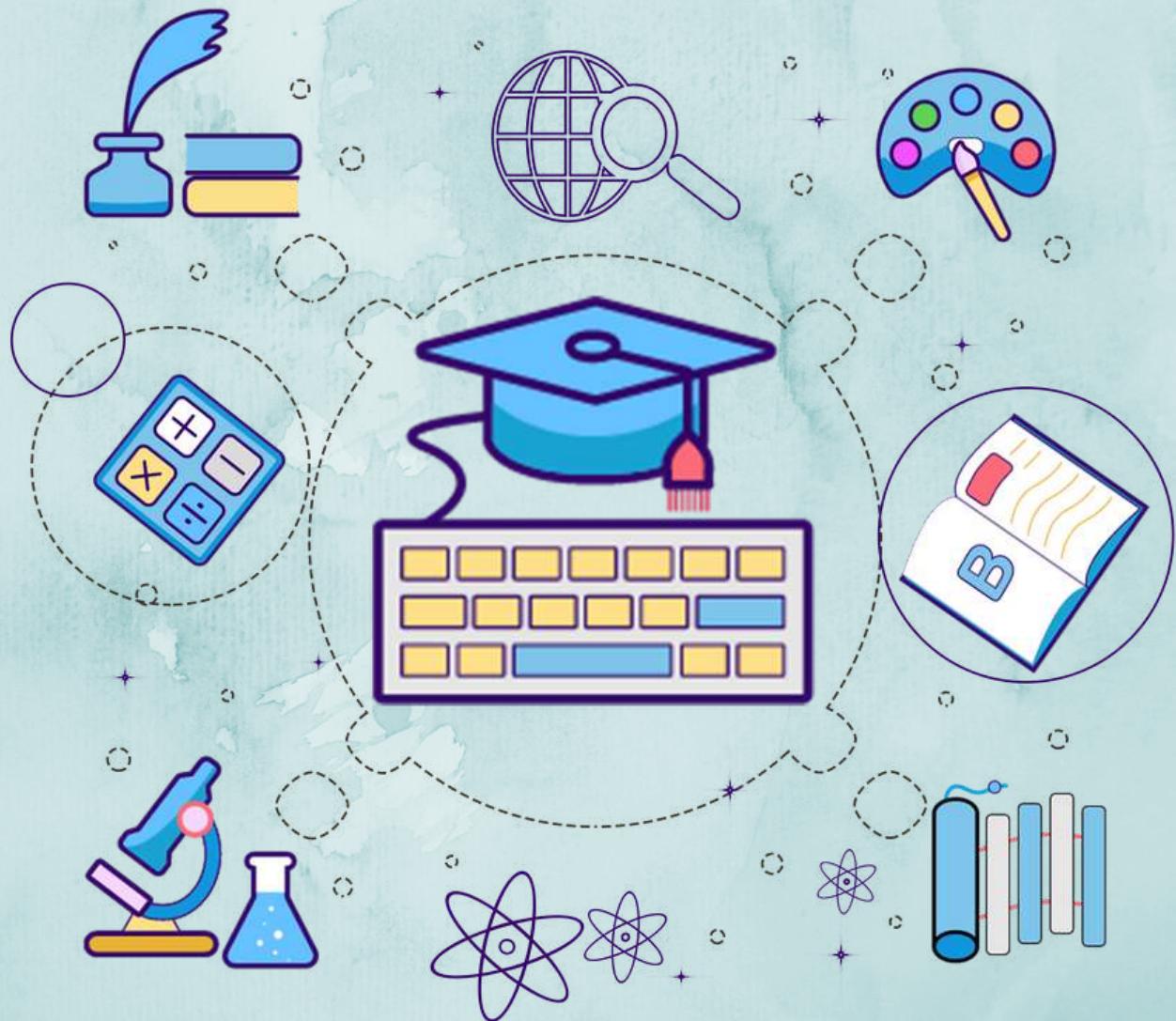


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CHAPTER – 1

OVER VIEW

Concepts & Definitions

- **Language Processor** : Computers, Interpreter, Assembler, normally an integrated product development environment
- **Phase of a compiler** : Using the divide & conquer strategy we break up the compiler into phases. Each phase transforms the source program into an intermediate form suitable for the next phase.

Phase	Intermediate form	Next Phase
1. Develop Program	Source program	Scanner
2. Scanner / Lexical Analyzer	Tokens	Syntax Analyzer
3. Syntax Analyzer	Syntax Tree	Semantic Analysis
4. Semantic Analyzer	Intermediate Forms (ex: quadruples)	Code Optimization
5. Code Optimization	Object Code	Linking, Loading & Execution

- **Machine Language** : First generation language programming done in binary.
- **Assembly Language**: Second Generation languages names or mnemonics given to numbers
- **High level Language**: Close to natural languages
- **Models in LP**: Automata, grammar, regular expressions, trees etc;
- **Code optimization**: undecidable in theory. In practice code as good as that written by an expert human programmer is produced.
- **Block structure**: Nesting & Blocks & naming conventions with scope rules
- **Parameter Parses**: Actual formal parameter correspondences
- **Aliasing** : Two or more names for the same object.
- **Grammar**: Define Hierarchical Structures uses terminal and meta symbols called non-terminals has a finite set of productions or rewriting rules.
- **Attributes** : Each rule of a grammar has an attribute

- **Lexical Analyzer:** Translates the source program to tokens isolates the lexemes in the source program.
- **Parsing:** Syntax or form is checked by the rules of a grammar, Normally a derivation tree is constructed.
- **Predictive parser :** The flow of parsing is unambiguous by looking at one symbol ahead.
- **Synthesized Attributes:** Deal with semantic analysis with bottom up parsing.
- **Intermediate code:** Abstract parsing Syntax tree or three address codes can be used
- **Symbol table:** The heart of the compiler
- **Concept :** A compiler can be considered to be a file conversion program
- **Concept:** The target code may be code in a HLL like on software conversion projects.
- **Boot strapping:** Compiler that generates newer versions of themselves.
- A good compiler generates correct, truthful code.
- A good compiler conforms to the source language standard
- A good compiler can handle program of any size
- A good compiler does not have quadratic or worse algorithms
- A compiler that can run on different platforms is called portable
- A compiler that produces target code for different platforms is re-targetable
- Target code optimizations are dangerous but useful
- Lot of compiler work is automatic generations from formalisms
- A context free grammar recovers the structure a linear programs hides.

GRAMMARS

- **Grammars :** Finite set of syntax rules with which we can define infinite number of sentences :

Ex: $S \rightarrow a S / S a / a$

Three simple rules define infinite strings like a,aa,aaa,.....

- Grammars can be classified into Ambiguous and Unambiguous grammars based on the number of parse trees that exist for a given input string.

- **Ambiguous grammar:** For a given input string if there exist more than one parse tree, then the grammar is called ambiguous grammar.

Exc: G: $S \rightarrow a S / S a / a$ is ambiguous for input string w= “aaa”

- There is no algorithm that converts an ambiguous grammar to unambiguous grammar.
- Grammars can be classified into two types.
 1. Left recursive
 2. Right recursive – based on the way the productions are defined

- **Left Recursive Grammar:** In a grammar, in any rule, if L.H.S non-terminal is same as left most non-terminal on R.H.S, then such a grammar is called left recursive grammar.

Exc: $A \rightarrow A\alpha / \beta$

- Problem with left recursive grammar is that if such a grammar is used by parser that uses LMD in deriving a string, then there is a danger of going into an infinite loop.
- The eliminate left recursion: For each rule of the form $A \rightarrow A\alpha / \beta$ rewrite as

$$A \rightarrow \beta A^1, A^1 \rightarrow \alpha A^1 / \epsilon$$

- This can be generalized as for each rule

$$A \rightarrow A\alpha_1, A\alpha_2 \dots / \beta_1 / \beta_2 \dots$$

Solution grammar which is free of left recursion is

$$A \rightarrow \beta_1 A^1 / \beta_2 A^1 / \dots$$

$$A^1 \rightarrow \alpha_1 A^1 / \alpha_2 A^1 / \dots / \epsilon$$

- **Left Factoring :** When a rule has common prefix at more than one alternative production, then the parser may not be able to decided which alternative to use.

Ex: $A \rightarrow \alpha\beta_1 / \alpha\beta_2$

- Solution to avoid such ambiguity is left factoring. Result of left factoring above grammar is

$$A \rightarrow \alpha A^1 \quad A^1 \rightarrow \beta_1 / \beta_2$$

- Left factoring avoids backtracking in parser
- Left factoring is not a solution for eliminating ambiguity

Example 1.1:

Consider the grammar

$$S \rightarrow a \text{ } S/Sa$$

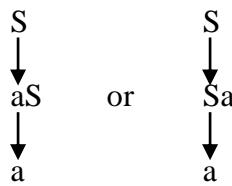
Is this grammar ambiguous ?

How many parse trees exist for an input string w=aaa

Sol: Ambiguity

Consider w=aa

i. The derivation trees are



ii. The leftmost derivations are

$$S \Rightarrow aS \Rightarrow aa$$

Or $S \Rightarrow aS \Rightarrow aa$

There are two leftmost derivations

iii. The right most derivations are

$$S \Rightarrow aS \Rightarrow aa$$

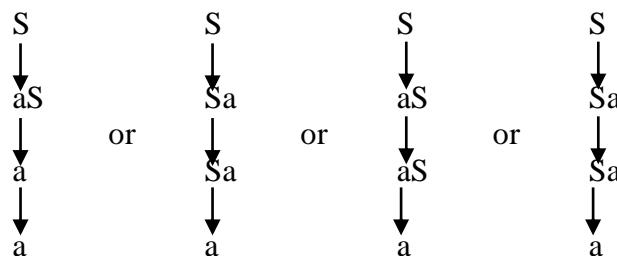
Or $S \Rightarrow aS \Rightarrow aa$

There are two rightmost derivations

So the grammar is ambiguous

Consider the sentence w=aaa

The parse trees are



There are 4 derivation trees for w=aaa

Example 1.2:

Consider the grammar

$$S \rightarrow aSbS/bSaS/\epsilon$$

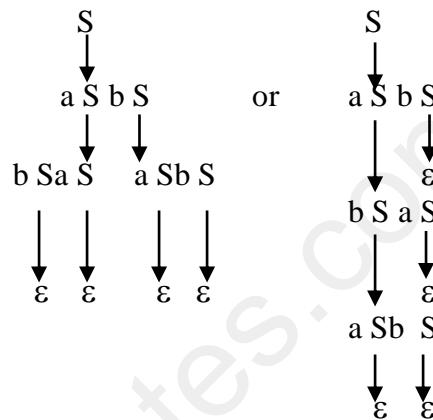
Is this grammar ambiguous ?

How many parse trees exist for $w=abab$

Sol: Ambiguity

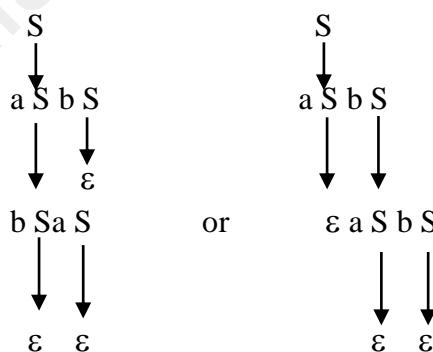
Consider the string $w=ababab$

i. The derivation trees are



There are two or more derivation trees for the sentence $w=abab$, so the grammar is ambiguous.

Parse trees for $w= abab$



There are two parse trees for the sentence $w=abab$

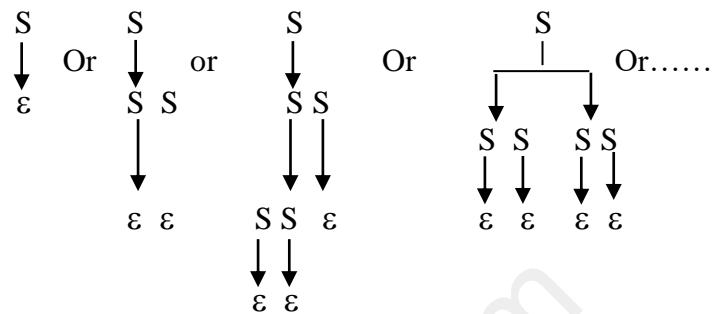
Example 1.3:

Check if G is ambiguous

S \rightarrow aSb/SS/ ε

Sol: Consider the string $w = \varepsilon$

The derivation trees are



There are an infinite number of derivation trees for $w = \varepsilon$.

So the grammar is ambiguous.

Example 1.4:

Eliminate left recursion in

$$A \rightarrow b/Bd$$

B → Bc/ Ac

Sol: Eliminate the left recursion from the B -rules

$$A \rightarrow b/Bd$$

B → AcZ

$$Z \rightarrow \varepsilon/cZ$$

Substituting $A \rightarrow b / AcZd$

B → AcZ

$$Z \rightarrow \varepsilon / c Z$$

Eliminate the self recursion from the A-rules

$$A \rightarrow bZ_1$$

$$Z_1 \rightarrow c Z_1 d Z_1 / \varepsilon$$

B → AcZ

$$Z \rightarrow \varepsilon/cZ$$

The grammar is not left recursive

Example 1.5:

Left factor the grammar

$$S \rightarrow a b c / a b d / a e f$$

Sol: $S \rightarrow a [b c | b d | e f]$

Example 1.6:

Left factor the grammar

$$S \rightarrow a S S b S | a S a S b | a b b | b$$

Sol: $S \rightarrow a [S S b S | S a S b | b b] | b$

Or

$$S \rightarrow a X | b$$

$$X \rightarrow S [S b S | a S b] | b b$$

Or

$$S \rightarrow a X | b$$

$$X \rightarrow S Y | b b$$

$$Y \rightarrow S b S | a S b$$

Example 1.7:

Eliminates left recursion

$$S \rightarrow S O S / 1 0 1$$

Sol: $S \rightarrow 1 0 1 Z$

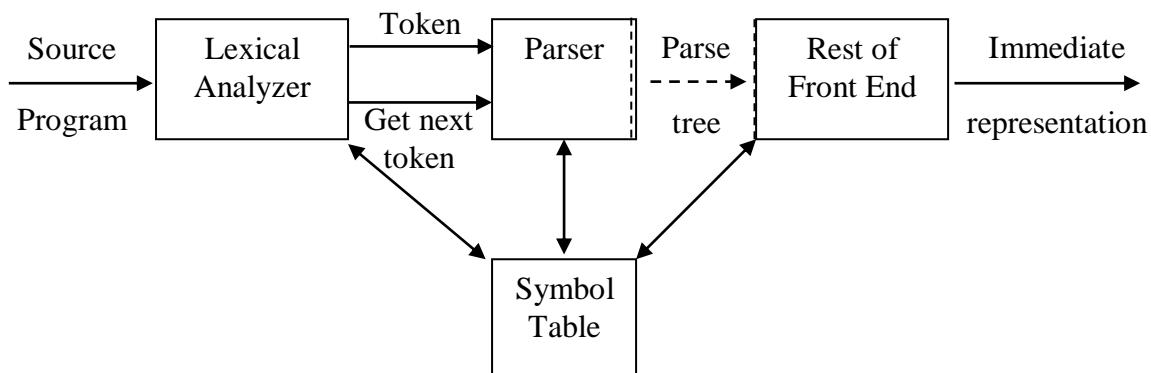
$$Z \rightarrow \epsilon / O S Z$$

The Role of the Parser

The parser obtains a string of tokens from the lexical analyzer, as shown in fig 1.1, and verifies that the string of token names can be generated by the grammar for the source languages.

Conceptually, for well –formed programs, the parser constructs a parse tree and passes it to the rest of the compiler for further processing. In fact, the parse tree need not be constructed explicitly, since checking and translation actions can be interspersed with parsing.

Thus, the parser and the rest of the front end could well be implemented by a single module.



Top-Down Parsing

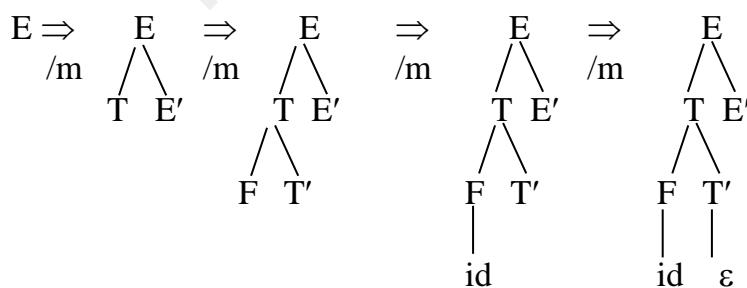
Top-down parsing can be viewed as the problem of constructing a parse tree for the input string, starting from the root and creating the nodes of the parse tree in preorder. Equivalently, top-down parsing can be viewed as finding a leftmost derivation for an input string.

Example : Consider the Grammar

$$\begin{aligned}
 E &\rightarrow T E' \\
 E' &\rightarrow + T E' \mid \epsilon \\
 T &\rightarrow + F T' \\
 T' &\rightarrow * F T' \mid \epsilon \\
 F &\rightarrow (E) \mid id
 \end{aligned}$$

Generating the top-down parse for the input string $id+id *id$.

The following sequence of trees corresponds to a leftmost(/m) derivation of the input



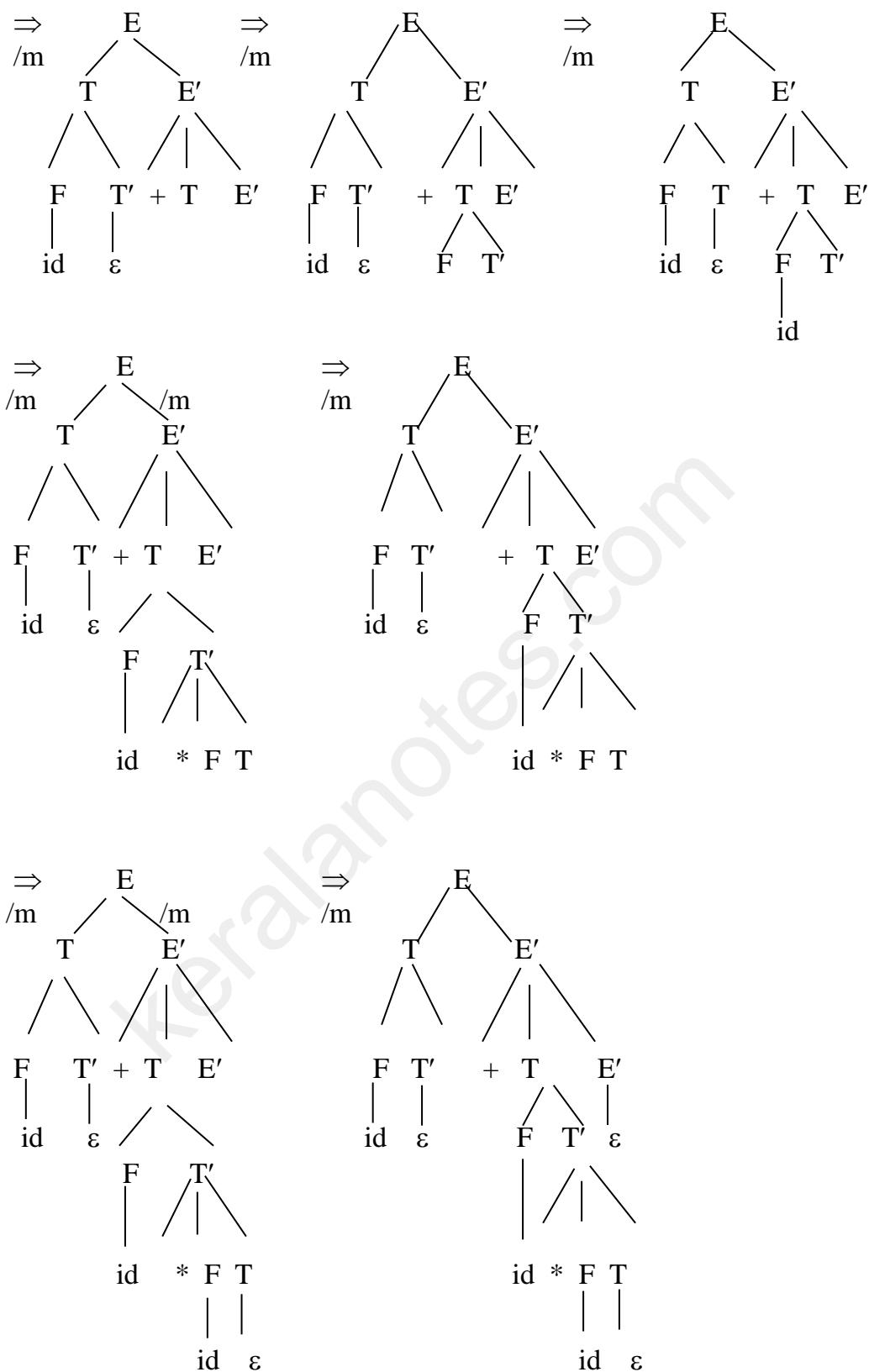


Fig 1.2: Top-down parse for $\text{id} + \text{id} * \text{id}$

Classification of Top-Down parsers:

- Recursive Descent Parsing
 - LL(I) Grammars
 - Non-recursive predictive parsing

Bottom Up Parsing

A bottom up parse corresponds to the construction of a parse tree for an input string beginning at the leaves (the bottom) and working up towards the root (the top). It is convenient to describe parsing as the process of building parse trees, although a front end may in fact carry out a translation directly without building an explicit tree.

Example: For above mentioned grammar generate the bottom up parse for input string id^*id .

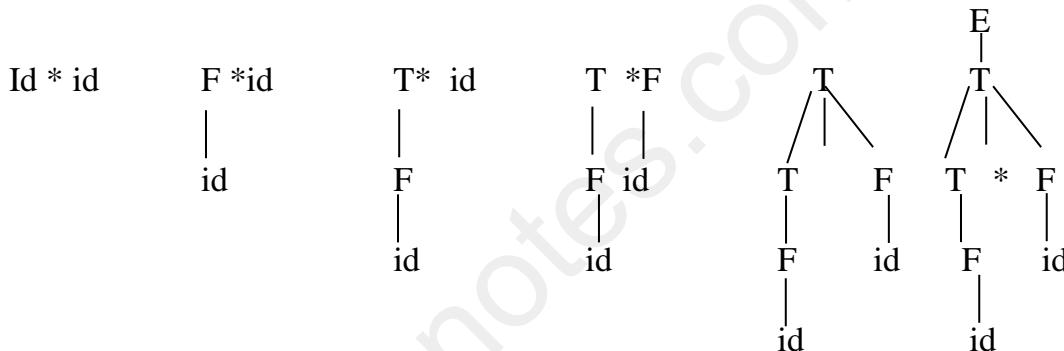


Fig 1.3: A bottom-up parse for id *id

Classification of Bottom up parsers:

- Shift Reduce parsing
 - Operator precedence parser
 - LR(k) Parsing
 - LR(O)
 - SLR(I)
 - LALR(I)
 - CLR(I)

SYNTAX DIRECTED TRANSLATION

- SDT – Generates code while parsing by following rules attached with each production.

Ex: $A \rightarrow \alpha \text{ Printf}(\text{"Hai"})$;

Whenever parser reduces by $A \rightarrow \alpha$ gives message "Hai"

- **Synthesized attribute:** – If attribute value at a node is defined in terms of attribute at its children, such an attribute is called synthesized attribute

Ex: $m A \rightarrow BC$, if $A.a = f(B.a \text{ or } C.a)$

Then attribute 'a' is synthesized

- **Inherited attribute:** – If attribute value at a node is defined in terms of attributes at its parent or sibling, such an attribute is called inherited attribute

Ex: In rule $A \rightarrow BC$, if

$B.I = f(A.i \text{ or } C.i)$; then 'i' is called inherited attribute

- **Annotated /Decorated parse tree:** – A parse tree which shows attribute values at each node is called annotated or Decorated parse tree

- **S-attribute definition:** An SDT which uses only synthesized attribute is called S-attributed definition.

- **L-attributed definition:** An SDT on which attribute are restricted to inherit either from parent or left siblings is called L-attributed definition.

- Every S-attributed definition is L-attributed

Example 1.8:

Write a SDT for evaluating an infix expression.

Sole: $E \rightarrow E_1 + E_2 \{ E.val = E_1.val + E_2.val \}$

Out (E.val)

$/E_1 * E_2 \{ E.val = E_1.val + E_2.val \}$

$/ (E_1) \{ E.val = E_1.val \}$

$/\text{num} \{ E.val = \text{num}.val \}$

Example 1.9:

Write a SDT to convert infix to postfix notation

Sole: $E \rightarrow E_1 + E_2 \{ E.val = E_1.val + E_2.val \}$

$/E_1 * E_2 \{ E.val = E_1.val E_2.val \}$

$/E_1 * E_2 \{ E.val = E_1.val E_2.val^* \}$

$/(E_1) \{ E.val = E_1.val \}$

$/id \{ E.val = id.name \}$

Intermediate Code Generation

- Advantage of intermediate code generation is
 - Retargeting is supported
 - Machine independent code optimization can be applied
- I.C.G. can be represented as
 - Syntax tree
 - DAG
 - Post-fix notation
 - Three address code
- Three address code can be implemented as
- **Quadruple** – a record with 4 fields (oprd, oprd1, oprd2, result)

Advantage - Statements can be moved around.

Disadvantage- wastage of space for temporaries

- **Triples** – a record with 3 fields (oprd, oprd1, oprd2)

Advantage - no wastage of space

Disadvantage- but statements cannot be moved around.

- **In-direct Triples** – maintains list of pointers to triples

Advantage -

1. Statements can be moved around
2. No wastage of space for temporaries

- **Types of 3 address statements**

1. $x = y \text{ op } x$
2. $x = \text{op } y$
3. $x = y [i]$

4. $x[i] = y$
5. $x = *y$
6. $x = &y$
7. if x reloop y goto L
8. goto L

Example 1.10:

While $a < b$ do

If $c < d$ then

$x := y + z$

else

$x := y - z$

Lbegin: if $a < b$ goto L1

goto Lenext

L1: if $c < d$ goto L2

goto L3

L2: $t1 := y + z$

$x := t1$

goto Lbegin

L3: $t2 := y - z$

$x := t2$

goto Lbegin

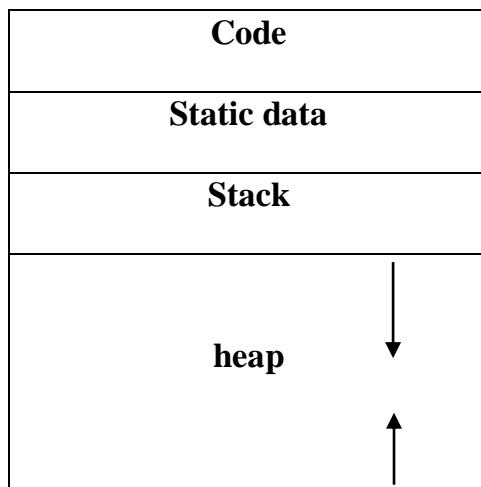
Lenext:

Runtime Environments

Static and Dynamic Notions

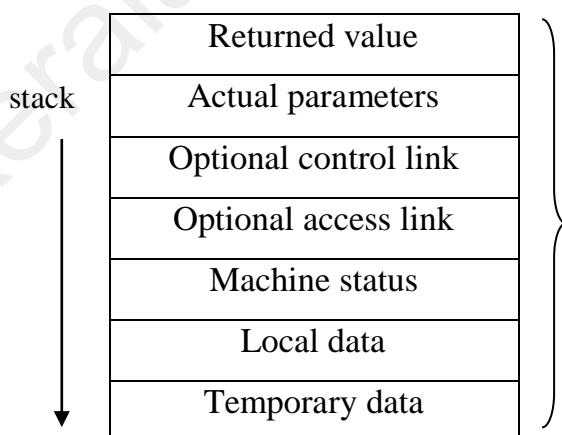
Static	Definition of a procedure Declaration of a name Scope of a declaration
Dynamic	Activation of a procedure Binding of a name Lifetime of a binding

Storage Organization



- Target code: static
- Static data objects: static
- Dynamic data objects: heap
- Automatic data objects: stack
- Information needed for each execution of a procedure is stored in a record called activation record.
- Activation record contains 7 fields

Activation Records



Size of each of this can be estimated at compile time

- Before Target code generation, we need to consider how programs can be implemented on target machines in particular
- How are data objects represented memory

- How is storage of data objects be allocated
- How are parameters passed to function
- Answers to this are very much depended on storage allocation strategies
- **Static Allocation:** Storage is allocated at compile time only
- Static storage has fixed allocation that does not change during program execution.
- As bindings do not change at runtime, no runtime support is required.

Disadvantages:

1. Size of data objects should be known at compile time
2. Recursion is not supported
3. Data structures cannot be created at runtime

Stack Allocation:

- is based on idea of control stack
- activation record are pushed and popped as activations begin and end respectively.
- Locals are always bound to fresh storage in each activation because a new activation is pushed onto stack when a call is made.
- Values of locals deleted as activation end.

Disadvantages

- Values of locals cannot be retained once activation ends.

Heap Allocation

- Storage can be allocated and de-allocated in any order
- So ever a time, it may contain pieces of alternate areas that are free and in use

Disadvantage: Heap manager overhead.

BASIC OF OPTIMIZATION

Optimization refers to technique used by compiler to improve the execution efficiency of the generated code.

- Every optimization should preserve semantics
- Actually after applying optimization there is no guarantee that generated code is optimal

- Optimizations that are restricted to one basic block are called **local optimizations**; Otherwise, they are called **global optimizations**. Compiler optimizations can be broadly divided into two categories.
- Machine dependent Optimizations- optimizations that can be applied on target code.
- Machine independent optimizations – optimizations that can be applied on three address code.

Examples of Machine independent optimizations are:

1. Loop optimization
2. Folding
3. Constant Propagation
4. Copy Propagation
5. Redundancy elimination
6. Strength reduction
7. Dead-Code eliminations
8. Algebraic simplifications

Examples of Machine dependent optimizations are:

- Peephole optimization
- Register allocation
- Effective usage of addressing modes.

Machine independent optimization

1. **Loop optimization:** Loops are principle sources of optimization as computer spends much time. These act on the statements which make up a loop, such as a for loop. Loop optimizations can have a significant impact because many programs spend a large percentage of their time inside loops.

For detecting loops control flow analysis (CFA) is used. In CFA, a program is represented by program flow graph (PFG) which shows how control is flowing in a program.

