

# Language Operations and a Structure Theory of $\omega$ -Languages

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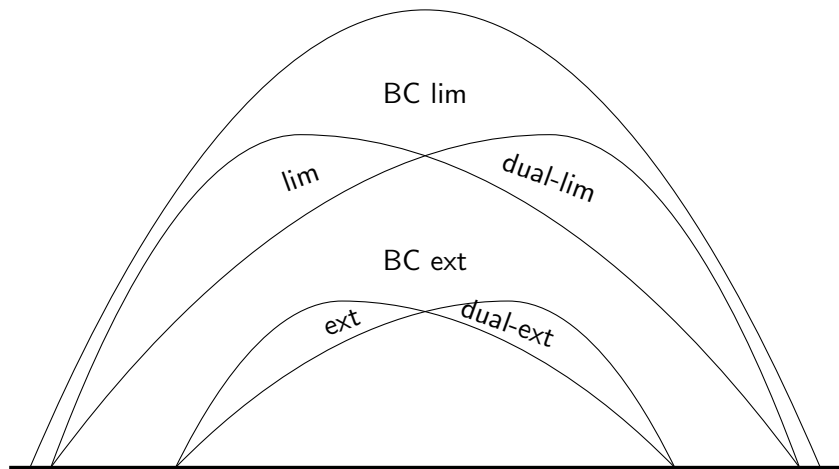
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# Introduction

- ▶ consider regular  $*$ -languages
- ▶ look at  $\mathcal{L}^* \rightarrow \mathcal{L}^\omega$  language operators like  $\lim$ ,  $\text{ext}$ , BC of them, etc.
  1.  $\text{ext}(L) := \{\alpha \in \Sigma^\omega \mid \exists n: \alpha[0, n] \in L\} = L \cdot \Sigma^\omega$
  2.  $\overline{\text{ext}}(L) := \{\alpha \in \Sigma^\omega \mid \forall n: \alpha[0, n] \in L\} = L \cdot \Sigma^\omega$
  3.  $\lim(L) := \{\alpha \in \Sigma^\omega \mid \forall N: \exists n > N: \alpha[0, n] \in L\} = \{\alpha \in \Sigma^\omega \mid \exists^\omega n: \alpha[0, n] \in L\}$
  4.  $\overline{\lim}(L) := \{\alpha \in \Sigma^\omega \mid \exists N: \forall n > N: \alpha[0, n] \in L\}$
  5. Kleene-Closure of  $\mathcal{K}$ :  $\bigcup_{i=1}^n U_i \cdot V_i^\omega$ ,  $U_i, V_i \in \mathcal{K}$
- ▶ compare the resulting  $\omega$ -languages

# Diagram



# Questions

- ▶ instead of regular  $\ast$ -languages, look at other  $\ast$ -language classes, e.g.  $\text{FO}[\leq]$  (starfree),  $\text{FO}[+1]$ , piece-wise testable, positive piece-wise testable, ...
- ▶ does it result in the same relations as in the diagram? are the enclosures strict?

## Some results

1.  $\text{BC ext } \mathcal{L}^*(\text{piece-wise testable}) = \text{BC lim } \mathcal{L}^*(\text{piece-wise testable})$
2.  $\mathcal{L}^\omega(\text{FO}[+1]) = \text{BC ext } \mathcal{L}^*(\text{FO}[+1])$
3.  $\mathcal{L}^\omega(\text{FO}[<]) = \text{BC lim } \mathcal{L}^*(\text{FO}[<])$
4.  $\text{BC ext } \mathcal{L}^*(\text{FO}[<]) \subsetneq \text{BC lim } \mathcal{L}^*(\text{FO}[<])$
5.  $\text{BC ext } \mathcal{L}^*(\text{locally testable}) \subsetneq \text{BC lim } \mathcal{L}^*(\text{locally testable})$
6.  $\text{BC ext } \mathcal{L}^*(\text{pos-PT}) = \text{BC lim } \mathcal{L}^*(\text{pos-PT})$
7.  $\text{BC ext } \mathcal{L}^*(\text{pos-PT}) = \text{BC ext } \mathcal{L}^*(\text{PT})$

## More general results

1.  $\forall L \in \mathcal{L}: L \cdot \Sigma^* \in \mathcal{L}$   
 $\Rightarrow \text{ext}(\mathcal{L}^*) \subset \text{lim}(\mathcal{L}^*)$
2.  $\exists \tilde{L} \in \text{lim } \mathcal{L}, \tilde{L} \neq \Sigma^\omega, \forall n: \tilde{L}[0, n] = \Sigma^n$   
 $\Rightarrow \tilde{L} \notin \text{BC ext}(\mathcal{L})$   
 $\Rightarrow \text{BC ext}(\mathcal{L}) \neq \text{BC lim}(\mathcal{L})$