

# Gear diagnostics under widely varying speed conditions

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**Abstract.** Order tracking is becoming more and more used for analysing signals from variable speed machines. However, even though order tracking removes the frequency modulation effects associated with variable speed, over wider speed ranges there are also amplitude modulations, which are not removed. It means for example that such gear signals cannot be sensibly enhanced by synchronous averaging, since this would give some sort of average amplitude, and if this “average” were subtracted from the total signal, it would not only leave the random part of the signal, but also variations of the deterministic part about the mean. This paper introduces a method to remove that part of the amplitude modulation due to passage of forcing functions, such as gearmesh frequencies, through resonances, but it is found that in some cases the modulation is due to other causes such as variations of torque with speed.

**Keywords:** gear diagnostics; order tracking; wide speed range; cepstral processing; resonance compensation

## 1 Introduction

Many diagnostic methods for machines such as gearboxes have been developed for constant speed or almost constant speed operation, such as when driven by an induction motor, or driving an induction generator [1]. Even when the speed variation is quite small like this, it is still necessary to perform “order tracking” before carrying out diagnostic procedures such as TSA (time synchronous averaging), in order to have exactly the same number of samples per revolution, and a fixed starting phase for each revolution. This has been well developed for many years. In recent years, there has been an attempt to extend these methods to much wider speed ranges, such as are typical with wind turbines, as much as  $\pm 30\%$ . However, even though the order tracking techniques can be extended to wider speed ranges such as this, and can now remove virtually all frequency modulation, the resulting order tracked signals often have considerable residual amplitude modulation, varying with the speed, so that the results of TSA would have dubious validity. For example, for almost constant speed machines, the deterministic part of the signal is made up of sinusoids, and if these are determined by TSA and subtracted, the residual signal would usually be stationary random. If however

the deterministic part is amplitude modulated, at a much lower frequency than the rotational speed (used to synchronize the TSA), the result of TSA will have some sort of “average” amplitude, and if this is repeated and subtracted from the overall signal, much of the residual will be the amplitude variation of the deterministic part, and no longer just the random part. A partial solution to this problem can be to search the record for sections with minimum speed variation [2], but this is not always possible.

In a companion paper, on bearing diagnostics [3], it was pointed out that much information about bearing faults is carried by resonances, and this should preferably be extracted while in the time domain, before transformation to the angle domain by order tracking. With gear faults, it is almost the opposite, in that they tend to be dominated by the forcing functions directly related to shaft orders, and resonances if anything tend to disturb these, as one of the possible sources of amplitude modulation is the **passage of strong forcing functions**, such as gearmesh frequencies, through resonances. In [3] it was shown how the cepstrum could be used to obtain an estimate of the resonances from the response signal, and in this paper it is proposed to compensate the overall signal in the time domain, using this estimate, to leave a residual dominated by the gear forcing functions, before carrying out order tracking.

## 2 Proposed Method and Results

### 2.1 Pre-processing - Resonance Compensation

The proposed method is first applied to signals from a gearbox (with one cracked tooth) with mean speed 6 Hz, and speed variations of  $\pm 10\%$  and  $\pm 25\%$  (from [4]), and the former signal is discussed here. However, it was difficult in [4] to make a diagnosis of the tooth crack fault, though the toothmesh demodulation method of McFadden [5] did appear to give an indication.

The proposed method for resonance compensation makes use of the cepstrum of frequency response functions, first developed by Oppenheim and Schaffer [6], and extended to modal analysis by Gao and Randall [7, 8]. The cepstrum is the inverse Fourier (or z-) transform of the log spectrum, and is called the complex cepstrum if the log spectrum includes the phase, and the real (or power) cepstrum if it is based on the log amplitude only, even though both are in fact real (since the log amplitude spectrum is even and the phase spectrum odd). **The phase spectrum has to be unwrapped to a continuous function of frequency**, and this is only possible for well-behaved functions such as frequency response functions (FRFs). For minimum phase functions, the phase is in fact the Hilbert transform of the log amplitude and does not have to be measured, so that the complex cepstrum can be obtained from an averaged power spectrum. However, it is not possible to unwrap the phase of stationary (or continuous non-stationary) signals, whose phase is either undefined between discrete frequencies, or random. However, it has recently been discovered that the amplitude spectrum of stationary signals can be edited using the real cepstrum and combined with the original phase of individual records to obtain edited time signals. This has been used to remove discrete frequency components prior to Operational Modal Analysis (OMA) [9]. This made use of

