Module 6

Branch and Bound-TSP

Branch and Bound (B& B)

- An enhancement of backtracking
  - **Similarity**
    - A state space tree is used to solve a problem.
  - **Difference**
    - The branch-and-bound algorithm does not limit us to any particular way of traversing the tree.
    - Used only for optimization problems (since the backtracking algorithm requires the using of DFS traversal and is used for non-optimization problems.)

Branch and bound

- A general algorithm for finding optimal solutions of various optimization problems.
- BFS like state space search in which the live nodes are maintained using a queue is called **FIFO**
- D-Search like state space search in which the live nodes are maintained using a stack is called **LIFO**.
- Bounding functions are used to avoid the generation of subtrees that do not contain an answer node.

Branch and Bound

- The idea:

Set up a **bounding function**, which is used to compute a **bound** (for the value of the objective function) **at a node** on a state-space tree and determine **if it is promising**

  – **Promising** (if the bound is better than the value of the best solution so far): expand beyond the node.

  – **Nonpromising** (if the bound is no better than the value of the best solution so far): not expand beyond the node (pruning the state-space tree).

Travelling Salesman Problem(TSP)

- Given a directed graph G=(V,E), the TSP problem is to find a tour or cycle that begins from a node, covers all the nodes of the graph exactly once and returns back to that node.

Traveling Salesman Problem

- Construct the state-space tree:
  - A node = a vertex: a vertex in the graph.
  - A node that is not a leaf represents all the tours that start with the path stored at that node; each leaf represents a tour (or non-promising node).
  - Branch-and-bound: we need to determine a **lower** bound for each node
    - For example, to determine a lower bound for node [1, 2] means to determine a lower bound on the length of any tour that starts with edge 1—2.
  - Expand each promising node, and stop when all the promising nodes have been expanded. During this procedure, prune all the nonpromising nodes.
    - Promising node: the node's lower bound is less than current minimum tour length.
    - Non-promising node: the node's lower bound is NO less than current minimum tour length.

- end

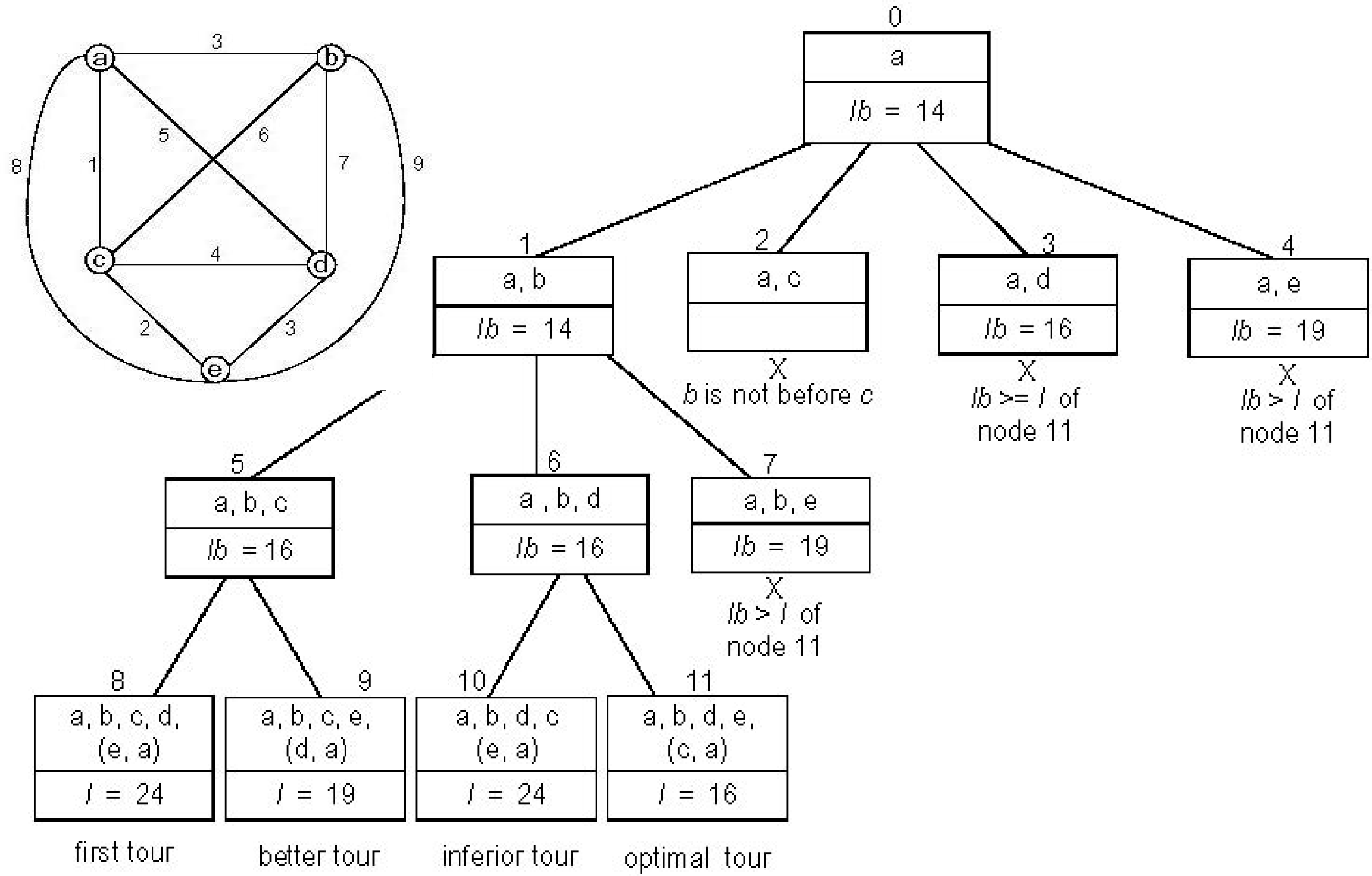
Traveling Salesman Problem—Bounding Function 1

