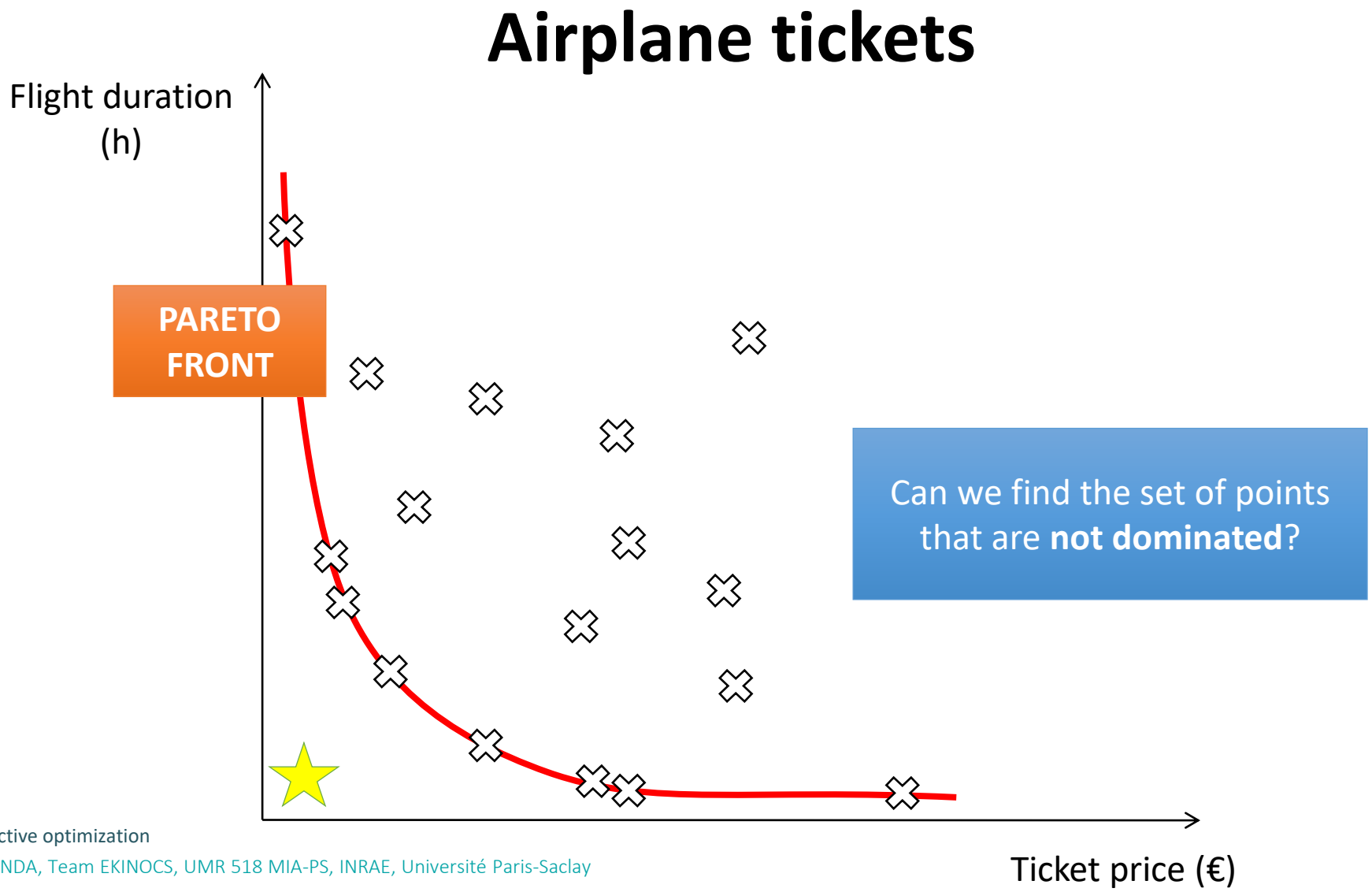
Multi-objective problems



Multi-objective problems



Multi-objective problems



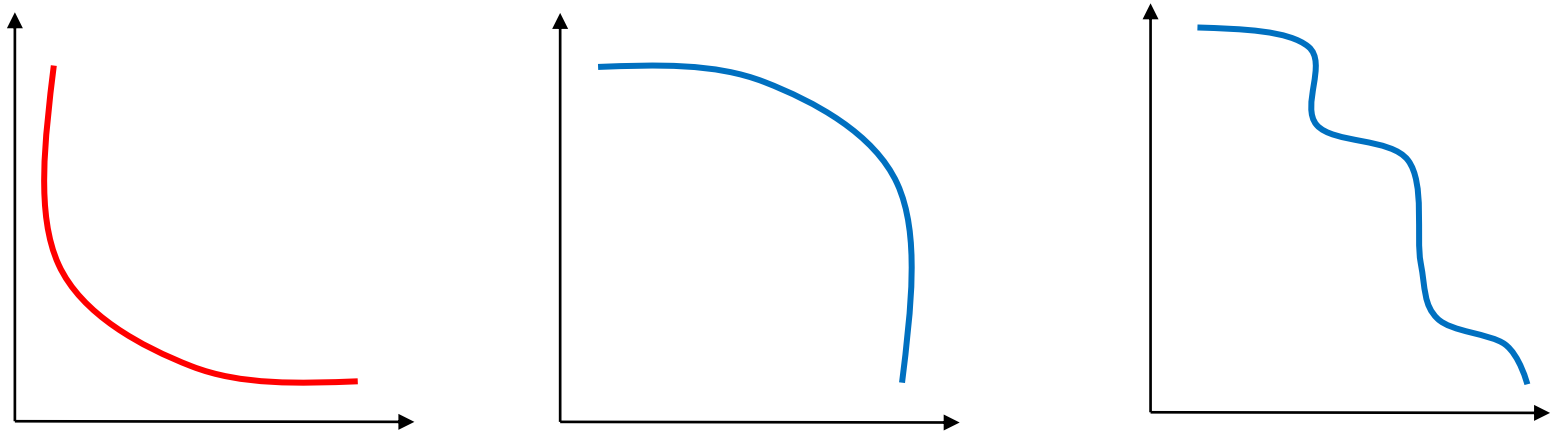
Multi-objective problems

- Pareto-optimality

x : solution $\quad f_i(x)$: fitness

$$\nexists x' : f_i(x') \geq f_i(x)\ \forall\ i, \exists j : f_j(x') > f_j(x),$$

- Pareto for *minimizing* or *maximizing*



Multi-objective problems

- Real-world problems are often MO
  - Often with A LOT of conflicting objectives
  - Plane tickets: seat position, airline, airport…
  - Production: energy, quality, price, …
  - Distribution: speed, cost, employment, …

Multi-objective optimization

- Single-objective optimization
  - Find ONE best solution

- Multi-objective optimization
  - Find THE PARETO FRONT (hard, maybe impossible)
  - Find as many non-dominated points as possible
  - Finding one point on the Pareto front is easy…
  - …but finding many is not!