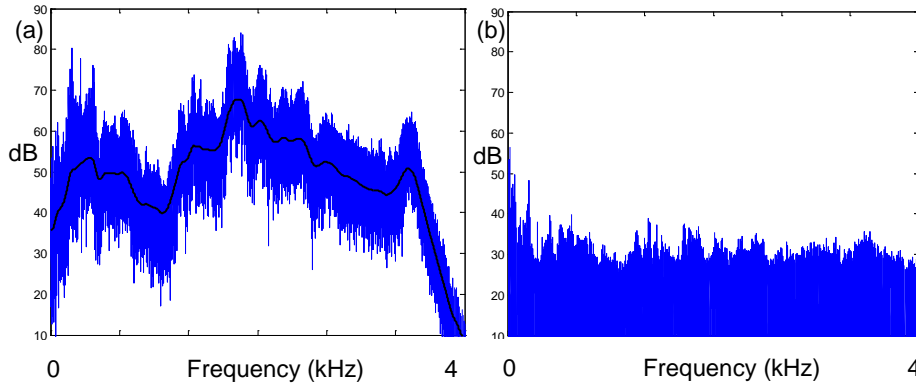
the fact that the cepstrum of an impulse response function (IRF) has the same mathematical form as the IRF (a complex exponential) further damped by multiplication by  $1/n$ , where  $n$  is time sample number (and where the time axis of the cepstrum is known as “quefrency”). Thus, as shown in [7], the complex cepstrum of a minimum phase function can be expressed as:

$$C(n) = 2 \sum_{j=1}^{Np/2} \frac{A_j^n}{n} \cos(\omega_j n) - 2 \sum_{k=1}^{Nz/2} \frac{A_k^n}{n} \cos(\omega_k n) \quad (1)$$

where the  $A_j$  are the moduli and  $\omega_j$  the frequencies of the poles in the z-plane; the  $A_k$  and  $\omega_k$  are the same for the zeroes, all inside the unit circle. Note that  $A_j^n$  and  $A_k^n$  are exponential decays which can be written as  $\exp(-\sigma_i n \Delta t)$ , where  $\sigma_i$  represents in rad/s half the 3 dB bandwidth of the corresponding resonance peak.  $\Delta t$  is the time sample spacing. Thus, if a cepstrum is multiplied by an exponential window  $\exp(-\sigma_0 n \Delta t)$ , the only effect on the modal properties is to add  $\sigma_0$  to the damping of all poles and zeroes in the FRF, and this can in principle be compensated for.

In [9] an exponential window was applied to remove as much information as possible except for the modal information, as a precursor to OMA, and this was also done in [3] as a precursor to bearing envelope analysis. In this case, as mentioned above, it is proposed to subtract the modal information in the cepstrum, leaving primarily the forcing function without distortion by the modal properties. It is anticipated that this should reduce the amplitude modulation effects of passage through resonances.

