HERMITIAN MODULAR FORMS FOR FIELDS OF LOW DISCRIMINANT

DIPLOMA THESIS
in Mathematics

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Contents

Chapter 1

Introduction

We develop an algorithm to compute Fourier expansions of Hermitian modular forms of degree 2 over $\mathrm{Sp}_2(\mathcal{O})$ for $\mathcal{O} \subseteq \mathbb{Q}(\sqrt{-\Delta})$, $\Delta \in \{3, 4, 8\}$.

In [PY07], spaces of Siegel modular cusp forms are calculated.

A similar algorithm is also [Rau12, Algorithm 4.3] for Jacobi forms.

We are doing the same for Hermitian modular forms.

Chapter 2

Preliminaries

$\mathbb{N}$ denotes the set $\{1, 2, 3, \ldots\}$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and $\mathbb{Z}$ are all **integers**. $\mathbb{Q}$ are all the **rational numbers**, $\mathbb{R}$ are the **real numbers** and $\mathbb{C}$ are the **complex numbers**. $\mathbb{R}^+ := \{x \in \mathbb{R} \mid x > 0\}$, $\mathbb{R}^\times$ and $\mathbb{C}^\times$ denotes all non-zero numbers.

Let $\mathrm{Mat}_n(R)$ be the set of all $n \times n$ **matrices** over some commutative ring R. Likewise, $\mathrm{Mat}_n^T(R)$ are the **symmetric** $n \times n$ matrices. X^T is the **transposed** matrix of $X \in \mathrm{Mat}_n(R)$. $\overline{Z}$ is the **conjugated** matrix of $Z \in \mathrm{Mat}_n(\mathbb{C})$. For $R \subseteq \mathbb{C}$, $\overline{R} \subseteq R$, the set of **Hermitian matrices** in R is defined as

$$\mathrm{Her}_n(R) = \left\{ Z \in \mathrm{Mat}_n(R) \,\middle|\, \overline{Z}^T = Z \right\}.$$

A matrix $Y \in \mathrm{Mat}_n(\mathbb{C})$ is greater 0 if and only if $\forall x \in \mathbb{C}^n - \{0\} : Y[x] := \overline{x}^T Y x \in \mathbb{R}^+$. Such symmetric matrices are called the **positive definitive matrices**, defined by

$$\mathcal{P}_n(R) = \{ X \in \mathrm{Mat}_n(R) \mid X > 0 \}$$

for $R \subseteq \mathbb{C}$. Note that $\mathcal{P}_n(\mathbb{C}) \subseteq \mathrm{Her}_n(\mathbb{C})$.

For $A, X \in \mathrm{Mat}_n(\mathbb{C})$, we define $A[X] := \overline{X}^T A X$. The **denominator** of a matrix $Z \in \mathrm{Mat}_n(\mathbb{Q})$ is the smallest number $x \in \mathbb{N}$ such that $xZ \in \mathrm{Mat}_n(\mathbb{Z})$. We also write $\mathrm{denom}(Z) = x$. $1_n \in \mathrm{Mat}_n(\mathbb{Z})$ denotes the **identity matrix**. We use the **Gauß notation** $[a, b, c] := \left(\begin{smallmatrix} a & b \\ b & c \end{smallmatrix} \right) \in \mathrm{Mat}_n(\mathbb{C})$.

The **general linear group** is defined by

$$\mathrm{GL}_n(R) = \{ X \in \mathrm{Mat}_n(R) \mid \det(X) \text{ is a unit in } R \}$$

and the **special linear group** by

$$\mathrm{SL}_n(R) = \{ X \in \mathrm{Mat}_n(R) \mid \det(X) = 1 \}.$$

The **orthogonal group** is defined by

$$\mathrm{O}_n(R) = \left\{ X \in \mathrm{GL}_n(R) \,\middle|\, X^T 1_n X = 1_n \right\} \subseteq \mathrm{GL}_n(R).$$