To obtain such graph, intermediate code is divided into basic blocks.

Control Flow Graphs

- A (control) flow graph is a directed graph
- The nodes in the graph are basic blocks
- There is an edge from B1 to B2 iff B2 immediately follows B1 in some execution sequence.
 - There is a jump from B1 to B2
 - B2 immediately follows B1 in program text
- B1 is a predecessor of B2, B2 is a successor of B1

Basic Blocks

- A basic block is a sequence of consecutive statements in which control enters at the beginning and leaves at the end without halt or possibility of branching except at the end.

Construction of Basic Blocks

- Determine the set of leaders
 - The first statement is a leader
 - The target of a jump is a leader
 - Any statement immediately following a jump is a leader
- For each leader, its basic block consists of the leader and all statements up to but not including the next leader or the end of the program.

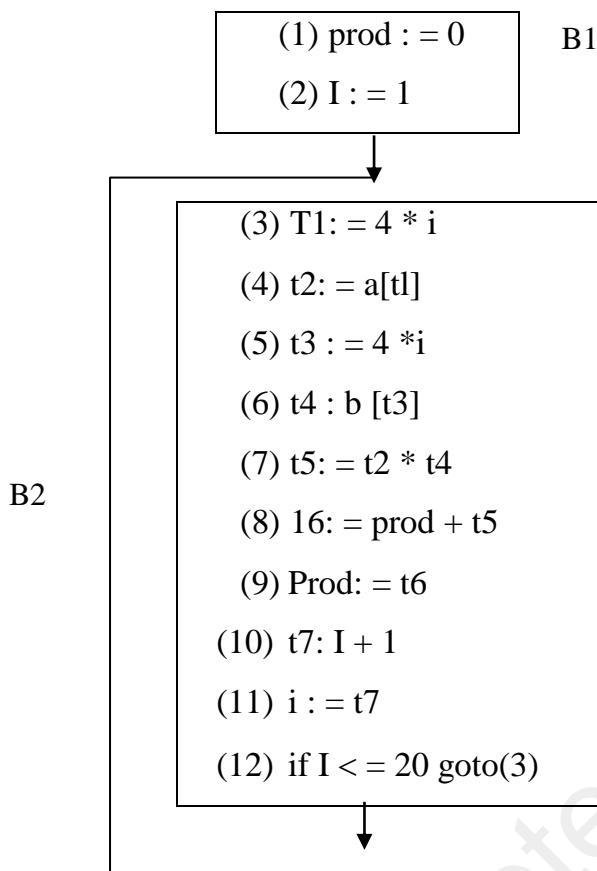
Representation of Basic Blocks

- Each basic block is represented by a record consisting of
 - a count of the number of statements
 - a pointer to the leader
 - a list of predecessors
 - a list of successors

An example

- 1) Proud:=0
- 2) i := 1
- 3) t1:= 4 * i
- 4) t2:= a[t1]
- 5) t3:= 4 * i
- 6) t4:= b[t3]
- 7) t5:= t2*t4
- 8) t6:=prod +t5
- 9) prod:=t6
- 10) t7:=i+1
- 11) I := t7
- 12) If i <= 20 goto (3)

PFG is



Loop detection:

A loop is a cycle in flow graph

Some of Loop optimizations are :

- (a) Code motion / loop invariant computation elimination / frequency reduction:

This involves moving loop invariant computation outside the loop as follows.

While (i <= 1000)

$$x = \text{SIN}(A) * \text{COS}(A) * i$$

as

$$t = \text{SIN}(A) * \text{COS}(A);$$

while (i < 1000)

$$x = t * i;$$

- (b) **Loop unrolling:** Unrolling duplicates the body of the loop multiple times, in order to decrease the number of times the loop condition is tested and the number of jumps, which hurt performance by impairing the instruction pipeline. Completely unrolling a loop eliminates all overhead, but requires that the number of iterations be known at compile time.

Ex: while (i <100)

```
{
x[i]=0;
i++;
}
```

as

```
while ( i <100)
```

```
{
x[i]=0;
i++;
x[i]=0;
i++;
}
```

There is clearly a trade off space Vs Time.

- (c) **Loop jamming** / Loop fusion or loop combining another technique which attempts to reduce loop overhead. When two adjacent loops would iterate the same number of times (whether or not that number is known at compile time), their bodies can be combined as long as they make no reference to each other's data.
- 2. **Folding / constant folding** : Replacing expressions consisting of constants (e.g. “3+5”) with their final value (“8”) at compile time, rather than doing the calculation in run-time. Used in most modern languages.
- 3. **Constant Propagation** : Replacing variables by their associated values known at compile time.

Ex: pi =3.1415

$$D_to_R=pi/180=3.1415/180$$

- 4. **Copy propagation** : Propagate copy may be useful in finding dead code.

Ex:

- a. x=y
- b. z=1.0+x! dependent on previous “x”

- Problem because no parallelism! no pipelining!
- Second statement depends on last

- Would be better to write
- $x = y$
- $z = 1.0 + y$

5 Dead Code Removal :

- Blocks that are never entered can be removed
- Not much use to programmer
- But during multiple passes compiler may generate dead code.

Dead Code elimination: Removes instructions that will not affect the behaviour of the program, for example definitions which have no uses, called dead code. This reduces code size and eliminates unnecessary computation.

- ## 6. Common sub expression elimination / Redundancy elimination :
- In the expression “ $(a+b) - (a+b)/4$ ”, “Common sub expression” refers to the duplicated “ $(a+b)$ ”. Compilers implementing this technique realize that “ $(a+b)$ ” won’t change, and as such, only calculate its value once.

Dags are also useful in redundancy elimination.

- ## 7 Strength Reduction :
- Replace a costlier operation by cheaper one.

- Classic one is exponentiation:
- $x^{**}2 = x*x$
- Exponentiation is expensive (software)
- Help the compiler here:

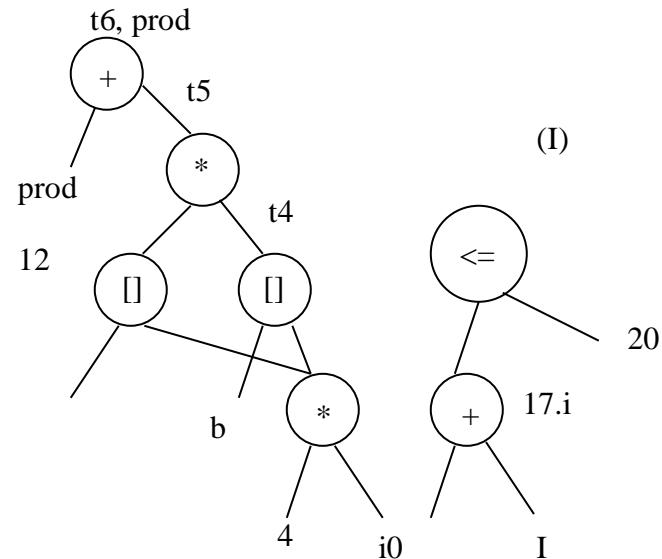
- ## 8. Algebraic simplification :
- eliminate statements like $x=x*1$ or $y = y + 0$;

DAG Representation of Blocks

- Easy to determine:
- Common sub expressions
- Names used in the block but evaluated outside the block
- Names whose values could be used outside the block
- Leaves labeled by unique identifiers
- Interior nodes labeled by operator symbols
- Interior nodes optionally given a sequence of identifiers, having the value represented by the nodes.

An Example

- (1) $t1 := 4*i$
- (2) $t2 := a[t1]$
- (3) $t3 := 4*i$
- (4) $t4 := b[t3]$
- (5) $t5 := t2 * t4$
- (6) $t6 := \text{prod} + t5$
- (7) $\text{prod} := t6$
- (8) $t7 := i + 1$
- (9) $i := t7$
- (10) if $i \leq 20$ goto(1)



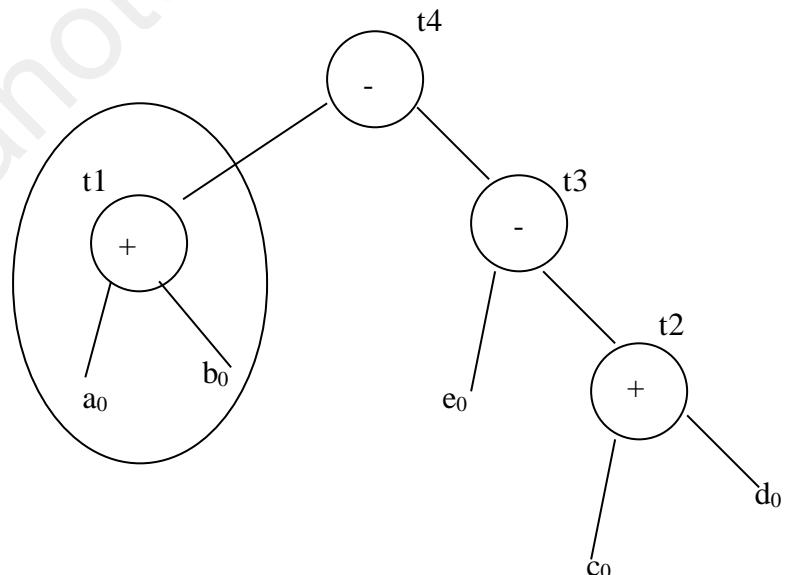
Generating Code from DAGs

```

t1 := a+b
t2 := c+d
t3 := e - t2
t4 := t1 - t3
1)    MOV a, R0
2)    ADD b, R0
3)    MOV c, R1
4)    ADD d, R1
5)    MOV R0, t1
6)    MOV e, R0
7)    SUB R1, R0
8)    MOV t1, R1
9)    SUB R0, R1
10)   MOV R1, t4

```

Only R0 and R1 available



Rearranging the order

$t2 := c + d$

$t3 := e - t2$

$t1 := a + b$

$t4 := t1 - t3$

1) MOV c, R0

2) ADD d, R0

3) MOV e, R1

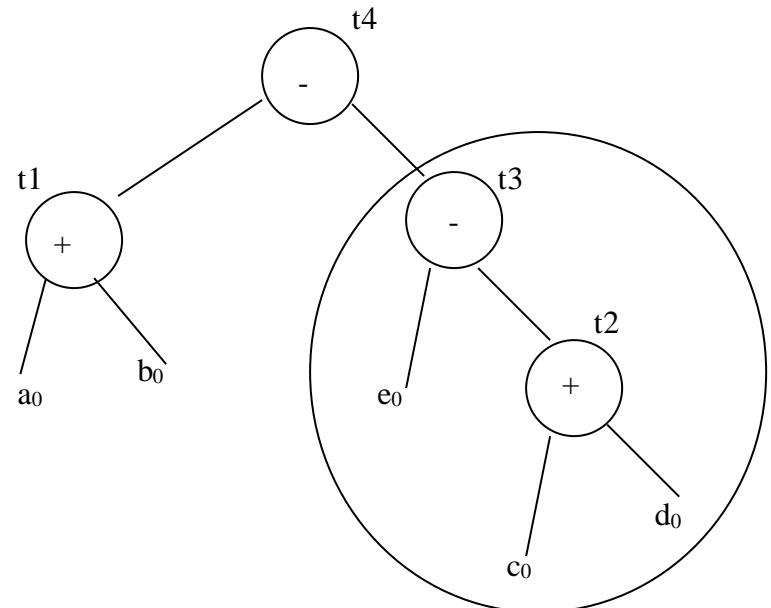
4) SUB R0, R1

5) MOV a, R0

6) ADD b, R0

7) SUB R1, R0

8) MOV R0, t4



Machine Dependent Optimization:

Peephole Optimization

- Improve the performance of the target program by examining and transforming a short sequence of target instructions.
- May need repeated passes over the code
- Can also be applied directly after intermediate code generation

Examples

(a) Redundant loads and stores

MOV R0, a

MOV a, R0

(b) Algebraic Simplification

$x := x + 0$

$x := x * 1$

(c) Unreachable code

#define debug 0

If (debug) (print debugging information)

If 0<>1 goto L1

Print debugging information

L1:

If 1 goto L1

Print debugging information

L1:

(d) Flow-of-control	optimization
goto L1	goto L2
...
L1: goto L2	L2.goto L2
goto L1	if a < b goto :2
...	goto L3
L1: if a < b	goto L2
...	
L3:	L3:

(e) **Reduction in strength:** replace expensive operations by cheaper ones

- $x^2 \Rightarrow x * x$
- fixed point multiplication and division by a power of 2 \Rightarrow shift
- floating-point division by a constant \Rightarrow floating-point multiplication by a constant.

(f) **Use of machine idioms:** hardware instructions for certain specific operations auto-increment and auto-decrement addressing mode (push or pop stack in parameter passing).

CHAPTER – 2

LEXICAL ANALYSIS

Concepts & Definitions

1. Tokens – The lexical analyzer (sometimes called scanner) is the front end of the compiler. It breaks up the source program into lexemes & outputs them as a stream of tokens. Wherever the syntax analyzer calls the scanner it gets a token in return.
2. Lexemes – Every token has an associated lexeme the sequence of input character that the token represents.
3. Buffers – It is sometimes necessary for the scanner to scan past the lexeme into the next lexeme. To facilitate this buffering is used.
4. Pattern- Each token has an associated pattern normally regular expressions are model patterns.
5. Regular expression-An algebra which has union, concatenation & kleene closure as operators used to model the structure of lexemes & tokens.
6. Regular Definition – Regular grammar giving the set of rules of the formation of tokens.
7. Extended Regular Expressions – includes + for one or more & ? for zero or one of operation.
8. Transition diagrams – formed from regular expression also called finite automata with ϵ moves.
9. Finite automata – Model the scanner
10. Deterministic Finite automata – No choice in any situation. Accept the regular sets described by regular expression.
11. Non Deterministic Finite automata – have choice & move naturally model regular definitions.
12. Lex-A'C' tool for automatic generation of lexical analyzer.

2.1 Definition

The scanner or lexical analyzer is the front end of the compiler. It automatically generates a listing by printing out whatever is read in. Preprocessors by macros are handled by lexical analyzer.

2.2 Concept

The scanner breaks up the input into lexemes. These are mapped into tokens.

2.3 Strategy

The structure of identifiers, reserved words, operators, number & comments can be described by a regular expression / regular grammar.

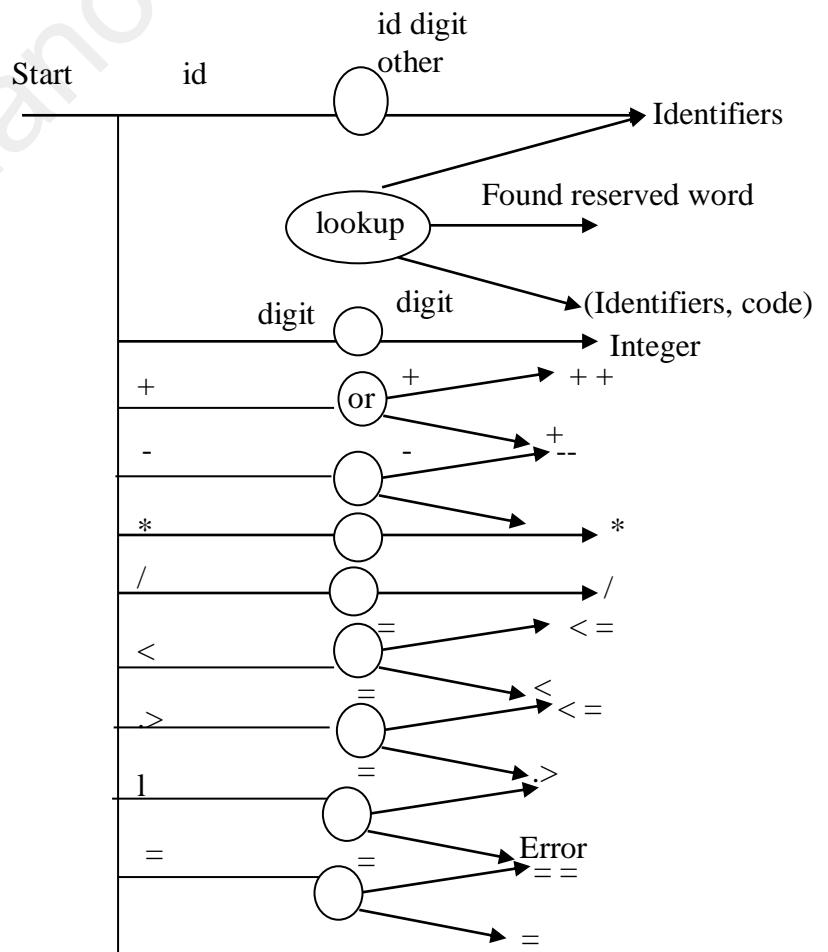
2.4 Concept

A lexical analyzer is modeled by a transition diagram of a finite automata .

2.5 Concept

One can specify the structure of lexemes by a regular expression & then from the regular expressions build a deterministic finite automata which is the lexical analyzer.

2.6 Structure of the lexical analysis



2.7 We will use a running example throughout the various phases of the compiler as follows:

1. Description of the running example – quick sort
2. Breakup of the running example into lexemes
3. Breakup of the running example into tokens
4. A cfg for the running example
5. A parse tree for the running example
6. The SDTS for the running example
7. Translation of the running example to intermediate 3-address code
8. Translation of the running example to optimized 3 – address code
9. The code for the running example in a hypothetical machine.

2.8 The running example chosen.

Void quicksort (int m, int n)

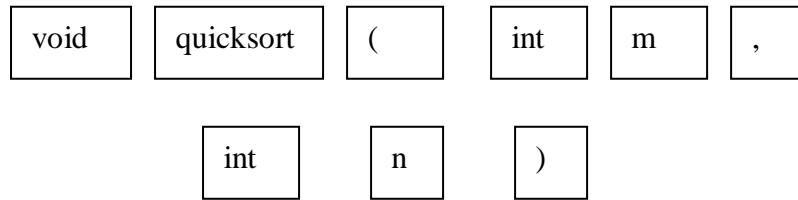
/*recursively sort the elements a[m] through a[n]*/

```

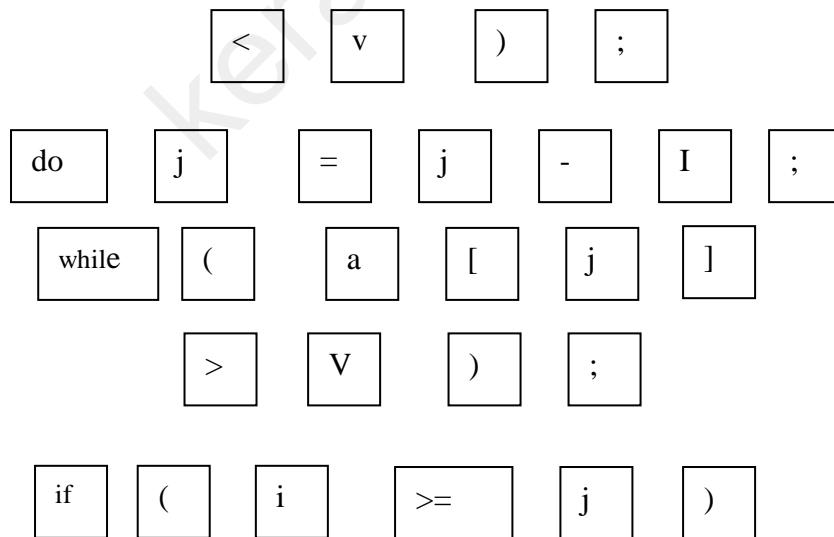
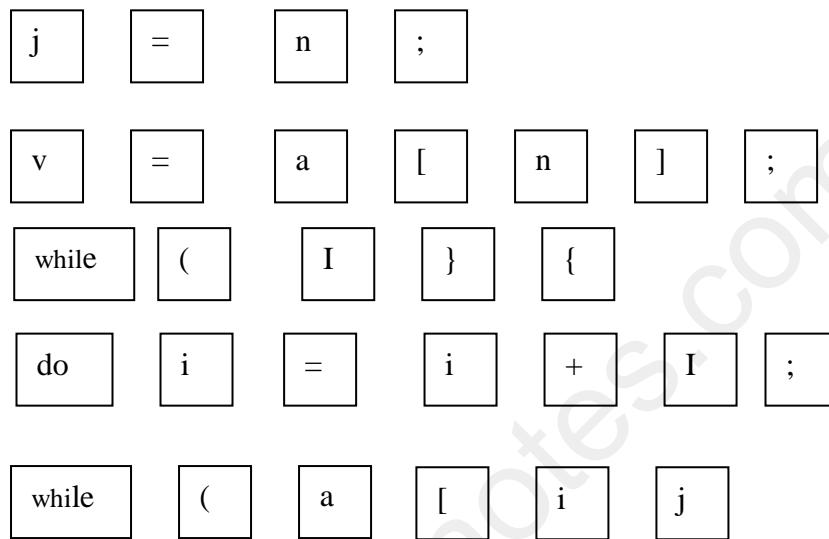
{
    int I, j;
    int v, x;
    if(n<=m) return;
    I = m - 1; j = n; v = a[n];
    While (I) {
        do i = I + 1; while (a[i] < v);
        do j = j - I; while (a[j] > v);
        if (i >= j) break;
        x = a[i], a[i] = a[j]; a[j] = x;
        /*swap a[i], a[j]*/
    }
    x = a[i]; a[i] = a[n]; a[n] = x;
    /*position partition element */
    Quicksort(m, j); quicksort(i+1, n);
}

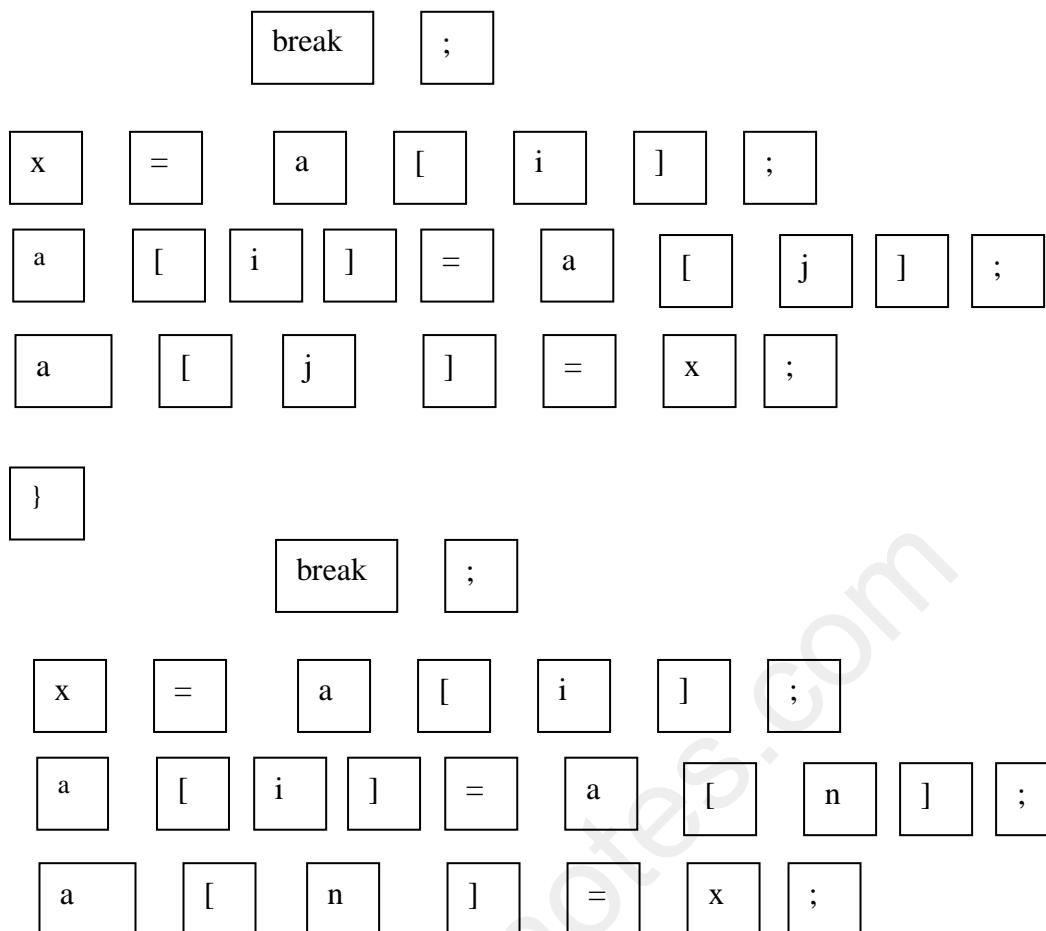
```

2.9 Classification of a running example into lexemes & tokens

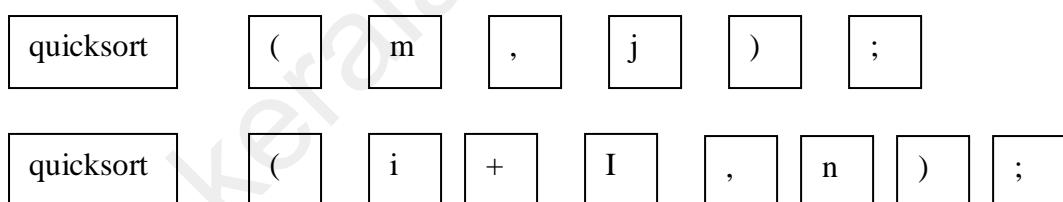


Comment is removed





Comment is removed



2.10 Symbol table

	Name	Type	Width
1	i	int	4
2	j	int	4
3	v	int	4
4	x	int	4
5	n	int	4
6	a	Array	
7	Quicksort	Function	Parameters
8	m	int	4
9	n	Int	4

2.11 Constant table

	Name	Value
1	“1”	I

2.12 Reserved words:

10	Void	
11	If	I
12	do	
13	while	
14	Return	
15	Break	
16	Int	

2.13 token string of the running example

10 7 (16 var I, 17 var 5)
 {
 16 var I, var 2;
 16 var 3, var 4;
 11 (var n <= var 8) 14;
 Var 1 = var 8 – cons 1; var 2 = var 5;
 Var3 = var 6 [var 5] ;
 13 (var 1) { 12 var 1 = var 1 + const 1;
 13 (var 6 [var 1] < var3);
 12 var = var 2 + const1;
 13 (var 6 [var 2 ≥ var 3);
 11 (var 1>= var 2) 15;
 var 4 = var 6 [var 6 [var 2];
 var 6 [var 2] = var 4;
 }

var 4 = var 6 [var 1];
 var 6 [var 1] = var [var 5];
 var 6 [var 5] =v ar 4;
 var 7 (var 8, var 2) ;
 var 7 (var 1+const 1, var 5);

Example 2.1:

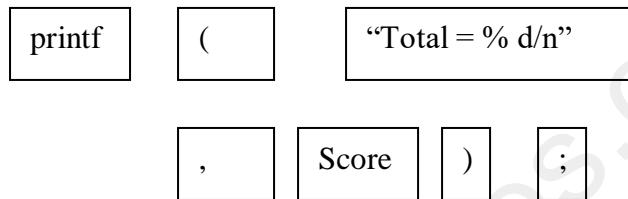
How many lexemes are there in the following statement ?

Print (“total = %d/n “, score);

- (a) 4 (b) 5 (c)6 (d) 7

Ans: (d)

Sol :



Example 2.2:

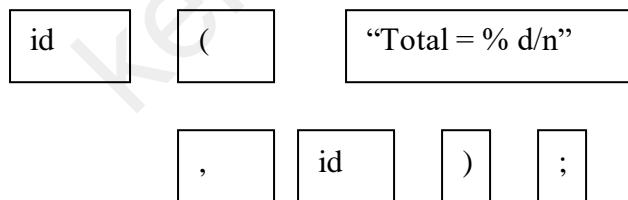
How many different tokes are there in the following statement ?

Print (“total = %d/n “, score);

- (a) 4 (b) 5 (c)6 (d) 7

Ans: (c)

Sol :



Example 2.3:

How many lexemes are there in the following FORTRAN statement ?

DO 51 = 1.25

- (a) 8 (b) 5 (c)3 (d) 4

Ans: (c)

Sol :



Example 2.4:

How many lexemes are there in the following FORTRAN statement ?

DO 51 = 1.25

- (a) 5 (b) 3 (c) 8 (d) 7

Ans: (d)

Sol :



Example 2.5:

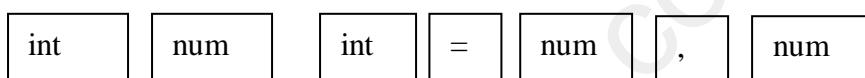
How many different kinds of tokens are there in the following FORTRAN statement ?

DO 5J = 1,250

- (a) 5 (b) 6 (c) 4 (d) 8

Ans: (c)

Sol :



Token types

- (1) int (2) num (3) = (4) ,

Class Practice Questions

01. Comments are normally detected from the source program during

a) Lexical analysis	b) Syntax analysis
c) Semantic analysis	d) Code generation
02. The macro -expansion or expansion of preprocessor statements take place during

a) Lexical analysis	b) Syntax analysis
c) Semantic analysis	d) Code optimization
03. Keywords are recognized in a compiler during

a) Lexical analysis	b) Syntax analysis
c) Semantic analysis	d) Code generation
04. Type checking is done during

a) Lexical analysis	b) Syntax analysis
c) Semantic analysis	d) Code generation

- 05 Compiler can diagnose the following errors ?
a) Grammatical errors b) Logical errors
c) Runtime errors d) All of the above
- 06 A compiler that may run on one machine and produces target code for another machine is called _____
a) Bootstrapping b) Direct Compiler
c) Indirect compiler d) Cross compiler
- 07 The object code which is obtained from Assembler is
a) Assembly language code b) Relocatable object code
c) Executable object code d) None
- 08 Choose the correct statement ?
a) A compiler must compulsorily keep lexical analysis separate from the syntax analysis.
b) The syntax analysis phase can be expanded to do lexical analysis also
c) Macro expansion is done during syntax analysis
d) Macro expansion is done during semantic analysis.
- 09 Choose the correct statement ?
a) The symbol table is constructed in the lexical analysis phase
b) The symbol table is constructed during the syntax phase.
c) The symbol table is not needed for semantic analysis
d) The symbol table is not needed for code generation.
- 10 Choose the correct statement ?
a) Most of the time for compilation is spent in lexical analysis
b) Most of the time for compilation is spent in semantic analysis
c) Most of the time for compilation spent during code optimization phase
d) Most of the time for compilation is spent during the code generation phase.
- 11 Choose the correct statement ?
a) The lex utility generates a lexical analyzer
b) The lex utility cannot handle all regular expressions
c) The lex utility cannot handle all regular definitions
d) None of the above.