Multi-objective optimization

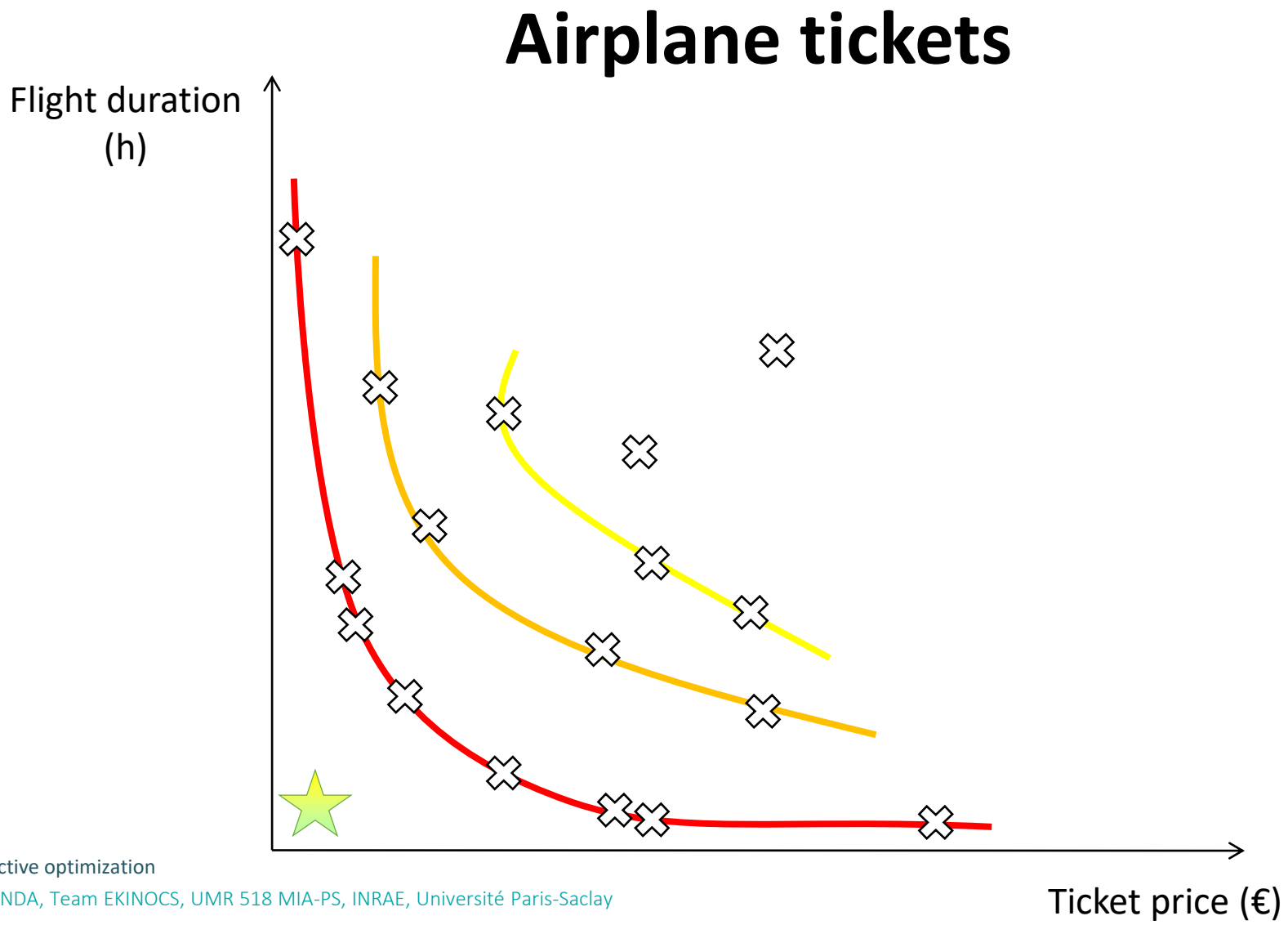
- Techniques to deal with MO
  - Assign weights to objectives, adjust weights
  - Some only work on (differential) equations
  - Multi-objective EAs (**state-of-the-art**)

- EAs are particularly suited
  - Population of solutions -> lots of points!
  - Black-box optimization -> easy to adopt!

Multi-objective optimization

- MOEAs (general idea)
  - Create population, evaluate
  - Create offspring
  - Find Pareto front
  - Remove individuals in Pareto front
  - Recompute Pareto front (iterate)
  - Obtain list of fronts
  - Kill individuals starting from worst fronts

Multi-objective optimization



Example: Influence in social networks

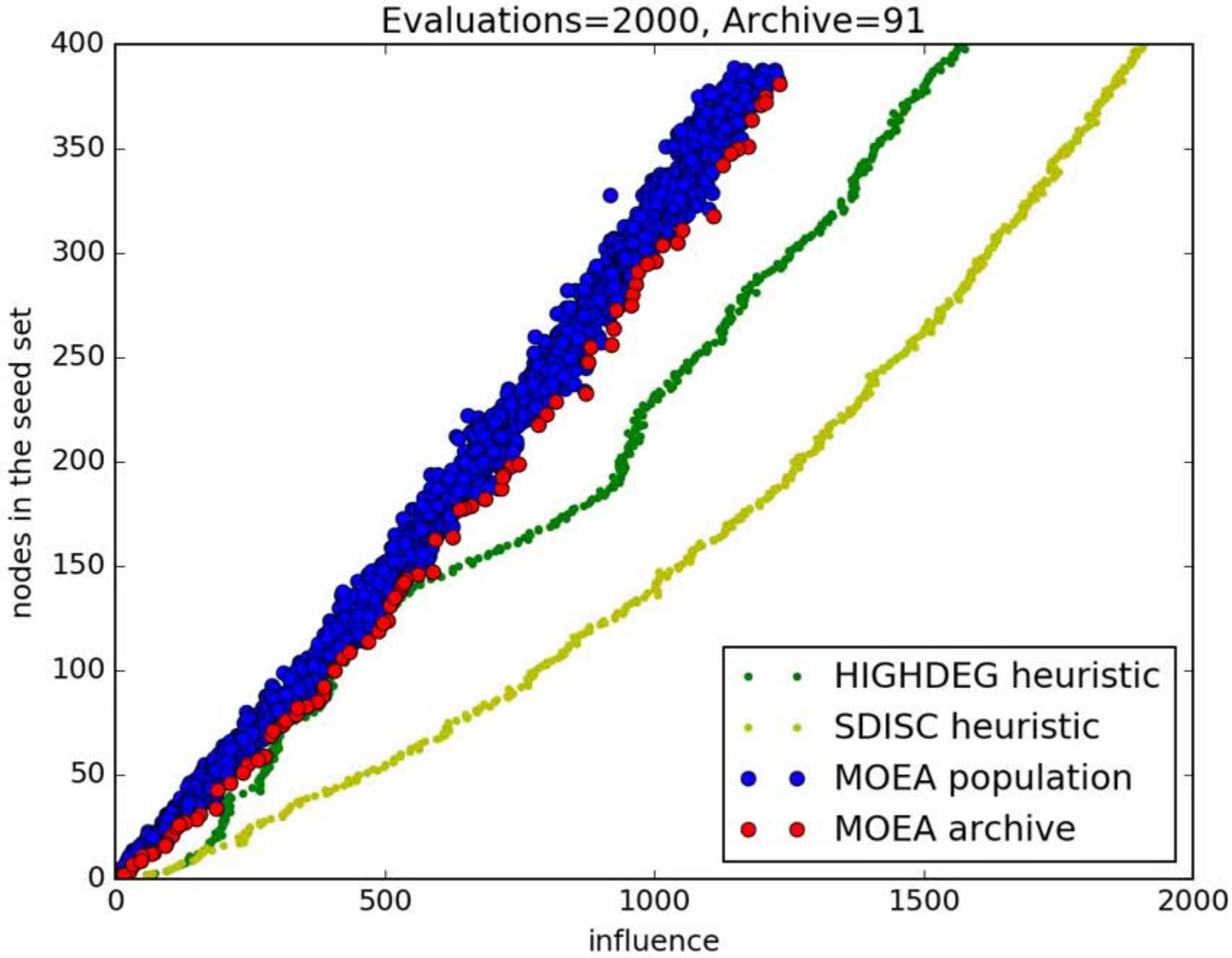
- Advertise products in social networks
  - Use influencers (lots of followers)
  - How to choose influencers? (following overlap)
  - Spend as little as possible

- Multi-objective problem
  - Minimize influencers
  - Maximize influence

Example: Influence in social networks

- Genome (candidate solution)
  - Set of nodes taken from a graph
  - Vector of integers of different size
  - String of bits (1=influencer, 0=not)

- Fitness function
  - (Max) influence spread in the network
  - (Min) number of nodes/influencers


Evaluations=2000, Archive=91
nodes in the seed set
influence
0
50
100
150
200
250
300
350
400
0
500
1000
1500
2000
HIGHDEG heuristic
SDISC heuristic
MOEA population
MOEA archive