The **symplectic group** is defined by

$$\mathrm{Sp}_n(R) = \left\{ X \in \mathrm{GL}_{2n}(R) \,\middle|\, \overline{X}^T J_n X = J_n \right\} \subseteq \mathrm{GL}_{2n}(R) \subseteq \mathrm{Mat}_{2n}(R)$$

where $J_n := \left(\begin{smallmatrix} 0 & -1_n \\ 1_n & 0 \end{smallmatrix} \right) \in \mathrm{SL}_{2n}(R)$ (as in [Der01]). (Note that some authors (e.g. [PY07]) define J_n negatively.) $\mathrm{Sp}_n(R)$ is also called the **unitary group**. Note that [Der01] uses $\mathrm{U}_n(R) = \mathrm{Sp}_n(R)$. Also note that $M = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix} \right) \in \mathrm{Sp}_1(\mathbb{Z}) \Leftrightarrow ad - bc = 1 \Leftrightarrow M \in \mathrm{SL}_2(\mathbb{Z})$. Thus, $\mathrm{Sp}_1(\mathbb{Z}) = \mathrm{SL}_2(\mathbb{Z})$.

For $Z \in \mathrm{Mat}_n(\mathbb{C})$, we call

$$\Re(Z) = \frac{1}{2} \left(Z + \overline{Z}^T \right) \in \mathrm{Mat}_n(\mathbb{C})$$

the **real** part and

$$\Im(Z) = \frac{1}{2i} \left(Z - \overline{Z}^T \right) \in \mathrm{Mat}_n(\mathbb{C})$$

the **imaginary** part of Z and we have $Z = \Re(Z) + i\Im(Z)$. Note that we usually have $\Re(Z), \Im(Z) \notin \mathrm{Mat}_n(\mathbb{R})$ but we have $\Re(Z), \Im(Z) \in \mathrm{Her}_n(\mathbb{C})$.

We say that some function $f\colon \mathcal{A} \to \mathcal{B}$ with $\mathcal{A} \subseteq \mathrm{Mat}_n(R), \mathcal{B} \subseteq R$ is k-**invariant** under some $\mathcal{X} \subseteq \mathrm{Mat}_n(R)$ where $\mathcal{A}[\mathcal{X}] \subseteq \mathcal{A}$ if and only if $\det(U)^k f(T[U]) = f(T)$ for all $T \in \mathcal{A}, U \in \mathcal{X}$.

2.1 Siegel modular forms

Siegel modular forms aren't directly used in this work. However, the idea of this work is inspired by [PY07] and they are using them. Also, they are a generalization of Elliptic modular forms.

Let $\mathcal{H}_n := \left\{ Z \in \mathrm{Mat}_n^T(\mathbb{C}) \,\middle|\, \Im(Z) > 0 \right\}$ be the **Siegel upper half space**. We call $\mathrm{Sp}_n(\mathbb{Z})$ the **Siegel modular group**.

A **Siegel modular form** of degree $n \in \mathbb{N}$ with weight $k \in \mathbb{Z}$ for some $\Gamma \subseteq \mathrm{Sp}_n(\mathbb{Z})$, Γ subgroup of $\mathrm{Sp}_n(\mathbb{Z})$, is a holomorphic function

$$f\colon \mathcal{H}_n \to \mathbb{C}$$

with

(1) $f\left((AZ + B) \cdot (CZ + D)^{-1} \right) = \det(CZ + D)^k \cdot f(Z) \quad \forall \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix} \right) \in \Gamma, Z \in \mathcal{H}_n$

(2) for $n = 1$: $f(Z) = O(1) \quad$ for $Z \to i\infty$

Note that many authors define the transformed function $f|M$ for $M = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix} \right) \in \Gamma$ with

$$(f|M)(Z) := f(M \cdot Z) \cdot \det(CZ + D)^{-k}$$

with $Z \in \mathcal{H}_n$, where $M \cdot Z := (AZ + B) \cdot (CZ + D)^{-1}$. Then the first property of Siegel modular forms can be written as

$$f|M = f \quad \forall M \in \Gamma.$$

$\mathcal{M}_k^{\mathcal{H}_n}(\Gamma)$ denotes the vector space of such Siegel modular forms.

