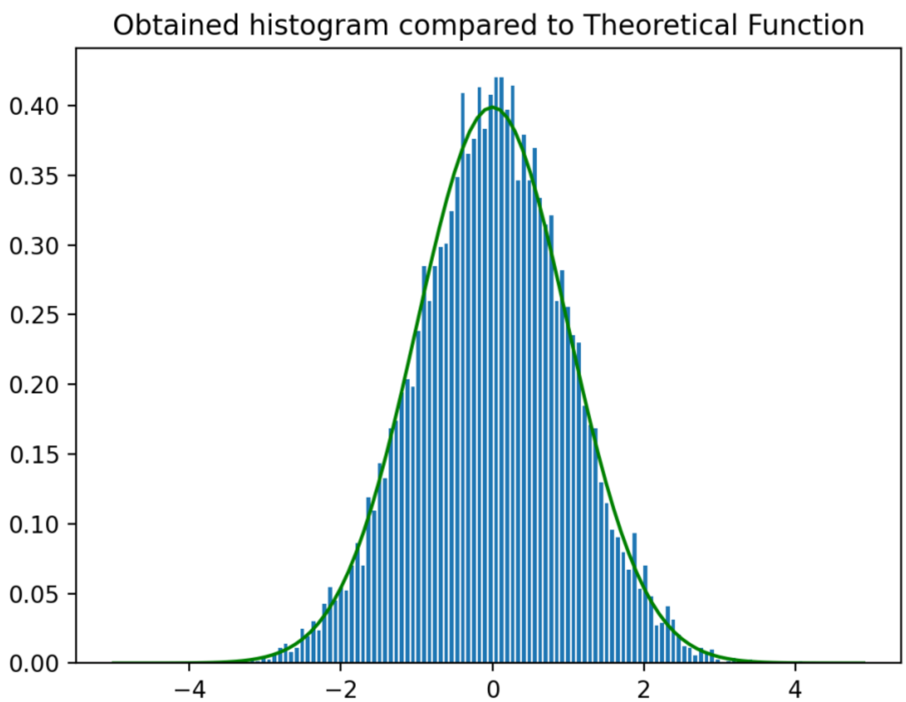
Albert Wiryawan
Avw2@illinois.edu
673431511

Random Number Generator

Problem #1: Pseudo-random Numbers

a) For a given *m* in LCG, the longest sequence one can generate before it begins to repeat itself is given by the value m since c and x_0 are only relevant to the initialization before cycling

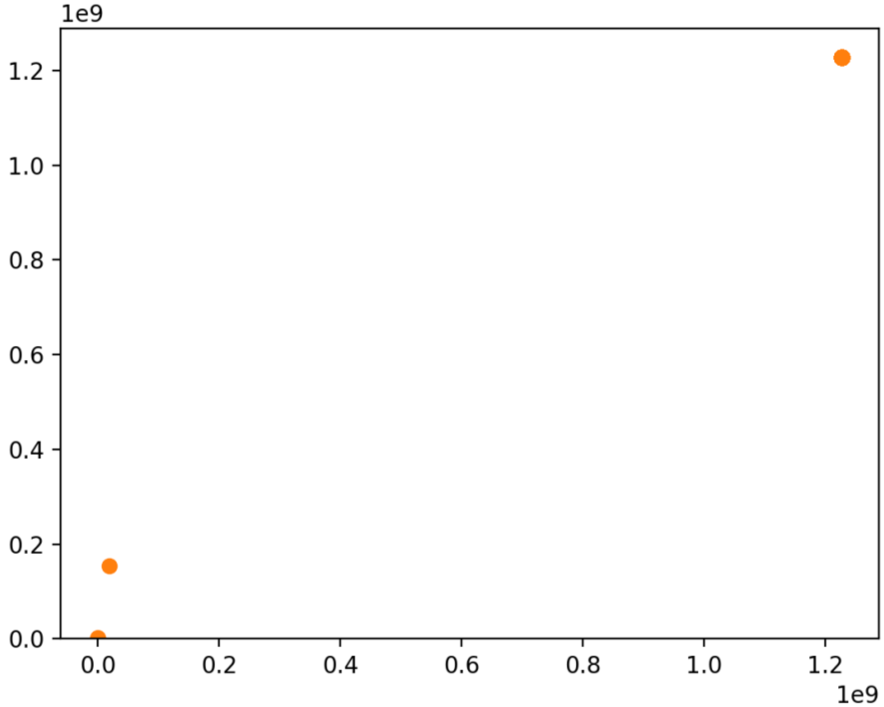
b)