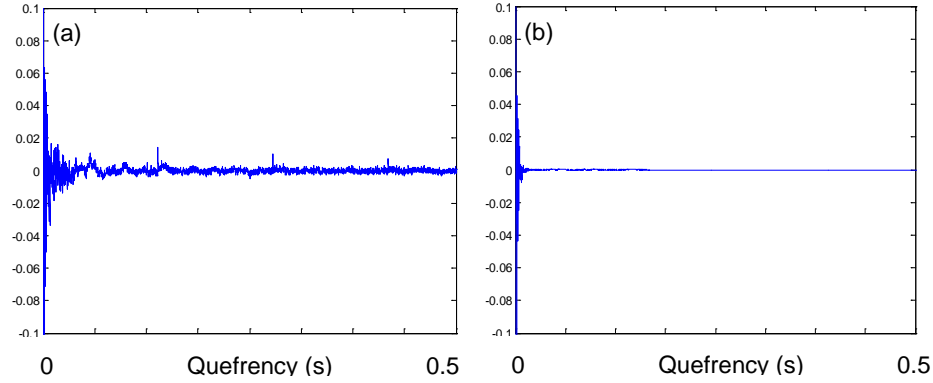
Figure 1(a) shows (on a dB scale) the spectrum of the signal from [4] with  $\pm 10\%$  speed variation over the full frequency range, as well as the modal information from applying an exponential window (lifter, in cepstrum terminology) to its cepstrum. Figure 1(b) shows the spectrum obtained by removing the modal information in the cepstrum (amplitude scaling then arbitrary). The FRF would modify both amplitude



**Fig. 1.** Spectra of the original (a) and “liftered” (b) signal. The black curve in (a) was produced using an exponential lifter in the cepstrum

and phase when passing through a resonance, but it is thought that the amplitude effect would dominate. Future work will investigate if a modal model could be fitted to the black curve in Fig. 1(a), so that phase effects could also be taken into account.

Figure 2(a) and (b) show the original and liftered cepstra corresponding to the original and smoothed spectra in Fig. 1(a).

