Problem set 5

Due on October 31, 2022, at 11:15am

Exercise 1

Consider an option on a stock that has an initial price $S_0 = 100$. The expiration is $T = 1$ (year), but the option can be exercised at four possible dates: at $t_1 = 0.25$, $t_2 = 0.5$, $t_3 = 0.75$, $t_4 = T = 1$. The payout is similar to the payout of a call option, but depends on the average of the stock price over the previous exercise dates: the payout is

$$(S(t_1) - K)^+ \qquad \text{if exercised at } t_1 \tag{1}$$

$$\frac{S(t_1) + S(t_2)}{2} - K \qquad \text{if exercised at } t_2 \tag{2}$$

$$\frac{S(t_1) + S(t_2) + S(t_3)}{3} - K \qquad \text{if exercised at } t_3 \tag{3}$$

$$\frac{S(t_1) + S(t_2) + S(t_3) + S(t_4)}{4} - K \qquad \text{if exercised at } t_4 \tag{4}$$

Use $K = 98$, $r = 0$, $q = 0.02$.

Use the Least-Square Monte Carlo method. Define the process A such that $A(t_1) = S(t_1)$, $A(t_2) = \frac{S(t_1)+S(t_2)}{2}$, $A(t_3) = \frac{S(t_1)+S(t_2)+S(t_3)}{3}$, $A(t_4) = \frac{S(t_1)+S(t_2)+S(t_3)+S(t_4)}{4}$ At each exercise date, regress the continuation value on S, S^2, S^3, A, A^2, A^3 and a constant.

- (100 points) Use a constant volatility equal to 23%. For this part, you can build your paths by simulating the stock price only at the 4 exercise dates.

- Quote your final result for the option price and its standard error.