2.2   Elliptic modular forms

These are functions over $\mathcal{H}_1 = \{z \in \mathbb{C} \mid \Im(z) > 0\} \subseteq \mathbb{C}$ which is called the **Poincaré upper half plane**.

We have Γ as a subgroup of $\mathrm{SL}_2(\mathbb{Z})$. A **Elliptic modular form** with weight $k \in \mathbb{Z}$ over Γ is a holomorphic function

$$f \colon \mathcal{H}_1 \to \mathbb{C}$$

with

(1) $f\left(\dfrac{a\tau + b}{b\tau + c}\right) = (c\tau + d)^k \cdot f(\tau) \quad \forall \; \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma, \tau \in \mathcal{H}_1$

(2) $f(\tau) = O(1) \quad$ for $\tau \to i\infty$

Note that most authors write $M\tau := \frac{a\tau+b}{b\tau+c}$.

$\mathcal{M}_k(\Gamma)$ denotes the vector space of such Elliptic modular forms.

Note that we have $\mathrm{SL}_2(\mathbb{Z}) = \mathrm{Sp}_1(\mathbb{Z})$. We can see that Elliptic modular forms are Siegel modular forms of degree $n = 1$. Thus we have $\mathcal{M}_k(\Gamma) = \mathcal{M}_k^{\mathcal{H}_1}(\Gamma)$.

In this work, we use a specific subgroup of $\mathrm{Sp}_1(\mathbb{Z})$. We define

$$\Gamma_0(l) := \left\{ \left(\begin{array}{cc} a & b \\ c & d \end{array}\right) \in \mathrm{Sp}_1(\mathbb{Z}) \;\middle|\; c \equiv 0 \pmod{l} \right\} \subseteq \mathrm{Sp}_1(\mathbb{Z}) \subseteq \mathrm{Mat}_2(\mathbb{Z})$$

as a subgroup of $\mathrm{Sp}_1(\mathbb{Z})$.

2.3   Hermitian modular forms

Let $\mathbb{H}_n := \{Z \in \mathrm{Mat}_n(\mathbb{C}) \mid \Im(Z) > 0\}$ be the **Hermitian upper half space**. Note that these matrices are not symmetric as the Siegel upper half space $\mathcal{H}_n$ but we have $\mathcal{H}_n \subseteq \mathbb{H}_n$ and $\mathcal{H}_1 = \mathbb{H}_1 \subseteq \mathbb{C}$.

Let $\Delta \in \mathbb{N}$ so that we have the imaginary quadratic number field $\mathbb{Q}(\sqrt{-\Delta})$ where $-\Delta$ is the fundamental discriminant. Then, let $\mathcal{O} \subseteq \mathbb{Q}(\sqrt{-\Delta})$ be the maximum order. We call $\mathrm{Sp}_n(\mathcal{O})$ the **Hermitian modular group**. Let Γ be a subgroup of $\mathrm{Sp}_n(\mathcal{O})$. Let $\nu \colon \Gamma \to \mathbb{C}^\times$ be an abel character of $\mathrm{Sp}_n(\mathcal{O})$.

A **Hermitian modular form** of degree $n \in \mathbb{N}$ with weight $k \in \mathbb{Z}$ over Γ and ν is a holomorphic function

$$f \colon \mathbb{H}_n \to \mathbb{C}$$

with

(1) $f((AZ + B) \cdot (CZ + D)^{-1}) = \nu(M) \det(CZ + D)^k f(Z), \quad M = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in \Gamma, Z \in \mathbb{H}_n,$

(2) for $n = 1$: f is holomorphic in all cusps.

Again as for Siegel modular forms, most authors write $M \cdot Z := (AZ + B) \cdot (CZ + D)^{-1}$.

$\mathcal{M}_k^{\mathbb{H}_n}(\Gamma, \nu)$ denotes the vector space of such Hermitian modular forms.

In this work, we will concentrate on Hermitian modular forms of degree $n = 2$. We will start with $\Delta \in \{3, 4, 8\}$.