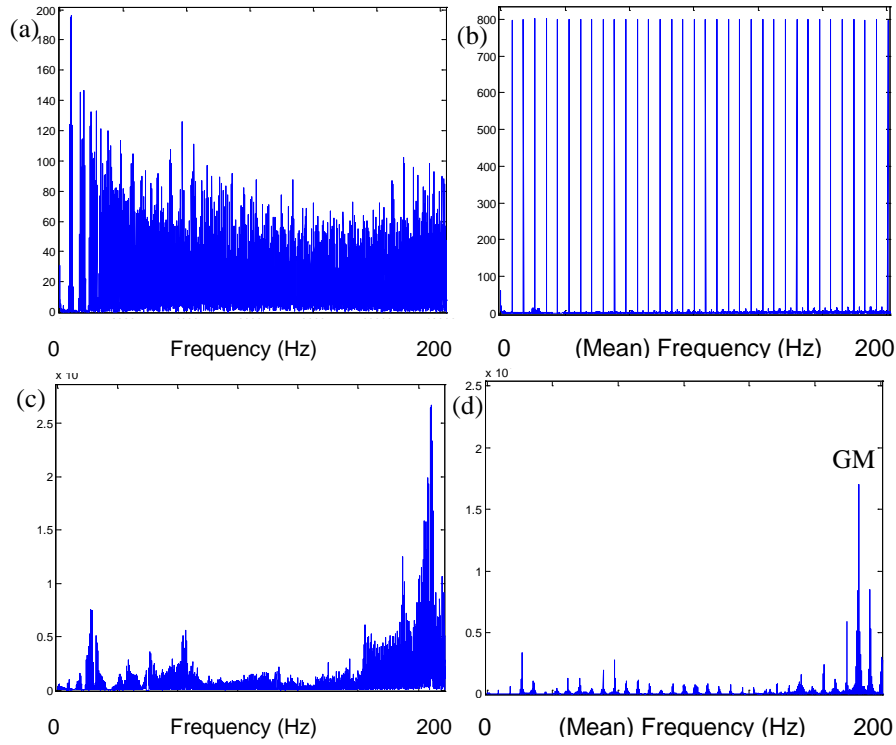


**Fig. 2.** Cepstra corresponding to Fig. 1(a). (a) Original (b) Liftered

## 2.2 Order Tracking

The order tracking is achieved using the phase demodulation based method described in [4]. A phase (rotational angle) vs time map is obtained by phase demodulation of a reference signal phase-locked to machine speed. This can be a once-per-rev tachometer (usually required for the first stage) or a multiple pulse per rev encoder signal. These can sometimes be extracted from the vibration signal itself, eg the first order, or a gearmesh component, as long as these are sufficiently separated from other spectrum components, and the mechanical system response time is short with respect to the rate of speed change. The advantage of the phase demodulation method is that as long as the spectrum of the reference order, including modulation sidebands, is separated in the spectrum, the phase/time curve can be determined to any degree of resolution. It is then used to find sampling times at constant increments of phase, which can be used to resample signals initially sampled at constant time intervals, using cubic spline interpolation. The first order will be separated in the spectrum for speed variations up to  $\pm 30\%$ , with an additional limitation given by rate of speed change [4], but once the major part of the speed variation is removed in the first stage, further iterations can be made using higher orders for more accuracy [4].

Figure 3 compares the spectra of the tachometer signal, used as reference, and the response signal, before and after two stages of order tracking for the gear with a “crack” fault (actually a slot machined on a  $45^\circ$  angle at the root of one tooth), and with  $\pm 10\%$  speed variation. It is seen in Fig. 3(a) that both the first and second orders are separated and could have been used for order tracking, while in Fig. 3(c) only the third order is strong enough, but is not well separated. Fig. 3(b) shows that frequency modulation is well removed up to the highest order shown, but Fig. 3(d) shows that there are still sidebands

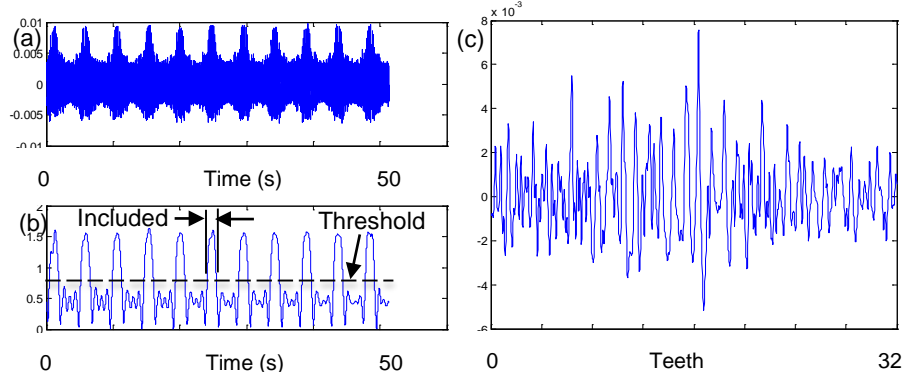


**Fig. 3.** Spectra (a, c) before order tracking (b, d) after 2 stages of order tracking (a, b) Tacho (c, d) Acceleration GM indicates gearmesh (order 32)

around the gearmesh frequency (GM) in the response, which must be due to amplitude modulation.

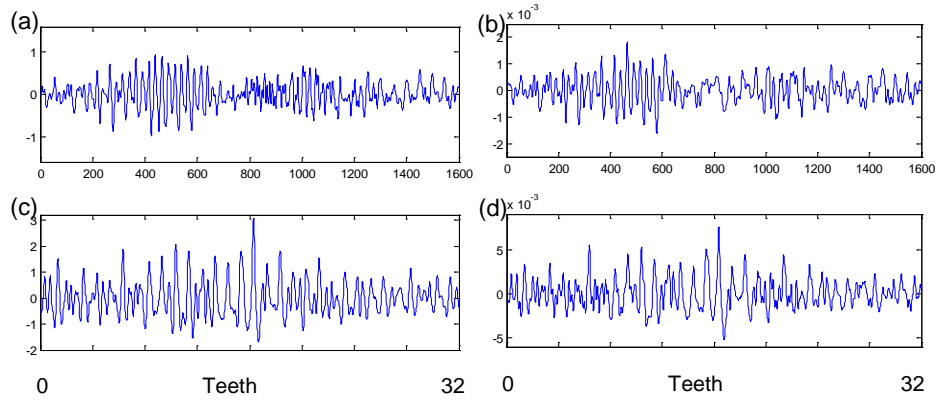
### 2.3 Time Synchronous Averaging (TSA)