- 12 The output of the Lex compiler is
- a) Lex, yy
 - b) Lex, yy.c
 - c) a.out
 - d) Tokens
- 13 Consider the following error recovery operations in lexical analysis
- 1) Delete one character from the remaining input
 - 2) Insert a missing character into the remaining input
 - 3) Replace a character by another character
 - 4) Transpose two adjacent character
 - 5) If parenthesis mismatch delete the offending parenthesis
- a) 1,2,3,4 are valid at the lexical analysis phase
 - b) 4,5 cannot be done at the lexical analysis phase
 - c) 1,2 are not valid at the lexical analysis phase
 - d) Only 5 is not valid at the lexical analysis phase.
- 14 for (I = 0; <10; i++)
 printf("I = %d", i);
- The number of tokens in the above 'C' statement is _____
- 15 printf ("Total= %d", sum);
- The number of tokens in the above 'C' statement is _____
- 16 Find number of tokens in the following program ?
- If (x>=y) (x=x+y; } else { x=x-y; };
- a) 23
 - b) 24
 - c) 25
 - d) None
- 17 Which of the following is not a token of 'C' program ?
- a) .02e+2
 - b) MAX
 - c) 123.33
 - d) None
- 18 Match the following according to input (from the left column) to the compiler phase (in the right column) that processes it:
- | | |
|---------------------------------|-------------------------|
| (P) Syntax tree | (i) Code generator |
| (Q) Character stream | (ii) Syntax analyzer |
| (R) Intermediate representation | (iii) Semantic analyzer |
| (S) Token stream | (iv) Lexical analyzer |

- a) P →(ii), Q→ (iii), R → (iv), S → (i)
- b) P → (ii), Q → (i), R → (iii) S → (iv)
- c) P → (iii), Q → (iv), R → (i), S→ (ii)
- d) P → (i), Q → (iv), R → (ii), S → (iii)

KEY

01.(a) 02.(a) 03.(a) 04.(c) 05.(a) 06.(d) 07.(b)
08.(d) 09.(d) 10.(b) 11.(a) 12.(a) 13.(b) 16.(b)
17.(d) 18.(c)

CHAPTER – 3

PARSING TECHNIQUES

Concepts & Definitions

- Parser – A parser takes the stream of tokens given by the lexical analyzer and builds the derivation trees for the source program in accordance with the rule of a context free grammar.
- Context Free Grammar – Also called BNF are a formation to model the syntax or form of most HLLs.
- Terminals – Symbols – in a context free grammar actual occurrence in the language being described.
- Non-Terminals – Meta symbols of a context free grammar used to describe the HLL
- Parse Trees - also called derivation trees and are in accordance with the rules of a context free grammar
- Ambiguity – If two distinct parse trees exist for a string in the language then the grammar is said to be ambiguous.
- Top – down parsing – simulates the left most derivation of a sentence in a context – free grammar.
- Bottom – up parsing – simulates the reverse of a right most derivation in a context – free grammar.
- Design of grammars – Top down parsing grammar must eliminate left recursion and are harder to design.
- Recursive Decent parser – Top down with recursive decent.
- LL(I) Parser – Top down parser with one symbol lookahead
- Shift –reduce parsers –bottom up parser with shift meaning a push operation on the parse start & reduce being a pop.

3.1 Concept

In syntax analysis we are interested in determining if the source language is in accordance with the rules of the programming language.

3.2 Concept :

We model (a superset of) the correct programs by a context –free grammar.
The problem of syntax analysis reduces to the membership problem of CFLs.

3.3 Theorem:

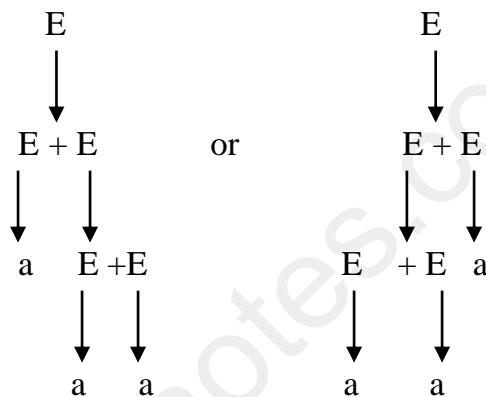
An arbitrary CFG can be put into the Chomsky Normal Form & the membership problem, i.e. syntax analysis can be done by the CYK algorithm. The CYK algorithm is a dynamic programming algorithm of time complexity $O(n^3)$.

3.4 Definition

An arbitrary CFG may be ambiguous

For example the grammar $E \rightarrow E + E / a$ is ambiguous

- a) The sentence $a + a + a$ has two derivation



If any sentence in the language generated by the grammar has more than one derivation tree then the grammar is ambiguous

- b) The sentence $a+a+a$ has two leftmost derivations in the grammar

$$\begin{aligned}
 E &\Rightarrow E + E \Rightarrow E + E + E \\
 &\Rightarrow a + E + E \Rightarrow a + a + E \\
 &\Rightarrow a + a + a
 \end{aligned}$$

$$\begin{aligned}
 E &\xrightarrow{\text{leftmost}} a + a + a \\
 \text{Or}
 \end{aligned}$$

$$\begin{aligned}
 E &\Rightarrow E + E \Rightarrow a + E \\
 &\Rightarrow a + E + E \Rightarrow a + a + E \\
 &\Rightarrow a + a + a
 \end{aligned}$$

$$\begin{aligned}
 E &\xrightarrow{\text{leftmost}} a + a + a
 \end{aligned}$$

If a sentence in the language generated by the grammar has more than one right most derivation tree then the grammar is ambiguous

3.5 Definition

A grammar is said to be ambiguous if there is more than one derivation tree for a sentence or there is more than one left most derivation for some sentence or there is more than one rightmost derivation for some sentence.

3.6 Definition

The grammar $S \rightarrow SS/a/\epsilon$ has

- a) an infinite number of derivation trees for the sentence ϵ
- b) an infinite number of leftmost derivations for the sentence ϵ
- c) an infinite number of rightmost derivations for the sentence ϵ

3.7 Definition

A context -free language is said to be inherently ambiguous if every grammar for it is ambiguous

Example: The language $L = \{a^n b^n c^n d^m | n, m \geq 1\}$ is inherently ambiguous.

3.8 Definition

Passing is the systematic construction of a derivation tree to determine if a given sentence is I the language.

3.9 Definition

The grammar has a rule $A \rightarrow A \dots \dots \dots$

Where A is the non-terminal on the left hand side & the rule is also the first symbol o the right hand side of the rule then the grammar is said to be left recursive.

3.10 Definition

If a grammar has a rule $A \rightarrow A$ where A is the non-terminal on the left hand side of the rule is also the last symbol on the right hand side of the rule then the grammar is said to be right recursive.

3.11 Definition

If a grammar has no ϵ - rules i.e. rules of the form $A \rightarrow \epsilon$ and no rule of the form $A \rightarrow \dots BC\dots$ where B, C & A are non-terminals then the grammar is said to be an operator grammar.

3.12 Definition

Given an arbitrary CFG then the parsing of a string can be done in exponential time with back up.

3.13 Definition

Parsing techniques are classified as top down parsing techniques (or) bottom up parsing techniques depending on the way the syntax tree is built.

3.14 Definition

Top down parsing algorithms build the derivation tree top down, starting from the root and working down to the leaves. Top down parsing algorithms simulate a leftmost derivation.

3.15 Definition

Bottom up parsing algorithms build the derivation tree bottom up, starting from the leaves & working to the root. Bottom up parsing algorithm simulate the reverse of a rightmost derivation.

3.16 Concept

An arbitrary CFL suffices from three difficulties when constructing a parser algorithm.

1. Non-determinism – choice of right parts of rules
2. Left recursion
3. Ambiguity – more than one derivation tree for some sentence.

3.17 Theorem:

It is un-decidable if a CFG is ambiguous.

3.18 Definition

Practical parsing algorithm must parse the input in linear time & to do this are empty deterministic context free languages (or a subset of them)

3.19 Definition

Practical parsing algorithm use a deterministic push down automata as the parser.

TOP DOWN PARSING

3.20 Introduction

We will consider the language of expression in C with +&*. The following are expressions.

$$\begin{aligned} &i + i \\ &i + i * i \\ &(i + i) * i \end{aligned}$$

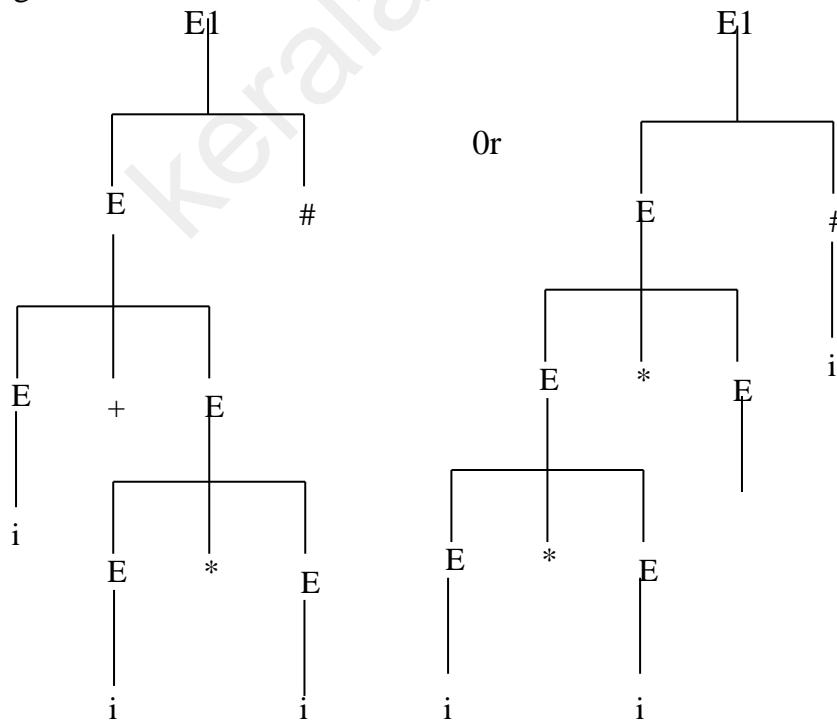
There are an infinite number of valid expressions in C. Each expression is finite. We need a finite description for this infinite set of expression. We will use a context free grammar (CFG)

3.21 Attempt I:

$$\begin{aligned} E1 &\rightarrow E \# \\ E &\rightarrow E + E \\ &/ E * E \\ &/ (E) \\ &/ i \end{aligned}$$

The above is called a context free grammar. The meta symbols or non -terminals are {E1, E} the terminal set is [i, + , *, (,) #]. (Assume # is used to symbol the end of the input).

Now let us consider a sample expression $i + i * i \#$. The expression have the two following derivation trees.



As there are two distinct derivation trees for the sentence $i + i * i$, the grammar is ambiguous. We can represent the derivation by two leftmost derivations.

$$E1 \Rightarrow E \# \Rightarrow E + E \# \Rightarrow i + E \#$$

$$\Rightarrow i + E^* \# \Rightarrow i + i^* E \#$$

$$\Rightarrow i + i^* i \#$$

Or

$$E1 \Rightarrow E \# \Rightarrow E^* E \# \Rightarrow E^* E \#$$

$$\Rightarrow i + E^* \# \Rightarrow i + i^* E \#$$

$$\Rightarrow i + i^* i \#$$

There are two distinct leftmost derivations. The grammar is ambiguous.

We can construct rightmost derivations.

$$E1 \Rightarrow E \# \Rightarrow E + E \# \Rightarrow E + E^* E \#$$

$$\Rightarrow E + E^* \# \Rightarrow E + i^* i \#$$

$$\Rightarrow i + i^* i \#$$

Or

$$E1 \Rightarrow E \# \Rightarrow E^* E \#$$

$$\Rightarrow E^* i \# \Rightarrow E + E^* i \#$$

$$\Rightarrow E + i^* i \# \Rightarrow i + i^* i \#$$

3.22 Strategy:

First the ambiguity has to be removed. With the ambiguity it is not known as to whether $+$ has a higher precedence than $*$ or the way around.

By trial & error, the following grammar has been answered for the string $i + i^* i$.

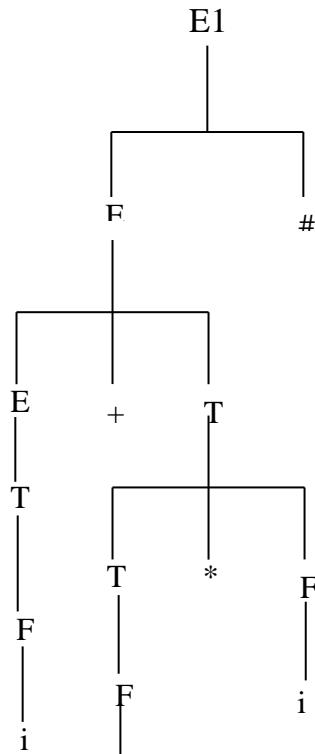
$$E1 \rightarrow E \#$$

$$E \rightarrow E + T/T$$

$$T \rightarrow T * F/F$$

$$F \rightarrow (E) / i$$

If we consider sentence $i + I * I \#$ there is a unique derivation tree for the sentence.



In the above grammar '*' occurs lower than '+'. In a bottom up reduction the operator '*' is handled first & then the operator '+'. This means '*' has a higher priority than '+'.

3.23 Theorem:

This grammar can be shown to be unambiguous

Problem:

The grammar $E1 \rightarrow E \#$

$E \rightarrow E + E \mid E^* E \mid (E) \mid a$ is left recursive. If we try to build a derivation tree top down, we may end up in the tree.

The infinite tree does not terminate. Top down parsing algorithms do not tolerate left recursive.

3.24 Theorem:

Any context free grammar can be put into the Griebach Normal Form (GNF) which is free from left recursive.

Example 3.1:

$E_1 \rightarrow E \#$

$E \rightarrow +E \mid E * E \mid (E) \mid i$

Convert to the Griebach Normal Form

Sole: $E_1 \rightarrow E \#$

$E \rightarrow i Z \mid (E) Z$

$Z \rightarrow +E Z \mid * E Z$

Example 3.2:

$E_1 \rightarrow E \#$

$E \rightarrow +T \mid T / T$

$T \rightarrow * F \mid F / F$

$F \rightarrow (E) \mid i$

Convert to avoid left recursion.

Sole: $E_1 \rightarrow E \#$

$E \rightarrow T + E \mid T$

$T \rightarrow F * \mid T / F$

$F \rightarrow (E) \mid i$

3.25 Concept

Recursion can be replace by iteration.

LL (I) PARSERS

- LL (I) is most widely used Top down parser
- LL (I) parser parse the given input string with help of LL (I) parsing table
- Given a grammar, two functions First () and Follow () are used to construct LL (I) parsing table.

First (α): Gives set of terminals that begin in strings derived from α

- First (α) is computed using 3 rules

1. If α is terminal, $\text{First}(\alpha) = \{\alpha\}$
2. If α is non-terminal
 - a) and has ϵ rule like $\alpha \rightarrow \epsilon$, then $\text{First}(\alpha) = \{\epsilon\}$
 - b) and has non null production like $\alpha \rightarrow X_1X_2X_3$ then

$$\begin{aligned}\text{First}(\alpha) &= \text{First}(X_1X_2X_3) \\ &= \text{First}(X_1) \text{ if } X_1 \text{ does not derive } \epsilon \\ &= \text{First}(X_1) - \{\epsilon\} \cup \text{First}(X_2X_3) \quad \text{if } \Rightarrow \epsilon\end{aligned}$$

Follow (A) : Gives set of terminals that may immediately follow to the right of A in any sentential form

- Follow (A) is computed using 3 rules
 1. If A is start symbol of G,
 $\text{Follow}(A) = \{\$\}$
 2. If $X \rightarrow \alpha A \beta$ as in G,
 $\text{Follow}(A) = \text{First}(\beta) - \{\epsilon\}$
 3. If $X \rightarrow \alpha A$ or $X \rightarrow \alpha A \beta$ $\{\beta \Rightarrow \epsilon\}$ is in G,
 $\text{Then follow}(A) = \text{follow}(X)$

LL(I) parsing table construction:

For each rule $A \rightarrow \alpha$ in G, apply following steps:

1. For each terminal a in $\text{First}(\alpha)$, Add $A \rightarrow \alpha$ to M [A, a] where M is parsing table.
 2. If $\text{first}(\alpha)$ contains ϵ , add $A \rightarrow \alpha$ to M [A, b] for each b in $\text{follow}(A)$
- To construct an LL(I) parser, there is a restriction on grammar. The grammar has to be left factored and should be free of left recursion.
 - Given a grammar, construct an LL(I) parsing table using the above procedure, If the LL(I) table has no multiple entries, then the grammar is called LL(I)
 - Left recursive grammar cannot be LL(I)

Example 3.3:

Consider the following grammar. Compute FIRST & FOLLOW sets for each non-terminal. Check whether the grammar is LL(I)

$$S \rightarrow ACB / CbB/Ba$$

$$A \rightarrow da / BC; B \rightarrow g/\epsilon; C \rightarrow h/\epsilon$$

Sol: FOLLOW (B) = { a} \cup {#} = { a, #}

FIRST (B) = {g} \cup FOLLOW (B) = {g,a, ϵ }

FIRST (C) = {h} \cup FOLLOW (C)

$$= \{h\} \cup \{b\} \cup \text{FOLLOW}(C)$$

$$= \{h, b, g, a, \epsilon\}$$

FIRST (A) = {d} \cup { FIRST (B) }

$$= \{d, g, a, \epsilon\}$$

FOLLOW (A) = FIRST (C) = { h,b, g, a, ϵ }

FIRST (C) = {b} \cup FIRST (B)

$$= \{g, a, b, \epsilon\}$$

Let us construct the predictive parsing table for the grammar

	a	b	d	g	h	\$
S	S \rightarrow ACB		S \rightarrow ACB	S \rightarrow ACB		S \rightarrow ACB
A		S \rightarrow CbB			S \rightarrow CbB	S \rightarrow CbB
B						
C						
A						

So the grammar is not LL(I)

Example 3.4:

Compute FIRST and FOLLOW for each non-terminal in the grammar,. Check if the grammar is LL(I)

$$S \rightarrow aABb, A \rightarrow c/\epsilon; B \rightarrow d/\epsilon$$

Sol: FIRST (S) = A

FOLLOW (S) = \$

FIRST (B) = {d} \cup FOLLOW (b)

$$= \{ d, b \}$$

FOLLOW (B) = {b}

FIRST (A) = {c} \cup FOLLOW (A)

$$= \{ c, b \}$$

FIRST (A) = {b}

The predictive parsing table, so the grammar is LL (I)

	a	b	d	d	\$
S	S \rightarrow aABb				
A		A \rightarrow ϵ	A \rightarrow c		
B		B \rightarrow ϵ		B \rightarrow d	

Example 3.5:

Construct the LL(i) parsing table and verify whether it is LL(I) or not

S \rightarrow aBDh

B \rightarrow cC

C \rightarrow bC/ ϵ

D \rightarrow EF

E \rightarrow g/ ϵ

F \rightarrow f/ ϵ

Sol: FIRST (S) = { a }, FIRST (B) = c

FIRST (F) = {f} \cup FOLLOW (F)

$$= \{ f \} \cup \text{FOLLOW}(D)$$

$$= \{ f, h \}$$

$$\text{FIRST}(E) = \{g\} \cup \text{FOLLOW}(E)$$

$$= \{g\} \cup \text{FIRST}(F)$$

$$= \{f, g, h\}$$

$$\text{FIRST}(C) = \{b\} \cup \text{FOLLOW}(C)$$

$$= \{b\} \cup \text{FOLLOW}(B)$$

$$= \{b\} \cup \text{FIRST}(D)$$

$$= \{b\} \cup \text{FIRST}(E)$$

$$= \{b, f, g, h\}$$

$$\text{FIRST}(D) = \text{FIRST}(E) = \{f, g, h\}$$

The predictive parsing table

	a	b	c	f	g	h	\$
S	$S \rightarrow aBDh$	--	--				
B			$B \rightarrow c$				
D		$C \rightarrow bC$		$C \rightarrow \epsilon$	$E \rightarrow \epsilon$	$C \rightarrow \epsilon$	
D				$D \rightarrow EF$	$D \rightarrow EF$	$D \rightarrow EF$	
E				$E \rightarrow \epsilon$	$E \rightarrow g$	$E \rightarrow \epsilon$	
F				$F \rightarrow f$		$F \rightarrow \epsilon$	

So the grammar is indeed LL(1)

Example 3.6:

Construct the LL(1) parsing table and verify whether it is LL(1) or not

$$S \rightarrow AaAb/BbBa$$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

$$\text{Sol: } \text{FIRST}(A) = \text{FOLLOW}(A)$$

$$= \{a, b\}$$

$$\text{FIRST}(B) = \text{FOLLOW}(B)$$

$$= \{a, b\}$$

$$\text{FIRST}(S) = \text{FIRST}(A) \cup \text{FIRST}(B)$$

$$= \{a, b\}$$

The Predictive parsing table

Example 3.7:

Construct the LL(I) parsing table and verify whether it is LL(I)

$$S \rightarrow 1AB/\epsilon$$

$$A \rightarrow IAC/OC$$

$$B \rightarrow OS$$

$$C \rightarrow 1$$

Sol: FOLLOW(S) = \$

$$\text{FIRST}(S) = \{I, \$\}$$

$$\text{FIRST}(A) = \{0, 1\}$$

$$\text{FIRST}(B) = \{0\}$$

$$\text{FIRST}(C) = \{1\}$$

The predictive parsing table

	0	1	\$
S		$S \rightarrow 1AB$	$S \rightarrow \epsilon$
A	$A \rightarrow OC$	$A \rightarrow 1AC$	
B	$B \rightarrow OS$		
C		$C \rightarrow 1$	

The grammar is LL(I)

Example 3.8:

Check whether the following grammar is LL (I)

$$S \rightarrow A$$

$$A \rightarrow aB/Ad$$

$$B \rightarrow bBC/f$$

$$C \rightarrow g$$

Sol: The left recursive rule $A \rightarrow Ad$ makes the grammar unsuitable for top down parsing. So the grammar is not LL (I)

Example 3.9:

Convert the following grammar to an unambiguous grammar. Check whether the resulting grammar LL(I)

$$R \rightarrow R + R/R \quad R^*/(R) / a/b$$

Sol: We will have the order of precedence $+, -, #$ An unambiguous grammar is

$$R \rightarrow R + T/T$$

$$T \rightarrow TF/F$$

$$F \rightarrow F^*/(R)/a/b$$

Factoring yields

$$R \rightarrow T(+T)$$

$$T \rightarrow F(F)$$

$$F \rightarrow F^*/(R)/a/b$$

Converting the BNF to standard CFLs

$$R \rightarrow TA$$

$$A \rightarrow \epsilon/+TA$$

$$T \rightarrow FB$$

$$B \rightarrow \epsilon/FB$$

$$F \rightarrow F^*/(R)/a/b$$

Removing the left recursive F – rule

$$R \rightarrow TA$$

$$A \rightarrow \epsilon/+TA$$

$$T \rightarrow FB$$

$$B \rightarrow \epsilon/FB$$

$$F \rightarrow (R) Z/aZ/bZ$$

$$Z \rightarrow \epsilon/*Z$$

Computing the FIRST and FOLLOW sets

$$\begin{aligned}
 \text{FIRST (F)} &= \{ (, a, b) \} \\
 \text{FIRST (T)} &= \text{FIRST (F)} = \{ (, a, b) \} \\
 \text{FIRST (R)} &= \text{FIRST (T)} = \{ (, a, b) \} \\
 \text{FIRST (A)} &= \{ + \} \cup \text{FOLLOW (A)} \\
 &= \{ +, \#,) \} \\
 \text{FIRST (B)} &= \text{FIRST (F)} \cup \text{FOLLOW (B)} \\
 &= \text{FIRST (F)} \cup \text{FOLLOW (T)} \\
 &= \text{FIRST (F)} \cup \text{FIRST (A)} \\
 &= \{ (, A, B, +, \#) \}
 \end{aligned}$$

The predictive parsing table is

The above is LL (I)

Example 3.10:

Check whether the following grammar is LL (I)

$$S \rightarrow I E t SSI / a$$

$$SI \rightarrow \epsilon / eS$$

$$E \rightarrow b$$

Sole : $\text{FIRST (E)} = b$

$$\text{FIRST (S)} = \{ i, a \}$$

$$\text{FIRST (SI)} = \{ e \} \cup \text{FOLLOW (SI)} = \{ e, \#, i \}$$

The predictive parsing table is

	i	t	e	a	b	#
S	$S \rightarrow iEiSSI$			$S \rightarrow a$		
SI			$SI \rightarrow Es$		$SI \rightarrow \epsilon$	
E			$SI \rightarrow \epsilon$		$E \rightarrow b$	

The grammar is not LL (I)

Example 3.11:

Check whether the following grammar is LL (I)

$$E \rightarrow aE / (E)$$

$$A \rightarrow +E / *E/\epsilon$$

Sole : FIRST (E) = { a, (}

$$\text{FIRST}(A) = \{+, *\} \cup \text{FOLLOW}(A)$$

$$= \{+, *, \#,)\}$$

The predictive parsing table is

	a	+	*	()	#
E	$E \rightarrow aA$			$E \rightarrow (E)$		
A			$A \rightarrow +E$	$A \rightarrow *E$		$A \rightarrow \epsilon$

The grammar is LL (I) as there is no conflict in the table.

Example 3.12:

Check whether the following grammar is LL (I)

$$S1 \rightarrow S\#$$

$$S \rightarrow aAa/\epsilon$$

$$A \rightarrow abS/c$$

Sole : FOLLOW(S) = { # }

$$\begin{aligned}\text{FIRST}(S) &= \{a\} \cup \text{FOLLOW}(S) \\ &= \{a, \#\}\end{aligned}$$

The predictive parsing table follows

	a	b	c	#
S1	$S1 \rightarrow S\#$			$S1 \rightarrow S\#$
S	$S \rightarrow aAa$			$S \rightarrow \epsilon$
A	$A \rightarrow abS$		$A \rightarrow c$	

The grammar is LL (I)

Example 3.13:

Check whether the following grammar is LL (I)

$$\begin{aligned} S &\rightarrow A\# \\ A &\rightarrow Bb/Cd \\ B &\rightarrow aB/\epsilon \\ C &\rightarrow cC/\epsilon \end{aligned}$$

Sole : FOLLOW(B) = { b }
 FOLLOW(C) = { d }
 = { a, # }

The predictive parsing table is

	a	b	c	d	#
S	$S \rightarrow A\#$	$S \rightarrow A\#$	$S \rightarrow A\#$	$S \rightarrow A\#$	
A	$A \rightarrow Bb$	$A \rightarrow Bb$	$A \rightarrow Cd$	$A \rightarrow Cd$	
B	$B \rightarrow aB$	$B \rightarrow \epsilon$			
C			$C \rightarrow cC$	$C \rightarrow \epsilon$	

The grammar is LL (I)

Example 3.14:

Can a LL(I) parser be constructed for the following grammar

$$\begin{aligned} S &\rightarrow xSx / Tx \\ T &\rightarrow Tx / x \end{aligned}$$

Sole : The rule $T \rightarrow Tx / x$ is left recursive, so a LL (1) parser cannot be constructed for the grammar.

Example 3.15:

Can a LL(I) parser be constructed for the following grammar

$$\begin{aligned} S &\rightarrow (S) \\ S &\rightarrow SS / () \end{aligned}$$

Sole : The grammar is left recursive and so is unsuitable for LL(I) parsing technique.

Dangling else problem

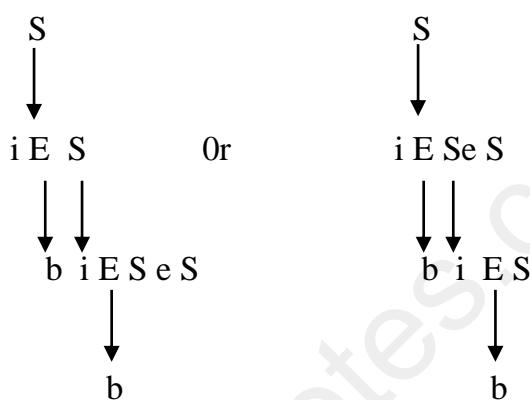
- 3.26. Consider a HLL which allows both the if & if else statement
 $\langle \text{statement} \rangle ::= \text{if } \langle \text{expression} \rangle \text{ then } \langle \text{statement} \rangle \mid \text{if } \langle \text{expression} \rangle \text{ then } \langle \text{statement} \rangle \text{ else } \langle \text{statement} \rangle$
 $\langle \text{statement} \rangle ::= a$

The above can be encoded in C as

$$S \rightarrow i E S \mid I E Se S \mid a$$

$$E \rightarrow b$$

Consider the construct $i E I E Se S$. This has two derivation trees in the grammar.



This is called the dangling else problem

The grammar is not LL(1) as there are two choices for S

- 3.27. $S \rightarrow i E S S^1$

$$S^1 \rightarrow e S / \epsilon$$

$$E \rightarrow b$$

➤ FIRST & FOLLOW Sets

$$\text{FIRST}(S) = \{ i \}$$

$$\text{FIRST}(E) = \{ b \}$$

$$\text{FIRST}(S^1) = \{ e \} \cup \text{FOLLOW}(S^1)$$

$$= \{ e \} \cup \{ \# \}$$

$$= \{ e, \# \} \setminus$$

- The predictive parsing table

	i	e	b	#
S	$S \rightarrow iESS$			
S^1		$S^1 \rightarrow eS$ $S^1 \rightarrow \epsilon$		$S^1 \rightarrow \epsilon$
E	Conflict		$E \rightarrow b$	

Bottom Up Parsers

- Bottom up parsers are also known as SR parsers.
- There are two types of Bottom up parsers.
 1. Operator precedence parsers
 2. LR parsers.