2.3.1 Properties

Because $-\Delta$ is fundamental, we have two possible cases:

1. $\Delta \equiv 3 \pmod 4$ and Δ is square-free, or

2. $\Delta \equiv 0 \pmod 4$, $\Delta/4 \equiv 1, 2 \pmod 4$ and $\Delta/4$ is square-free.

And for the **maximum order** $\mathcal{O}$, we have (compare [Der01])

$$\mathcal{O} = \mathbb{Z} + \mathbb{Z} \frac{-\Delta + i\sqrt{\Delta}}{2},$$
$$\mathcal{O}^{\#} = \mathbb{Z} \frac{i}{\sqrt{\Delta}} + \mathbb{Z} \frac{1 + i\sqrt{\Delta}}{2}.$$

From now on, we will always work with Hermitian modular forms of degree $n = 2$. We also use $\Gamma = \mathrm{Sp}_2(\mathcal{O})$ for simplicity.

Chapter 3

Theory

Lemma 3.1. *Let $f\colon \mathbb{H}_2 \to \mathbb{C}$ be a Hermitian modular form of weight k. Let $S \in \mathcal{P}_2(\mathcal{O})$. Then,*
$f(S\tau)\colon \mathbb{H}_1 \subseteq \mathbb{C} \to \mathbb{C}$ is an Elliptic modular form of weight $2k$ to $\Gamma_0(l)$, where l is the denominator
of S^{-1}.

Proof. Define $\Gamma^H := \operatorname{Sp}_2(\mathcal{O})$ as the translation group for f. Then, we can verify that

$$M := \begin{pmatrix} 1_2 & B \\ & 1_2 \end{pmatrix} \in \Gamma^H, \quad B \in \operatorname{Her}_2(\mathcal{O}).$$

Let $\tau \in \mathbb{H}_1$. With $S \in \mathcal{P}_2(\mathbb{C})$ we have

$$
\begin{aligned}
\Im(S\tau) &= \frac{1}{2i}\left(S\tau - \overline{S}^T \overline{\tau}\right) \\
&= \frac{1}{2i} S(\tau - \overline{\tau}) \\
&= \frac{1}{2i} S \cdot 2i\Im(\tau) \\
&= S\Im(\tau) > 0,
\end{aligned}
$$

thus $S\tau \in \mathbb{H}_2$.

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \operatorname{SL}_2(\mathbb{Z})$. We have

$$
\begin{aligned}
& S\frac{a\tau + b}{c\tau + d} \\
=\ & (a(S\tau) + bS) \cdot ((cS^{-1})(S\tau) + d)^{-1} \\
=\ & \begin{pmatrix} a1_2 & bS \\ cS^{-1} & d1_2 \end{pmatrix} \cdot S\tau.
\end{aligned}
$$

And with $S = \overline{S}^T$ and $ad - bc = 1$ we have

$$\overline{\begin{pmatrix} a1_2 & bS \\ cS^{-1} & d1_2 \end{pmatrix}}^T J_2 \begin{pmatrix} a1_2 & bS \\ cS^{-1} & d1_2 \end{pmatrix}$$

$$= \begin{pmatrix} (-acS^{-1} + ac\overline{S^{-1}}^T) & (-ad1_2 + cb\overline{S^{-1}}^T S) \\ (-bc\overline{S}^T S^{-1} + ad1_2) & (-bd\overline{S}^T + bdS) \end{pmatrix}$$

$$= J_2,$$

thus we have

$$M' := \begin{pmatrix} a1_2 & bS \\ cS^{-1} & d1_2 \end{pmatrix} \in \mathrm{Sp}_2(\mathbb{C}).$$

We also have $cS^{-1} \in \mathrm{Mat}_2(\mathcal{O})$, thus we have $M' \in \Gamma^H = \mathrm{Sp}_2(\mathcal{O})$.

Thus, because f is a Hermitian modular form, we have

$$f\left(S \frac{a\tau + b}{c\tau + d} \right)$$

$$= f\left(M' \cdot S\tau \right)$$

$$= \nu(M') \cdot \det(cS^{-1}S\tau + d1_2)^k \cdot f(S\tau)$$

$$= (c\tau + d)^{2k} \cdot f(S\tau).$$

$\square$

Lemma 3.2. *Prop 7.3. von Poor für herm Modulformen.* $\Gamma(\mathcal{L}) \supseteq \Gamma_0(l)$ *for* $l \in \mathbb{Z}^+, ls^{-1} \in \mathcal{P}_n(\mathcal{O})$.

