Fraleigh Problems

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0 Sets and Relations

0.1 Important Info

0.2 Problems

1. $\{-\sqrt{3}, \sqrt{3}\}$
2. $\varnothing$
3. $\{-60, -30, -20, -12, -10, -6, -5, -3, -2, -1, 1, 2, 3, 5, 6, 10, 12, 20, 30, 60\}$
4. $\{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.
5. Not well-defined. $\{n \in \mathbb{Z}^+ | n > 100\}$
6. Well-defined. $\varnothing$
7. Well-defined. $\varnothing$
8. Not well-defined.
9. Well-defined. $\mathbb{Q}$
10. Well-defined. $\{n | 4n \in \mathbb{Z} \vee 3n \in \mathbb{Z}\}$

1 Groups and Subgroups

1.1 Introduction and Examples

Read this and didn't find much interesting

1.2 Binary Operations

Read this and didn't find much interesting

1.3 Isomorphic Binary Structures

1.3.1 Important Info

A **binary structure** is a set S combined with a binary operation $*$, and is represented as $\langle S, * \rangle$.

Definition 1.1 (Isomorphism)**.** A one-to-one function ϕ mapping $\langle S, *\rangle$ onto $\langle S', *'\rangle$ such that $\phi(x * y) = \phi(x) *' \phi(y)$ for all $x, y \in S$.

If this function isn't one-to-one, then this is a **homomorphism**.

Theorem 1.2. If $\langle S, *\rangle$ has an identity element e, and $\langle S', *'\rangle$ is isomorphic with function ϕ, then $\phi(e)$ is the identity for $\langle S', *'\rangle$.

1.3.2 Exercises

1. We would need to confirm that our relation is injective, surjective, and homomorphic.
3. No, as 3 in the second set would have no corresponding value in the first set, so surjectivity fails.
8. No, as two matrices can have the same determinant, the homomorphism quality fails.
9. Yes. As the determinant of a 1×1 matrix is just the value inside, the relation must be injective and surjective. Moreover, as $\det(A \cdot B) = A_{11} \cdot B_{11} = \det A \cdot \det B$, this mapping is a homomorphism. Therefore, ϕ is an isomorphism between $\langle M_2(\mathbb{R}), \cdot\rangle$ and $\langle \mathbb{R}, \cdot\rangle$.
11. No, as it is not injective because all constant functions map to 0.
16. We see that ϕ is a bijection from $\mathbb{Z}$ to $\mathbb{Z}$, Only $n-1$ can map to n, implying injectivity, and n is guaranteed to be taken from $n-1$, implying surjectivity.

    For scenario (a), we need for $(a+1) * (b+1)$ to equal $a+b+1$, which can be done by just setting $a * b = a + b - 1$.

    For scenario (b), we need for $(a+1) + (b+1)$ to equal $a * b$, which would require $a * b = a + b + 2$.
21. A function $\phi : S \to S'$ is an *isomorphism* if and only if ϕ is bijective and $\phi(a) *' \phi(b)$.
22. No correction needed.