Initial analysis indicated that there was very strong amplitude modulation of all signals, at the rate of speed variation 0.2 Hz (a period of 5s) as compared with the average rotational speed of 6 Hz (see Figure 4(a) for a typical case). To reduce the effect of this on the resulting time averages, for rotation cycles, it was decided to limit the records going into the average, based on the local signal strength. This was determined from the envelope of the gearmesh component, as depicted in Figure 4(b), where it is seen that the amplitude fell to near zero every cycle. As indicated there, the records included were only those where the envelope was between 50% and 100% of its maximum value. The final synchronous average in that case (fault, filtered signal) is shown in Figure 4(c). It should be noted that even for a filtered signal such as this, there is still a great deal of amplitude variation with speed, indicating that the reason for the amplitude modulation in this case is not the passage of important harmonics through resonances, so other data will be sought where this is the case. It is possible that the reason here is load variation from acceleration and deceleration.



**Fig. 4.** Selection of records for TSA. Lifted signal, crack fault (a) Overall time record (b) Envelope of GM component, showing included sections (c) TSA over one gear rotation

The final TSA results for four cases are shown in Figure 5 (note differences in scales).



**Fig. 5.** Results of TSA (a, b) No fault (c, d) Fault (a, c) Original (b, d) Lifted

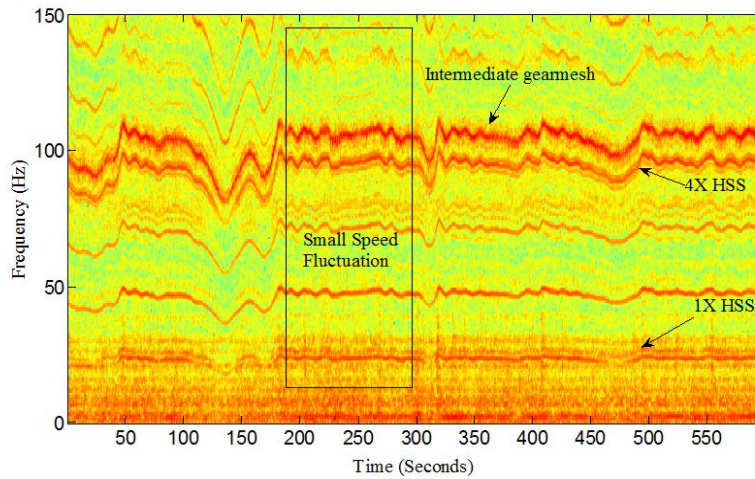
Presumably because of the lack of sensitivity of the shape of the signal to passage through resonances, there is not a great difference between the results with and without liftering, although the lifted signal of Fig. 5(b) is somewhat smoother than the original signal in Fig. 5(a). Note that the case of Fig. 5(c) corresponds very well to the equivalent case depicted in Figure 29(a) of Ref. [4].

Another point of interest is that there is a considerable amount of amplitude modulation within each rotation, which shows up clearly in the averaged results. It evidently has no relation to the amplitude modulation every speed cycle (as seen in Fig. 4(a)) and could be due to the alignment of the gears giving a variation in tooth loading every revolution. Note that the position of the tachometer pulses used to synchronize the averages is not aligned with the same teeth for the cases with and without a crack fault, though the lifted and non-lifted cases had the same alignment. It is interesting that the lower strength signals with no fault are dominated by the second harmonic of gearmesh frequency, whereas the signals with the faults are dominated by the first harmonic.

