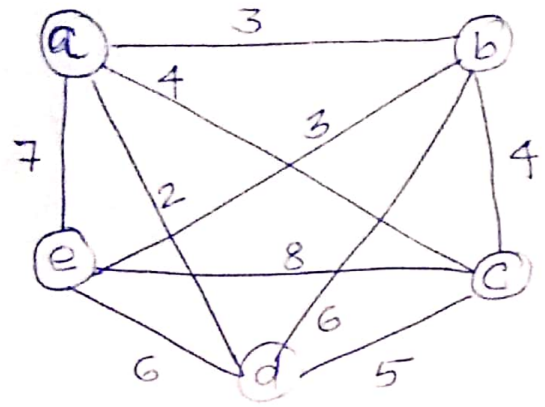


Solution

Step 1 : Write the initial cost matrix and reduce it

	A	B	C	D	E
A	∞	3	4	2	7
B	3	∞	4	6	3
C	4	4	∞	5	8
D	2	6	5	∞	6
E	7	3	8	6	∞

Reduce it

$$\begin{bmatrix} \infty & 3 & 4 & 2 & 7 \\ 3 & \infty & 4 & 6 & 3 \\ 4 & 4 & \infty & 5 & 8 \\ 2 & 6 & 5 & \infty & 6 \\ 7 & 3 & 8 & 6 & \infty \end{bmatrix} \begin{matrix} 2 \\ 3 \\ 4 \\ 2 \\ 3 \end{matrix}$$

After row reduction →

$$\begin{bmatrix} \infty & 1 & 2 & 0 & 5 \\ 0 & \infty & 1 & 3 & 0 \\ 0 & 0 & \infty & 1 & 4 \\ 0 & 4 & 3 & \infty & 4 \\ 4 & 0 & 5 & 3 & \infty \end{bmatrix} \begin{matrix} 2 \\ 3 \\ 4 \\ 2 \\ 3 \end{matrix}$$

$$\begin{matrix} - & - & 1 & + & - \end{matrix}$$

⇓ after column reduction

$$\begin{bmatrix} \infty & 1 & 1 & 0 & 5 \\ 0 & \infty & 0 & 3 & 0 \\ 0 & 0 & \infty & 1 & 4 \\ 0 & 4 & 2 & \infty & 4 \\ 4 & 0 & 4 & 3 & \infty \end{bmatrix}$$

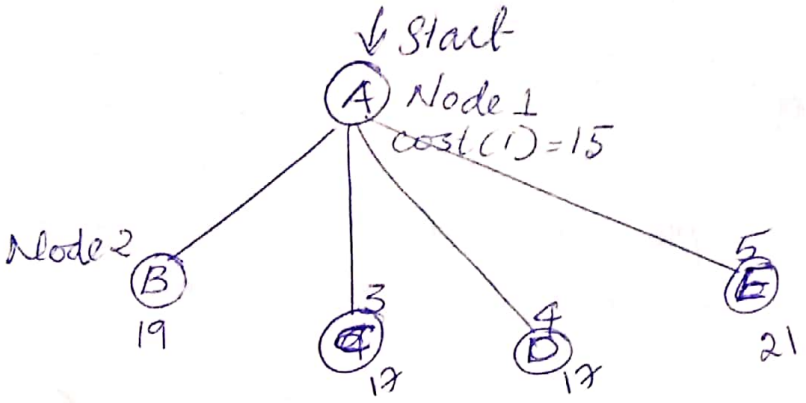
Reduced matrix

Row reduction

row wise reduction

if any row consist of '0' no reduction required, else reduce it

cost of node1 (A) is, cost(1) = 2+3+7+2+3+1
= 15



Step 2 : Choosing to goto vertex B : Node 2 (path A → B)

- From the reduced matrix of step 1, M[A,B] = 1
- Set row A and column B to ∞
- Set M[B,A] = ∞
- Now, resulting cost matrix is