Operator Precedence Parsing

- **Operator precedence parser :** To design an operator precedence parser, the grammar
- **Operator grammar:** A grammar that does not contain any ϵ -rules and does not contain two adjacent non-terminals on right side of any production is called operator grammar

Ex: $E \rightarrow E AE / id$; $A \rightarrow +/*/-//$ is not operator grammar equivalent operator grammar is

Ex: $E \rightarrow E+E | E*E | E-E | E/E | id$

- **Parsing algorithm:** If a is top of stack & b is symbol pointed by input pointer.
 1. If $a = b = \$$ successful completion of parsing
 2. If $a < b$ or $a = b$ then push b onto stack and advance input pointer to next symbol.
 3. if $a >$ then repeat pop stack until the top of stack is related by $<$ to the terminal most recently popped.

3.28 An operator grammar should not have ϵ productions. An operator grammar should not have two adjacent non-terminals in the right hand side of any rule.

3.29 Operator grammar for expressions

$$E \rightarrow E+E \mid E^*E \mid (E) \mid id$$

3.30 Operator Precedence Table

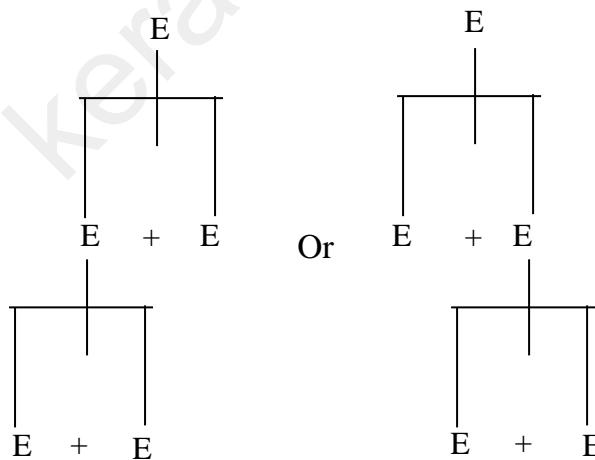
Though an operator grammar may be ambiguous, a deterministic shift, reduce parser can be constructed for it. Conflicts resulting in ambiguity are resolved using the Associativity and precedence of operators.

3.31 In the reverse of a rightmost derivation, the string (sentential form) that is replaced by the left hand side of the rule is called the handle.

3.32 Construction

	+	*	()	Op	#
+						
*						
(
)						
id						

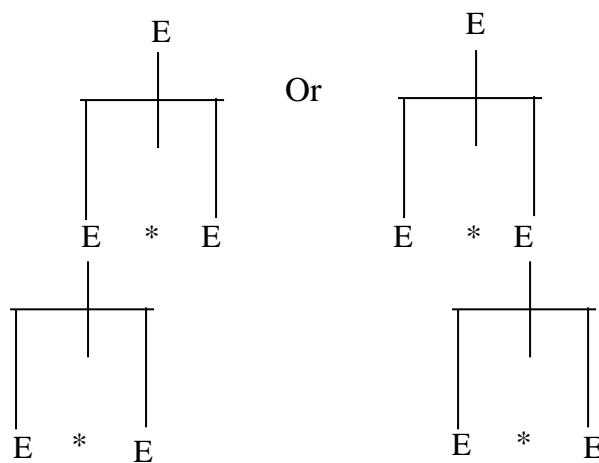
- $E \Rightarrow E+E \Rightarrow E+E+E$



The rule of a language like C say + is left associative. The first tree should be chosen.

So, $+ \bullet > +$

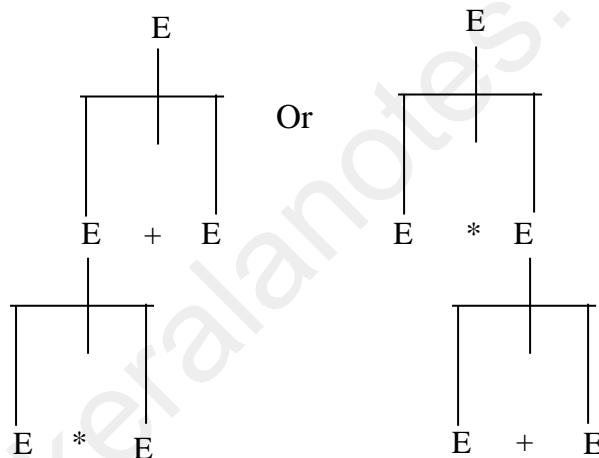
- $E \Rightarrow E + E \Rightarrow E * E * E$



The rule of a language like C demand that * be left associative. So the second of the trees invalid. So, $\# \cdot > *$

- $E \Rightarrow E + E \Rightarrow E + E * E$ or $E \Rightarrow E * E \Rightarrow E + E * E$

The above derivations yield the derivation trees.

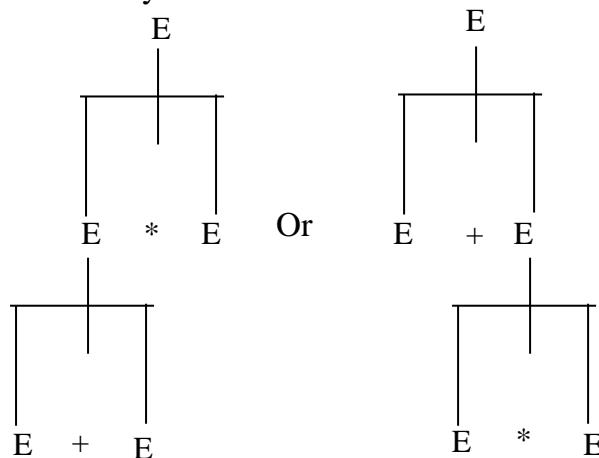


The rules of a language like C require a higher precedence for * than +, So the second tree valid.

So, $+ < \cdot *$

- $E \Rightarrow E * E \Rightarrow E * E + E$ or $E \Rightarrow E + E \Rightarrow E * E + E$

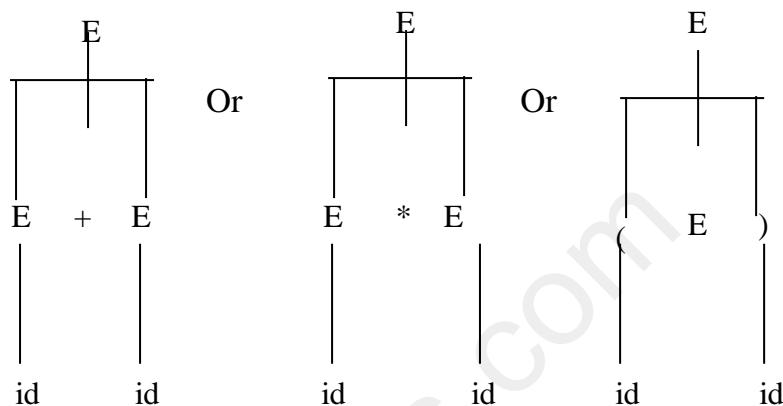
The above derivations yield the derivation trees.



The rules of a language like C require a higher precedence for * than +, So the second tree valid.

So, $* > +$

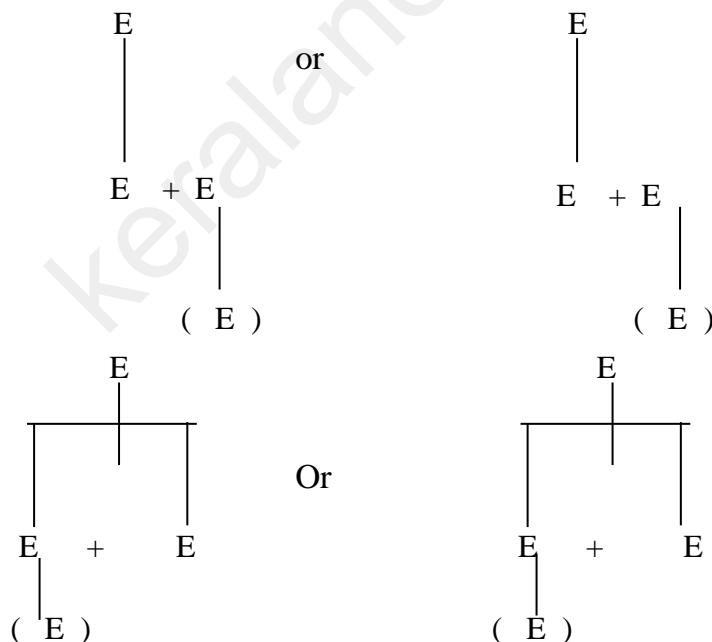
- The rules of a language like C do not allow the string id(),
So id no relation id, id no relation ()
- Also there is only one end of file symbol.
So, # no relation #
- The trees



Yield the relations

$\text{id} \rightarrow +, \text{id} \rightarrow *, \text{id} \rightarrow)$

- Consider the trees



The above trees yield the relations

$+ < ., * < .(,) > +,) > *$

- The operator precedence table

	id	+	*	()	#
id	-	•>	•>	-	-	•>
+	< •	•>	< •	< •	•>	•>
*	< •	•>	•>	< •	•>	•>
(< •	< •	< •	< •	=0	-
)	-	•>	•>	-	•>	•>
#	< •	< •	< •	< •	-	-

Example 3.16:

Convert the following grammar to an operator grammar

$$S \rightarrow SAS/a$$

$$A \rightarrow bSb/b$$

Sole : Substitution the A productions

$$S \rightarrow SbSbS/SbS/a$$

$$A \rightarrow bSb/b$$

The above grammar is an operator grammar.

Example 3.17:

Convert the following grammar to an operator grammar

$$P \rightarrow SR/S$$

$$R \rightarrow bSR/bS$$

$$S \rightarrow WbS/W$$

$$W \rightarrow L^*W \mid L$$

$$L \rightarrow id$$

Sole: Only the rule $P \rightarrow SR$ violates the operator grammar form. Substituting the R-Production.

$$P \rightarrow SbSR/SbS/S$$

$$R \rightarrow bSR/bS$$

$$S \rightarrow WbS/W$$

$$W \rightarrow L^*W \mid L$$

$$L \rightarrow id$$

Though not essential we can eliminate the unit productions

$$\begin{aligned}
 P &\rightarrow SbSR/SbS/S \\
 R &\rightarrow bSR/bS \\
 S &\rightarrow WbS/L^*W/L \\
 W &\rightarrow L^*W/id \\
 L &\rightarrow id
 \end{aligned}$$

Or

$$\begin{aligned}
 P &\rightarrow SbSR/SbS/S \\
 R &\rightarrow bSR/bS \\
 S &\rightarrow WbS/L^*W/id \\
 W &\rightarrow L^*W/id \\
 L &\rightarrow id
 \end{aligned}$$

Or

$$\begin{aligned}
 P &\rightarrow SbSR/SbS/WbS / L^*W/id \\
 R &\rightarrow bSR/bS \\
 S &\rightarrow WbS/L^*W/id \\
 W &\rightarrow L^*W/id \\
 L &\rightarrow id
 \end{aligned}$$

Example 3.18:

Prepare the precedence table and parse tree for $id * id b id^* id S$

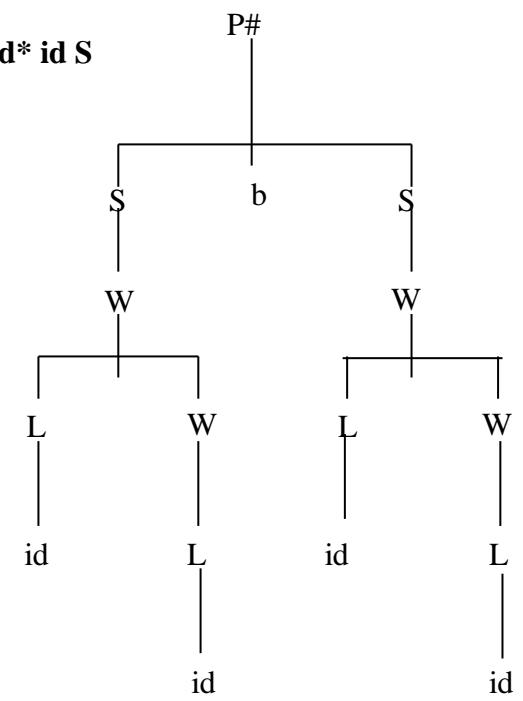
Slo:

$$\begin{aligned}
 P &\rightarrow SbSR/SbS/R \\
 R &\rightarrow bSR/bS \\
 S &\rightarrow WbS/W \\
 W &\rightarrow L^*W | L \\
 L &\rightarrow id
 \end{aligned}$$

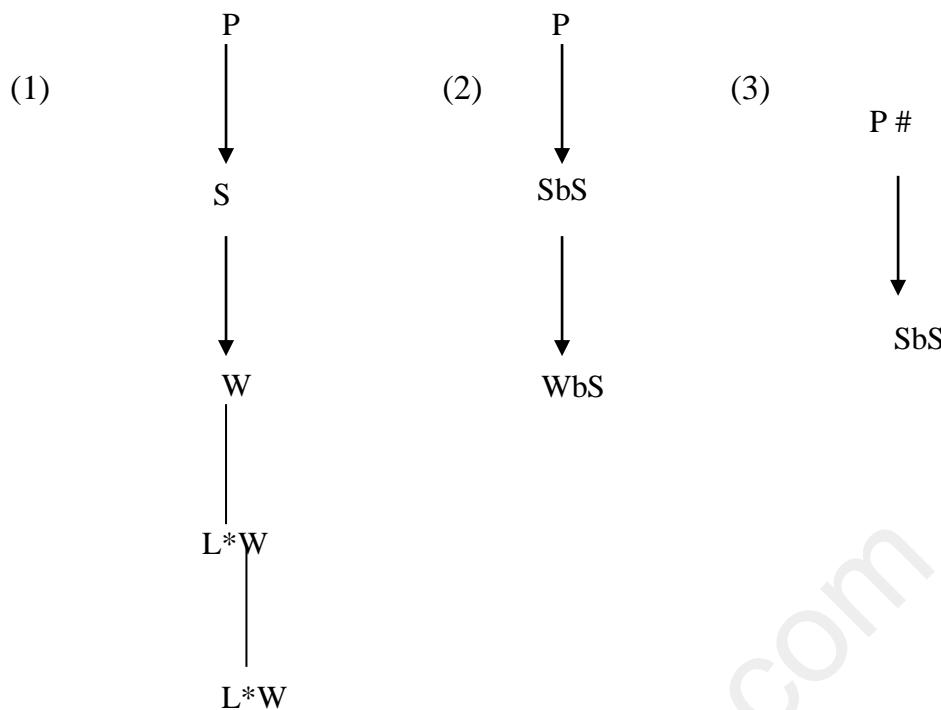
The parse tree

The precedence table

	b	*	Id	#
b	•>	< •	< •	•>
*	•>	•>	< •	•>
Id	•>	•>		•>
#				



To obtain the above precedence table we consider a number of syntax trees



The LR Parsing Techniques

- **LR Parsers :** are of three types
 1. SLR (1)
 2. LALR(1)
 3. CLR (1)
- **SLR (1) :** Simple LR (I) parser is constructed with LR(O) items
- **LALR (I) & CR (I) :** Look ahead LR(I) and canonical LR (I) parsers are constructed with LR(I) items.
- **Parsing algorithm:** If s is state on top of stack and is a symbol pointed by input pointer, initially push 0 (initial state of DFA) on top of stack and advance input pointer to next symbol.
 1. if action $[s,a] = s_i$ push i onto stack and advance input pointer to next symbol.
 2. if action $[s, a] = r_i$ if r_i is $A \rightarrow \beta$, pop off $2x / \beta$ / symbol from stack replace by
 - A. if s^1 is state below, a then push go to $[s^1, A]$ on to stack. Output production $A \rightarrow \beta$,
 3. if action $[s,a] = \text{accept}$, successful completion
 4. if action $[s, a] = \text{blank}$, error.

- **SLR (1) parsing table construction:**
 1. Create canonical collection of LR (0) items
 2. Draw DFA and convert to SLR (I) table
- To create canonical collection of LR (0) items, we require two functions closure () & Goto ()
 1. Create canonical collection of LR (0) items

Closure (I) : gives set of items by applying following two rules

1. Initially add every item from 1 to closure I
2. if $A \rightarrow \alpha. B\beta$ is in I, then add $B \rightarrow r$ to closure I. Repeat this step for every newly added item until no more items can be added.

Ex: $E \rightarrow E + T$ closure of ($E^1 \rightarrow .E$) is

/T	$I_0 E^1 \rightarrow .E$
$T \rightarrow T^* F$	$E \rightarrow .E + T/.T$
/F	$T \rightarrow .T^* F/.F$
$F \rightarrow id$	$F \rightarrow .id$

Goto (I): Closure of set of all items $A \rightarrow \alpha x .\beta$ such that $A \rightarrow \alpha x \beta$ is in I.

Ex: Goto (I_0, E) in above example is

$$E^i \rightarrow E; \quad E \rightarrow E. + T$$

- **Constructing SLR Parsing table:**

1. Construct canonical collection of set of items $C = \{ I_0, I_1, \dots, I_a \}$ for G^1 .
 2. State I in table is constructed from each I_i in C. parsing actions for state I are given by.
 - a) if $\text{goto}(I_i a) = I_j$, then set action $[i, a] = S_j$
 - b) if $\text{goto}(I_i A) = I_j$, then set action $[i, A] = j$
 3. If $A \rightarrow \alpha$ is in I_i , then set action (I_i, a) to reduce by $A \rightarrow \alpha$ for all a in follow (A)
 4. if $S^1 \rightarrow S.$ is in I_i , then set action $[i, \$]$ to accept.
- The same procedure is followed for constructing parsing tables LALR (I) & CLR (I) but with LR(I) items.
 - The main difference between SLR (I) & CLR (I) lies in reduce entries.

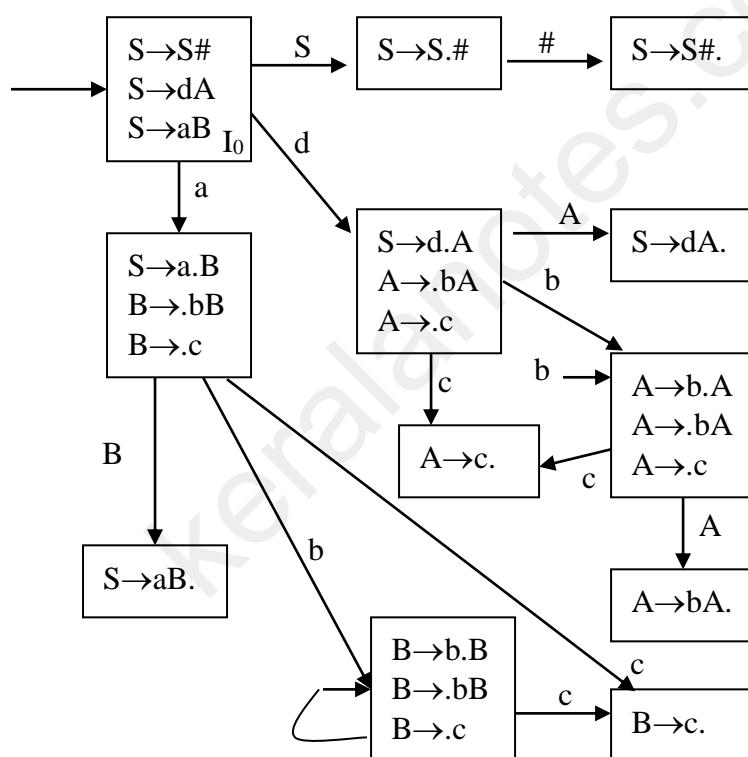
- In LALR (I) & CLR (i) Tables whenever a state I_i contains LR(I) item $A \rightarrow a$, $\$/a/b$
- $LR(0) \subseteq SLR(I) \subseteq LALR(I) \subseteq CLR(I)$
- Every $LR(0)$ grammar is $SLR (I)$

Example 3.19

Check whether the following grammar is $LR(0)$

$$\begin{aligned} S &\rightarrow S \# \\ S &\rightarrow dA / aB \\ A &\rightarrow bA/c \\ B &\rightarrow bB/c \end{aligned}$$

Sol: The $LR(0)$ machine is constructed



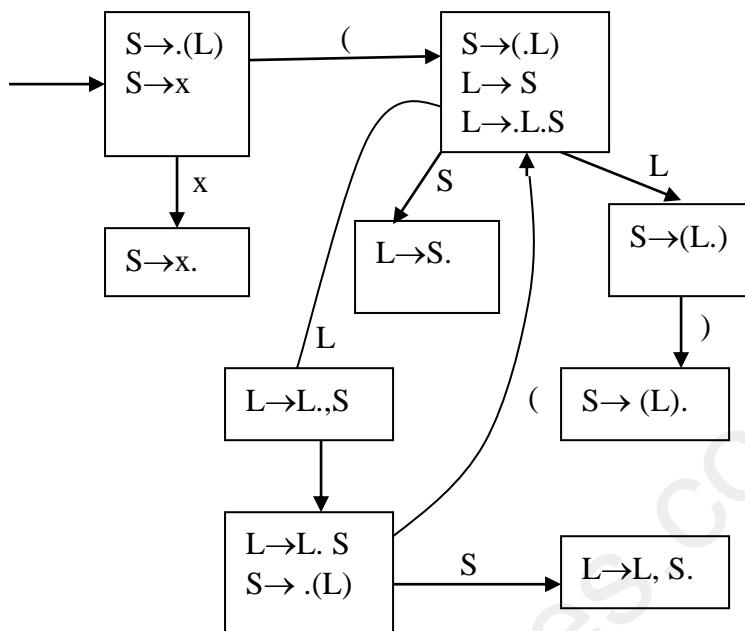
The grammar is $LR(0)$

Example 3.20

Check whether the following grammar is LR(0)

$$S \rightarrow (L), S \rightarrow x, L \rightarrow S, L \rightarrow L, S$$

Sol: The LR(0) machine follows



The grammar is LR(0)

Example 3.21

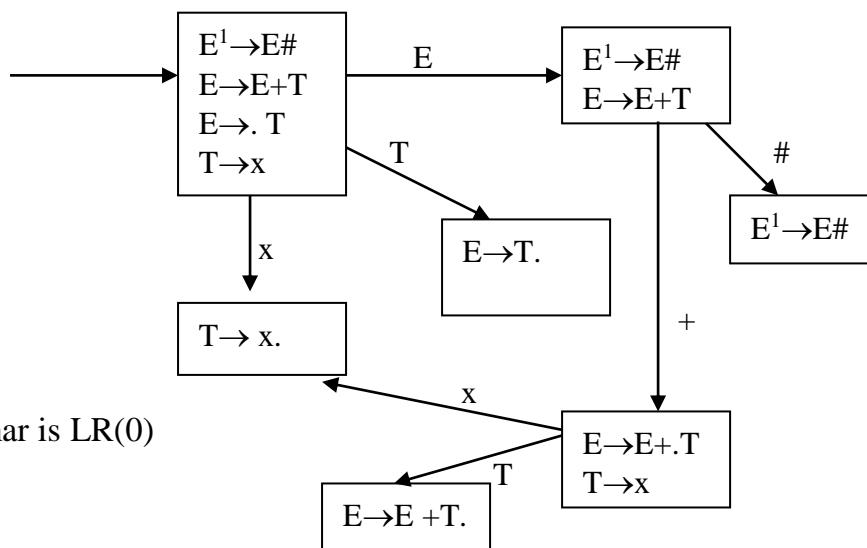
Check whether the following grammar is LR(0)

$$E \rightarrow E \#$$

$$E \rightarrow E + T \mid T$$

$$T \rightarrow x$$

Sol: The LR(0) machine is constructed



The grammar is LR(0)

Example 3.22

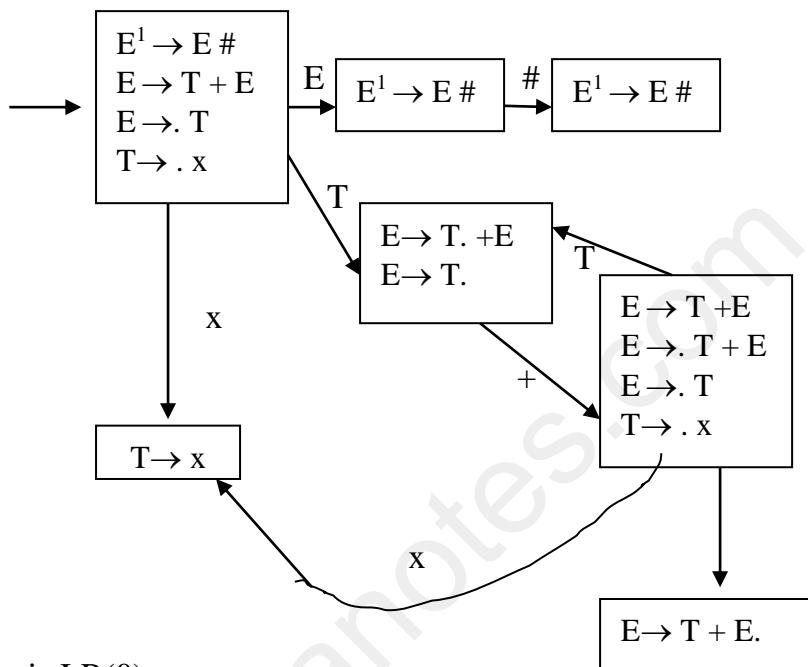
Check whether the following grammar is LR(0), SLR(I)

$$E^1 \rightarrow E \#$$

$$E \rightarrow T + E/T$$

$$T \rightarrow x$$

Sol: construct the LR(0) machine



The grammar is LR(0)

$\text{FOLLOW}(e) = \{\#\}$

So we can use this to resolve the inadequate state. So the grammar is SLR (I)

Example 3.23

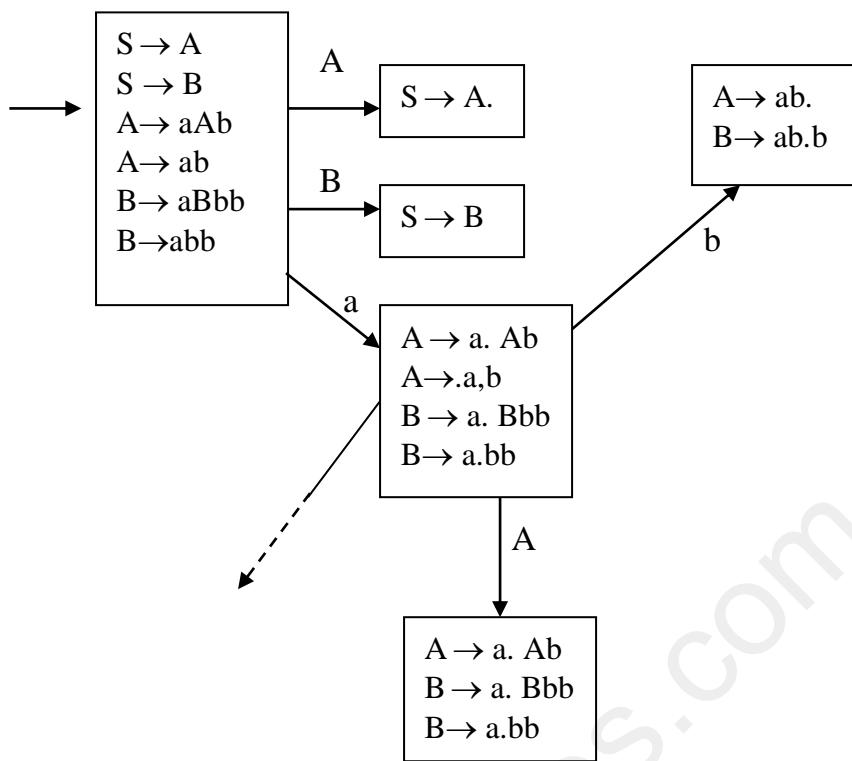
Check whether the following grammar is LR(0), SLR(1)

$$S \rightarrow A/B$$

$$A \rightarrow a A b/ab$$

$$B \rightarrow a B b b/abb$$

Sol: Construct LR(0) machine



The grammar is not LR(0). As the incoming symbol 'b' as the FOLLOW symbol cannot resolve a conflict the grammar is not SLR (I)

Example 3.24

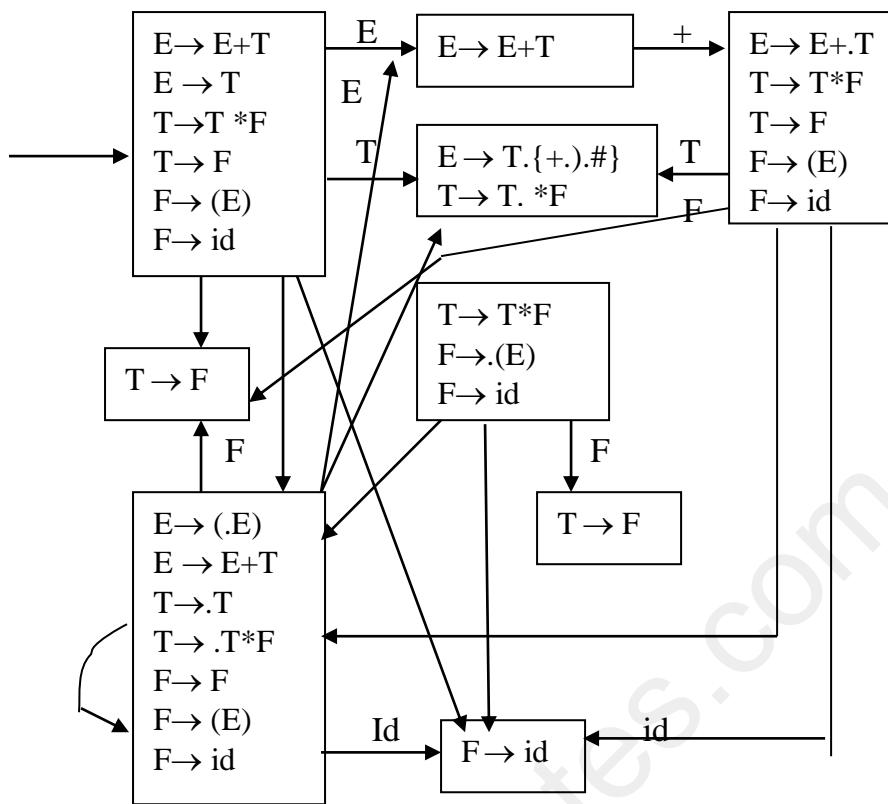
Check whether the following grammar is LR(0), SLR(1)

$$E \rightarrow E + T / T$$

$$T \rightarrow T * F / F$$

$$F \rightarrow (E) / \text{id}$$

Sol: Construct the SLR (I) machine



The grammar is SLR (i) & not LR (0)]

Example 3.25

Check whether the following grammar is LR(0), SLR(1)

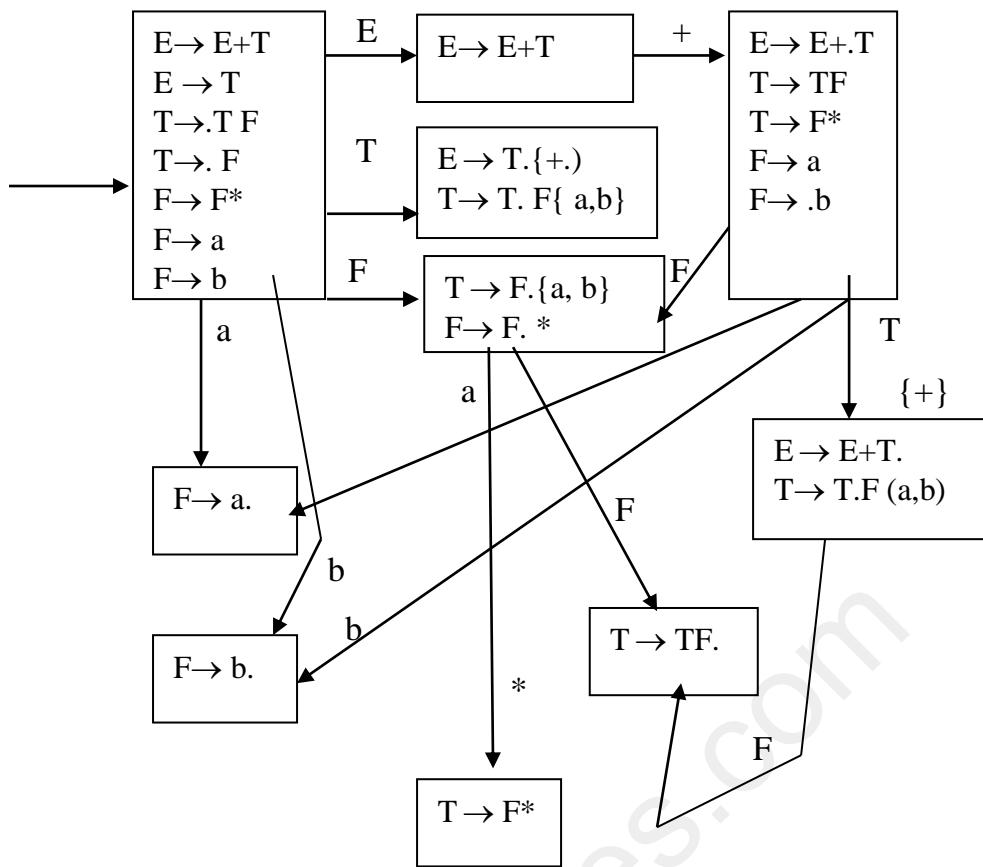
$$E \rightarrow E + T / T$$

$$T \rightarrow T \ F / F$$

$$F \rightarrow F^* / a / b$$

Sol: This is a grammar for regular expression which is SLR (i) & not LR (0) as shown earlier.