- Because a tour must leave every vertex exactly once, **a lower bound** on the length of a tour is **b (lower bound) minimum cost of leaving every vertex**.
  - The lower bound on the cost of leaving vertex v_1 is given by the minimum of all the nonzero entries in row 1 of the adjacency matrix.
  - …
  - The lower bound on the cost of leaving vertex v_n is given by the minimum of all the nonzero entries in row n of the adjacency matrix.
- **Note**: This is not to say that there is a tour with this length. Rather, it says that there can be no shorter tour.
- Assume that the tour starts with v1.

Traveling Salesman Problem—Bounding Function 2

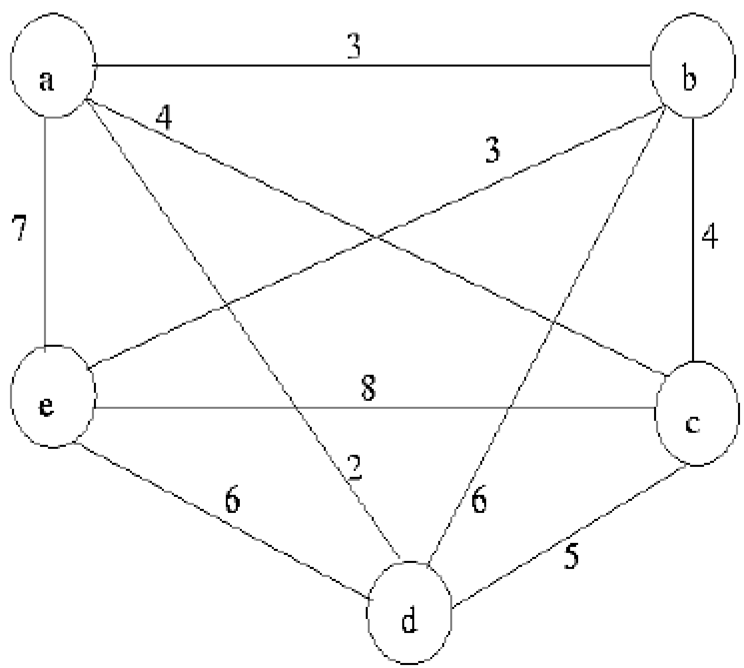
- Because every vertex must be entered and exited exactly once, a lower bound on the length of a tour is **the sum of the minimum cost of entering and leaving every vertex.**
  - For a given edge (u, v), think of half of its weight as the exiting cost of u, and half of its weight as the entering cost of v.
  - The total length of a tour = the total cost of visiting(entering and exiting) every vertex exactly once.
  - The lower bound of the length of a tour = the lower bound of the total cost of visiting (entering and exiting) every vertex exactly once.
    - Calculation:
      - for each vertex, pick top two shortest adjacent edges (their sum divided by 2 is the lower bound of the total cost of entering and exiting the vertex);
      - add up these summations for all the vertices.
- Assume that the tour starts with vertex a and that b is visited before c.
- The worst case complexity of TSP problem is $O(n^2 2^n)$.

