Homework 2 Advanced Derivatives

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Exercise 1

Using the standard BS option pricing formula, we obtain the present value of equity $PV = 60.42$. The analytical price of the compound option with the given parameters is

$$C(t, V_t) = V_t N_2(a_1, b_1, \frac{\sqrt{\tau - t}}{\sqrt{T - t}}) - De^{-r(T-t)} N_2(a_2, b_2, \frac{\sqrt{\tau - t}}{\sqrt{T - t}}) - Ke^{-r(\tau - t)} N(a_2)$$

where

$$\begin{cases} a_1 = \frac{\ln(V_t/S^*) + (r + \frac{\sigma^2}{2})(\tau - t)}{\sqrt{\tau - t}} \\ b_1 = a_1 - \sigma\sqrt{\tau - t} \\ a_2 = \frac{\ln(V_t/D) + (r + \frac{\sigma^2}{2})(T - t)}{\sqrt{T - t}} \\ b_2 = a_2 - \sigma\sqrt{T - t} \end{cases}$$

and S^* can be obtained by using the newton-rhapson method on the equation

$$S^* N(d_1) - e^{-r(T-\tau)} DN(d_2) - K = 0$$

where

$$\begin{cases} d_1 = \frac{\ln(S^*/D) + (r + \frac{\sigma^2}{2})(T - \tau)}{\sqrt{T - \tau}} \\ d_2 = d_1 - \sigma\sqrt{T - \tau} \end{cases} .$$

Solving this problem with the given values of τ and K gives us the compound option prices seen in table 1 and BS implied volatilities of the firms equity in table 2. A plot of the implied volatility as a function of strikes can be seen in figure 1.

	$0.6PV$	$0.8PV$	PV	$1.2PV$
2	30.0926	22.8413	17.1531	12.7956
4	35.4864	29.7341	24.9633	21.0060
6	39.9217	35.1416	31.0398	27.5004
8	43.7923	39.7555	36.1944	33.0350

Table 1: Compounded option price: The leftmost column represents the respective values of τ, and the top row represents the respective values of the strikes K. The other cells represent the compounded option price with the given combinations of τ and K.

	$0.6PV$	$0.8PV$	PV	$1.2PV$
2	0.4807	0.4673	0.4569	0.4487
4	0.4962	0.4812	0.4697	0.4605
6	0.5153	0.4982	0.4852	0.4746
8	0.5405	0.5201	0.5045	0.4920

Table 2: Implied volatility: The leftmost column represents the respective values of τ, and the top row represents the respective values of the strikes K. The other cells represent the implied volatility of the firms equity with the given combinations of τ and K.

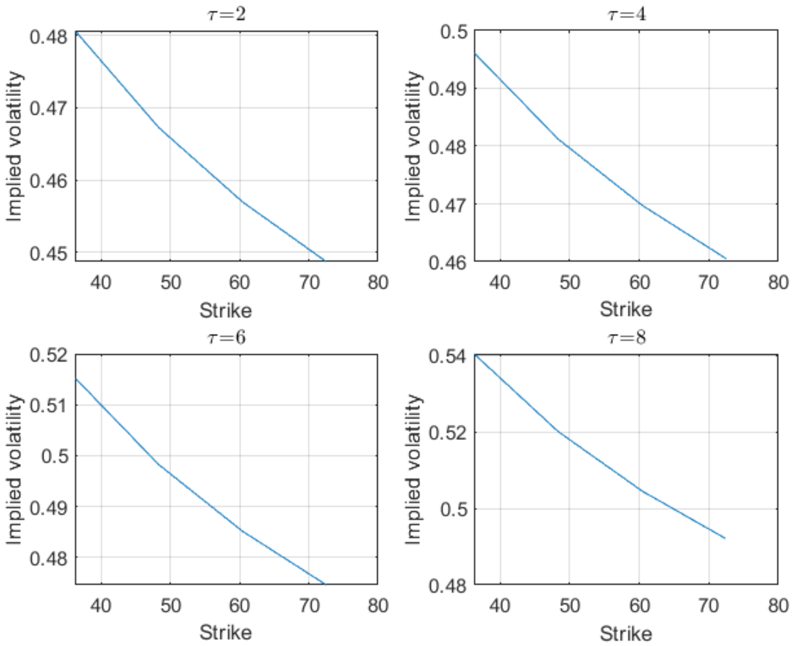

Figure 1: Implied volatility as a function of strikes for each value of τ.

Exercise 2

To solve this problem, we use the Merton Jump-Diffusion Option pricing formula

$$C = \sum_{j=0}^{\infty} P(N_T = j) BS_{call}(S(1+\gamma)^j, K, T, \sigma, r, q + \gamma\lambda^Q$$

where

$$P(N_T = j) = e^{-\lambda^G T} \frac{(\lambda^Q T)^j}{j!}.$$

Due to fast convergence, we do not need an infinite number of terms, 10 will be more than enough. We then proceed to calulate the BS implied volatility the same way as in the previous exercise, that is with the Newton-Rhapson method. This results in the IV vs strike plots seen in figure 2.

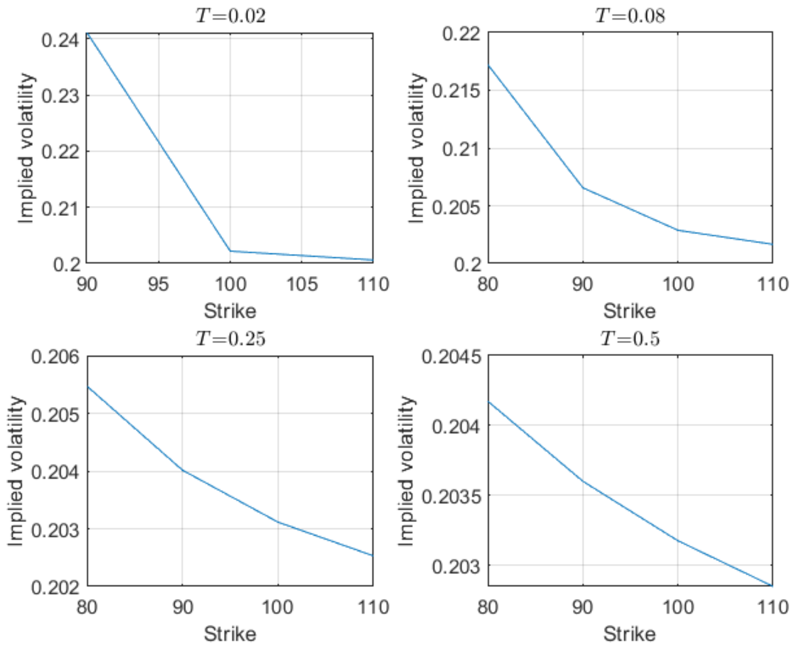

Figure 2: Implied volatility as a function of strikes for each value of T.