Example: Ecosystem services

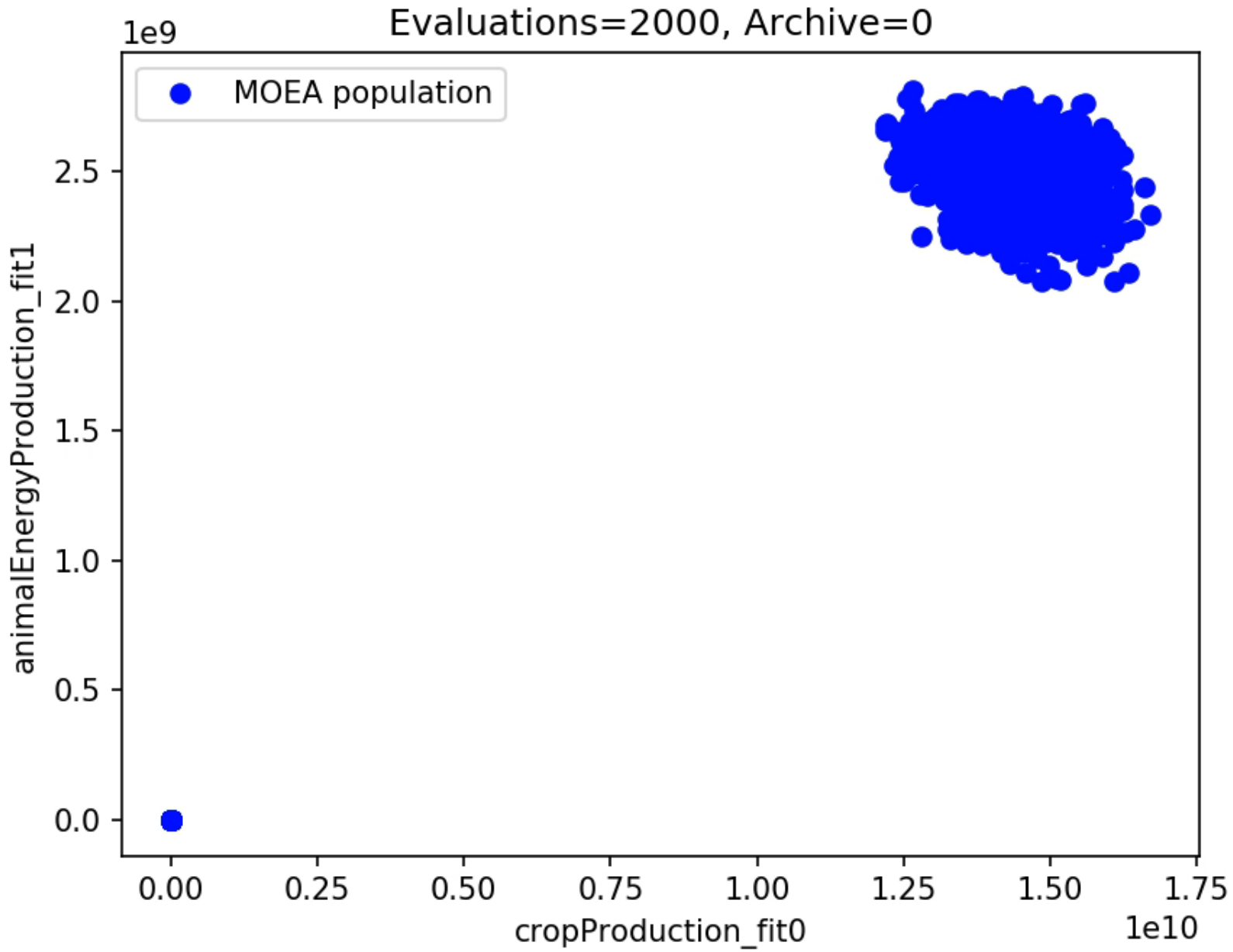
- Optimize land use in agricultural regions
  - Percentage of land assigned to each use
  - Animal feed, crops, forests (carbon sequestration)

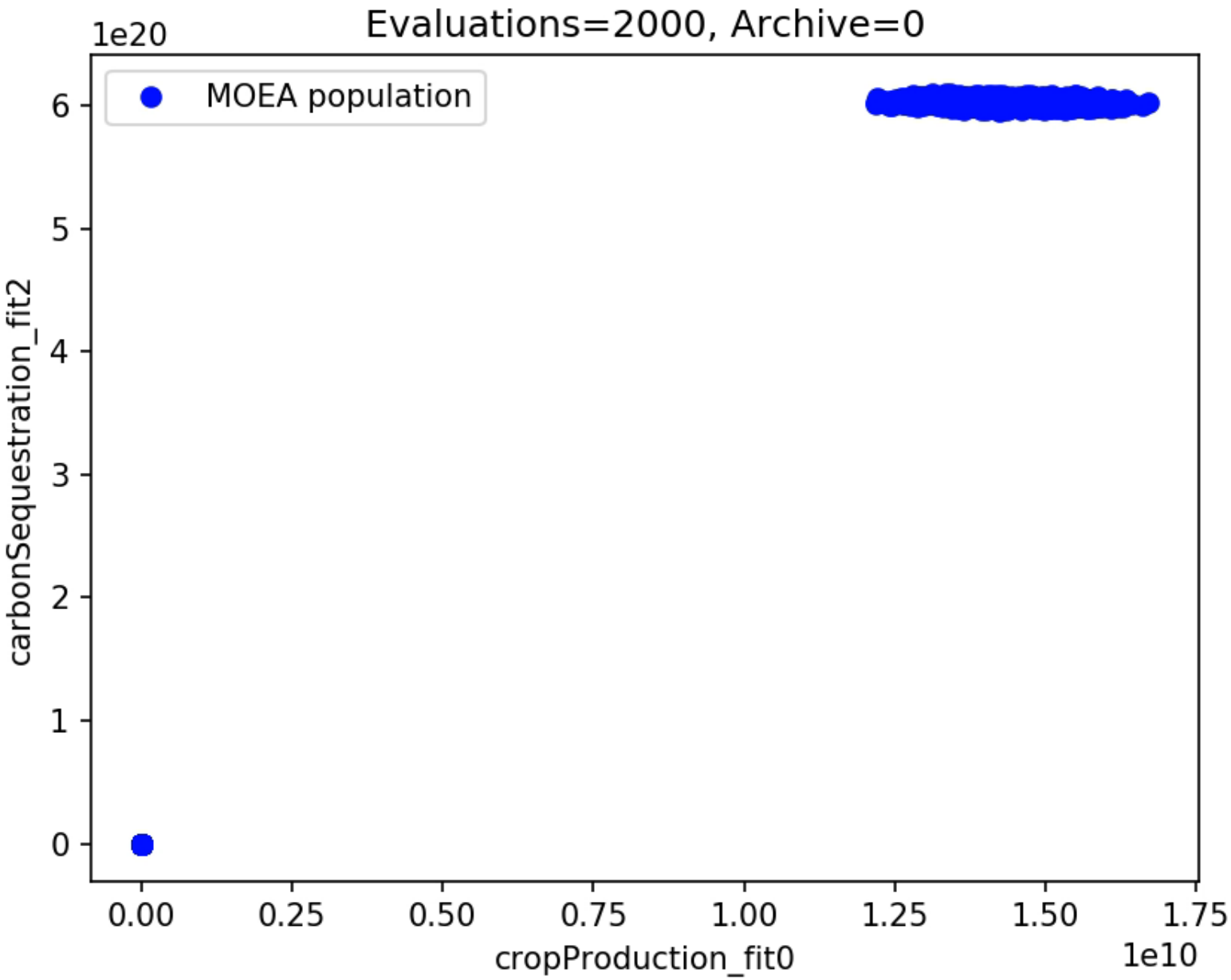
- Multi-objective problem
  - Maximize animal energy production
  - Maximize crop production
  - Maximize carbon sequestration