Traveling salesman example 2



- Suppose we have a graph with 4 nodes ie, n=4 then the different possibilities is shown in the state space tree S below:

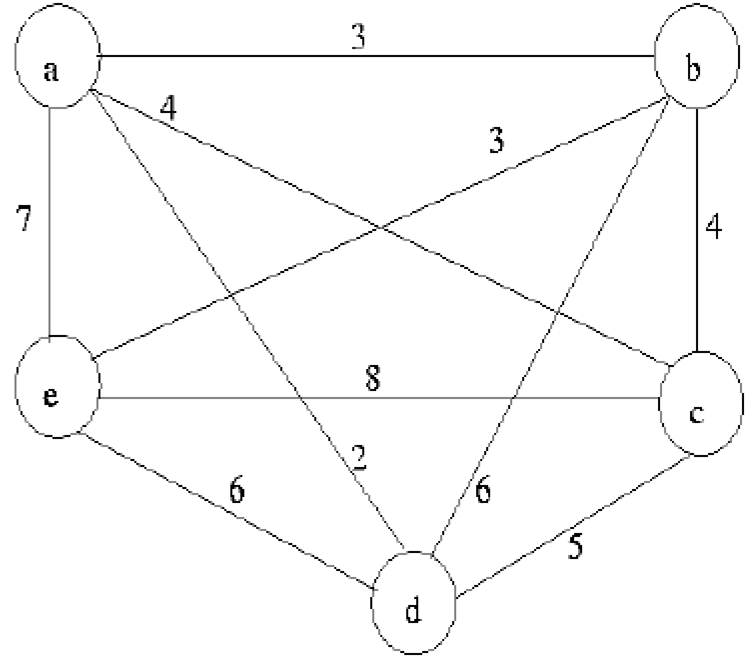
Problem

- Solve Travelling Salesman Problem using Branch and Bound Algorithm in the following graph-
- 

Step-01:

- Write the initial cost matrix and reduce it-

$$
\begin{array}{cc}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\begin{pmatrix}
\infty & 3 & 4 & 2 & 7 \\
3 & \infty & 4 & 6 & 3 \\
4 & 4 & \infty & 5 & 8 \\
2 & 6 & 5 & \infty & 6 \\
7 & 3 & 8 & 6 & \infty
\end{pmatrix}
\end{array}
$$



Rules

- To reduce a matrix, perform the row reduction and column reduction of the matrix separately.
- A row or a column is said to be reduced if it contains at least one entry ‘0’ in it.

Row Reduction-

- Consider the rows of the matrix one by one.

$$
\begin{array}{c|ccccc}
 & A & B & C & D & E \\
\hline
A & \infty & 3 & 4 & 2 & 7 \\
B & 3 & \infty & 4 & 6 & 3 \\
C & 4 & 4 & \infty & 5 & 8 \\
D & 2 & 6 & 5 & \infty & 6 \\
E & 7 & 3 & 8 & 6 & \infty
\end{array}
$$

- If the row already contains an entry ‘0’, then-There is no need to reduce that row.
- If the row does not contains an entry ‘0’, then- Reduce that particular row.
- Select the least value element from that row.
- Subtract that element from each element of that row.
- This will create an entry ‘0’ in that row, thus reducing that row.

Column Reduction-

- Consider the columns of above row-reduced matrix one by one.
  - If the column already contains an entry ‘0’, then- There is no need to reduce that column.
  - If the column does not contains an entry ‘0’, then-Reduce that particular column.
- Select the least value element from that column.
- Subtract that element from each element of that column.
- This will create an entry ‘0’ in that column, thus reducing that column.

- Following this, we have-
- Reduce the elements of row-1 by 2.
- Reduce the elements of row-2 by 3.
- Reduce the elements of row-3 by 4.
- Reduce the elements of row-4 by 2.
- Reduce the elements of row-5 by 3.

- There is no need to reduce column-1.
- There is no need to reduce column-2.
- Reduce the elements of column-3 by 1.
- There is no need to reduce column-4.
- There is no need to reduce column-5.

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{pmatrix}
\infty & 3 & 4 & 2 & 7 \\
3 & \infty & 4 & 6 & 3 \\
4 & 4 & \infty & 5 & 8 \\
2 & 6 & 5 & \infty & 6 \\
7 & 3 & 8 & 6 & \infty
\end{pmatrix}
\end{array}
\begin{array}{c}
\\ 2 \\ 3 \\ 4 \\ 2 \\ 3
\end{array}
\xrightarrow[\text{reduction}]{\text{After row}}
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ \\
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{pmatrix}
\infty & 1 & 2 & 0 & 5 \\
0 & \infty & 1 & 3 & 0 \\
0 & 0 & \infty & 1 & 4 \\
0 & 4 & 3 & \infty & 4 \\
4 & 0 & 5 & 3 & \infty
\end{pmatrix} \\
\begin{array}{ccccc} & & 1 & & \end{array}
\end{array}
$$

After column reduction

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{pmatrix}
\infty & 1 & 1 & 0 & 5 \\
0 & \infty & 0 & 3 & 0 \\
0 & 0 & \infty & 1 & 4 \\
0 & 4 & 2 & \infty & 4 \\
4 & 0 & 4 & 3 & \infty
\end{pmatrix}
\end{array}
$$

- Performing this,

we obtain the following

row – column reduced matrix-

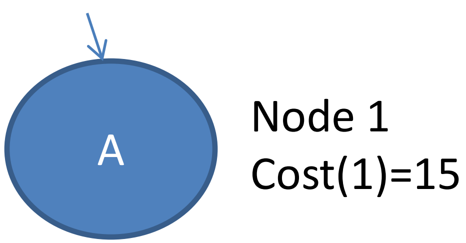
A to D

- Finally, the initial distance matrix is completely reduced.
- Now, we calculate the cost of node-1(A) by adding all the reduction elements.