The LR (0) & SLR (I) machines follow



The grammar is not LR (0) but the inadequate states can be resolved by the FOLLOW symbols, So the grammar is SLR (1)

Example 3.26

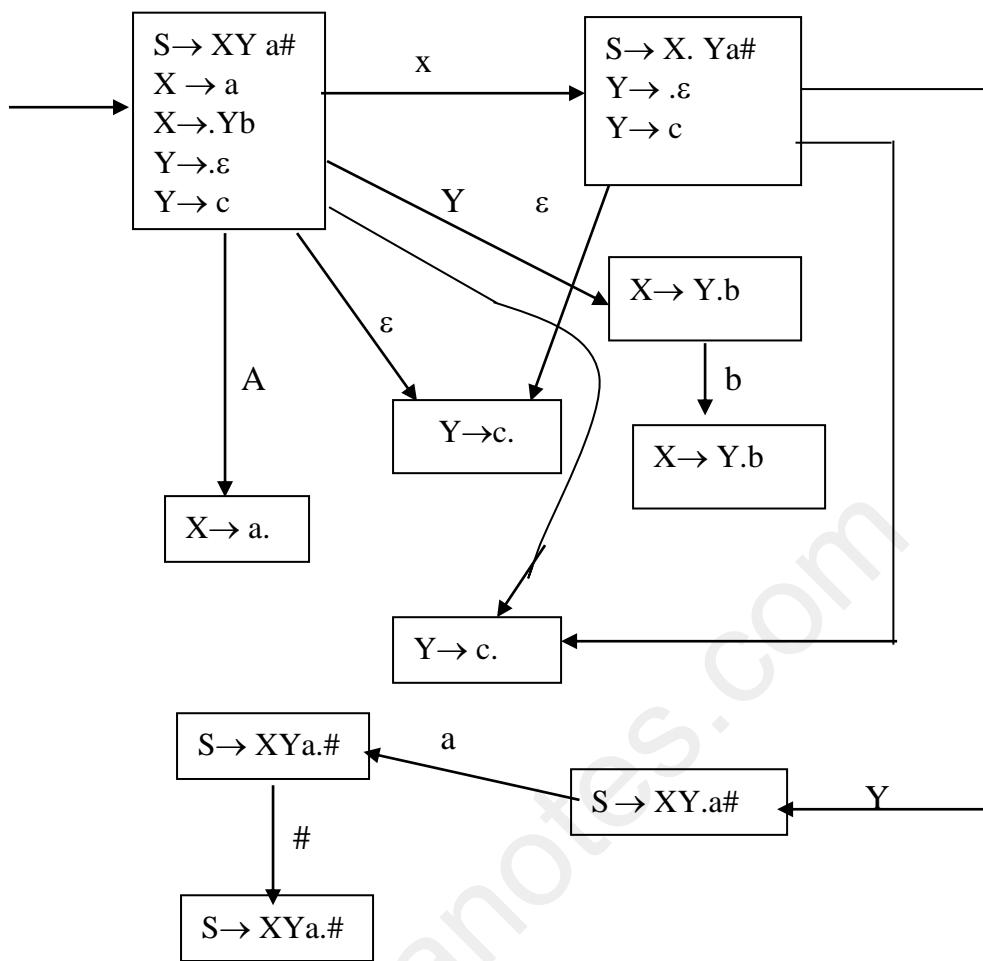
Check whether the following grammar is SLR(1)

$$S \rightarrow XY \text{ a}\#$$

$$X \rightarrow a / Yb$$

$$Y \rightarrow \epsilon / c$$

Sol: The LR (0) machine



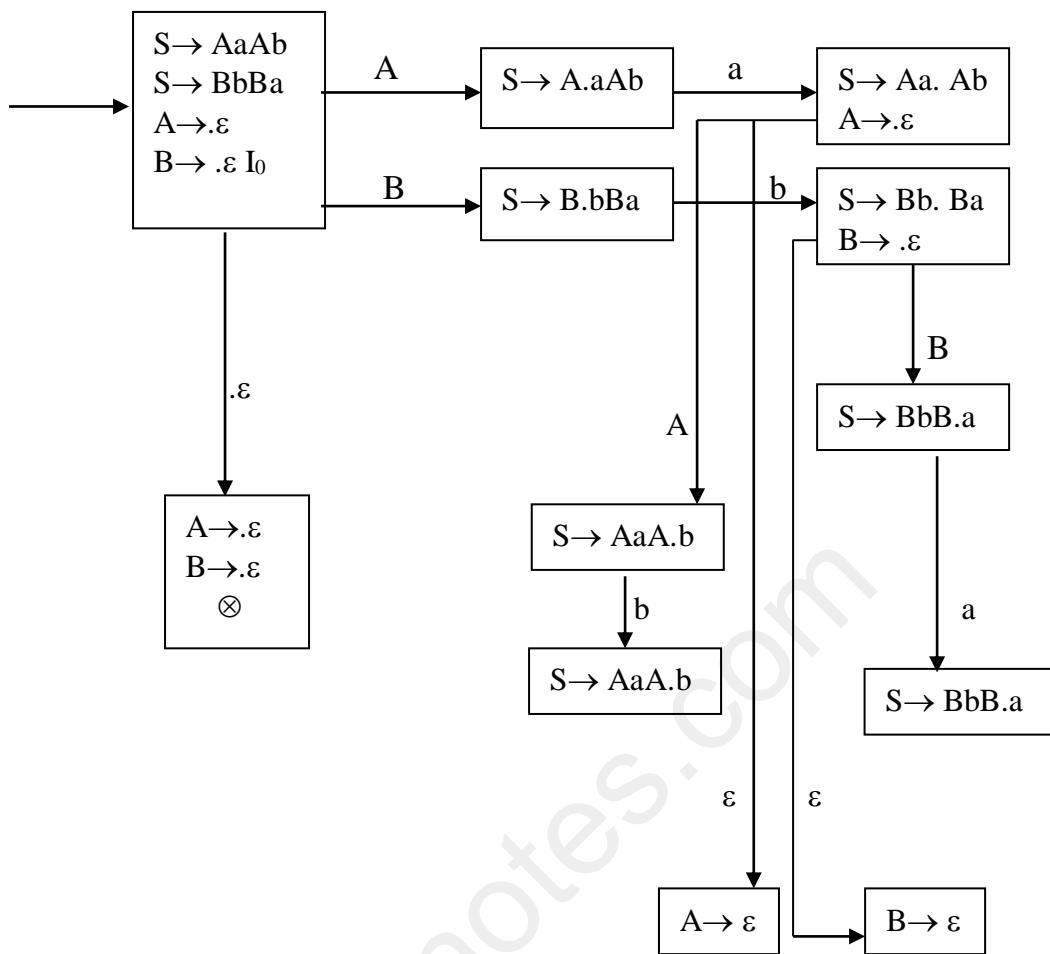
The grammar is SLR (1)

Example 3.27

Check whether the following grammar is LR (0), SLR (1), LALR(1) & LR (1)

$$S \rightarrow aAaB, \quad S \rightarrow BbBa, \quad A \rightarrow \varepsilon. \quad B \rightarrow \varepsilon$$

Sol: the LR (0) machine



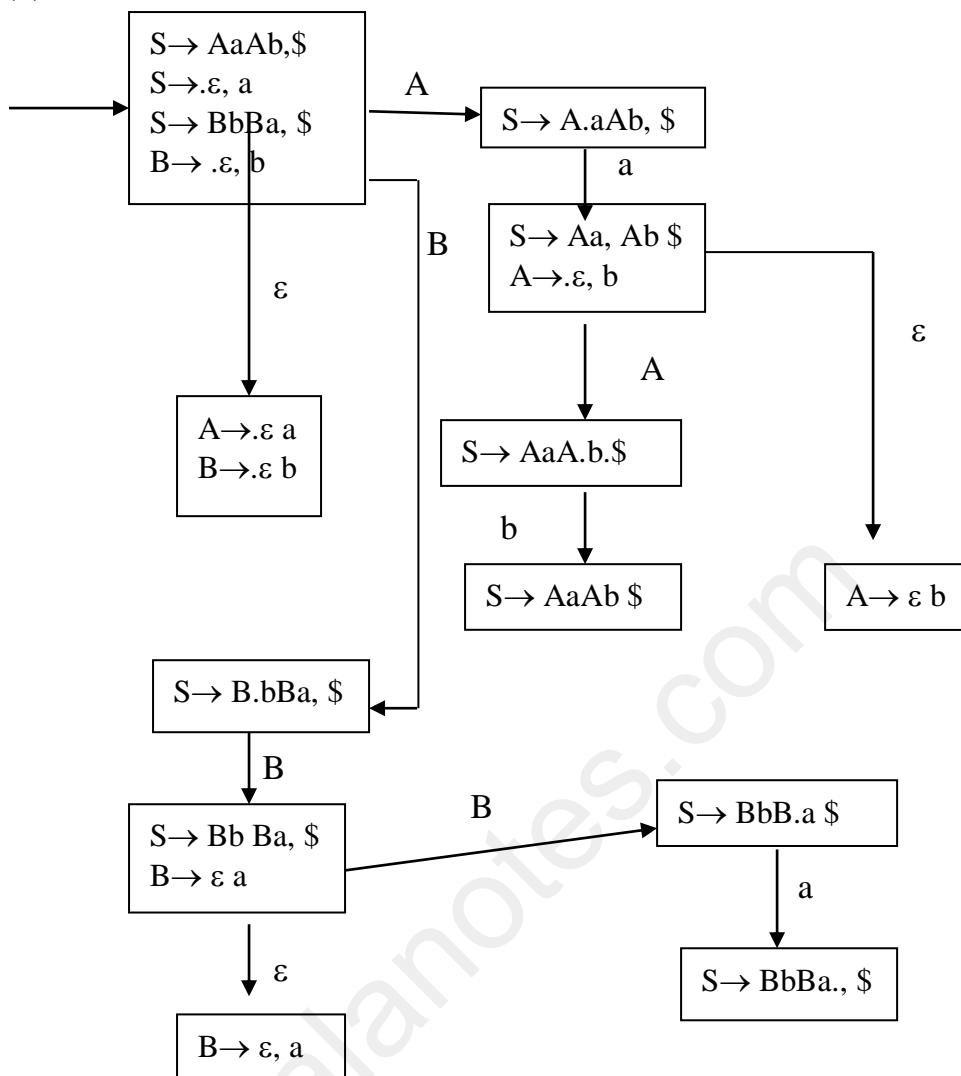
The grammar is not LR (0) as \otimes in an inadequate state.

$$\text{FOLLOW}(A) = \{a, b\}$$

$$\text{FOLLOW}(B) = \{a, b\}$$

So we cannot resolve the state \otimes based on the FOLLOW symbol. So the grammar is not LR (0)

The LR (1) machine



Example 3.28

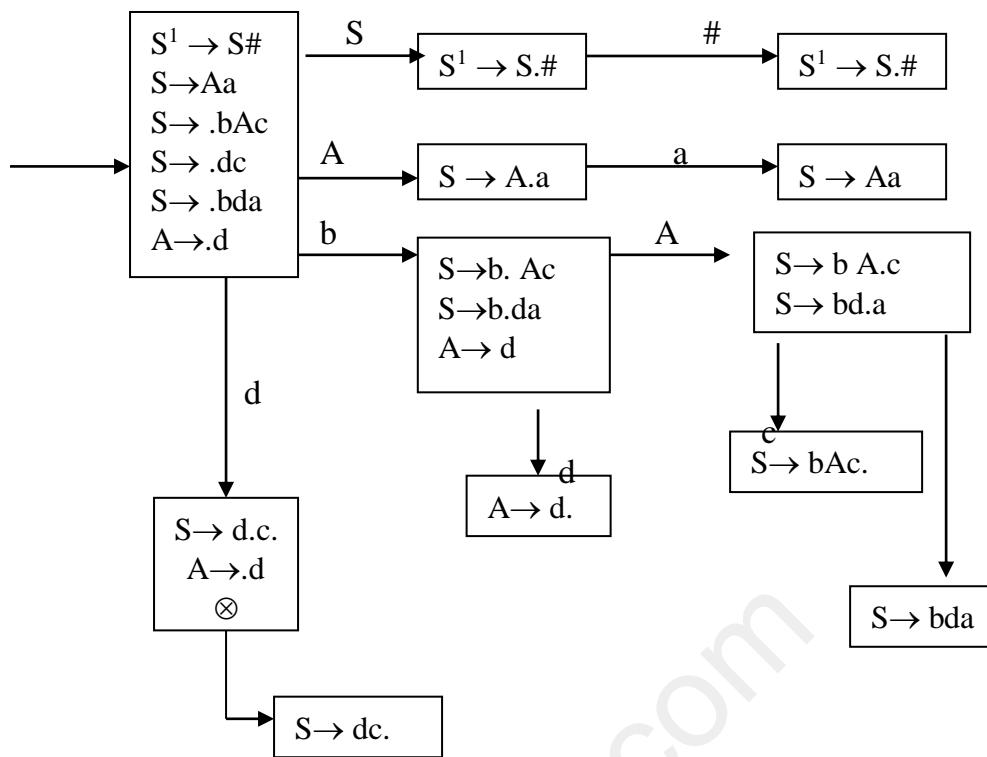
Check whether the following grammar is LR (0), SLR (1), LALR(1) & LR (1)

$$S^1 \rightarrow S \#$$

$$S \rightarrow Aa / bAc / d / bda$$

$$A \rightarrow d$$

Sol: the LR (0) machine



As there is a shit / reduce conflict in \otimes the grammar is not LR (0)

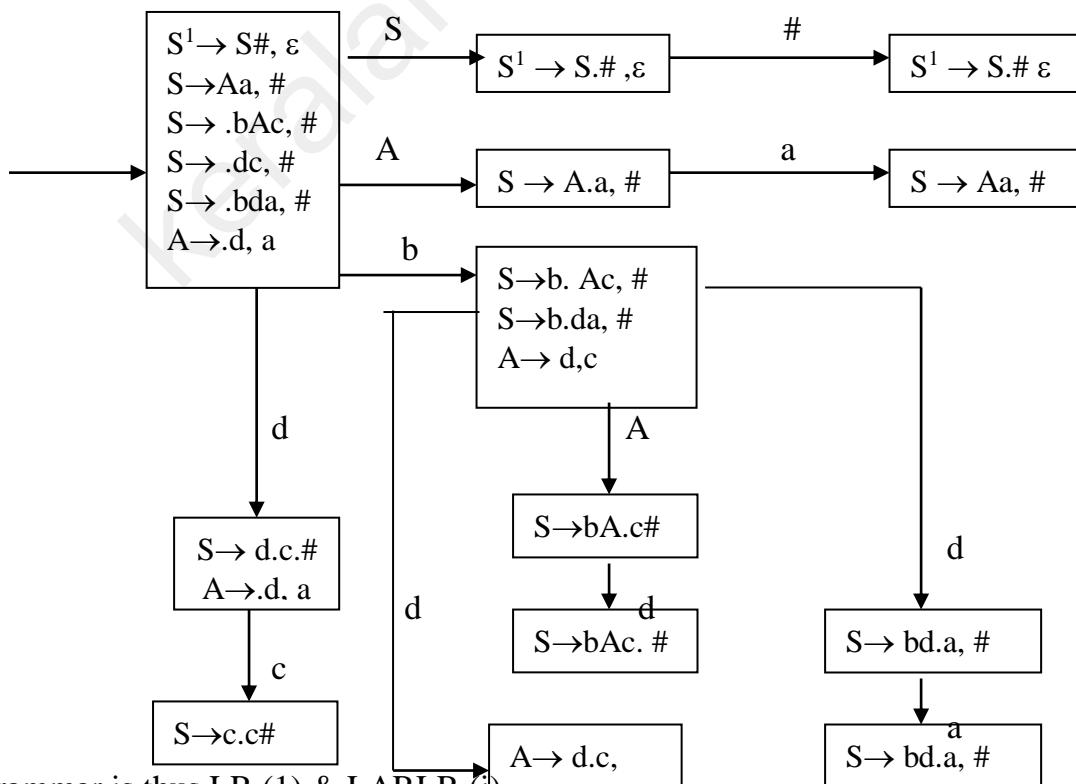
$\text{FOLLOW}(S) = \{ \# \}$

$\text{FOLLOW}(A) = \{ a, c \}$

So we cannot resolve the conflict in state \otimes based on the follow symbol.

So the grammar is not SLR (1)

The LRLR (1) machine for the grammar



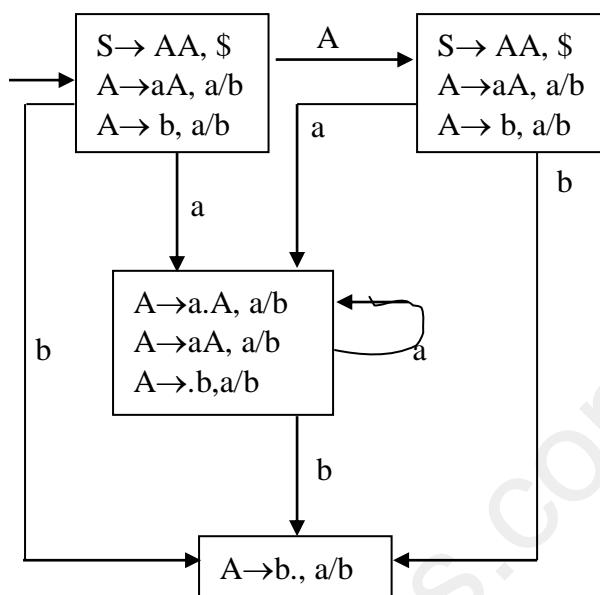
The grammar is thus LR (1) & LARLR (i)

Example 3.29

Check whether the following grammar is LALR(1) & LR (1)

$$S \rightarrow AA, A \rightarrow a A/b$$

Sol: the LR (1) machine



Example 3.30

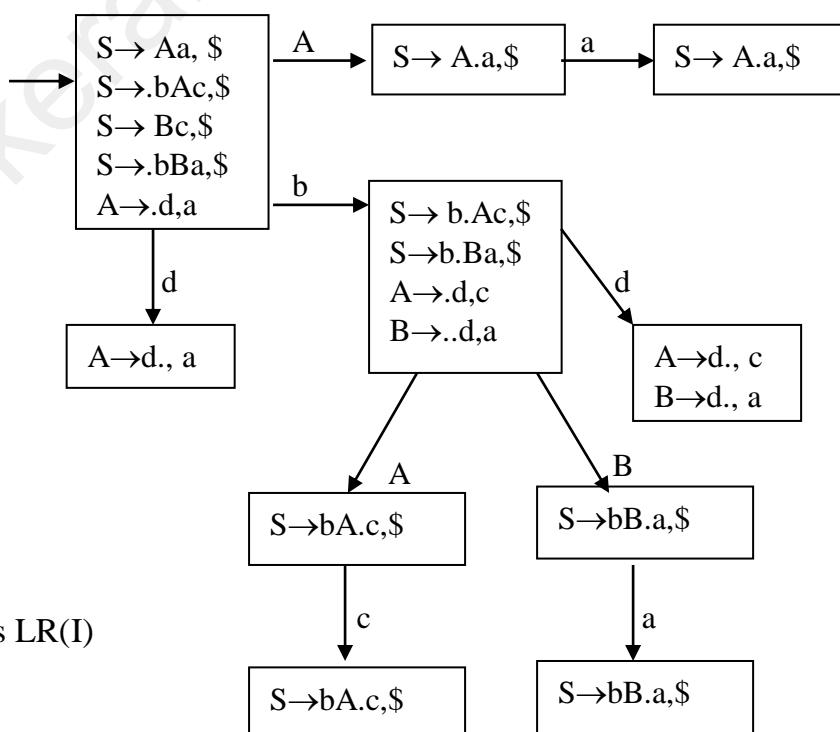
Check whether the following is LR (1)

$$S \rightarrow Aa/b Ac/ Bc / b Ba$$

$$A \rightarrow d$$

$$B \rightarrow d$$

Sol: the LR (1) machine for the grammar



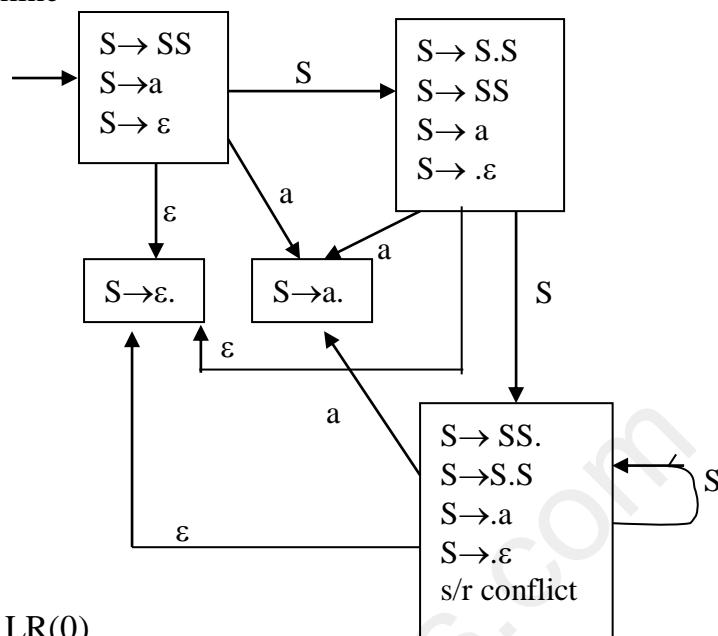
The grammar is thus LR(I)

Example 3.31

Check whether the following grammar is LR(0), SLR(1), LALR(1), LR(1)

$$S \rightarrow SS/a/\epsilon$$

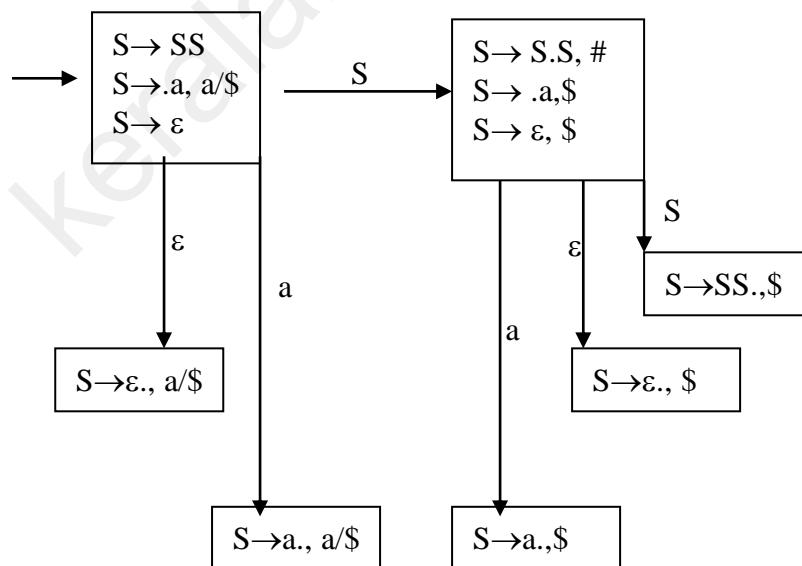
Sol: the LR (0) machine



The grammar is not LR(0)

FOLLOW(S) is {a, #} & does not resolve the inadequate state so the grammar is not SLR(1)

The LR (1) machine



It is not clear whether to use $S \rightarrow a$ or $S \rightarrow \epsilon$ as the look ahead symbols are the same. The grammar is neither LR(1) nor LALR(1). The grammar is ambiguous & no ambiguous grammar can be LR

Example 3.32

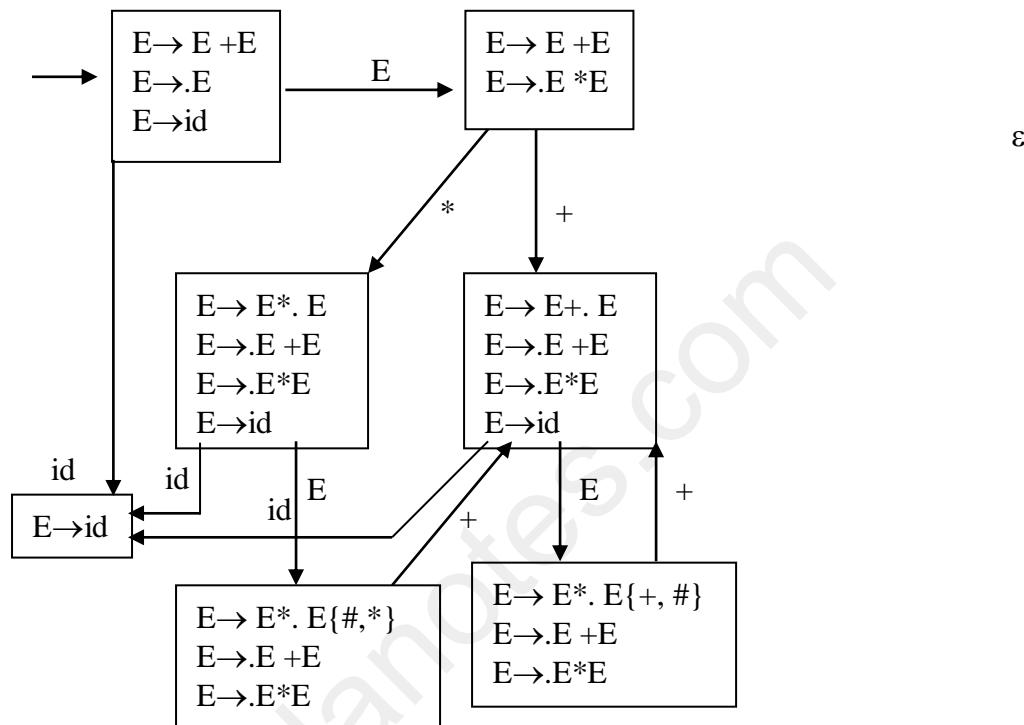
Design SLR Parser

$$E \rightarrow E + E$$

$$/ E * E$$

/ id

Sol: Construct the LR (0) machine & use follow symbols to resolve conflicts



In the case of

$$E \rightarrow E^* E^* E^* \dots$$

We use left associative of * and reduce.

In the case of $E \rightarrow .E^* E + E \dots$

We use the higher precedence of * and reduce

In the case of $E \rightarrow E + E + E \dots$

We use the left associativity of + and reduce

In the case of $E \rightarrow E + E^* E \dots$

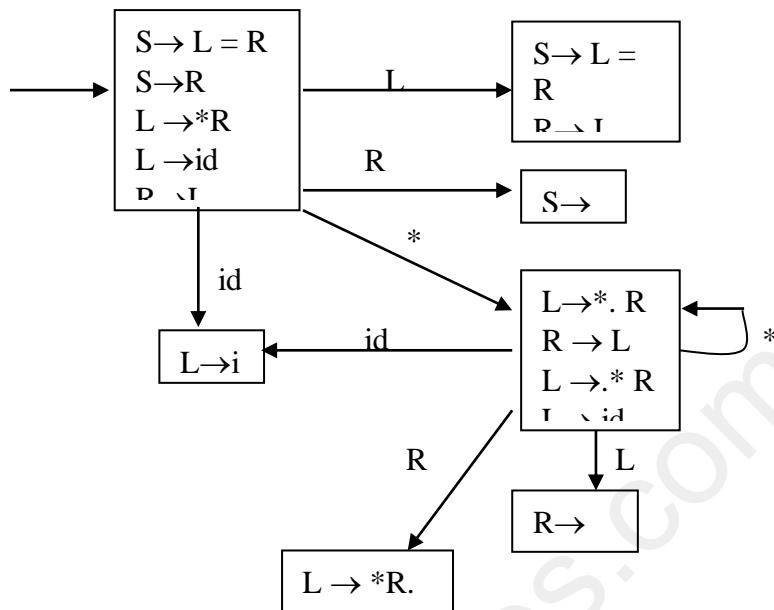
We use the higher precedence of * and shift.

