# Math 110 Lecture 12.

- try doing harder q's & studying proactively.
- Last lecture:
  - dual map
    setting $T \in \mathcal{L}(V,W)$, define $T' \in \mathcal{L}(W',V')$ : $\varphi \mapsto \varphi \circ T$
    Note that this is a linear operation.

## Annihilator

$S \subseteq V$, then annihilator of $S$ turns everything in $S$ to $0$.

ex: annihilator of $\{1 \cdot x^3\}$ is $p \mapsto p(1)$
(over $P_3$)
$p \mapsto p''(69697)$

$S^0 = \text{span}(\delta_1, \delta_1'')$ remember: $\delta_k$ means evaluating $p(k)$.

For any subset $S$, its annihilator is automatically a subspace of $V'$.

- 0 func. always in annihilator.
- for $\varphi, \psi \in S^0$, note that $a\varphi + \beta\psi = a\varphi(\vec{v}) + \beta\psi(\vec{v}) = 0$.

$\rightarrow$ must be a subspace.

- Actually, $S^0 = (\text{span } S)^0$ once again using linearity.

**Theorem.** $\dim U^0 = \dim V - \dim U$. Provided $U$ is subspace of $V$.

proof: start with an inclusion map: $i : U \hookrightarrow V$ !! FALSE !!

such that $i : \vec{u} \mapsto \vec{u}$. Consider its dual $i'$, where $i \in \mathcal{L}(U,V)$
then $i' \in \mathcal{L}(V', U')$.

1) what is Null $i'$?

$i'(\varphi) = \varphi \circ i$. So Null $i' = \{\varphi \in V' : i'(\varphi) = 0\}$.
$(\varphi \circ i)(\vec{u}) = 0 \ \forall \vec{u} \in U$. So $0 = \varphi(i(\vec{u})) = \varphi(\vec{u}) \ \forall \vec{u} \in U$.
So $\varphi$ must be in $U^0$ whenever $\varphi \in$ Null $i'$.
Therefore, Null $i' = U^0$.

2) what is range of $i'$?

$i' : V' \to U'$. So the range is a subspace of $U'$.
observation: A linear functional on $U$ has an extension to $V$.

So any functional on $U$ can be extended to a functional on $V$, then $i'$ will send this "extended" functional back to the original.

That is, any functional $\varphi \in U'$ can be realized as $\tilde{\varphi} \circ i$ where $\tilde{\varphi}$ is ext. of $\varphi$ to $V$.

range $i' = U'$
null $i' = U^0$
domain $i' = V'$.

apply fundamental thm. of lin alg.

$\dim U^0 + \dim U' = \dim V'$.
$\Rightarrow \dim U^0 + \dim U = \dim V$
$\Rightarrow \dim U^0 = \dim V - \dim U$.