- Cost(1)= Sum of all reduction elements

  = 2+3+4 + 2 + 3 + 1

  = 15



Step-02:

- We consider all other vertices one by one.
- We select the best vertex where we can land upon to minimize the tour cost.

Choosing To Go To Vertex-B: Node-2 (Path A → B)

- From the reduced matrix of step-01, M[A,B] = 1
- Set row-A and column-B to ∞
- Set M[B,A] = ∞

Now, resulting cost matrix is-

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} \infty & 1 & 1 & 0 & 5 \\ 0 & \infty & 0 & 3 & 0 \\ 0 & 0 & \infty & 1 & 4 \\ 0 & 4 & 2 & \infty & 4 \\ 4 & 0 & 4 & 3 & \infty \end{pmatrix} \end{array}
\qquad
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 3 & 0 \\ 0 & \infty & \infty & 1 & 4 \\ 0 & \infty & 2 & \infty & 4 \\ 4 & \infty & 4 & 3 & \infty \end{pmatrix} \end{array}
$$

- Now, We reduce this matrix.
- Then, we find out the cost of node-02.

Row Reduction-

- We can not reduce row-1 as all its elements are ∞.
- There is no need to reduce row-2.
- There is no need to reduce row-3.
- There is no need to reduce row-4.
- Reduce the elements of row-5 by 3
- Performing this,
  we obtain the row-reduced matrix

$$\begin{array}{ccc} & \begin{array}{ccccc} A & B & C & D & E \end{array} & \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 3 & 0 \\ 0 & \infty & \infty & 1 & 4 \\ 0 & \infty & 2 & \infty & 4 \\ 4 & \infty & 4 & 3 & \infty \end{pmatrix} & \begin{array}{c} \\ \\ \\ \\ 3 \end{array} \end{array} \xrightarrow[\text{reduction}]{\text{After row}} \begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 3 & 0 \\ 0 & \infty & \infty & 1 & 4 \\ 0 & \infty & 2 & \infty & 4 \\ 1 & \infty & 1 & 0 & \infty \end{pmatrix} \end{array}$$

Column Reduction-

- There is no need to reduce column-1.
- We can not reduce column-2 as
  all its elements are ∞.
- There is no need to reduce column-3.
- There is no need to reduce column-4.
- There is no need to reduce column-5
- Performing this,
  we obtain the column-reduced matrix

After column reduction

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 3 & 0 \\ 0 & \infty & \infty & 1 & 4 \\ 0 & \infty & 2 & \infty & 4 \\ 1 & \infty & 1 & 0 & \infty \end{pmatrix} \end{array}$$

- Finally, the matrix is completely reduced.
- Now, we calculate the cost of node-2.

Cost(2)= Cost(1) + Sum of reduction elements + M[A,B]

= 15 + 3 + 1

= 19

- **Choosing To Go To Vertex-C: Node-3 (Path A → C)**

- From the reduced matrix of step-01, M[A,C] = 1
- Set row-A and column-C to ∞
- Set M[C,A] = ∞
- Now, resulting cost matrix is-

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 1 & 1 & 0 & 5 \\ B & 0 & \infty & 0 & 3 & 0 \\ C & 0 & 0 & \infty & 1 & 4 \\ D & 0 & 4 & 2 & \infty & 4 \\ E & 4 & 0 & 4 & 3 & \infty \end{array}$$

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \infty & \infty & \infty & \infty \\ B & 0 & \infty & \infty & 3 & 0 \\ C & \infty & 0 & \infty & 1 & 4 \\ D & 0 & 4 & \infty & \infty & 4 \\ E & 4 & 0 & \infty & 3 & \infty \end{array}$$

- Now,We reduce this matrix.
- Then, we find out the cost of node-03.

Row Reduction-

- We can not reduce row-1 as all its elements are ∞.
- There is no need to reduce row-2.

  row-3, row-4 and row-5.
- Thus, the matrix is already row-reduced.

Column Reduction-

- There is no need to reduce column-1 and column-2.
- We can not reduce column-3 as all its elements are ∞.
- There is no need to reduce column-4 and column 5.