Example 3.33

Check if the following grammar is SLR (1) / LR (1)

$$S \rightarrow I = R / R, L \rightarrow * R / id, R \rightarrow L$$

Sol: The LR (0) machine for the grammar

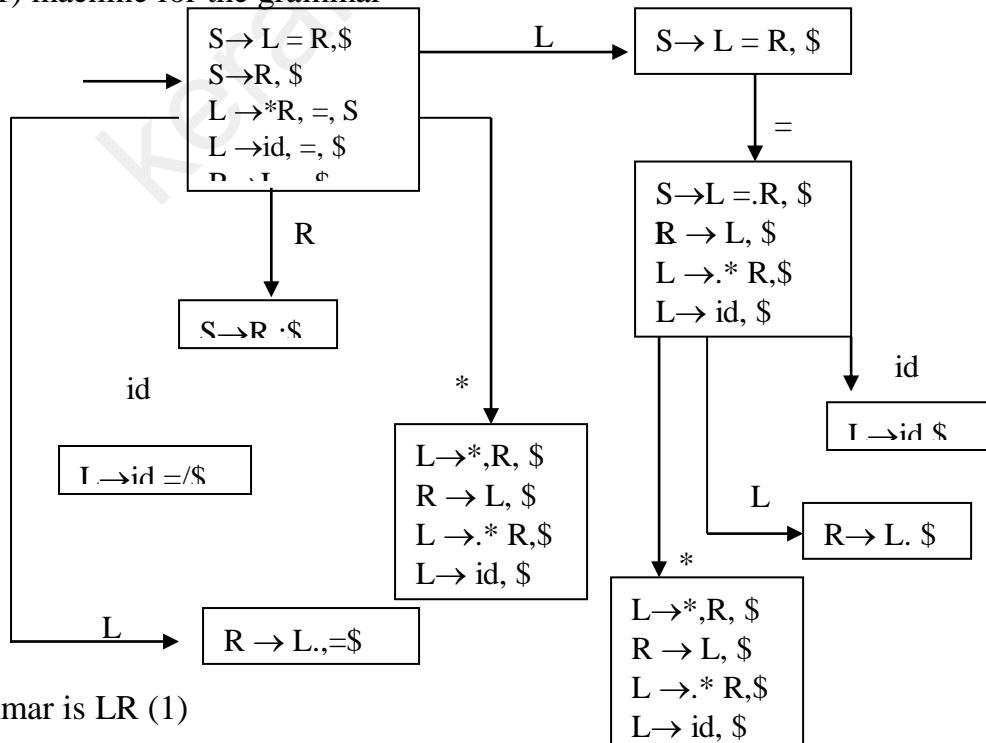


As there is s/r conflict in state \otimes , the grammar is not LR (0)

The FOLLOW (R) = { =, #}

So we cannot resolve the inadequate state \otimes based on the FOLLOW Symbol. So the grammar is not SLR (1)

The LR (1) machine for the grammar



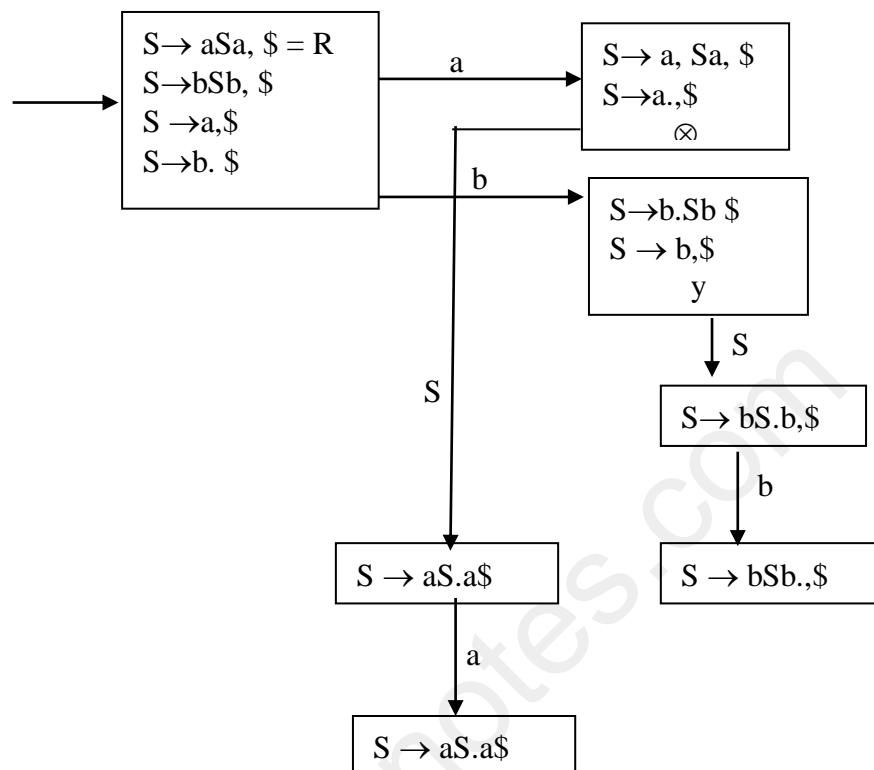
The grammar is LR (1)

Example 3.34

Check $L(G) = \{ww^R \mid w \in \{a,b\}^*\}$ is LR(1)

$$S \rightarrow aSa/b \quad Sb/a/b$$

Sol: construct the LR (I) machine



The inadequate states \otimes & \textcircled{Y} cannot be resolved by the lookahead set.

The grammar is not LR (1)

The language $L = \{ww^R / w \in (a+b)^*\}$ is the set of all palindromes over $\{a,b\}$. It is a CFL which is not a DCFL. LR techniques are restricted to DCFLs.

Example 3.35

How many conflicts occur in DFA with LR(1) items for the following grammar.

Sol: There are 2 inadequate states in the LR(I) machine.

Example 3.36

Find the closure of $E^1 \rightarrow E, \$$ in G

$$F \rightarrow E+ T/T$$

$$T \rightarrow T^* F / F$$

$$F \rightarrow \text{id}$$

Sol: The closure in

$$E^1 \rightarrow E, \$$$

$$E \rightarrow E + T, \$$$

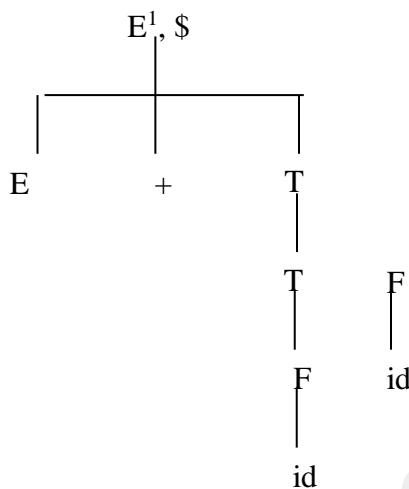
$$E \rightarrow T. \$ / +$$

$$T \rightarrow . T^* F, \$$$

$$T \rightarrow . F, \$ / *$$

$$F \rightarrow . id, \$, +, *$$

The tree

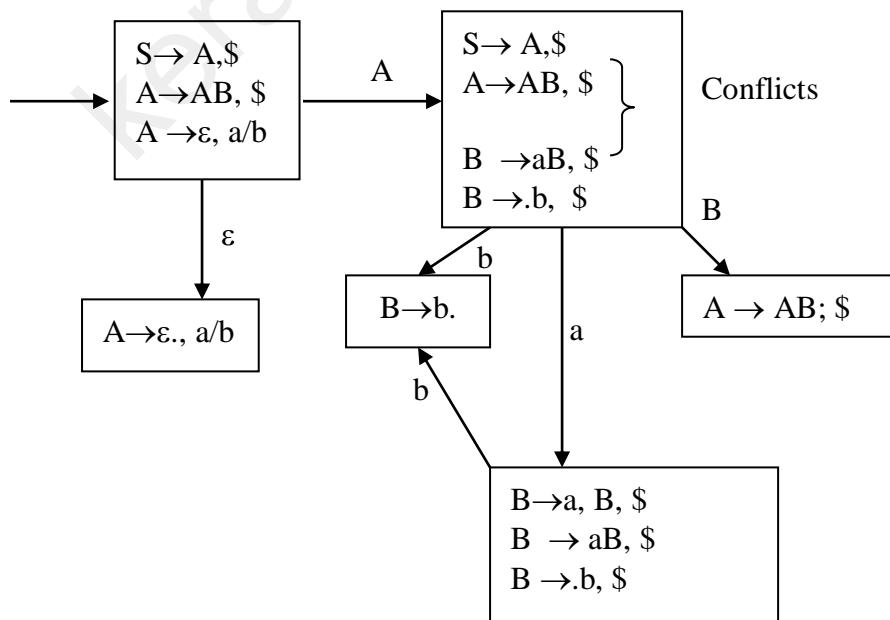


Example 3.37

Find the conflicts if any in the DFA for LR (1) items

$$S \rightarrow A, A \rightarrow AB/\epsilon, B \rightarrow aB/b$$

Sol:



There is one conflict in the LR(I) machine

Example 3.38

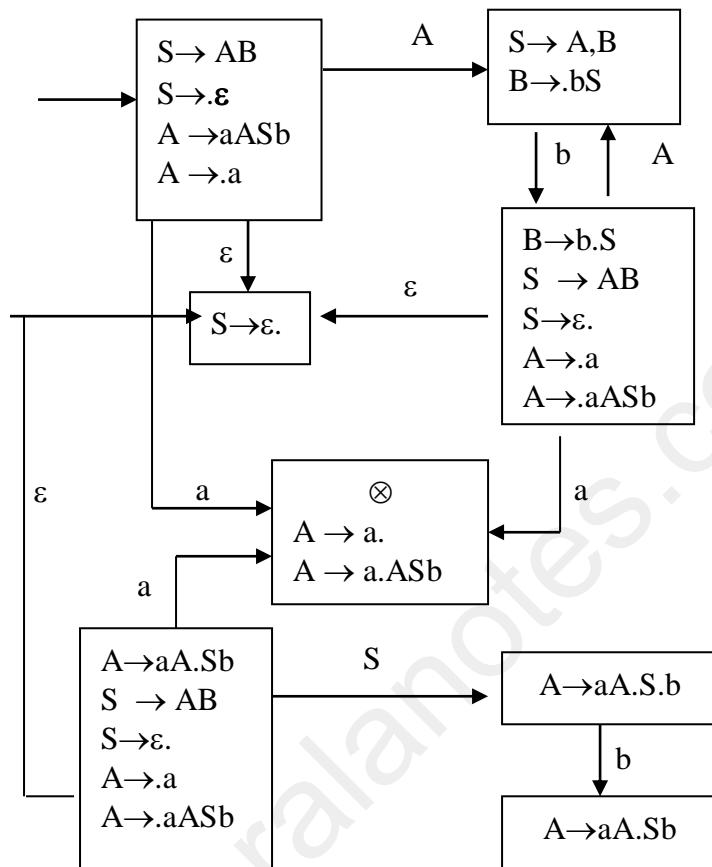
Check if G is SLR (I)

$$S \rightarrow AB/\epsilon,$$

$$A \rightarrow aASb/a$$

$$B \rightarrow bS$$

Sol:



The FOLLOW of A can be a or #. So we cannot resolve state \otimes . So the grammar is not SLR (I)

Example 3.39

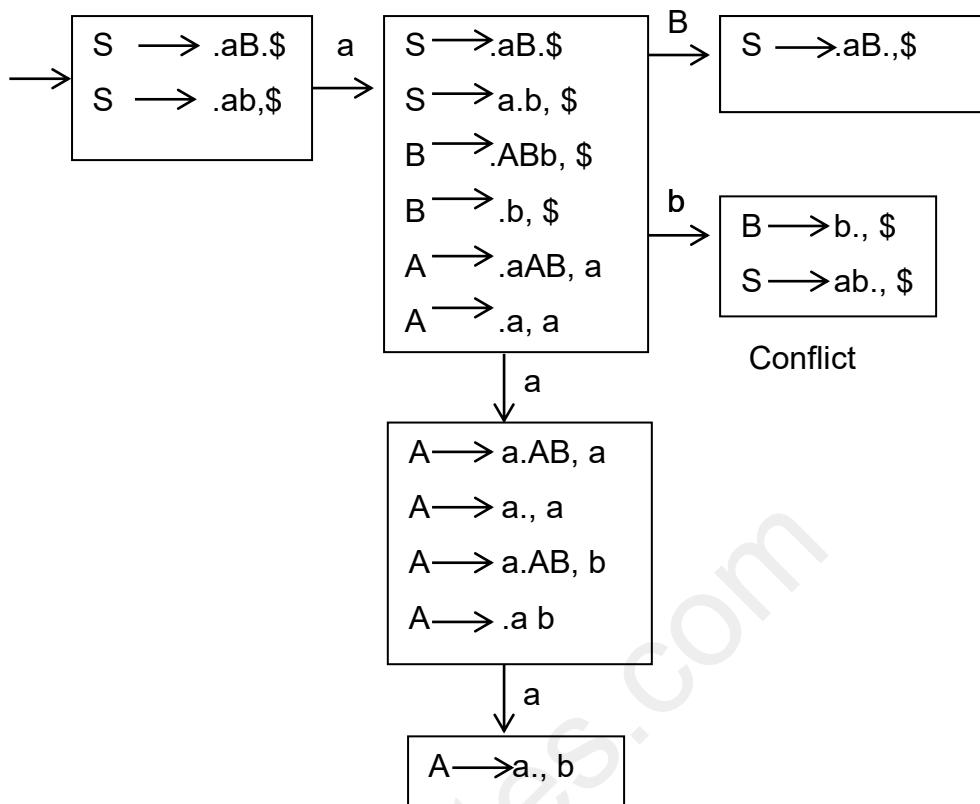
Check if G is SLR (I)

$$S \rightarrow aB/ ab;$$

$$A \rightarrow aAB/a;$$

$$B \rightarrow ABb/b$$

Sol : Construct the LR(1) machine



A conflicting state which cannot be resolved. Not LR(1)

Example 3.40 :

The LR(0) parser

Sample grammar

$$E_1 \rightarrow E \#$$

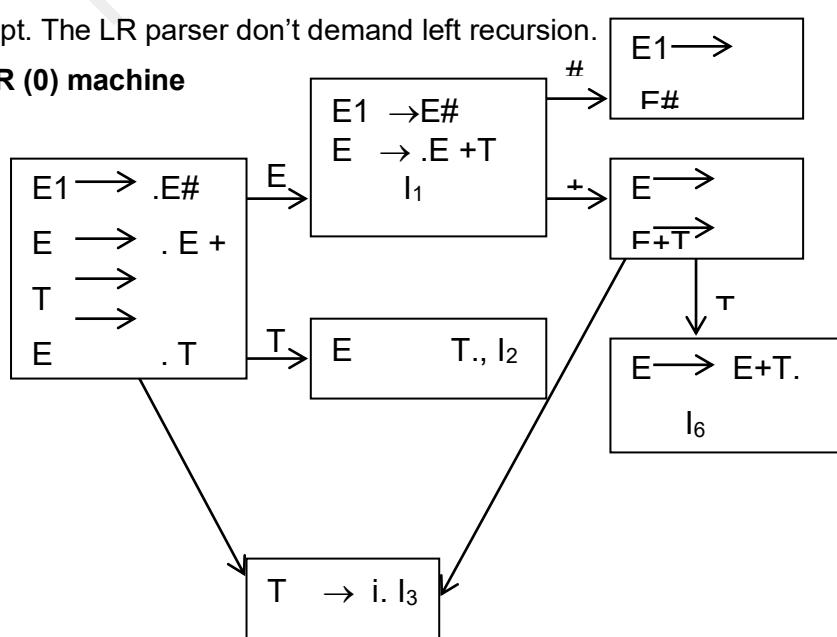
$$E \rightarrow E + T / T$$

$$T \rightarrow i$$

☞ Concept. The LR techniques demand unambiguous grammar

☞ Concept. The LR parser don't demand left recursion.

☞ **The LR (0) machine**



☞ The LR (0) parsing table

\downarrow	i	+	#	E1	E	T
0	S3				1	2
1			S5	S4		
2	Reduce unit	E \rightarrow T				
3	Reduce unit	T \rightarrow i				
4	Reduce unit	E1 \rightarrow E#				
5	S3	S6				
6	Reduce unit	E \rightarrow E + T				

As there are no shift – reduce or reduce – reduce conflicts the above grammar is LR(0)

☞ The predictive parsing table for the grammar

$$\begin{aligned} E1 &\rightarrow E \# \\ E &\rightarrow E + T / T \\ T &\rightarrow i \end{aligned}$$

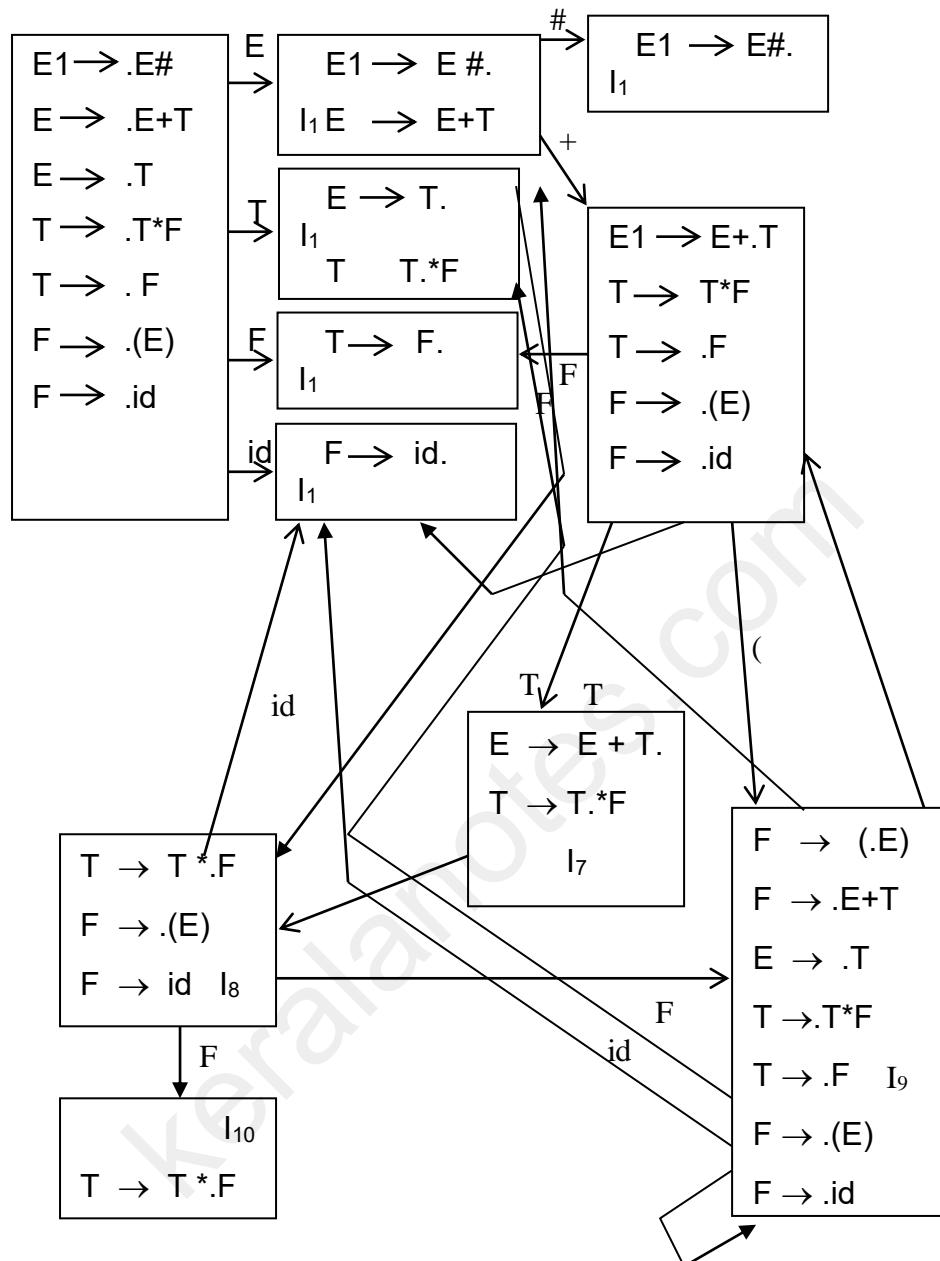
As the E – rules are left recursive the grammar isnot LL (0)

☞ A grammar that is not LR (0)

Consider the grammar

$$\begin{aligned} r1. \quad E &\rightarrow E \# \\ r2. \quad E &\rightarrow E + T / T \text{ (r6)} \\ r3. \quad T &\rightarrow T * F / F \text{ (r4)} \\ r4. \quad F &\rightarrow (E) \\ r5. \quad F &\quad id \end{aligned}$$

☞ The LR (0) machine for the above grammar



States 2 & 7 have a s/r conflict. So the grammar is not LR (0)

☞ The FOLLOW symbols

- r1. $E1 \longrightarrow E\#$
- r2. $E \longrightarrow E + T$
- r6. $E \longrightarrow T$
- r3. $T \longrightarrow T^*F$
- r7. $T \longrightarrow F$

$$r4. F \longrightarrow (E)$$

$$r5. F \longrightarrow id$$

$$FOLLOW(E1) = \epsilon$$

$$FOLLOW(E) = \{\#, +,)\}$$

$$FOLLOW(F) = FOLLOW(T) \cup FOLLOW(E) = \{\#, +, *, \}$$

$$FOLLOW(T) = \{*\} \cup FOLLOW(T) = \{\#, +, *, \}$$

☞ The SLR (1) parsing table follows

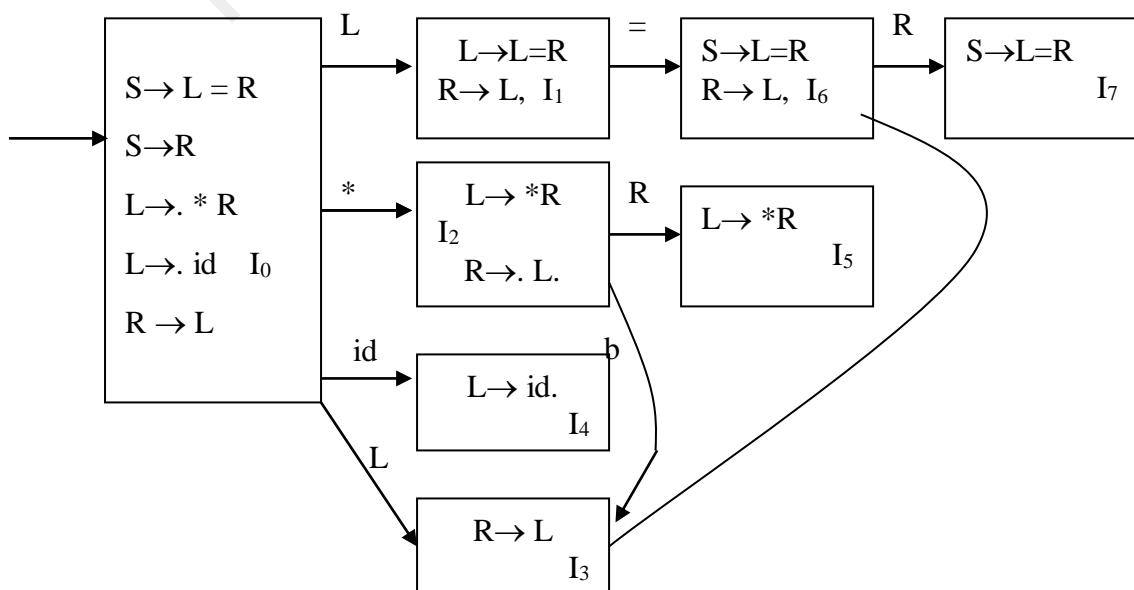
	Id	+	*	()	#	E1	E	T	F
0	S4							1	2	3
1		S6				S5				
2		r6	S8		r6	R6				
3										
4		r1	r9		r7	r7				
5	Accept									
6	S4			S9				7	3	
7		r2		S8						
8				S9						10
9				S9						2
10		r3	r3		r3	r3				
11										

Example 3.41 :

A grammar that is not SLR (1)

$$S \rightarrow L = R, S \rightarrow R, L \rightarrow *R, L \rightarrow id, R \rightarrow L$$

☞ The LR (0) machine for the grammar



Example 3.42 :
The LR (1) parsing technique
Example grammar

$r1. S^1 \rightarrow S$

$r2. S \rightarrow C \ C$

$r3. C \rightarrow c \ C$

$r4. C \rightarrow d$

⇒ The language generated by the grammar

$S \rightarrow S$

$S \rightarrow C \ C$

$C \rightarrow c \ C/d \Rightarrow C^* d \ C^* d$

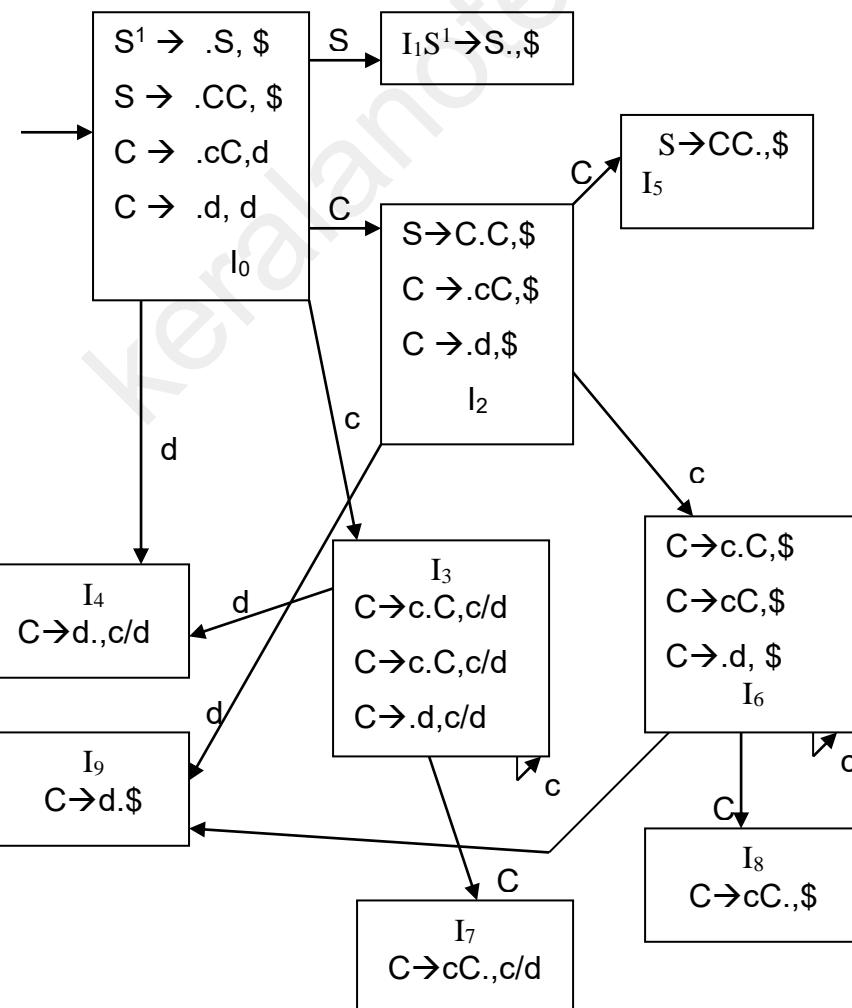
So the language generated by the grammar is a regular set.

⇒ The LR(1) machine for the grammar

$r1. S^1 \rightarrow S$

$r2. S \rightarrow C \ C$

$r3 \& r4. C \rightarrow c \ C/d$



☞ The LR(1) parsing table

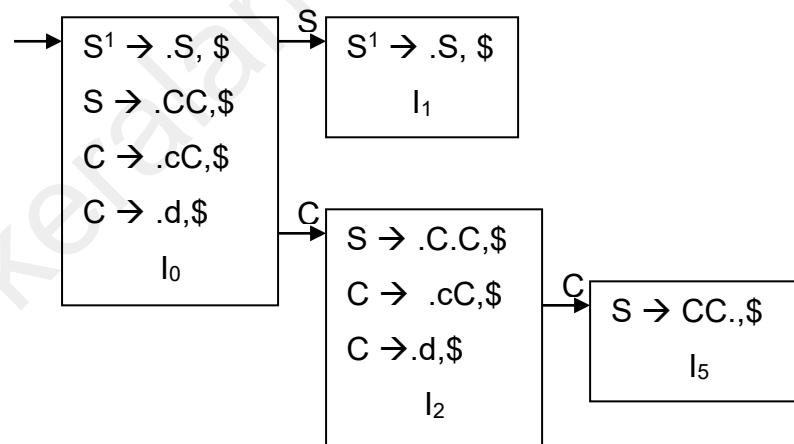
	c	d	\$	S^1	S	C
0	S3	S4			1	2
1				r1		
2						5
3	S3	S4				7
4	r4	r4				
5				r2		
6	S6	S9				8
7	r3	r3				
8				r3		
9					r4	

☞ The LALR(1) machine

r1 $S^1 \rightarrow S$

r2 $S \rightarrow C C$

r3 & r4 $C \rightarrow c C/d$



I₄₉ C → d, c|d|\$

S → C.C, c|d|\$
C → c.C, c|d|\$
C → .d, c|d|\$
I₃₆

C → cC., c, d|\$
I₇₈

- ☞ The LALR(1) Parsing table

	c	D	\$	S ¹	S	C
0			S49		1	2
1			r1			
2	S36		S49			5
36	S36	S49				78
49	r4	r4	r4			
5			r3			
78	r3	r3	r3			

Example 3-43 :

Consider

$$S^1 \rightarrow S$$

$$S \rightarrow a A d / b B d / a B c / b A c$$

$$A \rightarrow c$$

$$B \rightarrow c$$

- ☞ Is the grammar LL(1)?

$$\text{FIRST}(S) = \{a, b\}$$

If S is the goal & a or b are the incoming symbols, then there is a conflict of LL(1).