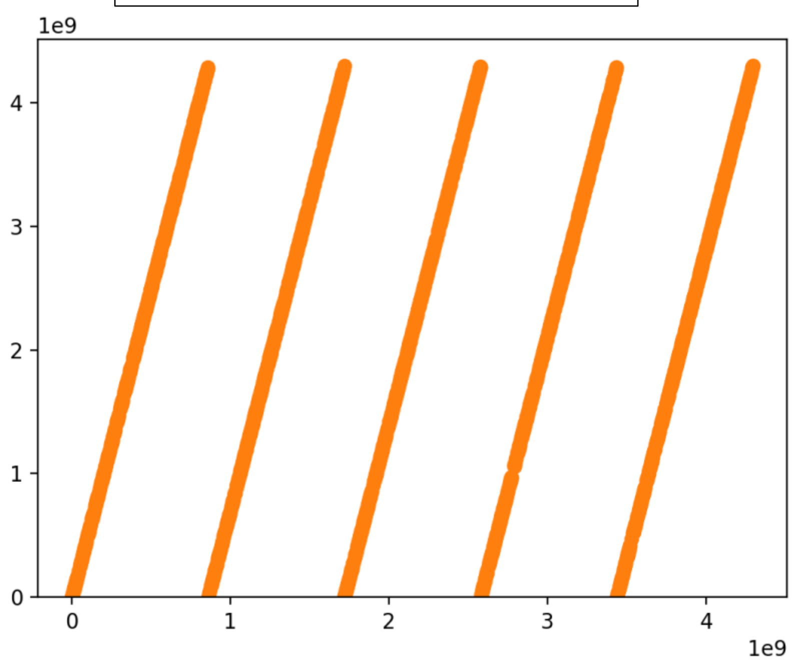
c) The value of the sample size was found to be the same as chi-squared test under LCG which was 6000, 3000, and 2000 for 1D, 2D, and 3D respectively. The values of chi-squared for the numbers produced from a number generator was 13.35, 3026.7, and 8040.049 for 1D, 2D, and 3D respectively.

d)

LCG (a =8)



LCG (a =5)



Problem #2: Numerical integration with rectangles

a) An interval of 5 is chosen to estimate the equation. This is because an increase in the interval beyond this point does not have a significant increase in the integral value (the increase in value was below 4 decimal places)

b) The minimum value of N chosen to be accurate within four digits is 20. In order to obtain this value, we find the first value of N where the written function is accurate up to 4 decimal places.

Problem #3: Monte Carlo Integration

a)

$$p(x) = \frac{1}{\sqrt{2\pi\alpha}} \exp\left(-\frac{x^2}{2\alpha}\right)$$

$$f(x) = \frac{\exp\left(-\frac{x^2}{2}\right)}{1+x^2}$$

Plugging into the equation below

$$\bar{X} = \frac{1}{N}\sum_{i=0}^{N} \frac{f(x)}{p(x)}$$

We obtain the equation which can be reduced to the final answer for the estimator of g(x)

$$\bar{X} = \frac{1}{N}\sum_{i=0}^{N} \frac{\frac{\exp\left(-\frac{x^2}{2}\right)}{1+x^2}}{\frac{1}{\sqrt{2\pi\alpha}}\exp\left(-\frac{x^2}{2\alpha}\right)} = \frac{\sqrt{2\pi\alpha}}{N}\sum_{i=0}^{N} \frac{\exp\left(\frac{x^2}{2\alpha}-\frac{x^2}{2}\right)}{1+x^2}$$

b)

utilizing the equation below the variance is v

$$Var(x) = \frac{1}{N}\sum_{i=0}^{N}\left(\frac{f(x)}{p(x)} - \bar{X}\right)^2$$

The equation below can be obtained by plugging in the values found from part a

$$\frac{1}{N}\sum_{i=0}^{N}\left(\frac{\sqrt{2\pi\alpha}\exp\left(\frac{x^2}{2\alpha}-\frac{x^2}{2}\right)}{1+x^2} - \frac{\sqrt{2\pi\alpha}}{N}\sum_{i=0}^{N}\frac{\exp\left(\frac{x^2}{2\alpha}-\frac{x^2}{2}\right)}{1+x^2}\right)^2$$