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 3 & 0 \\ 0 & \infty & \infty & 1 & 4 \\ 0 & \infty & 2 & \infty & 4 \\ 4 & \infty & 4 & 3 & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ - \\ 3 \end{matrix}$$

After row reduction →

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 3 & 0 \\ 0 & \infty & \infty & 1 & 4 \\ 0 & \infty & 2 & \infty & 4 \\ 1 & \infty & 1 & 0 & \infty \end{bmatrix}$$
$$- \quad - \quad - \quad - \quad -$$

After column reduction ⇒ same matrix

Thus cost of node 2 is

cost(2) = cost(1) + Reduction + M[A,B]
= 15 + 3 + 1
= 19

* Choosing to go to vertex C : Node 3 (path A → C)

- From reduced matrix of step 1, M[A,C] = 1
- Set row A & column C to ∞
- Set M[C,A] = ∞

- Now resulting cost matrix is

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 3 & 0 \\ \infty & 0 & \infty & 1 & 4 \\ 0 & 4 & \infty & \infty & 4 \\ 4 & 0 & \infty & 3 & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ - \\ - \end{matrix} \xrightarrow{\text{after row \& column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 2 & 0 \\ \infty & 0 & \infty & 0 & 4 \\ 0 & 4 & \infty & \infty & 4 \\ 4 & 0 & \infty & 2 & \infty \end{bmatrix}$$

$$\begin{matrix} - & - & - & 1 & - \end{matrix}$$

$$\text{cost}(2) = \text{cost}(1) + \text{Reduction} + M[A,C]$$

$$= 15 + 1 + 1 = 17$$

Choosing to go to vertex D : Node 4 (Path A → D)

* From reduced matrix of step 1, $M[A,D] = 0$
* set row A and column D to ∞
* set $M[D,A] = \infty$
* resulting cost matrix is

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & \infty & 0 \\ 0 & 0 & \infty & \infty & 4 \\ \infty & 0 & \infty & \infty & 4 \\ 4 & 0 & 4 & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ 2 \\ - \end{matrix} \Longrightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & \infty & 0 \\ 0 & 0 & \infty & \infty & 4 \\ \infty & 2 & 0 & \infty & 2 \\ 4 & 0 & 4 & \infty & \infty \end{bmatrix}$$

$$\begin{matrix} - & - & - & - & - \end{matrix}$$

$$\text{cost}(4) = \text{cost}(1) + \text{Reduction} + M[A,D] = 15 + 2 + 0 = 17$$

- Choosing to go to vertex E : Node 5 (Path A → E)

* $M[A,E] = 5$
* row A = column E = ∞
* $M[E,A] = \infty$

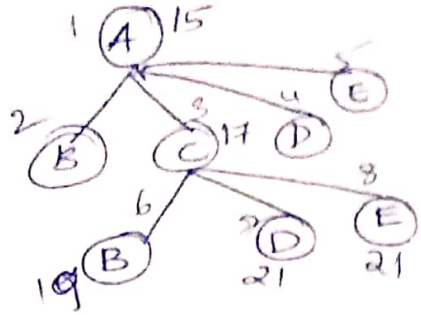
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & 3 & \infty \\ 0 & 0 & \infty & 1 & \infty \\ 0 & 4 & 2 & \infty & \infty \\ \infty & 0 & 4 & 3 & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ - \\ - \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & 2 & \infty \\ 0 & 0 & \infty & 0 & \infty \\ 0 & 4 & 2 & \infty & \infty \\ \infty & 0 & 4 & 2 & \infty \end{bmatrix}$$

$$\begin{matrix} \infty & - & - & 1 & - \end{matrix}$$

$$\text{cost}(5) = \text{cost}(1) + \text{Reduction} + M[A,E]$$

$$= 15 + 1 + 5$$

$$= 21$$



Thus we have
cost (2) 19
cost (3) - 17
cost (4) - 17
cost (15) - 21
we choose the node with lowest cost
we choose nodes in path A→C

Step 3 Now we will explore the vertices B, D & E from Node 3
- choose to go to vertex B (path A → C → B)

