Asian options: pricing, hedging, and market comparison

Executive summary

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GitHub: https://github.com/albesianor/Asian-options-hedging

Asian options are path-dependent derivatives

- widely used in commodities, energy, and currency markets (e.g. airlines for fuel prices),
- with payoff depending on the average of the underlying asset price,
- mitigating short-term volatility and manipulation risk.

Call type	**Payoff**
European	$\max(S_T - K, 0)$
Asian	$\max(\text{average}(S) - K, 0)$

The average can be arithmetic or geometric.

This project

- implements analytical pricing for geometric Asian options, Monte-Carlo simulation for arithmetic Asian options under GBM assumptions,
- explore delta-hedging strategies,
- compares model-based results with market data using realized volatilities.

1 Pricing

Geometric. Under GBM dynamics, the average price follows a log-normal distribution. This gives a closed-form price similar to Black–Scholes, with adjusted volatility and drift:

$$C_0 = S_0 e^{(b-r)t}\Phi(d_1) - Ke^{-rt}\Phi(d_2), \quad b = \frac{1}{2}\left(r - \frac{\sigma_{\text{adj}}^2}{2}\right), \quad \sigma_{\text{adj}} = \frac{\sigma}{\sqrt{3}}.$$

Arithmetic. Priced by Monte-Carlo simulation: option price is expected discounted payoff.

2 Hedging

- We use a conditional geometric delta as an approximate hedge for Asian options.
- This approach stabilizes the hedged P&L relative to drift, though not perfectly.

3 Market comparison

Exchange-traded Asian options are scarce, so Asian call prices are compared to hypothetical realized payoffs via backtesting, using realized volatility in the 21 days prior.

We compute the P&L distribution of buying calls at the price indicated by our models and holding them to maturity:

- for one geometric Asian call,
- with 1 week expiry,
- between Jan 1, 2019, and Dec 31, 2024.

4 Key findings

A side market comparison for European options guides us in the interpretation of the one for Asian options.

Observation	**Interpretation**
Model prices underprice calls vs. market	Missing volatility risk premium
Mean P&L positive but median about 0	Apparent profits due to underpricing bias
P&L VaR >> mean	Model apparent profitability not statistically/economically significant
Drift sensitivity reduced by hedging	Hedging is imperfect but partially effective

5 Limitations and future work

- **Lack of real Asian option data:** Obtain OTC or institutional Asian option quotes for a true market comparison.
- **Realized volatility:** Replace with implied volatility obtained from European calls data.
- **Simplified GBM dynamics:** Replace GBM with Heston or SABR to capture stochastic volatility and reduce systematic underpricing bias.
- **Limited understanding of market conventions:** Investigate how averaging conventions and fixing schedules are handled in real-world Asian option contracts.
- **Approximate delta for arithmetic options:** Use Monte-Carlo difference quotient methods to estimate deltas more accurately; apply also to geometric hedging to account for discrete averaging.
- **Incomplete hedging using only the underlying:** Explore more complete hedging strategies using additional financial instruments.
- **Limited asset scope:** Extend the analysis to commodity and FX markets, developing a full case study where Asian options play a more significant practical role.
- **High computing time:** Explore stochastic PDEs methods to replace Monte-Carlo simulations where possible.