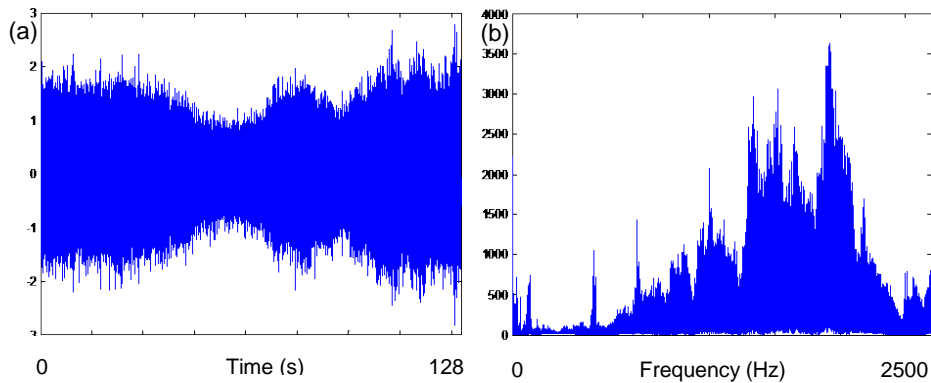
## 2.4 Application to a Wind Turbine Signal

Because the amplitude modulation of the data used to demonstrate the procedure was not dominated by passing through resonances, but apparently due to variations in the forcing function, it was decided to apply the same method to data from a wind turbine with speed varying over a wide range. A section of this data was used for blind identification of the numbers of teeth on the various gears [10] in the gearbox, but the section analysed was chosen carefully to have minimum speed variation (about 4%).

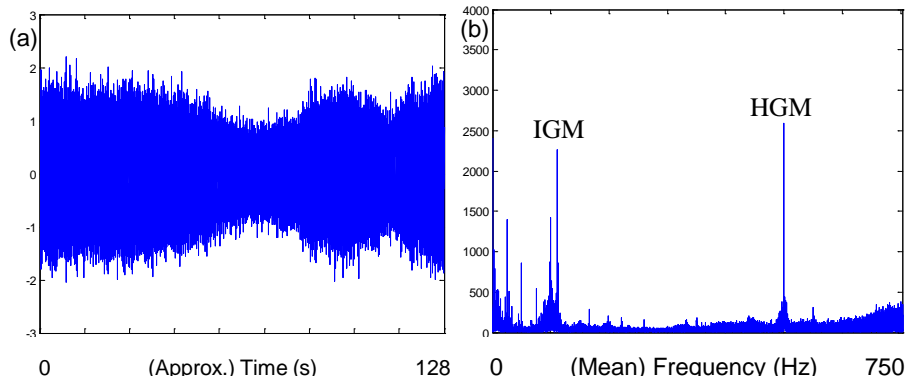
Figure 6 shows a spectrogram (from [10]) illustrating the chosen section, and the structure of the low frequency part of the spectrum, including the first four harmonics of the high speed shaft (HSS) speed and the intermediate gearmesh frequency, just above the fourth harmonic. It was decided to try to analyse the section immediately prior to that section (approx. 70-190 s) where the speed varied over a 35% range. The frequency range retained for the analysis was valid up to about 2500 Hz, and included five harmonics of the high gearmesh frequency. Figure 7 shows the time signal and its spectrum.



**Fig. 6.** Zoomed spectrogram in the lower frequency range (0-150 Hz)

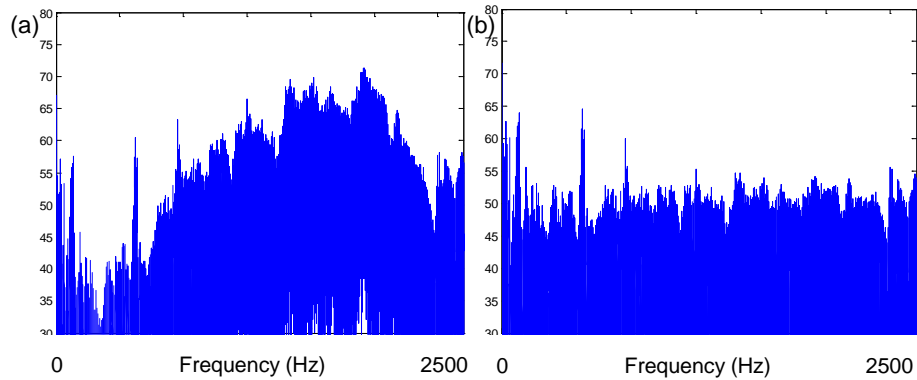


**Fig. 7.** (a) Variable speed time record (b) Spectrum of (a)



**Fig. 8.** (a) Order tracked signal (b) Order spectrum

Figure 8 shows the order tracked signal corresponding to Fig. 7(a) and the lower part of its order spectrum, showing that the four low harmonics of the HSS, the intermediate gearmesh frequency (IGM) and the high gearmesh frequency (HGM) are now separated and discrete.