- from reduced matrix of step 2 : M[C, B] = ∞
- set row C & column B to ∞
- set M[B, A] to ∞
- Now the resulting cost matrix is

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 4 \\ 4 & \infty & \infty & 2 & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ - \\ 2 \end{matrix}$$

cost (6) = cost (3) + Reduction M[C, B]
= 17 + 2 + 0
= 19

This matrix is already row & column reduced

- choose to goto vertex D (A → C → D)

M[C, D] = D
row [C & column D = ∞
M[D, A] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 4 & 2 & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ 4 \\ D \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & 2 \\ 4 & 0 & \infty & \infty & \infty \end{bmatrix}$$

cost (7) = cost (3) + Reduction M[C, D]
= 17 + 4 + 0
= 21

choose to go to vertex E (A → C → E)

M[C, E] = 4

row of C & C of E = ∞

M[E, A] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & 0 & \infty \end{bmatrix} \rightarrow$$

cost (8) = cost (3) + reduction + M[C, E]

= 17 + 0 + 4 = 21

- Choose to go to vertex B (A → C → B → D)

M[D → B] = ∞

so cost (8) = ∞

- choose to go to vertex E (A → C → D → E)

M[D, E] = 2

row D column E → ∞

M[E, A] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} = \text{cost}(9) = \text{cost}(7) + \text{red} + M[D,E]$$

= 21 + 0 + 2

= 25

Choose to goto vertex B (A → C → B → D)

M[B, D] = 2

$r_B = C_D = \infty$

M[D, A] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 4 \\ 2 & \infty & \infty & \infty & \infty \end{bmatrix}$$

2

cost (9) = cost (6) + reduction + M[B, D]

= 19 + 6 + 2

= 27

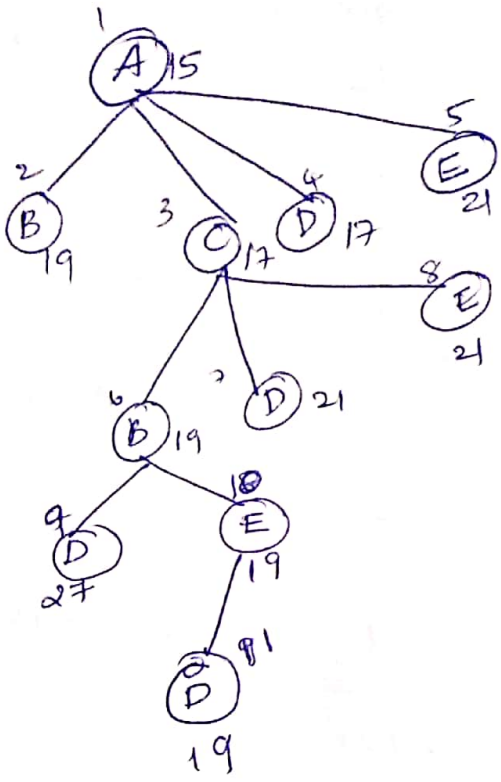
Choose to goto vertex E (A → C → B → E)

M[B, E] = 0 $r_B + C_E = \infty$

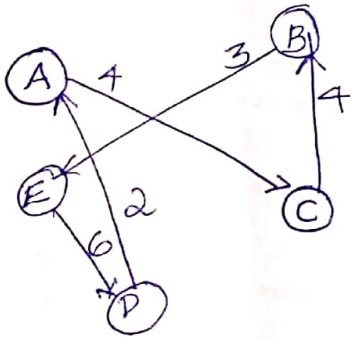
M[E, A] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & 0 & \infty \end{bmatrix}$$

cost (10) = 19 + 0 + 0 = 19



Path is A — C — B — E — D — A



Total cost = 19

Choose to go to vertex D (A → C → B → E → D)

M[E, D] = 0

M[D, A] = ∞

Y_E & C_D = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Cost (9) = Cost (10) + reduction + M(E, D) =

= 19 + 0 + 0 = 19