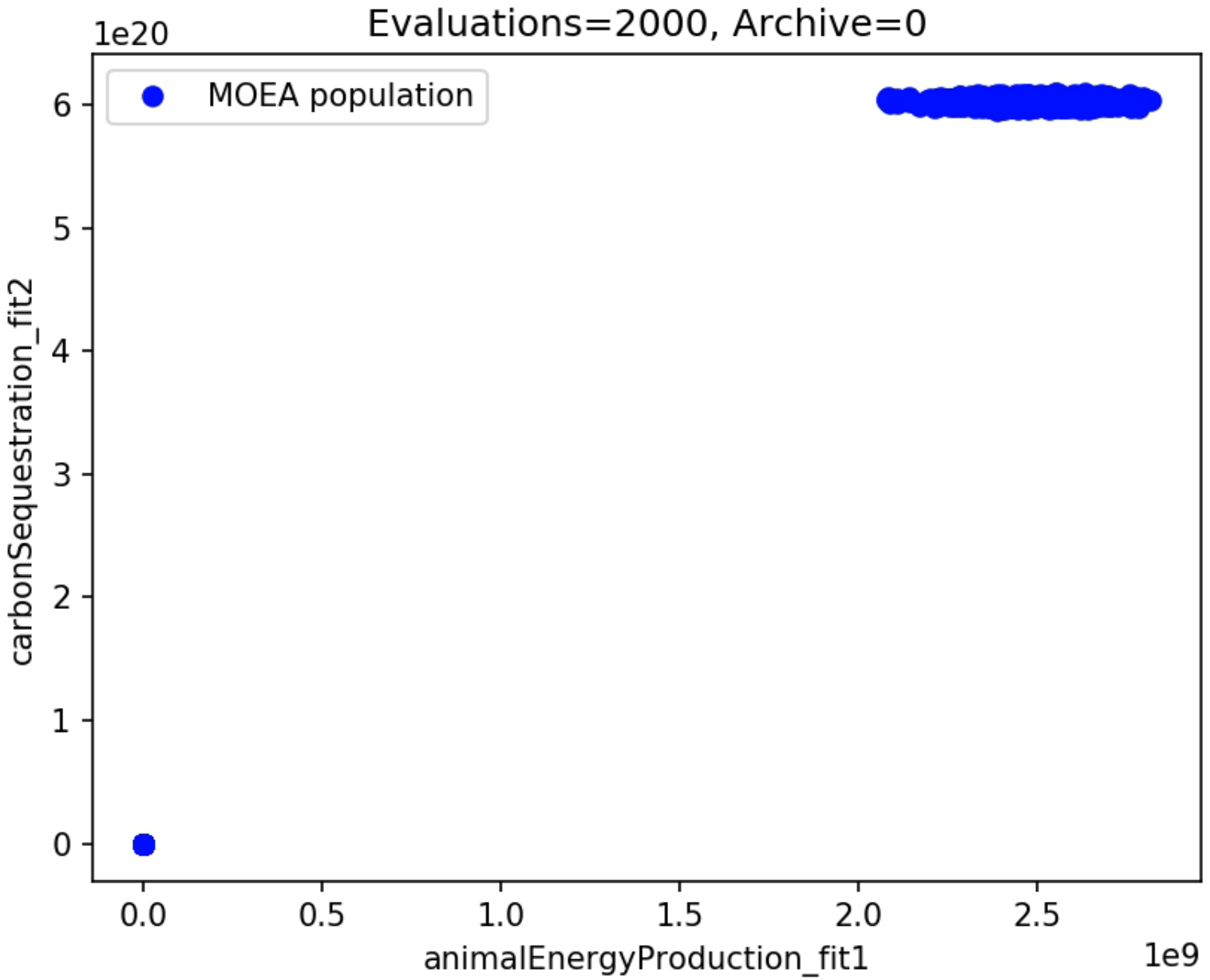
Example: Ecosystem services

- Genome (candidate solution)
  - Percentage of land assigned to each task
  - For each region! (~1500 variables for “massive central”)

- Fitness function
  - Model for animal energy production
  - Model for crop production
  - Model for carbon sequestration

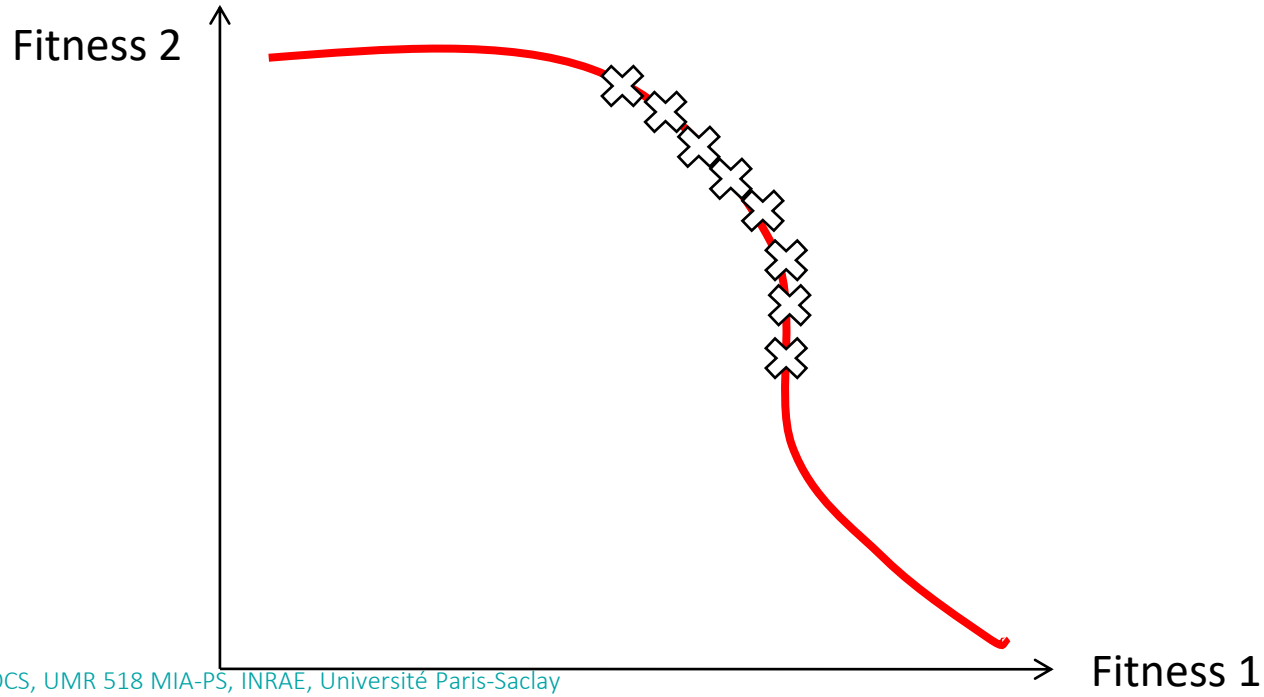

Evaluations=2000, Archive=0
1e9
MOEA population
2.5
2.0
1.5
1.0
0.5
0.0
animalEnergyProduction_fit1
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
cropProduction_fit0
1e10


Evaluations=2000, Archive=0
1e20
MOEA population
carbonSequestration_fit2
cropProduction_fit0
1e10
0
1
2
3
4
5
6
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75


Evaluations=2000, Archive=0
1e20
MOEA population
carbonSequestration_fit2
6
5
4
3
2
1
0
0.0
0.5
1.0
1.5
2.0
2.5
animalEnergyProduction_fit1
1e9

