$$\begin{array}{ll} \min & 0.5x^T Qx + c^T x \\ \text{s.t.} & Cx \geq d_1 \\ & Cx \leq d_2 \end{array}$$

Residuals

$$\begin{pmatrix} r_Q \\ r_{C_1} \\ r_{C_2} \\ r_{z_1} \\ r_{z_2} \end{pmatrix} = \begin{pmatrix} Qx + c - C^T z_1 + C^T z_2 \\ Cx - s_1 - d_1 \\ Cx + s_2 - d_2 \\ Z_1 S_1 - \sigma\mu e \\ Z_2 S_2 - \sigma\mu e \end{pmatrix}$$

Full system

$$\begin{pmatrix} Q & C^T & C^T & 0 & 0 \\ C & 0 & 0 & I & 0 \\ C & 0 & 0 & 0 & I \\ 0 & -S_1 & 0 & -Z_1 & 0 \\ 0 & 0 & S_2 & 0 & Z_2 \end{pmatrix} \begin{pmatrix} \Delta x \\ -\Delta z_1 \\ \Delta z_2 \\ -\Delta s_1 \\ \Delta s_2 \end{pmatrix} = \begin{pmatrix} -r_Q \\ -r_{C_1} \\ -r_{C_2} \\ -r_{z_1} \\ -r_{z_2} \end{pmatrix}$$

First reduction

$$\begin{pmatrix} Q & C^T & C^T \\ C & -Z_1^{-1} S_1 & 0 \\ C & 0 & -Z_2^{-1} S_2 \end{pmatrix} \begin{pmatrix} \Delta x \\ -\Delta z_1 \\ \Delta z_2 \end{pmatrix} = \begin{pmatrix} -r_Q \\ -r_{C_1} - Z_1^{-1} r_{z_1} \\ -r_{C_2} + Z_2^{-1} r_{z_2} \end{pmatrix}$$

$$\begin{aligned} \Delta s_1 &= -Z_1^{-1}(r_{z_1} + S_1 \Delta z_1) \\ \Delta s_2 &= -Z_2^{-1}(r_{z_2} + S_2 \Delta z_2) \end{aligned}$$

Second reduction

$$(Q + C^T(S_1^{-1} Z_1 + S_2^{-1} Z_2))C\Delta x = -r_Q + C^T(S_1^{-1} Z_1 r_{C_1} + S_1^{-1} r_{z_1} - S_2^{-1} Z_2 r_{C_2} + S_2^{-1} r_{z_2})$$

$$\begin{aligned} \Delta s_1 &= -Z_1^{-1}(r_{z_1} + S_1 \Delta z_1) \\ \Delta s_2 &= -Z_2^{-1}(r_{z_2} + S_2 \Delta z_2) \\ \Delta z_1 &= -S_1^{-1} Z_1 (C\Delta x + r_{C_1} + Z_1^{-1} r_{z_1}) \\ \Delta z_2 &= S_2^{-1} Z_2 (C\Delta x + r_{C_2} - Z_2^{-1} r_{z_2}) \end{aligned}$$