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REFORMULATION OF FOREST FIRE SPREAD EQUATIONS IN SI UNITS

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ABSTRACT

The basic fire spread equations published by Rothermel in 1972 are reformulated in the International System of Units.

KEYWORDS: fire spread, equations, Rothermel's model, the International System of Units

Rothermel's paper (1972) describing a mathematical model for predicting fire spread in wildland fuels is the basis for several fire management systems. That paper also defines fire parameters that are the subject of continuing research and refinement.

Van Wagner (1978) suggested a list of metric units and conversion factors of practical suitablity for forest fire operational work following the approved standards of the International System of Units (SI).

Heretofore, when SI units have been required of the Rothermel model, the input metric parameters were converted to British units, the calculations performed in the British standard, and then the output parameters reconverted to SI units--a cumbersome procedure.

This research note presents a reformulation in SI units of the basic fire spread equations summarized on pages 26 and 27 of Rothermel's original paper. The first list defines the input parameters in metric units as required and used in the succeeding list of fire spread equations. Also listed are the significant output parameters with their resulting metric units. Standard SI nomenclature for units and symbols is assumed (National Bureau of Standards 1975).

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INPUT/OUTPUT PARAMETERS FOR BASIC EQUATIONS IN METRIC FORM

Input

w_o	Ovendry fuel loading, kg/m^2
δ	Fuel depth, m
σ	Surface area:volume ratio, cm^{-1}
h	Fuel heat content, kJ/kg
ρ_p	Fuel particle density, kg/m^3
M_f	Fuel moisture content, dimensionless fraction
S_T	Fuel total mineral content, dimensionless fraction
S_e	Fuel effective mineral content, dimensionless fraction
U	Windspeed at midflame height, m/min
$\tan \phi$	Slope (verticle rise/horizontal run), dimensionless fraction
M_x	Fuel moisture of extinction, dimensionless fraction

Output

R	Spread rate, m/min
I_R	Reaction intensity, $kJ/(min \cdot m^2)$
I_B	Byram's intensity, kW/m
L_f	Flame length, m

SUMMARY OF BASIC FIRE SPREAD EQUATIONS

IN SI UNITS

Equation 52 Formulation is unchanged; the units for *spread rate* are meters per minute.

$$R = \frac{I_R \xi (1 + \phi_w + \phi_s)}{\rho_b \varepsilon Q_{ig}}$$

Equation 27 Formulation is unchanged; the units for *reaction intensity* are $(kJ/min)/m^2$.

$$I_R = \Gamma' w_n h \eta_M \eta_s .$$

For those who prefer kilowatts per square meter (kW/m^2) for units of reaction intensity, use

$$I_R = \frac{1}{60}\Gamma' w_n h \eta_M \eta_s .$$

However, when this form is used in equation 52 above, the units for spread rate are meters per second.

Equation 38 The *optimum reaction velocity* is unchanged in formula or units (min^{-1}). However, for easier calculation, some prefer the following:

$$\Gamma' = \Gamma'_{max}\left[\frac{\beta}{\beta_{op}} \exp(1 - \frac{\beta}{\beta_{op}})\right]^A .$$

Equation 36 The *maximum reaction velocity* units remain min^{-1}; the formula becomes

$$\Gamma'_{max} = (0.0591 + 2.926\sigma^{-1.5})^{-1} .$$

Equation 37 The *optimum packing ratio* is dimensionless; the formula becomes

$$\beta_{op} = 0.20395\sigma^{-0.8189} .$$

Equation 39 Remains dimensionless; the original Rothermel formulation becomes

$$A = (6.7229\sigma^{0.1} - 7.27) .$$

However, the (dimensionless) metric form used in the computer based library of fire behavior routines (Albini 1976) is

$$A = 8.9033\sigma^{-0.7913} .$$

Equation 29 The *moisture damping coefficient* (dimensionless fraction) is unchanged:

$$\eta_M = 1 - 2.59\frac{M_f}{M_x} + 5.11\left(\frac{M_f}{M_x}\right)^2 - 3.52\left(\frac{M_f}{M_x}\right)^3 .$$

Equation 30 The *mineral damping coefficient* (dimensionless fraction) is unchanged:

$$\eta_s = 0.174 s_e^{-0.19}$$

Equation 42 The *propagating flux ratio* is a dimensionless fraction; the metric formulation is

$$\xi = (192 + 7.9095\sigma)^{-1}\exp\left[(0.792 + 3.7597\sigma^{0.5})(\beta + 0.1)\right] .$$

Equation 47 The *wind coefficient* is dimensionless; the metric formula is

$$\phi_w = C(\overset{3.281}{\cancel{0.3048}}U)^B\left(\frac{\beta}{\beta_{op}}\right)^{-E} .$$

Equation 48 Becomes

$$C = 7.47 \exp(-0.8711\sigma^{0.55}).$$

Equation 49 Becomes

$$B = 0.15988\sigma^{0.54}.$$

Equation 50 Becomes

$$E = 0.715 \exp(-0.01094\sigma).$$

Equation 24 The *net fuel loading* units are kilograms per square meter; the preferred equation is now

$$w_n = w_o(1 - S_T).$$

Equation 51 The *slope factor* is dimensionless and unchanged:

$$\phi_s = 5.275\beta^{-0.3}(\tan \phi)^2.$$

Equation 40 The *ovendry bulk density* has no change in formula; the units are kilograms per cubic meter:

$$\rho_b = w_o/\delta$$

> If fuel depth, δ, is measured in centimeters, the alternative form for bulk density (in kilograms per cubic meter) is $\rho_b = 100w_o/\delta$.

Equation 14 The *effective heating number* is dimensionless; the metric form is

$$\varepsilon \doteq \exp(-4.528/\sigma).$$

Equation 12 The *heat of preignition* units are (kJ/kg); the metric formula is

$$Q_{ig} = 581 + 2594M_f.$$

Equation 31 Packing ratio is dimensionless and remains unchanged:

$$\beta = \rho_b/\rho_p.$$

The metric equation for Albini's formulation of *Byram's fireline intensity* may be of interest:

$$I_B = \frac{1}{60} I_R \, R(\overset{12.6}{\cancel{11700}}/\sigma).$$

The units of I_B are kilowatts per meter of fire line. (Note: The factor $\frac{1}{60}$ may be omitted if the alternative form of equation 27 is used.) His estimate of *flame length*, L_f, becomes

$$L_f = \underset{0.775}{\cancel{0.237}}\, I_B^{0.46} \text{ meters}.$$

PUBLICATIONS CITED

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E R R A T A

WILSON, RALPH. 1980. Reformulation of forest fire spread equations in SI units. USDA For. Serv. Res. Note INT-292, 5 p.

Errors in the equations for wind coefficient and Byram's fireline intensity have been discovered.

Specifically, at the bottom of page 3, equation 47 for the <u>wind coefficient</u> should read

$$\phi_w = C(3.281U)^B \, (\beta/\beta_{op})^{-E}$$

At the bottom of page 4, the equations for <u>Byram's fireline intensity</u> should read

$$I_B = (1/60)\, I_R \, R(12.6/\sigma) \quad \text{kilowatts/meter}$$

and <u>flame length</u> should be

$$L_f = 0.0775 \, I_B^{\,0.46} \text{ meters.}$$

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