Language Operations and a Structure Theory of ω-Languages

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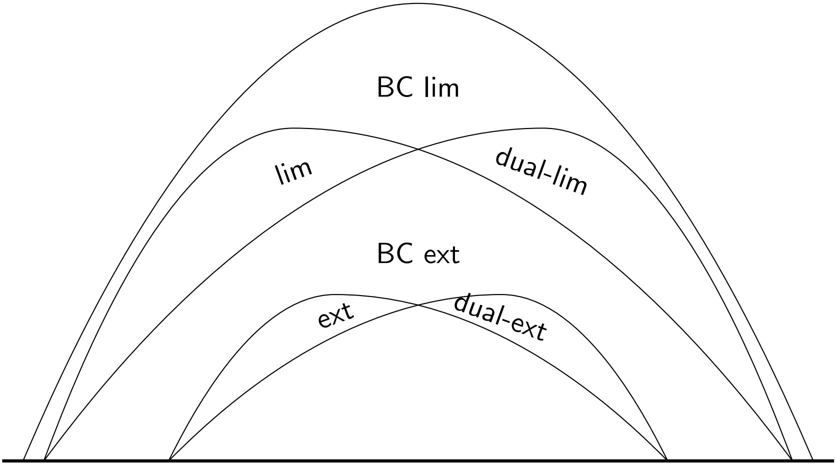
Introduction

- consider regular $*$-languages
- look at $\mathcal{L}^* \to \mathcal{L}^\omega$ language operators like lim, ext, BC of them, etc.
  1. $\mathrm{ext}(L) := \{\alpha \in \Sigma^\omega \mid \exists n \colon \alpha[0,n] \in L\} = L \cdot \Sigma^\omega$
  2. $\overline{\mathrm{ext}}(L) := \{\alpha \in \Sigma^\omega \mid \forall n \colon \alpha[0,n] \in L\} = L \cdot \Sigma^\omega$
  3. $\lim(L) := \{\alpha \in \Sigma^\omega \mid \forall N \colon \exists n > N \colon \alpha[0,n] \in L\} = \{\alpha \in \Sigma^\omega \mid \exists^\omega n \colon \alpha[0,n] \in L\}$
  4. $\overline{\lim}(L) := \{\alpha \in \Sigma^\omega \mid \exists N \colon \forall n > N \colon \alpha[0,n] \in L\}$
  5. Kleene-Closure of $\mathcal{K}$: $\bigcup_{i=1}^n U_i \cdot V_i^\omega$, $U_i, V_i \in \mathcal{K}$
- compare the resulting ω-languages

Diagram



Curiosity

- instead of regular $*$-languages, look at other $*$-language classes, e.g. FO[$<$] (starfree), FO[$+1$], $\text{piece-wise testable}$, positive $\text{piece-wise testable}$, ...
- does it result in the same relations as in the diagram? are the enclosures strict?

Some results

1. $\mathsf{BC}\,\mathsf{ext}\,\mathcal{L}^*(\text{piece-wise testable}) =$
   $\mathsf{BC}\,\mathsf{lim}\,\mathcal{L}^*(\text{piece-wise testable})$
2. $\mathcal{L}^\omega(\mathrm{FO}[+1]) = \mathsf{BC}\,\mathsf{ext}\,\mathcal{L}^*(\mathrm{FO}[+1])$
3. $\mathcal{L}^\omega(\mathrm{FO}[<]) = \mathsf{BC}\,\mathsf{lim}\,\mathcal{L}^*(\mathrm{FO}[<])$
4. $\mathsf{BC}\,\mathsf{ext}\,\mathcal{L}^*(\mathrm{FO}[<]) \subsetneqq \mathsf{BC}\,\mathsf{lim}\,\mathcal{L}^*(\mathrm{FO}[<])$
5. $\mathsf{BC}\,\mathsf{ext}\,\mathcal{L}^*(\text{locally testable}) \subsetneqq \mathsf{BC}\,\mathsf{lim}\,\mathcal{L}^*(\text{locally testable})$
6. $\mathsf{BC}\,\mathsf{ext}\,\mathcal{L}^*(\text{pos-PT}) = \mathsf{BC}\,\mathsf{lim}\,\mathcal{L}^*(\text{pos-PT})$
7. $\mathsf{BC}\,\mathsf{ext}\,\mathcal{L}^*(\text{pos-PT}) = \mathsf{BC}\,\mathsf{ext}\,\mathcal{L}^*(\text{PT})$

More general results

1. $\forall L \in \mathcal{L} \colon L \cdot \Sigma^* \in \mathcal{L}$
   $\Rightarrow \mathrm{ext}(\mathcal{L}^*) \subset \mathrm{lim}(\mathcal{L}^*)$
2. $\exists \tilde{L} \in \mathrm{lim}(\mathcal{L}, \tilde{L} \neq \Sigma^\omega, \forall n \colon \tilde{L}[0,n] = \Sigma^n$
   $\Rightarrow \tilde{L} \notin \mathrm{BC}\,\mathrm{ext}(\mathcal{L})$
   $\Rightarrow \mathrm{BC}\,\mathrm{ext}(\mathcal{L}) \neq \mathrm{BC}\,\mathrm{lim}(\mathcal{L})$