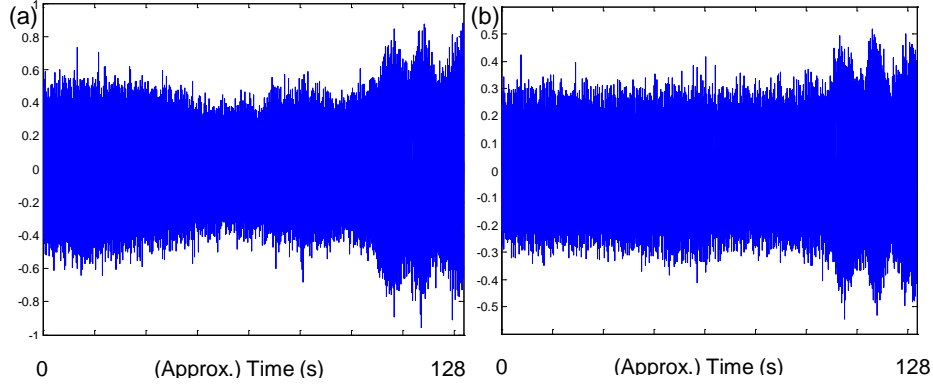


**Fig. 9.** (a) dB spectrum corresponding to 7(b) (b) Liftered spectrum

Figure 9 shows the dB spectrum corresponding to Fig. 7(b) and the liftered version after using an exponential lowpass lifter.

The liftered time record corresponding to Fig. 9(b) is shown in Fig. 10(a), and is more uniform than the original in Fig. 7(a), indicating that some of the amplitude variation was due to passing through resonances. Fig. 10(b) shows the result of lowpass filtering the signal just above the third harmonic of the high speed gearmesh frequency, and this is even more uniform over most of its length, indicating that the variation of these gearmesh components is much reduced with this compensation. It is interesting that the rather sudden variations near the end of the record correspond to where the speed was varying much more rapidly than in the rest of the record (see the spectrogram





**Fig. 10.** (a) Liftered time record over full bandwidth (b) LP filtered above  $3 \times \text{HS}$  gearmesh

near 190 s in Fig. 6. A possible explanation for this is that the acceleration and deceleration associated with this gave increased modulation through the load variation.

Finally, TSA was carried out on the various signals to see if any improvement was given by the cepstral liftering, but this did not give clear patterns even for the section of signal analysed in [10] so these are not reproduced here. It is not known if any faults were present in the gearbox, but suspected that there were none.

It will be necessary to search for data with large speed variation and a known existing fault in order to properly test the proposed method for application to wind turbines.

### 3 Conclusion

A method is proposed to aid with the diagnostics of gear faults under widely varying speed conditions. The differences with respect to bearing faults are pointed out in that much information about the latter is contained in resonances, which are fixed in frequency, whereas gear faults often change the forcing functions, which vary directly with speed. In both cases it is necessary to pre-process the signals in the time domain first, to compensate for resonances, before applying order tracking to compensate for speed variations. In the case of gears, it can be advantageous to remove modal information (determined using an exponential cepstral lifter) before order tracking to reduce the effect on forcing functions such as gearmesh frequencies passing through resonances. It was found that in one case this did not remove amplitude modulation, and it is suspected that this was because the amplitude variations were more likely due to load variations with speed. In another case on a wind turbine gearbox, the proposed method was more successful in greatly reducing the amplitude modulation, but since there was no known gear error, it could not be confirmed that an improvement in diagnostic ability was achieved, although it does seem likely.

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