We want to calculate a generating set for the Fourier expansions of Hermitian modular forms. Now we will formulate the main algorithm of our work.

Algorithm 3.3. We have the Hermitian modular form degree $n = 2$ fixed, as well as some Δ (for now, $\Delta \in \{3, 4, 8\}$). Then we select some form weight $k \in \mathbb{Z}$ ($k \in \{1, \ldots, 20\}$ or so), let $\mathcal{O} \subseteq \mathbb{Q}(\sqrt{-\Delta})$ be the maximum order (see chapter 2.3.1) and some subgroup Γ of $\mathrm{Sp}_2(\mathcal{O})$. Then we select an abel character $\nu \colon \Gamma \to \mathbb{C}^\times$ of $\mathrm{Sp}_2(\mathcal{O})$.

We define the index set

$$\Lambda := \left\{ 0 \leq \begin{pmatrix} a & b \\ \overline{b} & c \end{pmatrix} \in \mathrm{Mat}_2(\mathcal{O}^\#) \,\middle|\, a, c \in \mathbb{Z} \right\}.$$

Fix $B \in \mathbb{N}$ as a limit. Select a precision

$$\mathcal{F} := \left\{ \left(\begin{array}{cc} a & b \\ \bar{b} & c \end{array} \right) \in \Lambda \,\middle|\, 0 \leq a, c < B \right\} \subseteq \Lambda.$$

1. We start with $l = 1$ and increase it but only use the square-free numbers.

2. Set $\mathcal{S} = \{\}$,

3. Enumerate matrices $S \in \mathrm{Mat}_2^T(\mathbb{Z})$, and set $\mathcal{S} \leftarrow \mathcal{S} \cup \{S\}$ and for each time you add a new matrix perform the following steps.

4. We set

$$\mathcal{M}_{k,\mathcal{S},\mathcal{F}}^H := \left\{ (a[S])_{S \in \mathcal{S}} \,\middle|\, a \in \mathbb{Q}^{\mathcal{F}} \text{ is } \mathrm{GL}_2(\mathcal{O}) \text{ invariant} \right\} \subseteq \bigoplus_{S \in \mathcal{S}} \mathbb{Q}^{\mathcal{F}(S)},$$

where

$$a[S] := \mathbb{N}_0 \to \mathbb{Q}, n \mapsto \sum_{T \in \Lambda, \mathrm{tr}(ST)=n} a(T),$$

The elements $a \in \mathbb{Q}^{\mathcal{F}}$ are Fourier expansions of Elliptic modular forms ($\mathbb{H}_1 \to \mathbb{C}$) and $a(T) \in \mathbb{Q}$ for $T \in \mathcal{F} \subseteq \mathrm{Mat}_2(\mathcal{O}^{\#})$ are the Fourier coefficients. Recall that a being invariant under $\mathrm{GL}_2(\mathcal{O})$ means that we have

$$\det(U)^k a(T[U]) = a(T) \,\forall\, U \in \mathrm{GL}_2(\mathcal{O}).$$

With $[a, b, c] \in \mathcal{F}$, we have $0 \leq a, c < B$, thus there are only a finite number of possibilities. Because $0 \leq [a, b, c]$, we get $ac - |b|^2 \geq 0$ and thus b is also always limited. Thus, $\mathcal{F}$ is finite but it might be huge for even small B. Restricting the elements in $\mathcal{F}$ by the $\mathrm{GL}_2(\mathcal{O})$-invariation makes the set $\{x \in \mathcal{F} \mid x \text{ is } \mathrm{GL}_2(\mathcal{O}) \text{ invariant}\} \subseteq \mathcal{F}$ much smaller and better to handle in computer calculations. We use this set to identify a base of the finite dimension vector space $\{x \in \mathbb{Q}^{\mathcal{F}} \mid x \text{ is } \mathrm{GL}_2(\mathcal{O}) \text{ invariant}\}$.