So no predictive parsing table exist without conflicts.

- ☞ The predictive parsing table

	a	b	c	d	\$
S ¹	S ¹ → S	S ¹ → S			
S	S → aAd	S → bBd			
		S → aBc	S → bAc		
A				A → c	
B					B → c

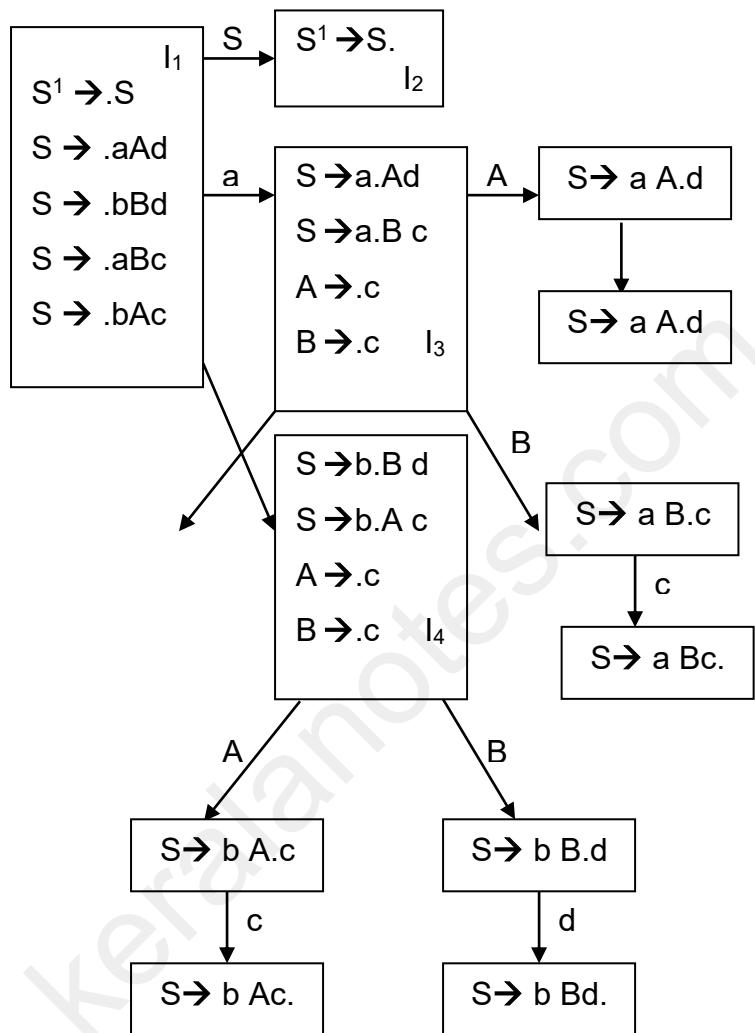
- ☞ The LR(0) parsing table for a sample grammar

$S^1 \rightarrow S$

$S \rightarrow aAd/ bBd/ aBc/ bAc$

$A \rightarrow c$

$B \rightarrow c$



The grammar is not LR(0)

- ☞ The SLR (1) parsing table for a sample grammar

$S^1 \rightarrow S$

$S \rightarrow aAd/ bBd/ aBc/ bAc$

$A \rightarrow c$

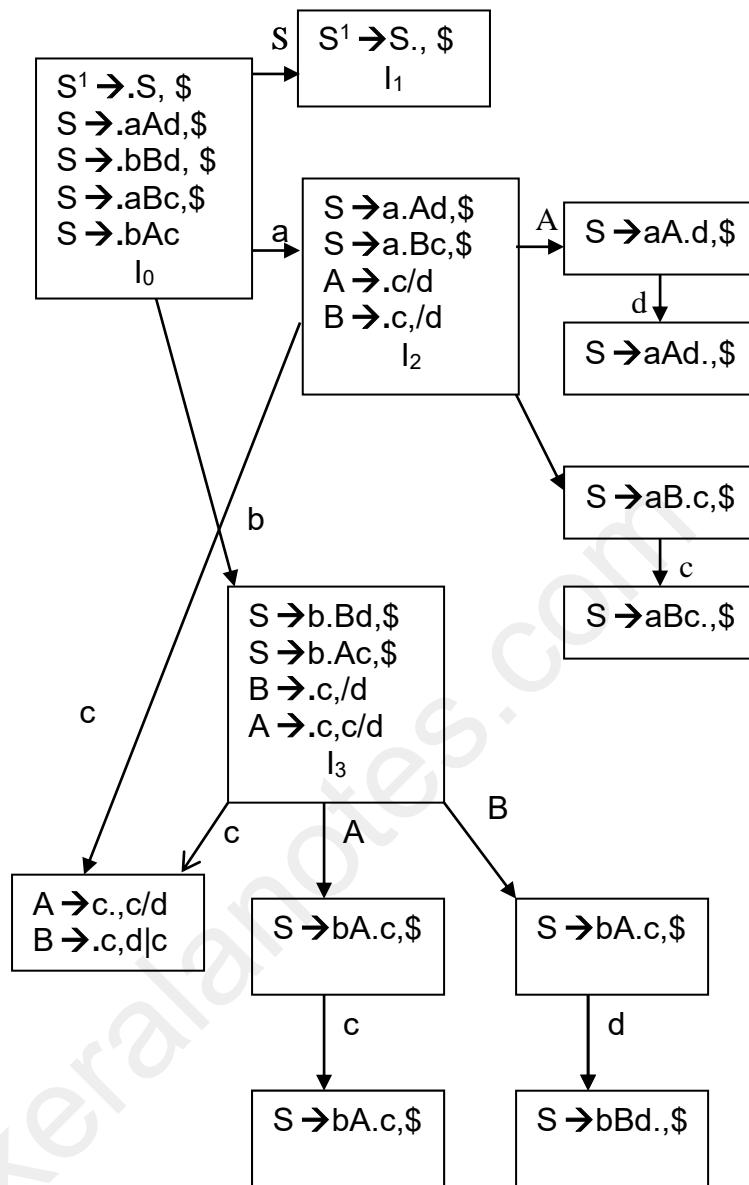
$B \rightarrow c$

The FOLLOW sets

$\text{FOLLOW}(S^1) = \{\$\}$, $\text{FOLLOW}(S) = \{\$\}$

$\text{FOLLOW}(A) = \{d, c\}$

The SLR (1) machine



The grammar is not SLR (1)

☞ The LR (1) parsing technique for a sample grammar

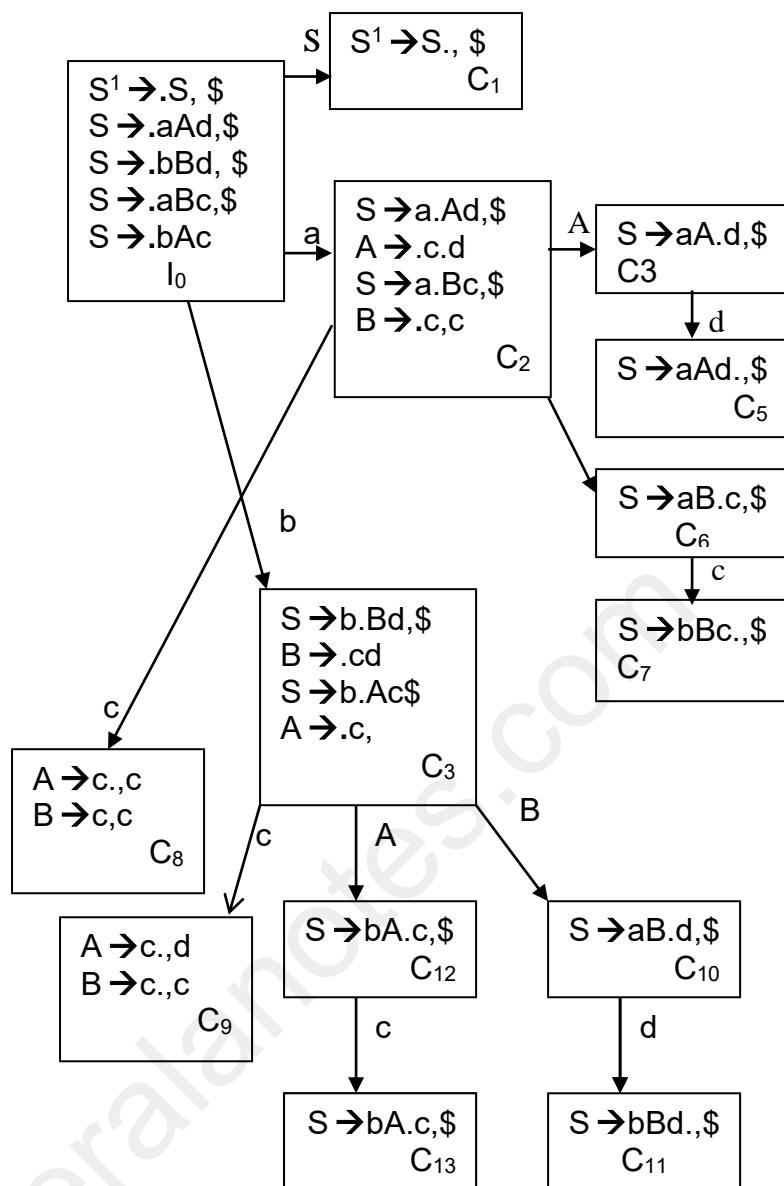
$r1 : S^1 \rightarrow S$

$r2, r3, r4, r5 : S \rightarrow aAd/ bBd/ aBc/ bAc$

$r6 : A \rightarrow c$

$r7 : B \rightarrow c$

The LR (1) machine



The grammar is LR(1) but not LALR (1)

⇒ The LR (1) parsing table

	a	b	c	d	\$	S'	A	B	S
0	S2	S4							1
1					r1				
2			S8				3	6	
3					S5				
4									
5			S9				12	10	
6			S7						
7					r4				
8		r7	r6						
9		r6	r7						
10				S11					
11					r2				
12			S13						
13					r4				

The grammar is LR (1).

As states 8 & 9 cannot be merged

The grammar is not LALR (1).

Example 3 -44 :

Consider the grammar

$$S \rightarrow Aa \mid bAc \mid dc \mid bda$$

$$A \rightarrow d$$

(a) The grammar is LL (1)

(b) The grammar is LR (0)

(c) The grammar is SLR (1)

(d) The grammar is LALR (1)

Ans : (d)

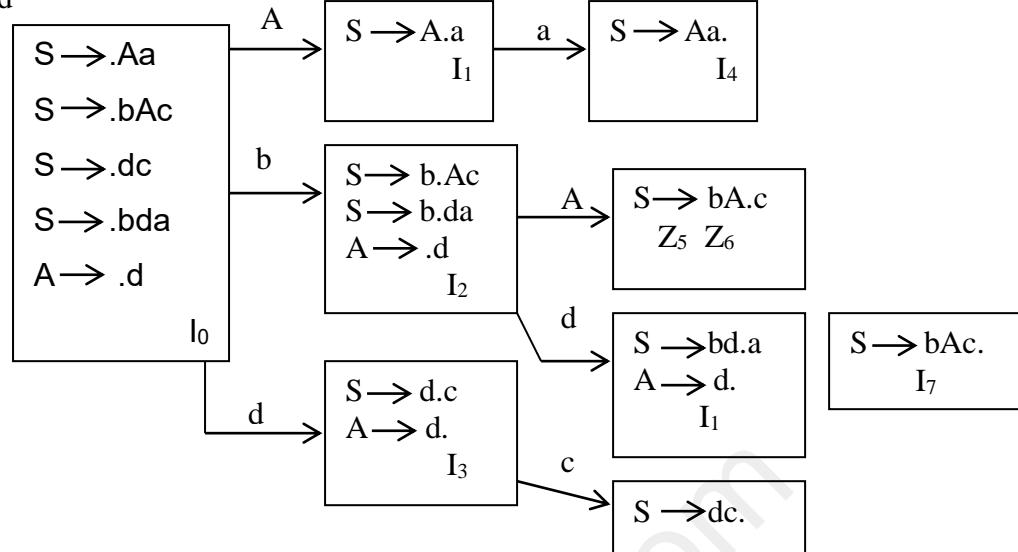
Sol : The rules $S \rightarrow Aa \mid bda$ show that a conflict arises when S is the goal and b is the incoming symbol. The grammar is thus not LL (1)

We will construct the LR (0) machine & show that the conflicts cannot be resolved based on the FOLLOW set. So the grammar is not SLR (1). We will construct a LR (1) machine for the grammar. We will merge sets with common cores & show that the resulting grammar LALR (1) has no conflicts.

The LR (0) machine for the grammar

$$S \rightarrow Aa/bAc/dc/bda$$

$$A \rightarrow d$$



State I_3 has a s/r conflict, and state I_7 has a r/r conflict. So the grammar is not LR(0).

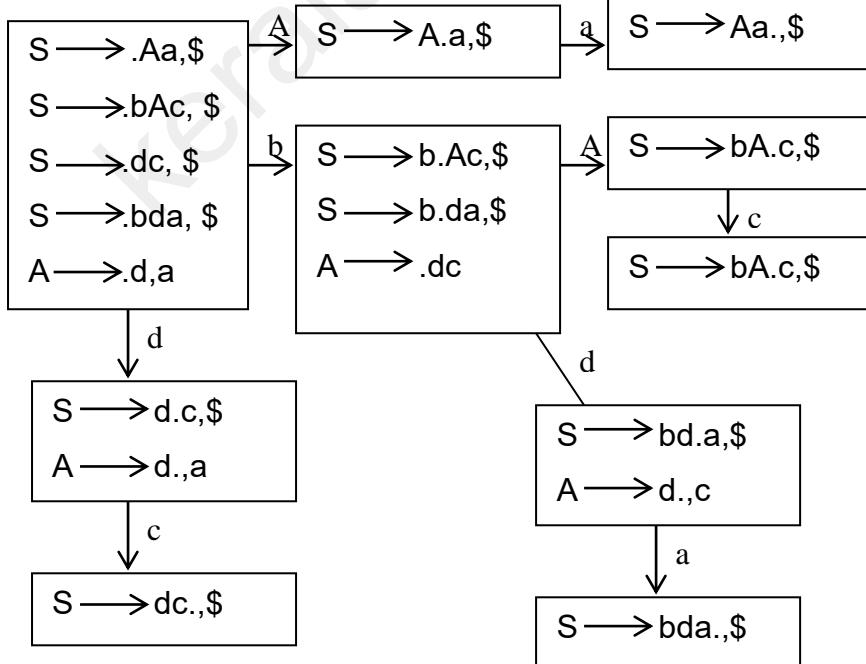
Can the grammar be SLR (1)?

$\text{FOLLOW}(S) = \{\$\}$ and this does not resolve the conflicts I_7 . So the grammar is not SLR(1)

The LR(1) machine for the grammar

$$S \rightarrow Aa/bAc/dc/bda$$

$$A \rightarrow d$$



The grammar is LR (1) and as there are no two states with a common core, the grammar is also LALR(1).

Example 3.45 :

Consider the grammar

$$S \rightarrow Aa/bAc/Bc/bBa$$

$$A \rightarrow d$$

$$B \rightarrow d$$

- (a) The grammar is LL (1)
- (b) The grammar is LR (0)
- (c) The grammar is SLR (1)
- (d) The grammar is LR (1) but not LALR(1)

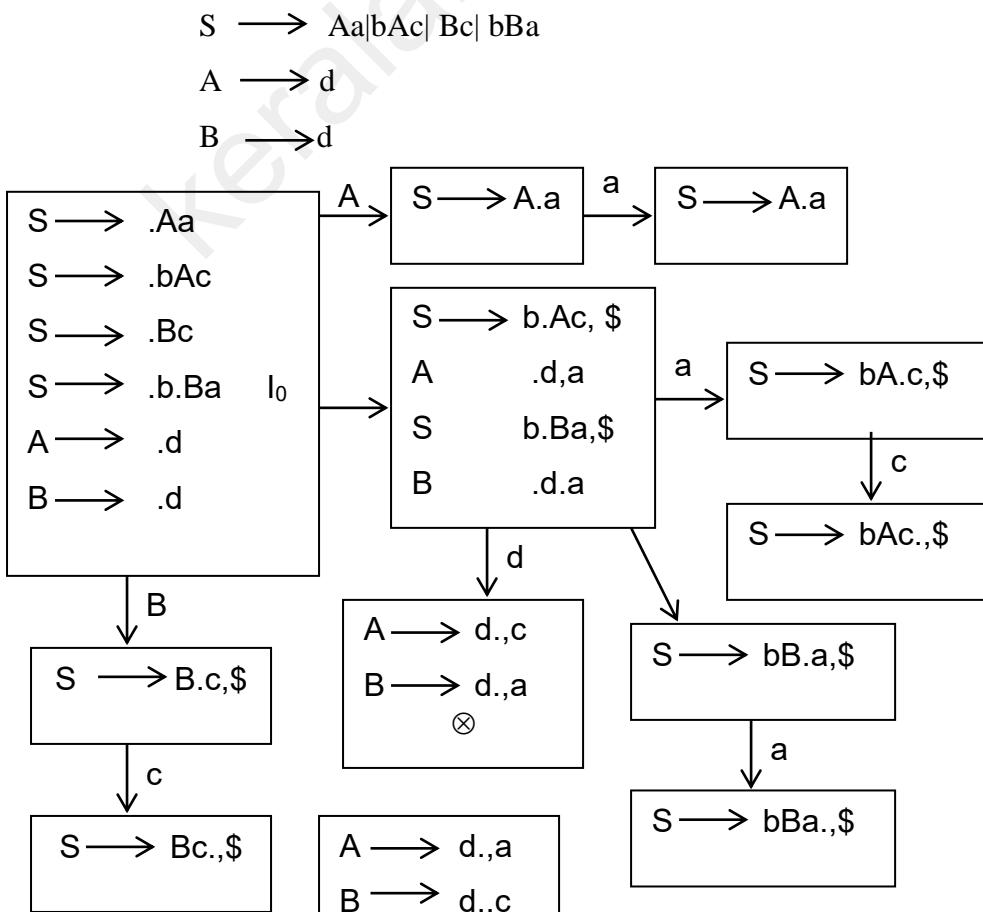
Ans : (d)

Sol : The rules $S \rightarrow bAc \mid bBa$ show that when S is the goal & b is the incoming symbol we cannot choose the correct rule. So the grammar is not LL (1).

We will construct the LR(0) machine & show that it has conflicts. So the grammar cannot be LR(0). The inadequate states cannot be restricted by the FOLLOW sets, so the grammar is not SLR(1).

The grammar will be shown to be LR(1) by constructing a LR(1) machine for the same. Merging state with common cores leads to conflicts to the grammar will not be LALR(1).

The LR(0) machine.



As the LR (1) machine has no conflicts, the grammar is LR(1). If state is \otimes & \otimes which have a common core are merged, conflicts arise. So the given grammar is not LALR (1).

Example 3.46 :

Consider the following grammar

$$E \longrightarrow T \{ + T \}$$

$$T \longrightarrow a$$

Which one of the following is the proper recursive descent parser for the grammar?

- (a) void E ()


```

      {
          T ();
          While (input = '+')
          Scan (); T();
      }
      Void T ( )
      {
          Is (input? = a) error ();
          Scan (); T();
      }
      Is (input? = a) error ();
      Scan ();
  }
```
- (b) void E ()


```

      {
          T (); T ();
          Scan ();
      }
      Void T ( )
      {
          Scan ()
      }
```
- (c) void E ()


```

      {
          If (input = '+') scan;
          T (); T ();
      }
      Void T ();
      {
          Scan ();
      }
```
- d) None of the above

Ans : (a)

Sol : Trace the syntax graph.

Example 3.47 :
The Chomsky Normal Form grammar for

$$S \rightarrow (L)/a; \quad L \rightarrow L, S/S$$

- | | |
|--------------------------|--------------------------|
| (a) $S \rightarrow AB/a$ | (b) $S \rightarrow LL/a$ |
| $B \rightarrow LC$ | $L \rightarrow LS/$, |
| $A \rightarrow ($ | |
| $C \rightarrow)$ | |
| $L \rightarrow AB$ | |
| $L \rightarrow LD$ | |
| $D \rightarrow ES$ | |
| $E \rightarrow ,$ | |
| (c) $S \rightarrow AB/a$ | (d) None of the above |
| $B \rightarrow LD$ | |
| $A \rightarrow ($ | |
| $C \rightarrow)$ | |
| $L \rightarrow AB$ | |
| $L \rightarrow LD$ | |
| $E \rightarrow .$ | |

Ans : (a)

Sol :

$S \rightarrow AB/a$	$S \rightarrow AB/C$
$A \rightarrow ($	$A \rightarrow ($
$B \rightarrow L)$	$C \rightarrow)$
$L \rightarrow LD$	$B \rightarrow LC$
$D \rightarrow , S$	$L \rightarrow AB$
$L \rightarrow AB$	$L \rightarrow LD$
	$D \rightarrow ES$
	$E \rightarrow ,$

Example 3.48 :
Consider the grammar

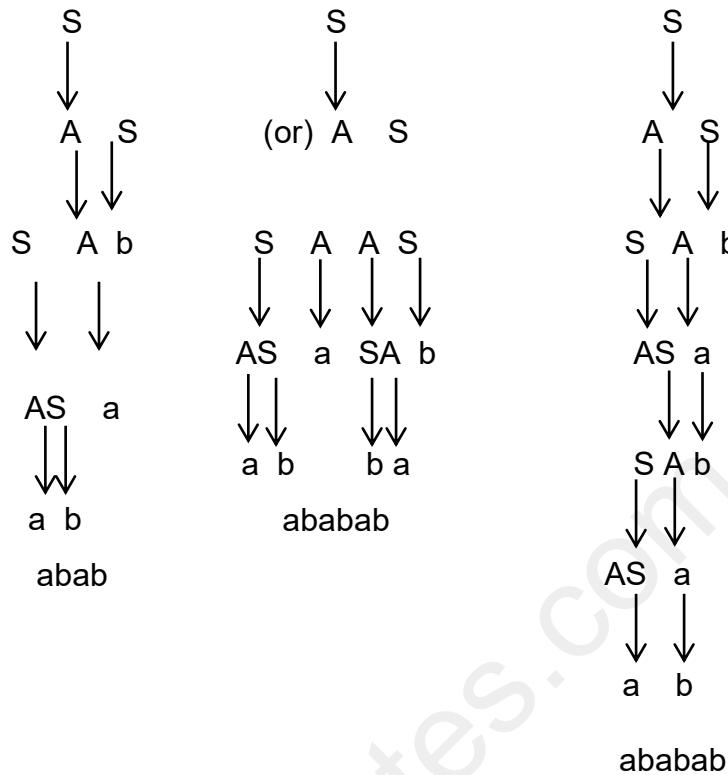
$$S \ A \ S/b$$

$$A \ S \ A/a$$

- (a) The grammar is ambiguous
- (b) The grammar is LL (1)
- (c) The grammar is SLR (1) but not LR (0)
- (d) The grammar is LR (1) but not LALR(1)

Ans : (a)

Sol : Consider the string abab



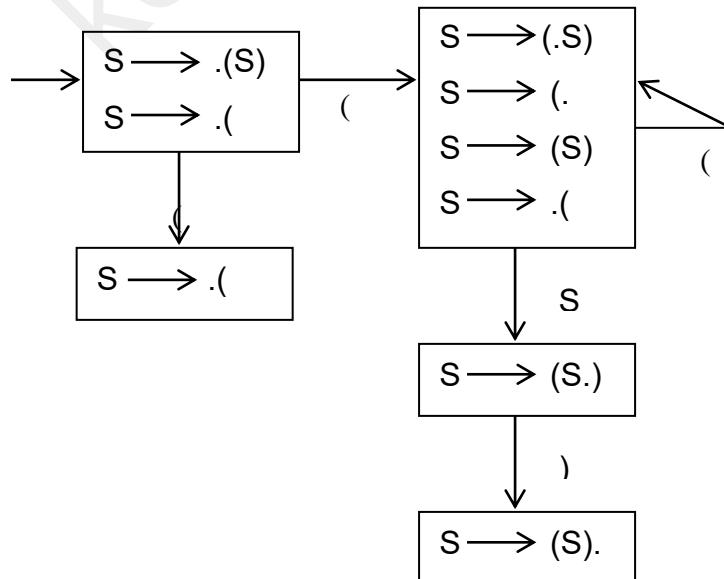
Example 3.49 :

Compute the LR (0) item states & then transitions for the grammar

$$S \rightarrow (S)$$

$$S \rightarrow (S)$$

Sol :



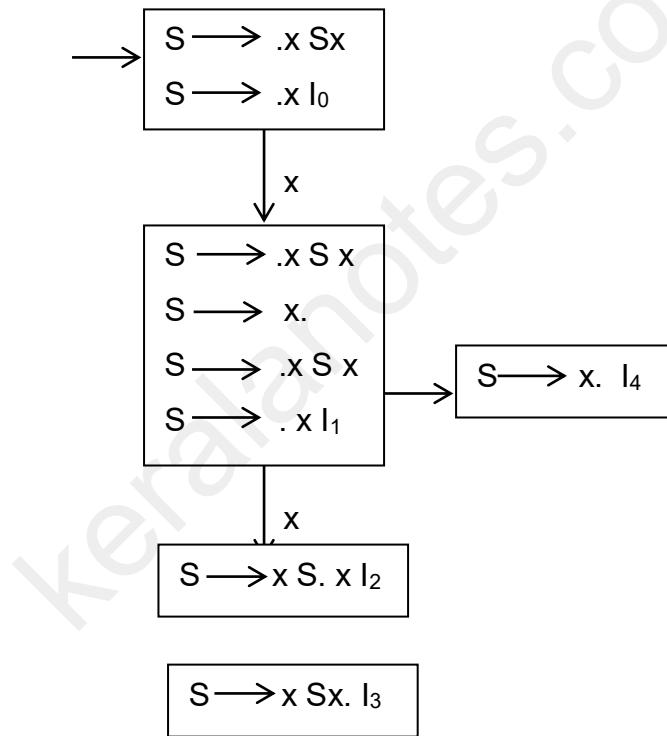
The LR(0) item sets are

- | | |
|-------------------------------|-----------------------------|
| 1) $S \longrightarrow .(S)$ | 2) $S \longrightarrow .$ |
| $S \longrightarrow .($ | |
| 3) $S \longrightarrow (.S)$ | 4) $S \longrightarrow (S.)$ |
| $S \longrightarrow (.$ | |
| $S \longrightarrow .(S)$ | |
| 5) $S \longrightarrow (S.)$ | |
| $S \longrightarrow .($ | |
| $S \longrightarrow x S x / S$ | |

Example 3.50 :

Construct the LR (0), SLR(1), LR(1) & LALR(1) automats for the grammar.

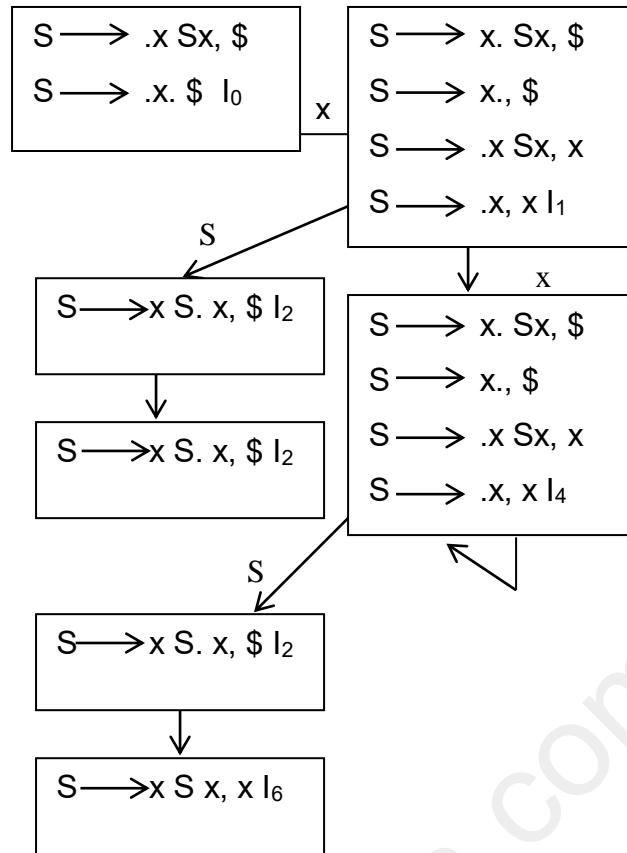
Sol : The LR (0) machine



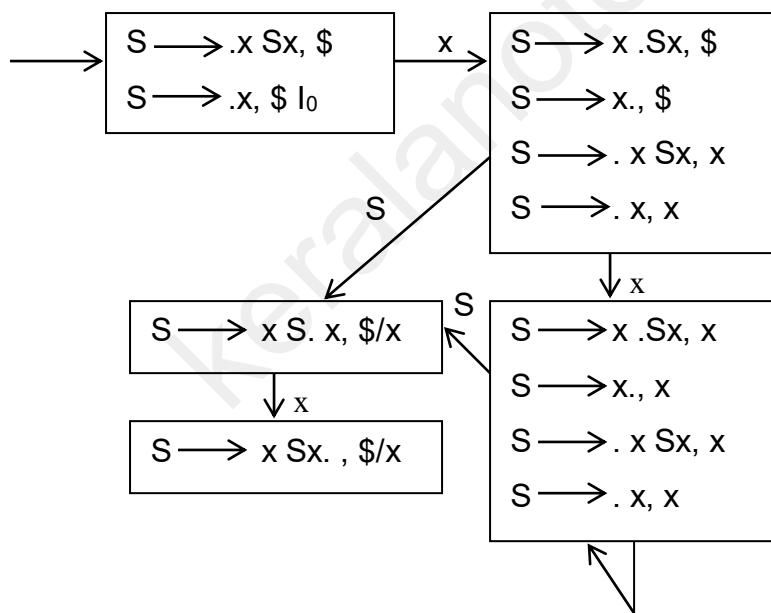
There is a S/R conflict in I_1 . This can not be resolved by the FOLLOW symbol x . So the grammar is not SLR (1).

