Contributions to the structure theory of ω-languages

Overview

1. (regular) $*$-languages
2. $\mathcal{L}^* \to \mathcal{L}^\omega$ operators
3. Some results about ω-languages

$*$-languages

1. regular
2. FO[$<$] / starfree
3. FO[+1] / locally threshold testable
4. piece-wise testable
5. positive piece-wise testable
6. (k-) locally testable
7. dot-depth-n
8. locally modulo testable
9. R-trivial
10. finite / co-finite
11. endwise testable

$$\mathcal{L}^* \to \mathcal{L}^\omega$$

1. $\mathrm{ext}(L) := L \cdot \Sigma^\omega$
2. $\lim(L) := \left\{\alpha \in \Sigma^\omega \;\middle|\; \exists^\omega n \colon \alpha[0,n] \in L\right\}$
3. dual $\overline{L} := \left\{\alpha \in \Sigma^\omega \;\middle|\; \forall n \colon \alpha[0,n] \in L\right\}$
4. $\left\{\alpha \in \Sigma^\omega \;\middle|\; \exists N \colon \forall n > N \colon \alpha[0,n] \in L\right\}$
5. Kleene-Closure of $\mathcal{K}$: $\bigcup_{i=1}^{n} U_i \cdot V_i^\omega$, $U_i, V_i \in \mathcal{K}$

Some results

1. $\mathrm{BC}\,\mathrm{ext}\,\mathcal{L}^*(\text{piece-wise testable}) = \mathrm{BC}\,\mathrm{lim}\,\mathcal{L}^*(\text{piece-wise testable})$
2. $\mathcal{L}^\omega(\mathrm{FO}[+1]) = \mathrm{BC}\,\mathrm{ext}\,\mathcal{L}^*(\mathrm{FO}[+1])$
3. $\mathcal{L}^\omega(\mathrm{FO}[<]) = \mathrm{BC}\,\mathrm{lim}\,\mathcal{L}^*(\mathrm{FO}[<])$
4. $\mathrm{BC}\,\mathrm{ext}\,\mathcal{L}^*(\mathrm{FO}[<]) \subsetneqq \mathrm{BC}\,\mathrm{lim}\,\mathcal{L}^*(\mathrm{FO}[<])$
5. $\mathrm{BC}\,\mathrm{ext}\,\mathcal{L}^*(\text{locally testable}) \subsetneqq \mathrm{BC}\,\mathrm{lim}\,\mathcal{L}^*(\text{locally testable})$
6. $\mathrm{BC}\,\mathrm{ext}\,\mathcal{L}^*(\text{pos-PT}) = \mathrm{BC}\,\mathrm{lim}\,\mathcal{L}^*(\text{pos-PT})$
7. $\mathrm{BC}\,\mathrm{ext}\,\mathcal{L}^*(\text{pos-PT}) = \mathrm{BC}\,\mathrm{ext}\,\mathcal{L}^*(\mathrm{PT})$