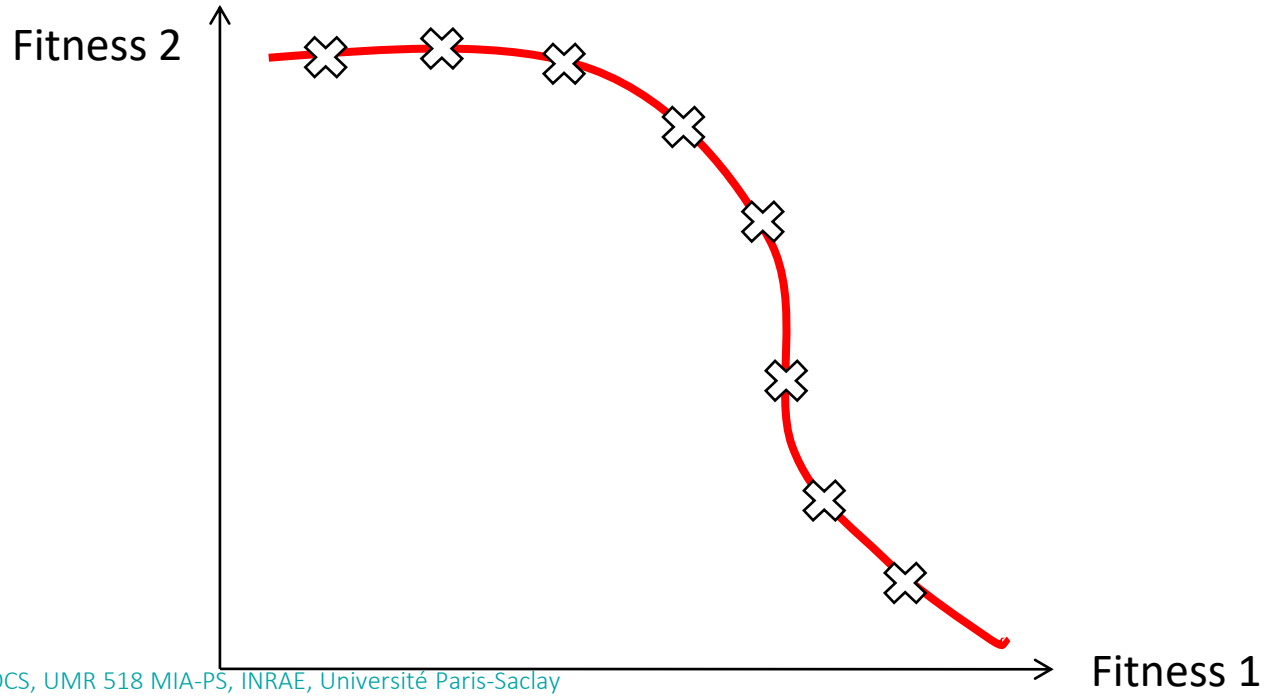
NSGA-II (Non-Sorting Genetic Algorithm II)

- *Crowding* can be an issue
  - Too many points too close together on the PF
  - Not really interesting…



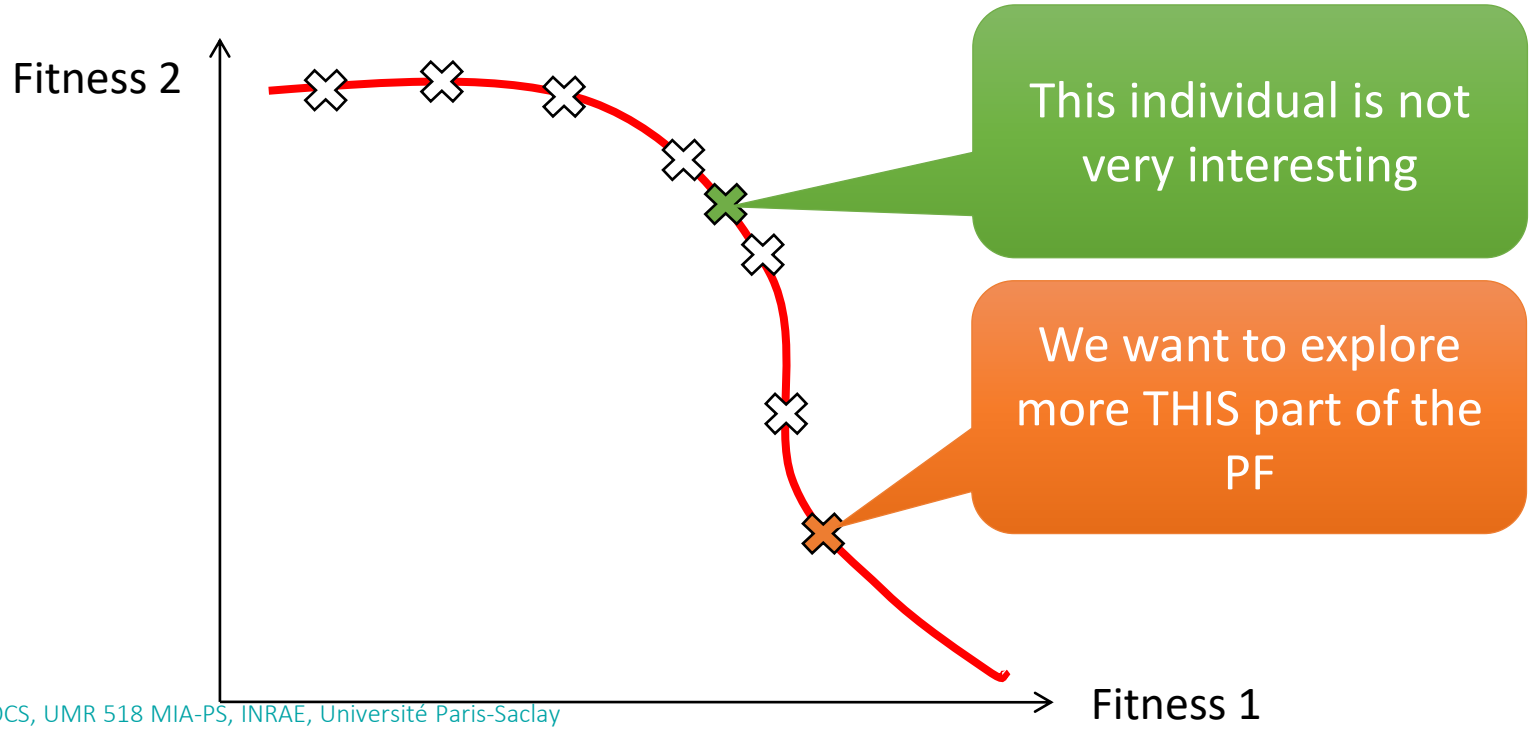
NSGA-II (Non-Sorting Genetic Algorithm II)

- *Crowding* can be an issue
  - Ideally, you would like to explore the PF
  - Distribute points “evenly” on the PF



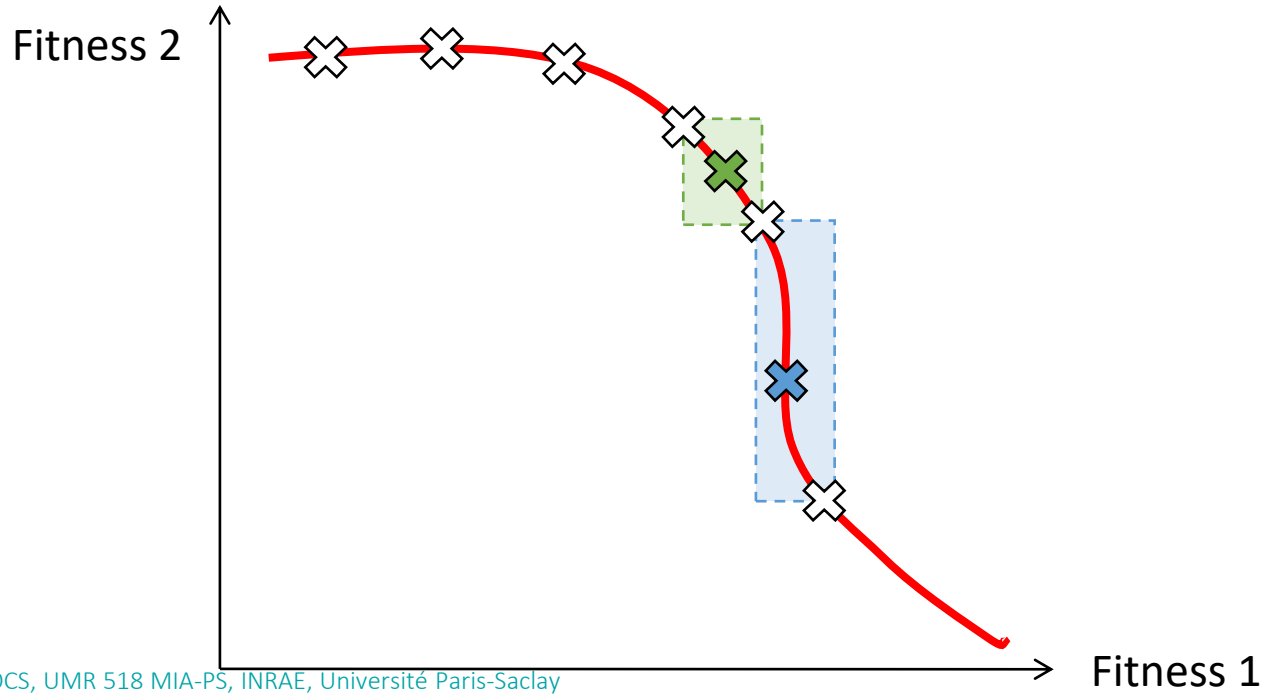
NSGA-II (Non-Sorting Genetic Algorithm II)