- Thus, the matrix is already column reduced.
- Finally, the matrix is completely reduced.

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 3 & 0 \\ \infty & 0 & \infty & 1 & 4 \\ 0 & 4 & \infty & \infty & 4 \\ 4 & 0 & \infty & 3 & \infty \end{pmatrix} \end{array} \xrightarrow[\text{reduction}]{\text{After row}} \begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 3 & 0 \\ \infty & 0 & \infty & 1 & 4 \\ 0 & 4 & \infty & \infty & 4 \\ 4 & 0 & \infty & 3 & \infty \end{pmatrix} \\ & \begin{array}{ccccc} & & & 1 & \end{array} \end{array}$$

$$\Big\downarrow \text{After column reduction}$$

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & 0 \\ \infty & 0 & \infty & 0 & 4 \\ 0 & 4 & \infty & \infty & 4 \\ 4 & 0 & \infty & 2 & \infty \end{pmatrix} \end{array}$$

- Now, we calculate the cost of node-3.
- Cost(3) = Cost(1) + Sum of reduction elements + M[A,C]

  = 15 + 1 + 1

  = 17

Choosing To Go To Vertex-D: Node-4 (Path A → D)

- From the reduced matrix of step-01, M[A,D] = 0
- Set row-A and column-D to ∞
- Set M[D,A] = ∞
- Now, resulting cost matrix is-

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} \infty & 1 & 1 & 0 & 5 \\ 0 & \infty & 0 & 3 & 0 \\ 0 & 0 & \infty & 1 & 4 \\ 0 & 4 & 2 & \infty & 4 \\ 4 & 0 & 4 & 3 & \infty \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & \infty & 0 \\ 0 & 0 & \infty & \infty & 4 \\ \infty & 4 & 2 & \infty & 4 \\ 4 & 0 & 4 & \infty & \infty \end{pmatrix} \end{array}$$

- Now, We reduce this matrix.
- Then, we find out the cost of node-04.

Row Reduction-

- We can not reduce row-1 as all its elements
- There is no need to reduce row-2,row 3.
- Reduce all the elements of row-4 by 2.
- There is no need to reduce row-5.
- Performing this, we obtain the row-reduced

Column Reduction-

- There is no need to reduce column-1.
- There is no need to reduce column-2.
- There is no need to reduce column-3 and co
- We can not reduce column-4 as all its elements are ∞.
- Thus, the matrix is already column-reduced
- Finally, the matrix is completely reduced.

$$
\begin{array}{c}
 \\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\left(\begin{array}{ccccc}
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & 0 & \infty & 0 \\
0 & 0 & \infty & \infty & 4 \\
\infty & 4 & 2 & \infty & 4 \\
4 & 0 & 4 & \infty & \infty
\end{array}\right)
\end{array}
\begin{array}{c}
 \\ \\ \\ \\ 2 \\ \\
\end{array}
\xrightarrow[\text{reduction}]{\text{After row}}
\begin{array}{c}
 \\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\left(\begin{array}{ccccc}
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & 0 & \infty & 0 \\
0 & 0 & \infty & \infty & 4 \\
\infty & 2 & 0 & \infty & 2 \\
4 & 0 & 4 & \infty & \infty
\end{array}\right)
\end{array}
$$

After column reduction

$$
\begin{array}{c}
 \\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\left(\begin{array}{ccccc}
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & 0 & \infty & 0 \\
0 & 0 & \infty & \infty & 4 \\
\infty & 2 & 0 & \infty & 2 \\
4 & 0 & 4 & \infty & \infty
\end{array}\right)
\end{array}
$$

- Now, we calculate the cost of node-4.

Cost(4)= Cost(1) + Sum of reduction elements
+ M[A,D]
= 15 + 2 + 0
= 17

Choosing To Go To Vertex-D: Node-5 (Path A → E)

- From the reduced matrix of step-01, M[A,E] = 5
- Set row-A and column-E to ∞
- Set M[E,A] = ∞
- Now, resulting cost matrix is-

$$
\begin{array}{c c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left(\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & 3 & \infty \\ 0 & 0 & \infty & 1 & \infty \\ 0 & 4 & 2 & \infty & \infty \\ \infty & 0 & 4 & 3 & \infty \end{array}\right) \\
 & \begin{array}{ccccc} & & & 1 & \end{array}
\end{array}
\xrightarrow[\text{and column reduction}]{\text{After row}}
\begin{array}{c c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left(\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & 2 & \infty \\ 0 & 0 & \infty & 0 & \infty \\ 0 & 4 & 2 & \infty & \infty \\ \infty & 0 & 4 & 2 & \infty \end{array}\right)
\end{array}
$$

- Now, We reduce this matrix.
- Then, we find out the cost of node-05.