We identify

$$\bigoplus_{S \in \mathcal{S}} \mathbb{Q}^{\mathcal{F}(S)} = \mathbb{Q}^N, \;\; N = \sum_{S \in \mathcal{S}} \mathcal{F}(S).$$

For a given $S \in \mathcal{S}$ and limit $B \in \mathbb{N}$ which restricts $\mathcal{F} \subset \Lambda$, $\mathcal{F}(S) \in \mathbb{N}_0$ is the limit such that for any $T \in \Lambda - \mathcal{F}$, $\mathrm{tr}(ST) \geq \mathcal{F}(S)$. Thus, for calculating the Fourier coefficients $T \in \Lambda$ with $\mathrm{tr}(ST) \in \{0, \ldots, \mathcal{F}(S) - 1\}$, it is sufficient to enumerate the $T \in \mathcal{F}$.

Let $S = [s, t, u]$ and $T = [a, b, c]$. Recall that $S \in \text{Mat}_2^T(\mathbb{Z})$. Then we have

$$\text{tr}(ST) = as + \bar{t}b + t\bar{b} + cu = as + cu + 2t\Re(b).$$

Because $T \geq 0$, we have $ac \geq |b|^2$ and thus $\Re(b) \leq \sqrt{ac} \leq \max(a, c)$. Thus, $2t\Re(b) \geq -2|t|\max(a, c)$. We also have $as + cu \geq \max(a, c)(s + u)$. Assuming $T \in \Lambda - \mathcal{F}$, we have $\max(a, c) \geq B$. For such T, we get

$$\text{tr}(ST) \geq B \cdot (s + u - 2|t|).$$

Given $S > 0$, we have $su > t^2$. Then we have

$$s + u - 2|t| > 0$$
$$\Leftrightarrow \quad su + u^2 - 2|t|u > 0$$
$$\Leftrightarrow \quad (t^2 + u^2 - 2|t|u) + (su - t^2) > 0$$
$$\Leftrightarrow \quad (|t| - u)^2 + (su - t^2) > 0.$$

All inequalities were sharp estimations[1], thus we get

$$\mathcal{F}(S) = B \cdot (s + u - 2|t|).$$

We want to calculate the matrix of the linear function

$$\{x \in \mathbb{Q}^{\mathcal{F}} \mid x \text{ is } \text{GL}_2(\mathcal{O}) \text{ invariant}\} \to \bigoplus_{S \in \mathcal{S}} \mathbb{Q}^{\mathcal{F}(S)}, \quad a \mapsto (a[S])_{S \in \mathcal{S}}.$$

The base of the destination room is canonical. The dimension is N. The base of the source room can be identified by $\{x \in \mathcal{F} \mid x \text{ is } \text{GL}_2(\mathcal{O}) \text{ invariant}\}$.

And we set

$$\mathcal{M}_{k,\mathcal{S},\mathcal{F}} := \bigoplus_{S \in \mathcal{S}} \mathcal{F}\mathcal{E}_{\mathcal{F}(S)}(\mathcal{M}_k(\Gamma_0(l_S)))$$

where $\mathcal{M}_k(\Gamma_0(l_S))$ is the vectorspace of Elliptic modular forms over $\Gamma_0(l_S)$.

5. If

$$\dim \mathcal{M}_{k,\mathcal{S},\mathcal{F}}^H \cap \mathcal{M}_{k,\mathcal{S},\mathcal{F}} = \dim \mathcal{M}_k^{\mathbb{H}_2}(\Gamma, \nu),$$

---

[1] For example, let $S = [2, -1, 1]$. Then you have $s + u - 2|t| = 1$. With $c = B$ and $a = b = 1$, you hit the limit $\text{tr}(ST) = 2 + B - 2 = B = \mathcal{F}(S)$.

then we are ready and we can reconstruct the Fourier expansion in the following way:

...

If not, then return to step 3, and enlarge $\mathcal{S}$.

Chapter 4

Implementation

In this chapter, we are describing the implementation.

Chapter 5

Conclusion

Blub

Chapter 6

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