- Crowding distance
  - Value associated to individuals
  - Used to select for reproduction/survival



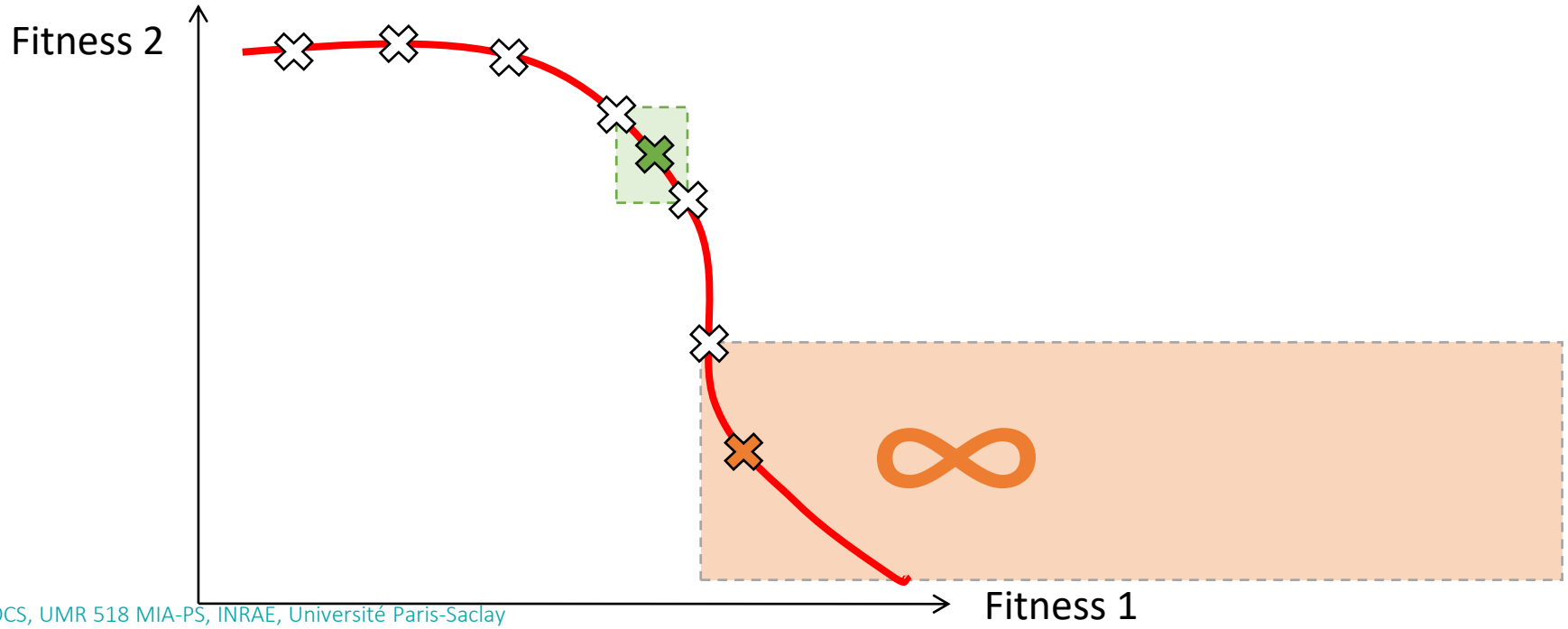
NSGA-II (Non-Sorting Genetic Algorithm II)

- Crowding distance
  - Value associated to individuals
  - Used to select for reproduction/survival



NSGA-II (Non-Sorting Genetic Algorithm II)

- Crowding distance
  - Value associated to individuals
  - Used to select for reproduction/survival



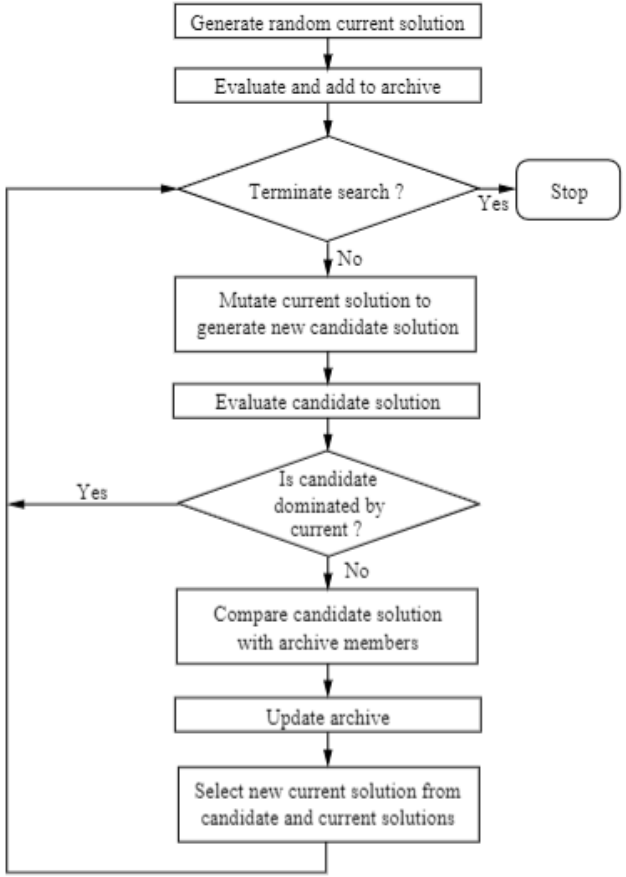
NSGA-II (Non-Sorting Genetic Algorithm II)

- NSGA-II (Non-Sorting Genetic Algorithm II)
  - Crowding distance is a volume for 3 objectives, hypervolume for 4+ objectives
  - For 2 or 3 objectives, it works *really well*

- Limitations
  - The more objectives, the less effective
  - In 10+ dimensions, all points have similar crowding distances