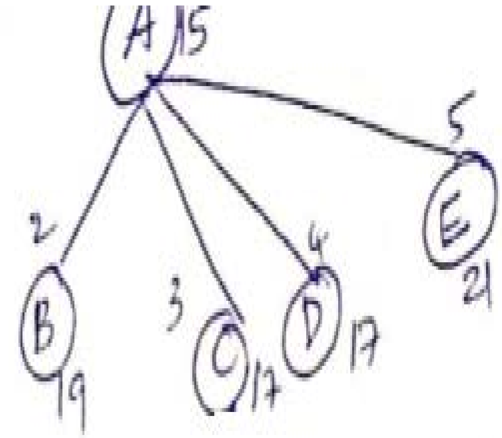
Cost(4)= Cost(1) + Sum of reduction elements

+ M[A,E]

= 15 + 1 +5 = 21

- **Thus, we have-**

- Cost(2) = 19 (for Path A → B)
- Cost(3) = 17 (for Path A → C)
- Cost(4) = 17 (for Path A → D)
- Cost(5) = 21 (for Path A → E)



- We choose the node with the lowest cost.
- Since cost for node-3 is lowest, so we prefer to visit node-3.
- Thus, we choose node-3 i.e. path **A → C**.

Step-03:

- We explore the vertices B,D and E from node-3.
- We now start from the cost matrix at node-3 which is-

Cost(3) = 17

Choosing To Go To Vertex-B: Node-6 (Path A → C → B)

- From the reduced matrix of step-02, M[C,B] = 0
- Set row-C and column-B to ∞
- Set M[B,A] = ∞
- Now, resulting cost matrix is-

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 4 \\ 4 & \infty & \infty & 2 & \infty \end{pmatrix} 2 \end{array}
\xrightarrow[\text{reduction}]{\text{After row and column}}
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 4 \\ 2 & \infty & \infty & 0 & \infty \end{pmatrix} \end{array}
$$

- Now, We reduce this matrix.
- Then, we find out the cost of node-6.

- Perform row and column reduction, the matrix is completely reduced.

$$\begin{array}{ccc} & \begin{array}{ccccc} A & B & C & D & E \end{array} & \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 4 \\ 4 & \infty & \infty & 2 & \infty \end{pmatrix} & \begin{array}{c} \\ \\ \\ \\ 2 \end{array} \end{array} \xrightarrow[\text{and column reduction}]{\text{After row}} \begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 4 \\ 2 & \infty & \infty & 0 & \infty \end{pmatrix} \end{array}$$

- Now, we calculate the cost of node-5.

Cost(6)= cost(3) + Sum of reduction elements + M[C,B]

= 17 + 2 + 0

= 19

Choosing To Go To Vertex-D: Node-7 (Path A → C → D)

- From the reduced matrix of step-02, M[C,D] = 0
- Set row-C and column-D to ∞
- Set M[D,A] = ∞

- Now, resulting cost matrix is-

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\left(\begin{array}{ccccc}
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & \infty & \infty & 0 \\
\infty & \infty & \infty & \infty & \infty \\
\infty & 4 & \infty & \infty & 4 \\
4 & 0 & \infty & \infty & \infty
\end{array}\right)
\end{array}
\begin{array}{c} \\ \\ \\ \\ 4 \\ \\ \end{array}
\xrightarrow[\text{and column reduction}]{\text{After row}}
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\left(\begin{array}{ccccc}
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & \infty & \infty & 0 \\
\infty & \infty & \infty & \infty & \infty \\
\infty & 0 & \infty & \infty & 0 \\
4 & 0 & \infty & \infty & \infty
\end{array}\right)
\end{array}
$$

- We reduce this matrix.
- Then, we find out the cost of node-7
- Cost(7)= cost(3) + Sum of reduction elements + M[C,D]

= 17 + 4 + 0

= 21

Choosing To Go To Vertex-E: Node-8 (Path A → C → E)

- From the reduced matrix of step-02, M[C,E] = 4
- Set row-C and column-D to ∞
- Set M[E,A] = ∞

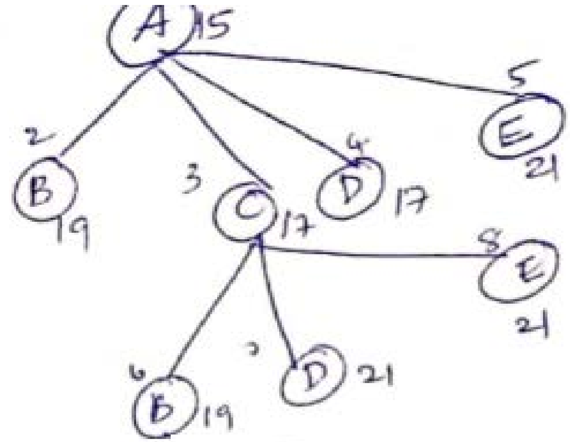
- Now, resulting cost matrix is-

$$
\begin{array}{c}
\begin{array}{c|c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left(\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 4 & \infty & \infty & \infty \\ \infty & 0 & \infty & 2 & \infty \end{array}\right) \\
 & \begin{array}{ccccc} & & & \;\;2 & \end{array}
\end{array}
\end{array}
\xrightarrow[\text{and column reduction}]{\text{After row}}
\begin{array}{c|c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left(\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 4 & \infty & \infty & \infty \\ \infty & 0 & \infty & 0 & \infty \end{array}\right)
\end{array}
$$

