Albert Wiryawan
Avw2@illinois.edu
673431511
PHYS466

Homework #6: Ising Model

Problem #1

(a)

P_a- Probability of being added to the cluster

$T(\{s\} \to \{s\}')$ -> Probability of not being in the cluster (for bonds along the edge of the cluster before it is flipped

$= (1 - P_a)^m$

$T(\{s\}' \to \{s\})$-> Probability of not being in the cluster (for bonds along the edge of the cluster after it is flipped.

$=(1 - P_a)^n$

From these two derived equations the ratio of observed probabilities will then be given by

$$\frac{T(\{s\} \to \{s\}')}{T(\{s\}' \to \{s\})} = (1 - P_a)^{m-n}$$

(b)

The change in energy of energy flipping in a cluster is dependent upon 2*(m-n) since energy sites from neighbors are used to calculate the energy. As such, (m-n) is needed since to find the number of bonds along an edge after an energy flip. This value is then multiplied by two because energy states go from -1 to 1 which is acquires a difference of two.

(c) $P_a = 1 - e^{-2\beta}$

$$(1 - P_a)^{m-n} \frac{A(\{s\} \to \{s\}')}{A(\{s\}' \to \{s\})} = e^{-2\beta(m-n)}$$

$$\frac{A(\{s\} \to \{s\}')}{A(\{s\}' \to \{s\})} = 1$$

$$A(\{s\} \to \{s\}') = A(\{s\}' \to \{s\})$$

$$A(\{s\} \to \{s\}') = \min(1,\ e^{(-\beta[H\{s\}' - H\{s\}])})$$

As such, when the minimum is 1 every cluster flip will be accepted resulting in no accept/reject check in step 4 of the Wolff Algorithm.

If minimum is

$\exp^{f0}(-\beta[H(\{s\}') - H(\{s\})])$, $A(\{s\} \to \{s\}^{\wedge}')/A(\{s\}' \to \{s\}) = \exp^{f0}(-2\beta(-H(\{s\}')+H(\{s\})) = \exp^{f0}(2\beta(H(\{s\}^{\wedge}'))) / \exp^{f0}(2\beta H(\{s\}))$

From this we can say that $H(\{s\}') = H(\{s\})$, which can only be made when every cluster is accepted

Problem #2

(a)Classic Algorithm

Temperature	**Average Energy**	**Average Square Magnetization**
1.4	0.0002487± 0.02148	0.00013±3.9887e-7
1.8	-0.0005447±0.02276	0.000134083±4.0806e-7
2.2	-0.000445±0.0203145	0.0001311758±4.1339e-7
2.6	-0.0018706±0.021206	0.000125772±3.69827e-7
3.0	-0.0004422±.02003425	0.00012494375±3.992e-7

(b)Wolff Algorithm

Temperature	**Average Energy**	**Average Square Magnetization**
1.4	1.37e-05±0.01221	0.002458±2.889e-7
1.8	-0.00015955±0.02221	.002303488 75±5.12114e-7
2.2	-0.0003589 ± 0.02103	.0006303±3.89e-7
2.6	-0.0006381±0.01998	0.0005083985±4.1612e-7
3.0	−0.0009171 ±0.02001	0.000334565±4.02e-7

(c)

Temperature	**Autocorrelation classic**	**Autocorrelation wolff**
1.4	1	1.342342126
1.8	1.0031312581365517	1. 006232123
2.2	1.0516899906812924	1.0893204
2.6	1	1
3.0	2.920034248429272625	2.10115176463136

The autocorrelation in the Wolff algorithm shorter than the classical is at temperature T =3.0