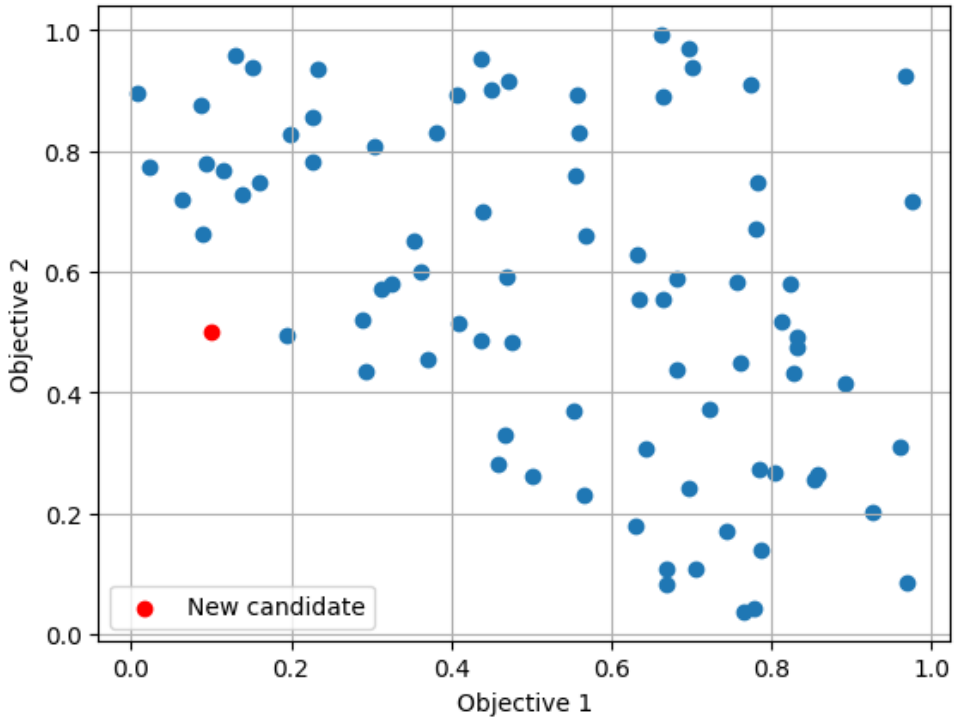
Pareto-Archived Evolution Strategy (PAES)

- (1+1)-ES
  - Evolution Strategy with μ=1, λ=1
  - Uses mutation, only
- Algorithm
  - Archive has max size
  - Archive is recomputed every time new candidate is added



Pareto-Archived Evolution Strategy (PAES)

- What about *crowding*?
  - PAES maintains a grid in objective space
  - Count how many candidates in same square
  - Grid size is a (hyper-) parameter (!)
  - So is the size of the archive



MOEA/D

- «D» for «Decomposition»
  - Take the multi-objective problem
  - Convert it into *N* single-objective problems
  - Solve the problems in parallel
- How to decompose the MOO problem?
  - Weighted sum, change weights
  - Tchebycheff approach
  - Boundary intersection

MOEA/D

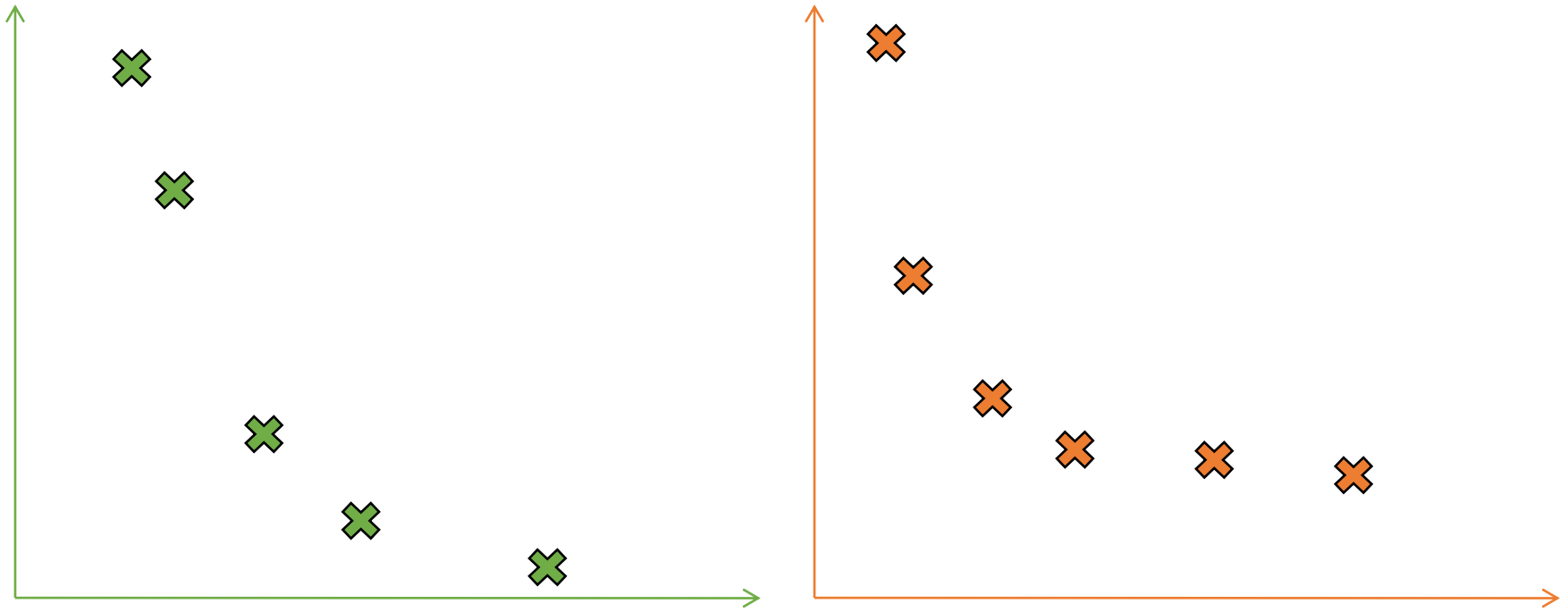
- Tchebycheff approach
  - Set reference points in objective space
  - Try to get as close as possible to the points
  - Objective: minimize distance to reference point(s)

Evaluation of MOEAs

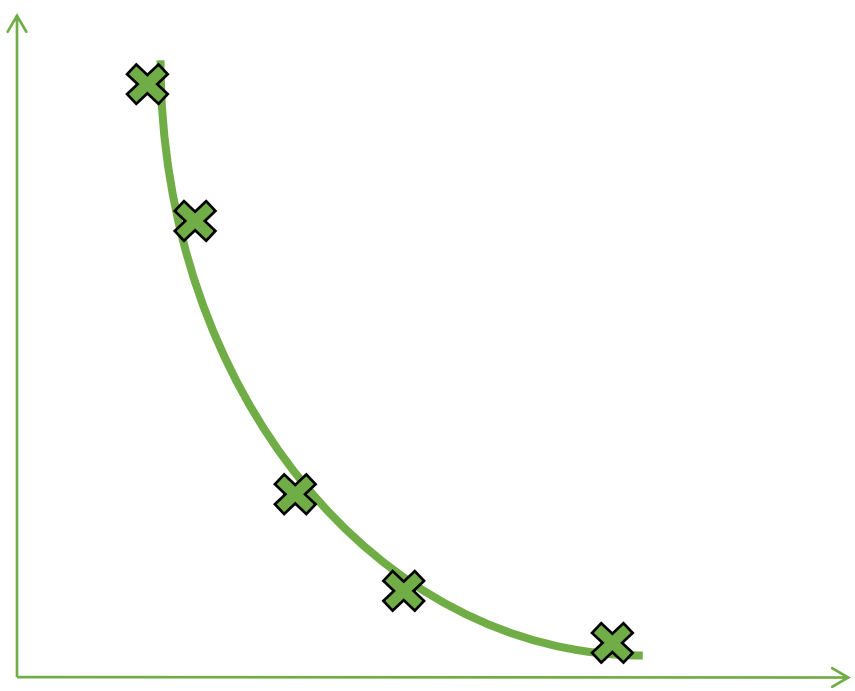
- How to evaluate different runs?
  - For single-objective optimization, best individual
  - Here we might have different Pareto fronts
  - We don’t know where the *true* front is