- We reduce this matrix.
- Then, we find out the cost of node-8
- Cost(8)= cost(3) + Sum of reduction elements + M[C,E]

= 17 + 2 + 4

= 23



- **Thus, we have-**
- Cost(6) =19 (for Path A → C → B)
- Cost(7) = 21 (for Path A → C → D)
- Cost(8) = 23 (for Path A → C → E)
- We choose the node with the lowest cost.
- Since cost for node-6 is lowest, so we prefer to visit node-6.
- Thus, we choose node-6 i.e. path **C → B**.

Step-04:

- We explore vertex B from node-6.
- We start with the cost matrix at node-6 which is-**Cost(6) = 25**

Choosing To Go To Vertex-B: Node-9 (Path A → C → B→ D)

- From the reduced matrix of step-03, M[B,D] = 2
- Set row-B and column-D to ∞
- Set M[D,A] = ∞
-

$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left(\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 4 \\ 2 & \infty & \infty & 0 & \infty \end{array}\right) \end{array} \longrightarrow \begin{array}{c c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} & \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left(\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 4 \\ 2 & \infty & \infty & \infty & \infty \end{array}\right) & \begin{array}{c} \\ \\ \\ 4 \\ 2 \end{array} \end{array}$$

$$\left(\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ 0 & \infty & \infty & \infty & \infty \end{array}\right)$$

- reduce this matrix,Then, we find out the cost of node-9

Cost(9)= cost(6) + Sum of reduction elements + M[B,D]

= 19 + 6 + 2 = 27

Choosing To Go To Vertex-B: Node-10 (Path A → C → B→E)

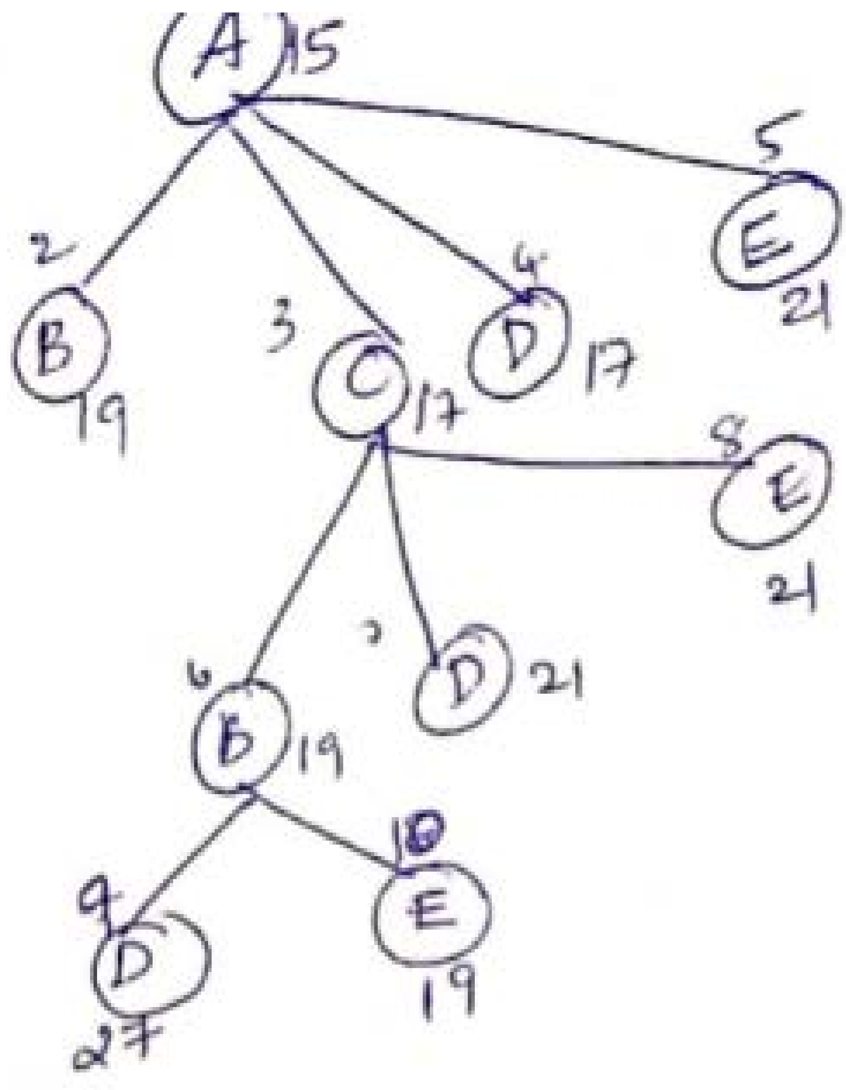
- From the reduced matrix of step-03, M[B,E] = 0
- Set row-B and column-E to ∞
- Set M[E,A] = ∞
-