This can then be reduced to

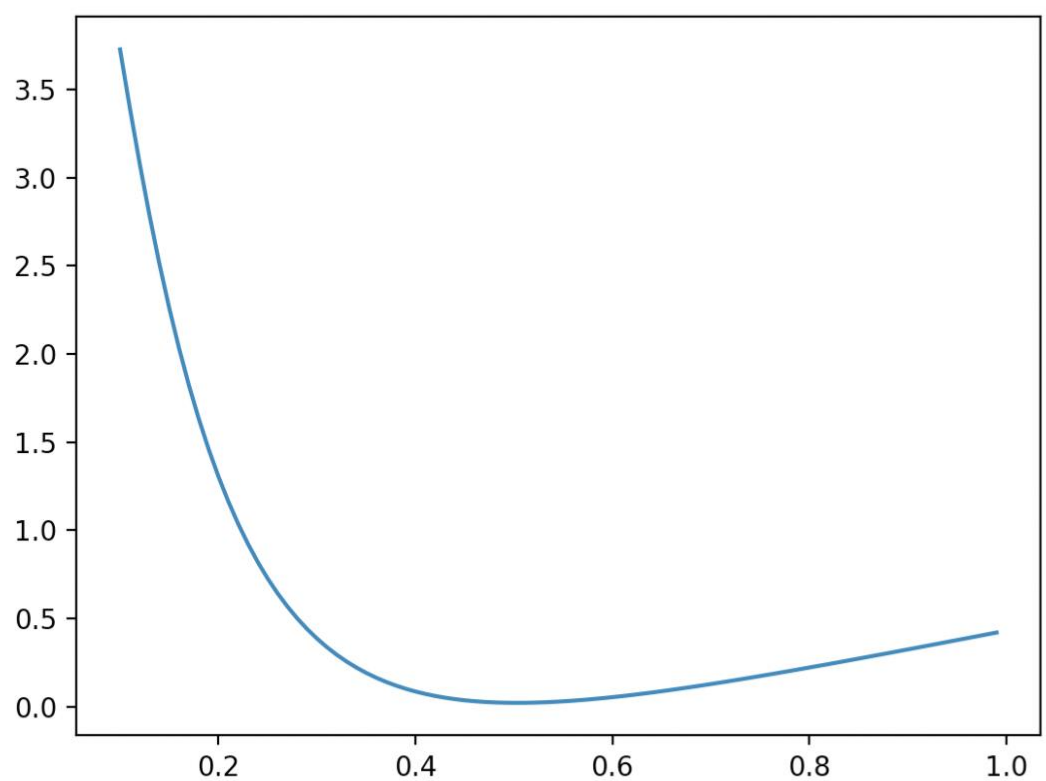
$$\frac{2\pi\alpha}{N}\sum_{i=0}^{N}\left(\frac{\exp\left(\frac{x^2}{2\alpha}-\frac{x^2}{2}\right)}{1+x^2}-\frac{1}{N}\sum_{i=0}^{N}\frac{\exp\left(\frac{x^2}{2\alpha}-\frac{x^2}{2}\right)}{1+x^2}\right)^2$$

Thus, representing an expression of variance for g(x)

c) The estimated mean is 1.63987 and the estimated variance is 1.44757. In order to find the standard error of the mean, the standard deviation (square root of the variance) is divided by the square root of the sample size. This value was found to be 0.03805.

Problem #4: Variance

a) Based on the expression from P3(b), the variance is infinite when α approaches 0.
b) The minimum value of variance was found to be 0. 0.02228 at an alpha value of 0.5029



c) Similar to the method of accuracy in part 2 b, an accurate estimate is found at a minimum of 50 samples.
d) In order to minimize variance, the probability distribution is supposed to have the least amount of difference with the function being approximated. Since the gaussian is being is not bound a student's T curve might work better as it gives more consideration for the

tails when the standard deviation is unknown. As a result, we can use this probability distribution to minimize the variance as it should follow the functional distribution more closely