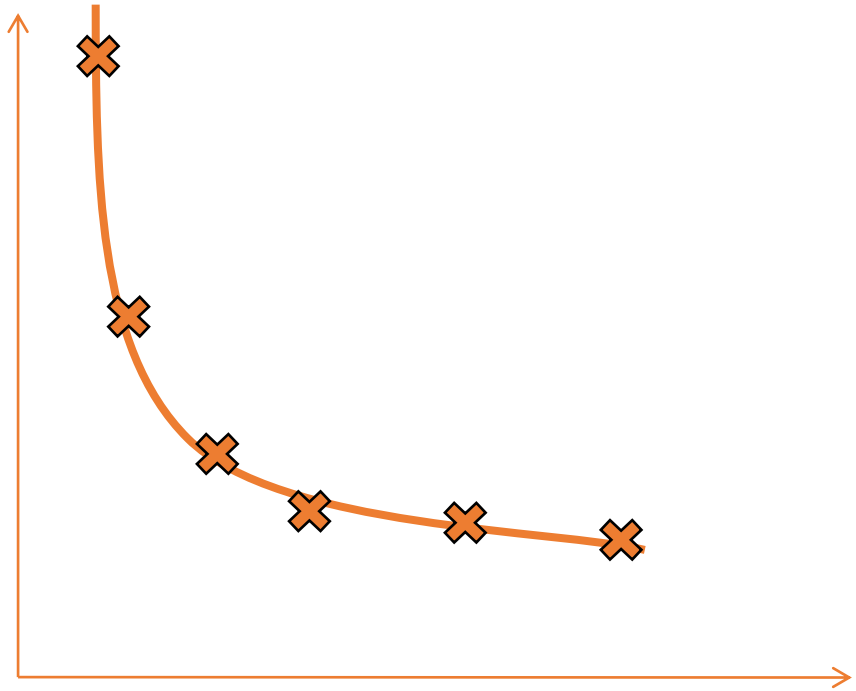
- We can compute the hypervolume of the current Pareto front, using a reference point

Evaluation of MOEAs

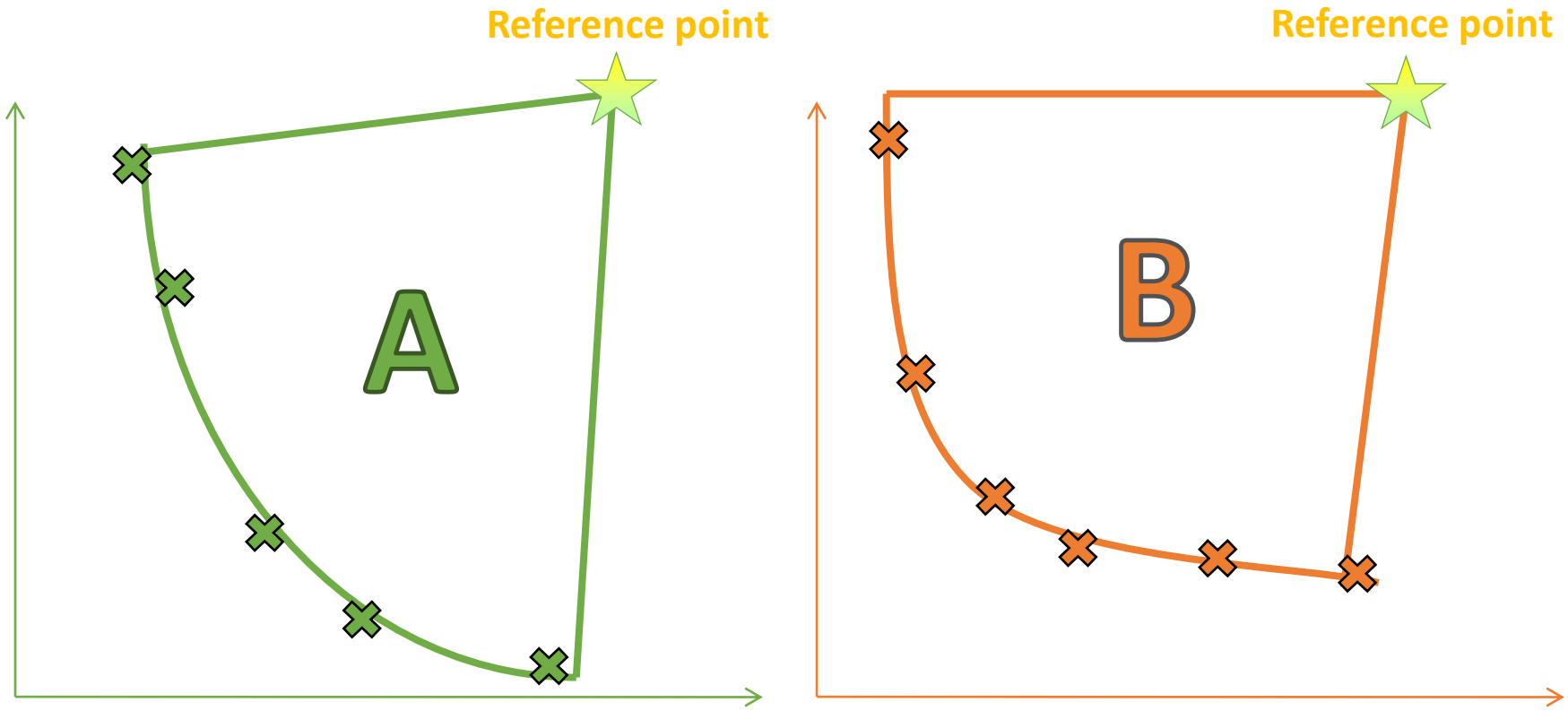


Evaluation of MOEAs





Evaluation of MOEAs



$A < B$?

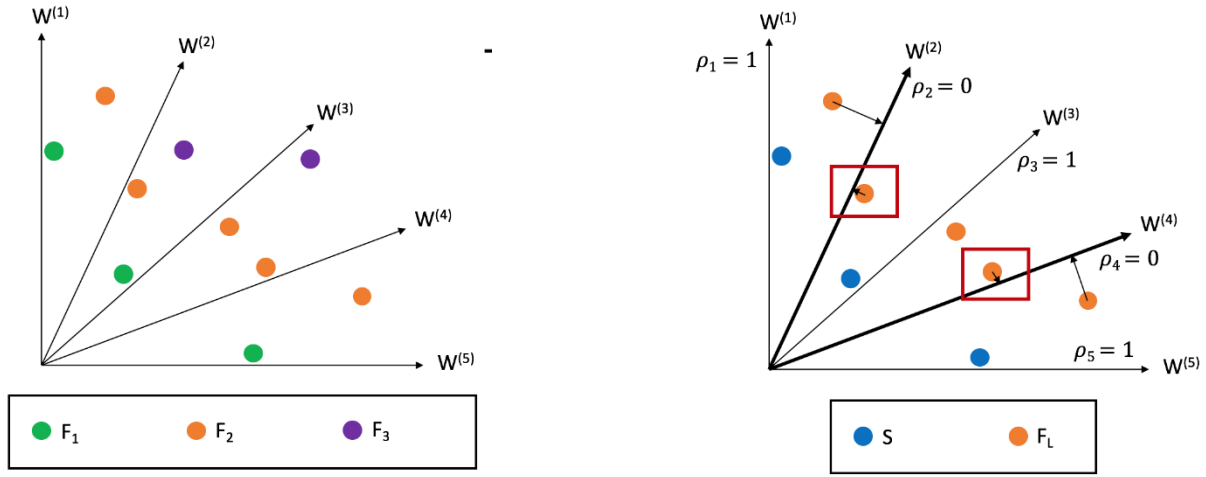
MANY-objective optimization…?

- Relatively recent research topic (2016+)
  - What do we do for 10+ objectives?
  - There's **no good answer** (yet)

- Clever ideas
  - Perform dimensionality reduction (NSGA-II+PCA)
  - Use individuals as references (NSGA-III)

NSGA-III

- Same procedure as NSGA-II for non-dominated sorting
  - However, user specifies reference solutions/ “directions”
  - When candidate solutions need to be killed in a front
  - …start by keeping at least one per reference direction
  - The closest to the reference vector



Alternatives to MOEAs?

- If all your objective functions are **linear** or **quadratic**
  - Linear programming or quadratic programming
  - Aggregate objectives in weighted sum, vary weights
  - Similar to MOEA/D
  - Guarantees finding optimal trade-offs

Questions?

Bibliography
- Deb, *Multi-objective Optimization using Evolutionary Algorithms*, 2011
- Deb & Jain, *An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems With Box Constraints*, 2013

Images: unless otherwise stated, I stole them from the Internet. I hope they are not copyrighted, or that their use falls under the Fair Use clause, and if not, I am sorry. Please don’t sue me.