$$
\begin{array}{cc}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\begin{pmatrix}
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & \infty & 2 & 0 \\
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & \infty & \infty & 4 \\
2 & \infty & \infty & 0 & \infty
\end{pmatrix}
\end{array}
\longrightarrow
\begin{array}{cc}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\begin{pmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & 0 & \infty
\end{pmatrix}
\end{array}
$$

- reduce this matrix,Then, we find out the cost of node-10

Cost(10)= cost(6) + Sum of reduction elements + M[B,E]

= 19 +0 +0 = 19



A 15
2
B 19
3
C 17
4
D 17
5
E 21
8
E 21
D 21
6
B 19
9
D 27
10
E 19

- **Thus, we have-**
- Cost(9) =27 (for Path A → C → B → D)
- Cost(10) = 19 (for Path A → C → B → E)
- we choose the node with the lowest cost.
- Since cost for node-10 is lowest, so we prefer to visit node-10.
- Thus, we choose node-10 i.e. path B **→ E**.

Step-05:

Choosing To Go To Vertex-B: Node-11 (Path A → C → B→E →D)

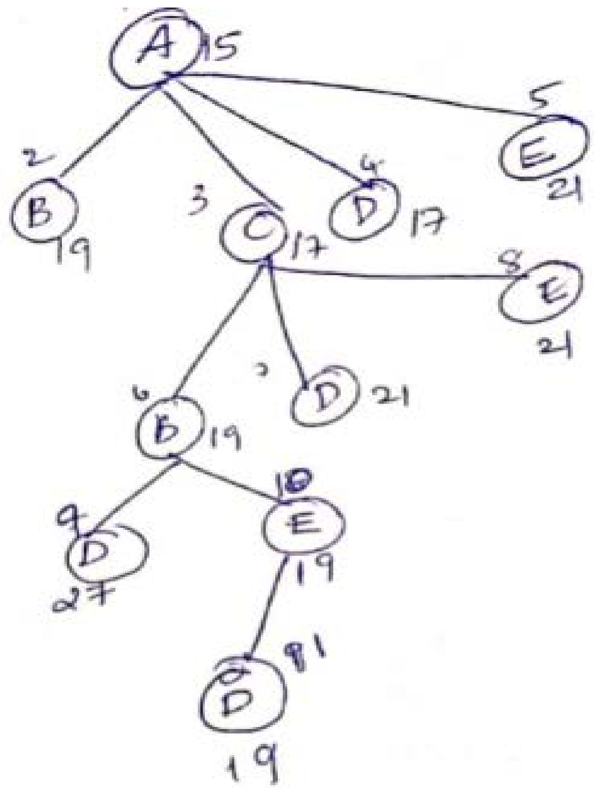
- From the reduced matrix of step-03, M[E,D] = 0
- Set row-E and column-D to ∞
- Set M[D,A] = ∞
-

$$
\begin{array}{c}
 \\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{pmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & 0 & \infty
\end{pmatrix}
\end{array}
\longrightarrow
\begin{array}{c}
 \\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{pmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty
\end{pmatrix}
\end{array}
$$

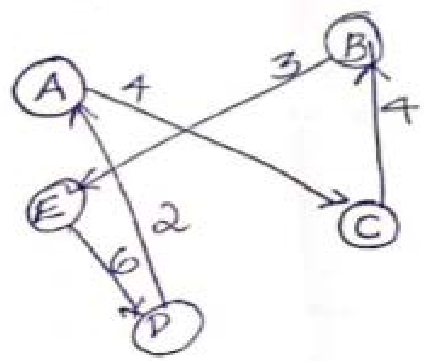
- Finally, the matrix is completely reduced.
- All the entries have become ∞.
- Now, we calculate the cost of node-11

Cost(11)= cost(10) + Sum of reduction elements + M[E,D]

= 19 +0 +0 = 19



Path is A — C — B — E — D — A



Total cost = 19

- Thus,
- Optimal path is: **A → C → B → E → D → A**
- Cost of Optimal path = **19**

- END