The LR (1) machine for the grammar

$$\begin{aligned} S &\longrightarrow x S x \\ S &\longrightarrow x \end{aligned}$$

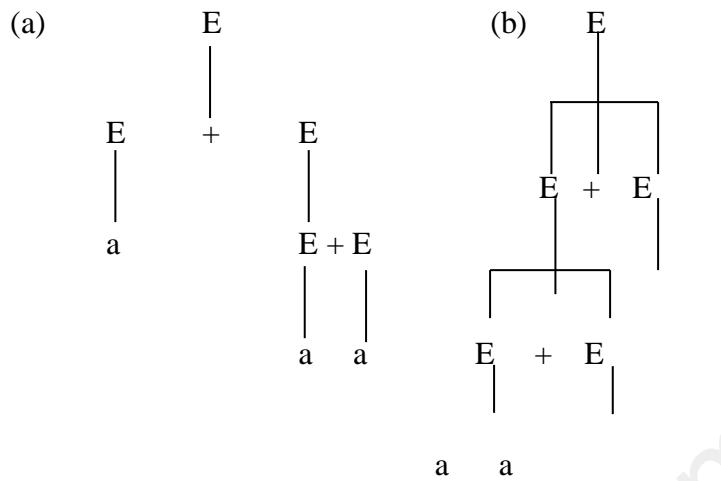


The LALR(1) machine



Classroom Practice Questions

01. Let + be a left associative operator then the syntax tree for $a + a + a$ is in the grammar $E \rightarrow E + E/a$ —



(c) Either (a) or (b)

(d) Neither (a) nor (b)

02. Consider the grammar given below

$$S \rightarrow SS|a|\epsilon$$

The sentence “aaa” has

- (a) a unique derivation tree in the grammar
- (b) two leftmost derivations in the grammar
- (c) ten rightmost derivations in the grammar
- (d) an infinite number of leftmost and rightmost derivations

03. A grammar that is both left and right recursive for a non-terminal is

- (a) Ambiguous
- (b) Unambiguous
- (c) Information is not sufficient
- (d) None

04. A grammar has the following productions :

$$S \rightarrow a S b / b S a$$

Which of the following sentences are in the language that is generated by this grammar

- (a) aaaaabb
- (b) aabbaabb
- (c) bbbaabbaa
- (d) ababbab

05. Consider the following grammar

$$E \rightarrow E + E / (E * E)/id$$

The number of parse trees are possible for an input string $w = id + id + id$ is _____.

06. How many parse trees exists for an input string $w = abab$ in the CFG

$$S \longrightarrow aSbS/bSaS/\epsilon$$

- (a) 4 (b) 3 (c) 2 (d) 5

07. Consider the left recursive grammar

$$A \longrightarrow Sa/b$$

$$S \longrightarrow Sc/Ad$$

Which of the following grammar is equivalent grammar with left recursion eliminated?

- | | |
|--|-------------------------------|
| (a) $A \longrightarrow Sa/b$ | (b) $A \longrightarrow Sa/b$ |
| $S \longrightarrow bbS'$ | $S \longrightarrow bdS'$ |
| $S' \longrightarrow cS'/adS'/\epsilon$ | $S' \longrightarrow cS'/adS'$ |
| (c) $S \longrightarrow bdS'$ | (d) none |
| $S' \longrightarrow S'/adS'/\epsilon$ | |
| $A \longrightarrow ScaA'/bA'$ | |
| $A' \longrightarrow daA'/\epsilon c$ | |

08. Consider the following expression grammar G :

$$E \rightarrow E - T \mid T$$

$$T \rightarrow T + F \mid F$$

$$F \rightarrow (E) \mid id$$

Which of the following grammars is not left recursive, but is equivalent to G?

- | | |
|-----------------------------------|--|
| (a) $E \rightarrow E - T \mid T$ | (b) $E \rightarrow TE'$ |
| $T \rightarrow T + F \mid F$ | $E' \rightarrow -TE' \mid \epsilon$ |
| $F \rightarrow (E) \mid id$ | $T \rightarrow T + F \mid F$ |
| | $F \rightarrow (E) \mid id$ |
| (c) $E \rightarrow TX$ | (d) $E \rightarrow TX \mid (TX)$ |
| $X \rightarrow -TX \mid \epsilon$ | $X \rightarrow -TX \mid +TX \mid \epsilon$ |
| $T \rightarrow FY$ | $T \rightarrow id$ |
| $F \rightarrow (E) \mid id$ | |

09. Left factoring the grammar helps in

- (a) Removing the Ambiguity (b) Avoiding going into infinite loop
 (c) Avoiding Back Tracking (d) None

10. Consider the following grammar

$$E \rightarrow E \uparrow T / T$$

$$T \rightarrow F + T / F$$

$$F \rightarrow id$$

- (a) \uparrow has higher precedence than $+$
- (b) $+$ has lower precedence than \uparrow
- (c) \uparrow has lower precedence than $+$
- (d) id has lower precedence than and $+$

11. Given the following grammar

$$E \rightarrow E^* F / F + E / F$$

$$F \rightarrow F - F / id$$

Which of the following is true?

- (a) $*$ > $+$
- (b) $-$ > $*$
- (c) $+$ = $-$
- (d) $+$ > $*$

12. Consider the following left – associative operators, in decreasing order of precedence
 $____, \$ * (exponent), ^, .$ The result of the expression.

$3 - 2^* 4 \$ 2^* 3 \$ 2$ is _____.

13. Evaluate the following expression considering the given rules :

$$3 - 2^* 4 \$ 1 * 2 \$$$

Where $\$$ mean exponentiation

- (a) $(-)$ highest followed by $(*)$ and then followed by $(\$)$ and all are left associative
- (b) $(-)$ highest followed by $(*)$ and then followed by $(\$)$ and all are right associative
- (c) $(\$)$ highest followed by $(-)$ and then followed by $(*)$ and all are left associative
- (a) $a < b < c$ (b) $c < a < b$ (c) $a < c < b$ (d) $b < c < a$

14. Choose the correct statement?

- (a) a bottom up parsing technique simulates a rightmost derivation
- (b) a top down parsing technique simulates the reverse of a leftmost Derivation
- (c) a bottom up parsing technique building the derivation tree in bottom up and simulates a rightmost derivation in reverse.
- (d) a top down parsing technique simulates building the derivations tree in top down and simulates a leftmost derivation in reverse.

15. What is FIRST (S) in

$$S \longrightarrow AC / Ca / Bb,$$

$$A \longrightarrow d$$

$$B \longrightarrow e / \epsilon$$

$$C \longrightarrow f / \epsilon$$

- | | |
|---------------------|----------------------------------|
| (a) {a, d, e, f} | (b) {a, e, f, ϵ } |
| (c) {a, b, d, e, f} | (d) {a, b, d, e, f, ϵ } |

16. Consider the following grammar for expressions :

$$\langle \text{expression} \rangle \rightarrow \text{atom } \langle \text{additional} \rangle \mid (\langle \text{expression} \rangle)$$

$$\langle \text{additional} \rangle \rightarrow +\text{expression}|^* \langle \text{expression} \rangle | \epsilon$$

The follow set for < additional > is

- | | |
|-------------------|------------------|
| (a) { atom, () } | (b) { [, \$] } |
| (c) { +, *, \$ } | (d) { \$ } |

17. $S \longrightarrow ABC$

$$A \longrightarrow aA|c$$

$$B \longrightarrow b|\epsilon$$

$$C \longrightarrow c$$

FIRST (A) \cap FOLLOW (A) is

- | | |
|---------|----------|
| (a) {a} | (b) {b} |
| (c) {c} | (d) none |

18. FOLLOW (B) in the following grammar

$$A \longrightarrow BCD$$

$$B \longrightarrow w|Bx$$

$$C \longrightarrow yCz|m$$

$$D \longrightarrow DB|a$$

- | | |
|-------------------|----------------------|
| (a) {x, y, m, \$} | (b) {x, y, w, \$} |
| (c) {x, w, m, \$} | (d) {x, y, m, w, \$} |

19. $S \longrightarrow [SX] | a$

$$X \longrightarrow \epsilon | +SY|Yb$$

$$Y \longrightarrow \epsilon | =SXc$$

Then FOLLOW (S) is

- | | |
|---------------------------|------------------------|
| (a) { \$, +, -,], c, b } | (b) { \$, _,], c, b } |
| (c) { \$, +, _,], b } | (d) { \$,], c, b } |

20. In the grammar $S \rightarrow TA,$
 $A \rightarrow +TA/\epsilon$
 $T \rightarrow a/\epsilon$

Follow (T) \cap First (S) is

- (a) {+, ϵ } (b) {+, \$}
(c) {+} (d) {a, +, ϵ }

21. Consider the following grammar

$$\begin{aligned} S &\rightarrow AaC/Bd \\ A &\rightarrow BC \\ B &\rightarrow bB/C \\ C &\rightarrow accS \end{aligned}$$

For which non terminals is follow set contains terminal ‘a’.

- (a) {C} (b) {A} (c) {A, B, C} (d) {A, B, C, S}

22. Consider the following grammar

$$\begin{aligned} <\text{expression}> &\quad <\text{factor}> <\text{rest}> \\ <\text{rest}> &\longrightarrow^* <\text{expression}> \mid \epsilon \\ <\text{factor}> &\longrightarrow \text{identifier} \end{aligned}$$

In the predictive parsing table M of the grammar the entries for $M[<\text{expression}>, \text{identifier}]$ and $M[<\text{rest}>, ^*]$ respectively are

- (a) [<expression> \longrightarrow <factor> <rest>] and [<rest> $\longrightarrow^* \epsilon$]
(b) [<expression> \longrightarrow <factor> <rest>] and []
(c) [<expression> \longrightarrow <factor> <rest>] and [<rest> $\longrightarrow^* <\text{expression}>$]
(d) [<factor> \longrightarrow identifier] and [<rest> $\longrightarrow^* \epsilon$]

23. Consider the grammar

$$S \rightarrow a \mid ab \mid abc$$

Choose the correct statement from the following :

- (a) The grammar is LL (1) (b) The grammar is LL (2)
(c) The grammar is LL (3) (d) None of the above

24. Consider the grammar

$$\begin{aligned} S &\longrightarrow \text{if expr then} \\ &\quad | \text{ if expr then stmt} \\ &\quad | \text{ if expr then stmt else} \\ &\quad | \text{ if expr then stmt else stmt} \end{aligned}$$

The grammar is

- (a) LL (1)
- (b) LL (4)
- (c) LL (5)
- (d) LL (6)

25. Consider the grammar, G1

$$E \longrightarrow E + E|E^*E|(E)|id$$

Consider the grammar G2,

$$E \longrightarrow E + T|T$$

$$T \longrightarrow T^*F|F$$

$$F \longrightarrow (E)| id$$

Choose the incorrect statement?

- (a) G2 generates the same language as G1
- (b) G2 is unambiguous but not LL (1) as it is left recursive
- (c) G2 gives a higher precedence to + over *
- (d) None

26. Choose the true statement?

- (a) If a grammar is left recursive it cannot be LL(1)
- (b) If a grammar is right recursive it cannot be LL (1)
- (c) An ambiguous grammar can sometimes be LL (1)
- (d) If a grammar is not context free then it sometimes can be LL (1)

27. $S \longrightarrow Aa$

$$A \longrightarrow b/\epsilon$$

If we construct a predictive parse table for the above grammar, the production A ϵ is added in 'A' row and _____.

- (a) '\$' column
- (b) ' ϵ ' column
- (c) 'a' column
- (d) 'b' column

28. $S \longrightarrow aSbS/bSaS/\epsilon$

In the predictive parse table M of the above grammar $M [S,a] = _____$

- (a) $S \longrightarrow aSbS$
- (b) $S \longrightarrow bSaS$
- (c) $S \longrightarrow \epsilon$
- (d) $S \longrightarrow aSbS, S \xrightarrow{\epsilon}$

29. $S \longrightarrow Aa/ Bb$

$$A \longrightarrow b$$

$B \longrightarrow b$ is

- (a) LL(1)
- (b) LL(2)
- (c) LR (0)
- (d) None

30. Consider a calculator modeled by the grammar

$$\begin{aligned} <\text{accumulated_sum}> &\longrightarrow <\text{accumulated_sum}> + \text{number} \\ &\quad | <\text{accumulated_sum}> * \text{number} \\ &\quad | \text{number} \end{aligned}$$

For an input “number + number*number”, the handles in the reverse of a rightmost derivation are

- (a) number, $<\text{accumulated_sum}> + \text{number}$.
 $<\text{accumulated_sum}> + \text{number} * \text{number}$
 - (b) number, $<\text{accumulated_sum}> + \text{number}$
 $<\text{accumulated_sum}> + <\text{accumulated_sum}> * \text{number}$
 - (c) number, number + number, number + number* number
 - (d) number, $<\text{accumulated_sum}> + \text{number}$, $<\text{accumulated_sum}> * \text{number}$
31. Which of the following is not an operator grammar?

I. $S \xrightarrow{\cdot\cdot} AB$ II. $S \xrightarrow{\cdot\cdot} AaB$

III. $S \xrightarrow{\cdot\cdot} a$ IV. $S \xrightarrow{\cdot\cdot} \epsilon$

- (a) only I (b) I and II
- (c) I, III and IV (d) None

32. Which of the following is not an operator grammar

(a) $S \xrightarrow{\cdot\cdot} AaB$ (b) $S \xrightarrow{\cdot\cdot} AaB$

$A \xrightarrow{\cdot\cdot} aA/b$ $A \xrightarrow{\cdot\cdot} a/b$

$B \xrightarrow{\cdot\cdot} bB/a$

- (c) $S \xrightarrow{\cdot\cdot} AaB$ (d) None
- $A \xrightarrow{\cdot\cdot} aA/b$
- $B \xrightarrow{\cdot\cdot} bB/\epsilon$

33. What is equivalent operator grammar for the following grammar?

$$S \xrightarrow{\cdot\cdot} AB, \quad A \xrightarrow{\cdot\cdot} c/d, \quad B \xrightarrow{\cdot\cdot} aAB/d$$

- (a) $S \xrightarrow{\cdot\cdot} Aa/Ab$ (b) $S \xrightarrow{\cdot\cdot} AaAB/Ad$
 $A \xrightarrow{\cdot\cdot} c/d$ $A \xrightarrow{\cdot\cdot} c/d$
 $B \xrightarrow{\cdot\cdot} aAB/d$
- (c) $S \xrightarrow{\cdot\cdot} AaS/Ab$ (d) None
 $A \xrightarrow{\cdot\cdot} c/d$
 $B \xrightarrow{\cdot\cdot} aS/b$

34. In operator precedence parsing, precedence relations are defined

 - (a) for all pairs of non – terminals
 - (b) for all pairs of terminals
 - (c) to delimit the handles
 - (d) both (b) and (c)

35. Consider the grammar given below

$E \rightarrow E+E|E^*E|E-E|E/E|E^{\wedge}E|(E)|id$

Assume that + and – have the same but least precedence, * and / have the next higher precedence but the same precedence and finally \wedge has the highest precedence. Assume + and – associate to the left like * and / and that \wedge associates to the right. Choose the correct statement in the operator precedence table constructed for the grammar the relations for the ordered pairs $(\wedge, \wedge), (\neg, \neg), (+, +), (*, *)$ are

36. $S \rightarrow AaBb$

A → Bc/D

B → e

In the operator grammar above lead (S) = _____

37. Consider the following operator grammar

$$E \rightarrow E + T/T$$

$$T \rightarrow F \uparrow T/F$$

$F \rightarrow id$

- | | |
|---|---|
| (a)  | (b)  |
| (c)  | (d)  |

38. Choose the correct statement?

- (a) All (1) grammar is compulsory SLR (1)
 - (b) An operator grammar can be ambiguous but a parser may be Possible
 - (c) An operator grammar is recursive LR(1) always
 - (d) An grammar may be LALR (1) but not LR(1)

39. Choose the correct statement?
- (a) A grammar that is LR(0) may not be LL (1)
 - (b) A grammar that is LR is necessarily LL (1) that is LR is necessarily LL (1)
 - (c) A SLR(1) grammar is always LR(0)
 - (d) None of the above
40. Choose the false statement?
- (a) An ambiguous grammar can never be LL (1)
 - (b) An ambiguous grammar can never be LR (0)
 - (c) An ambiguous grammar can never be SLR (1)
 - (d) An ambiguous grammar cannot have a shift reduce parser
41. Choose the false statement?
- (a) There exists a grammar that is LR(0) but not LL(1)
 - (b) There exists a grammar that is LL (1) but not LR (0)
 - (c) There exists a grammar that is LL(1) but not SLR (1)
 - (d) There exists an ambiguous grammar that is either LL(1) or LR(0)
42. Choose the false statement?
- (a) Top down parsing algorithms simulate a leftmost derivation
 - (b) Bottom up parsing algorithms simulate the reverse of a rightmost Derivation
 - (d) An LL(1) parser is a top down parser
 - (e) An LR (1) parser may sometimes exist for some ambiguous grammars
43. Choose the correct statement?
- (a) A grammar that is LL(1) is necessarily LR(0)
 - (b) A grammar that is LL(2) is necessarily LR(0)
 - (c) A grammar that is LL(1) is necessarily LL(2)
 - (d) None of the above
44. Choose the false statement?
- (a) There exists a grammar that is SLR(1) but not LR(0)
 - (b) There exists a grammar that is LALR(1) but not SLR(1)
 - (c) There exists a grammar that is LR(1) but not LALR (1)
 - (d) There doesn't exist a grammar that is LL(2) and LR(0)

45. Which of the following is incorrect?
- (a) every LR(0) is CLR(1)
 - (b) every LL(1) is LR(0)
 - (c) every LR(1) is CFG
 - (d) every SRL(1) is CLR(1)

46. Consider the grammar shown below

$$\begin{aligned} <\text{life}> &\rightarrow <\text{session}><\text{session}> \\ <\text{session}> &\rightarrow \text{play } <\text{session}>|\text{rest} \end{aligned}$$

In the LR (0) machine for the grammar consider the items

$$\begin{aligned} <\text{life}> &\rightarrow <\text{session}><\text{session}>, \\ <\text{session}> &\rightarrow \text{play. } <\text{session}>, <\text{session}> \quad \text{play} <\text{session}>. \end{aligned}$$

Choose the correct answer

- (a) No two of the items occur in the same state
- (b) All three of the items occur in same state
- (c) At least two of them occur in multiple states
- (d) None of the above

47. Consider the following grammar

$$\begin{aligned} <\text{expression}> &\rightarrow <\text{factor}><\text{rest}> \\ <\text{rest}> &\rightarrow^* <\text{expression}> \mid \epsilon \\ <\text{factor}> &\rightarrow \text{identifier} \end{aligned}$$

In the LR(0) machine the no two of the following LR(0) items appear in the same state

- (i) $<\text{expression}> \rightarrow <\text{factor}> <\text{rest}>$
 - (ii) $<\text{expression}> \rightarrow^* <\text{rest}>$
 - (iii) $<\text{factor}> \rightarrow \text{identifier}$
- | | |
|--------------------|-----------------------|
| (a) (i) and (ii) | (b) (i) and (iii) |
| (c) (ii) and (iii) | (d) None of the above |

48. $S \rightarrow SB / A$

$A \rightarrow a$

$B \rightarrow b$

Find the closure set of $S^1 \rightarrow^* .S$ for the LR(0) machine is _____.

- | | |
|---|---|
| (a) $\begin{array}{l} S^1 \rightarrow S \\ S \rightarrow .SB \\ S \rightarrow .A \\ A \rightarrow .a \\ B \rightarrow .b \\ \hline \rightarrow \end{array}$ | (b) $\begin{array}{l} S^1 \rightarrow S \\ S \rightarrow .SB \\ S \rightarrow .A \\ A \rightarrow .a \\ \hline \end{array}$ |
|---|---|

- (c) $S^1 \quad .S \quad (d)$ None

$S \rightarrow .SB$

$S \rightarrow .A$

49. $S \rightarrow AA$

$A \rightarrow aA/b$

In the LR(0) machine for the above grammar, the number of states are ____.

50. The LR(0) parser for the grammar

$E \rightarrow E+T|T$

$T \rightarrow T^*F|F$

$F \rightarrow id$

Contains _____ number of conflicts.

51. Consider the grammar

$\langle \text{statement} \rangle \rightarrow a \langle \text{statement} \rangle | b$

$\langle \text{statement} \rangle \rightarrow b|c$

Choose the false statement

$A \rightarrow A + A|i$

Choose the proper statement regarding the property of the above grammar.

- (a) The grammar is LR(1) but not LALR(1)
- (b) The grammar is LALR(1) but not LL(1)
- (c) The grammar is SLR(1) but not LR(0)
- (d) None of the above

53. Consider the grammar

$S^1 \rightarrow S, S \rightarrow aAd|bBd|aBe|bAe, A \rightarrow c, B \rightarrow c$

The grammar is

- (a) LR(1) but not LALR(1) (b) LALR(1) but not LL(1)
- (c) SLR(1) but not LR(0) (d) None of the above

54. Consider the grammar

$S \rightarrow Aa|bAc|Bc|bBa, A \rightarrow a, B \rightarrow b$

- (a) The grammar is LR(1) but not LALR(1)
- (b) LALR(1) but not LL(1)
- (c) SLR(1) but not LR(0)
- (d) None of the above

55. What is the conflict in LR(1) construction for the grammar

$$A \longrightarrow aA|Ab|d$$

- 56.** Consider the grammar

$$S \longrightarrow Aa|bAc|dc|bda$$

A → d

57. What is the closure of $S \rightarrow A, \$$ for the grammar $S \rightarrow A, A \rightarrow AB, B \rightarrow b$

(a) $S \rightarrow .A, \$b$ (b) $S \rightarrow .A, \$$

$A \rightarrow .AB\$/b$ $A \rightarrow .AB\$$

A → ..\$/b A → ..\$

B → .b, \$

(c) $S \Rightarrow .A, \$$ (d) None

A → .AB,\$/b

A → ...\$/h

58. The grammar $E \rightarrow E + T/T; T \rightarrow a$ is

- (a) not LR(0) (b) not SLR(1) but CLR(1)
(c) CLR(1) (d) not LALR(1)

59. What is the conflict in LR(1) construction for the grammar $S \rightarrow aS / c$ is

- (a) R-R conflict (b) S-R conflict
(c) no conflict (d) both RR and SR conflict

- $$60. \quad S \rightarrow L = R/R$$

$L \rightarrow * R/id$

R → L

In the LR (1) items of the above grammar, the closure set of items in $S^1 \Rightarrow S, \$$ contains

- (a) $L \xrightarrow{*} R, =$ (b) $L \xrightarrow{*} R, \$$
 (c) $L \xrightarrow{*} R, \equiv / \$$ (d) $L \xrightarrow{*} .id, \equiv$

61. Consider the grammars

- 1) $S \rightarrow L = R|R, R \rightarrow L \rightarrow *R|id$
 2) $S \rightarrow Aa|bAc|dc|dba, A \rightarrow$

The grammar are

62. The grammar

$$S \rightarrow (s)/\varepsilon \text{ is}$$

- (a) LL (1)
- (b) LR(0)
- (c) CLR (1)
- (d) both LL(1) and LR(1)

63. Consider the grammar

$\langle \text{statement} \rangle ::= [\langle \text{statement} \rangle] \mid \text{assignment} \mid \text{null}$

Let the Number of states in the SLR (1), LR(1) and LALR(1) parsers for the grammar be N1, N3 and N2 respectively. The following relationship holds good

- (a) $N1 < N2 > N3$
- (b) $N1 = N2 > N3$
- (c) $N1 = N2 = N3$
- (d) $N1 >= N3 >= N2$

64. Let there are '10' states for a grammar which is SLR(1), then the number of states in LALR(1) parser is ____.

65. The parser generator tool YACC uses ____ parsing table

- (a) LR(0)
- (b) SLR (1)
- (c) LALR (1)
- (d) LR (1)

Key

01. (b)	02. (d)	03. (a)	04.(d)	05. 2	06. (c)	07. (a)
08. (c)	09. (c)	10. (c)	11. (b)	12. 144	13. (b)	
14. (c)	15. (c)	16. (d)	17. (c)	18. (d)	19. (a)	
20. (c)	21. (d)	22. (d)	23. (c)	24. (c)	25. (c)	
26. (a)	27. (c)	28. (d)	29. (b)	30. (d)	31. (d)	
32. (c)	33. (c)	34. (d)	35. (d)	36. (d)	37. (b)	
38. (b)	39. (a)	40. (d)	41. (d)	42. (d)	43. (c)	
44. (d)	45. (b)	46. (a)	47. (b)	48. (b)	49. 7	
50.2	51.(a)	52.(d)	53. (a)	54. (a)	55.(b)	
56. (d)	57. (c)	58. (c)	59. (b)	60. (c)	61. (a)	
62. (d)	63. (b)	64. 10	65. (c)			

Syntax Directed Translation Scheme

Concepts & Definitions

- S- attributes – Synthesized attributes entered to bottom up parsing .
- Semantic Routines – A router attached to each rule of context free grammar for purposes of semantics.
- Semantic stacks- stacks parallel to the syntax stack for semantic routing.
- Display – A replication of the static chain for speeding up scope of variables.
- Dynamic Chain- the order in which procedure are called.
- Static chain – the static scoping of variables.
- Row major order – Storing an array row wise.
- Column Major order storing an array column wise

4.1 Concept

Syntax deals with form & Semantics with Meaning

4.2 Concept

The ‘syntax of a HLL is specified by a context free grammar (Also known as the Backus Normal form or Backus Normal form or Backus Noun Form). The Syntax specification of a Hill yields a superset of the correct HLL Programs.

4.3 The Semantics of a HLL can be specified by context- sensitive language. but this will be a cumbersome specification with a lot of undecidability results.

4.4 Concepts

The Semantics of HLL are normally given in a natural language like English.

4.5. Concept

The Semantic specification is a superset of the programs specified by the Syntax specification and a proper subset of the set of correct programs.

4.6. Concept

The undecidability of the halting problem of Turing machine shows that semantic specifications only yields statically correct programs.

4.7 Concept

The concept of static is related to regular functions & the concept of dynamic to R.E sets

4.8 Concept

The language $L = \{ww/w \dots (a+b)^+ \}$ models the semantic requirement of having specification & declarations occurs before use. It is a context – sensitive feature as L is a CSL & cannot be expressed by a context free grammar.

4.9. Concept

The language $L = \{a^i b^j \mid a^i b^j \geq 1\}$ is a context – sensitive language that is not context – free . It models the requirement that specification & use of function with relation to order and number of parameters match. It is a semantic requirement that is not context – free.

4.10 Corollary

Fortran is a context – sensitive language as are C, C ++ & Java .

4.11 Definition

Translation is made in the front end of a compiler to an intermediate code

4.12 Definition

Translation is made in the front end of a compiler to an intermediate code is not a must. Translation from the Syntax analysis phase using Semantic rules can directly yield the machine code. however the Semantic routines will become very bulky. To cut down the complexity of the Semantic routines & to facilities machine independent optimization, a translation to the intermediate code is preferred

4.13 Varieties of intermediate code

Let us take the code for the expression

$a + b * c / b * c * f$

i) Postfix notation

$abc*b/c*f +$

ii). Quadruples or 3 – address code

1. $(\#, b, c, T_1)$
2. $(/, T_1, b, T_2)$
3. $(*, T_2, c, T_3)$
4. $(*, T_3, f, T_4)$
5. $(+, a T_4, T_5)$

iii) Triples for $a + b \uparrow C * f$

1. $(\uparrow, b, c) \backslash$
2. $(*, (1), (1))$

3. $(*, (2), f)$
 4. $(+, a, (3))$
- iv. Indirect triples (form the triples)

$((1))$

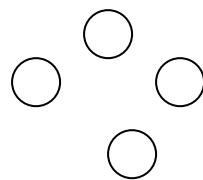
$((2))$

$((3))$

$((4))$

(v) **Trees**

Consider the expression $a+b*c$



vi. DAGs(directed Analysis Graph)

This is a representation that is useful to isolate common sub-expressions.

DAGs can be constructed using the same techniques as those used to construct Syntax trees.

A DAG has atomic operands & interior nodes that correspond to operators just like a Syntax tree. However a node N in a DAG may have more than one parent if N is a common sub-expression . Thus a DAG represent expressions succinctly.

Example of DAG

$a + a*(b-c) + (b-c)* d$

.....

.....

- 4.14. Translation to intermediate 3- address code. A sample snipped of coe void quick sort (int m, int n) /*recursively sort a (m) through a(n) */

int i, j ;

int, v, x;

if (n<=m)return;

/* fragrant begin here */

i= m - 1; j =n; v=a(n)

While (i)

```

do i= I+1 ; while (a[i]< v );
do j=j -1 ; while (a[j]>v
if (i > =j ) break ;
x = a[i]; a(i)= a(j) = a[j] = x ; /* swap a(i), a(n) */

/* fragrant ends here */
quick sort (m,j); quick sort (i+ 1, n )*

```

The program fragment deals with the famous quick sort sorting algorithm . Here a sub- array A(M) TO A(N) is to be sorted. The algorithm proceeds by selected arbitratrarily apportion elements a(v) in the subarray. The elements are then arranged around the partition element, the losses over to the left & the greater one's to the right. this partitioning step is recursively applied to the left & right sub arrays using the Divide & Conquer strategy.

The translation of the fragrant to the intermediate code is discussed in detail.

- void quick sort (int m, int n)

the function quick sort is only a procedure m & n are passed by value and define the subarray a(m). . . a(n) that is to be sorted. Let it be assumed that a is an integer array.

- int. I, j ;

i is a pointer which moves left to right from m to n. It skips over all elements that are smaller than the pivot element. j is a pointer which moves right to left from n to m. It skips over all elements that are larger than the pivot element .

- in v, x ;

a(v) is the pivot element for the partition step. It can be arbitrarily chosen. It is chosen. It is chosen here to be the rightmost elements of the partition. a[X] is temporary value used for swapping purposes .

if($n \leq m$)return;

If $= m$ there is only one element in the range a(m) to a(n) & it is trivially sorted. If $n < m$ then the partition a(n) contains null elements & is trivially sorted . this expresses the terminal condition for the Divide & Conquer step. If there is only 0 or 1 elements to be sorted then the subarray is trivially sorted.

- i=m-1;

Here the left pointer is set up.

j = n

Here the right pointer is set up.

- $v = a(n)$

Here the pivot is chose as the right most elements

- $\text{while } (1)$

Repeat the above loop continuously till the pointer cross .

- $\text{do } i = i + 1; \text{ while } (a(i) < n$

Advances the left pointer step by step by step to the right skipping over large /small elements.

- $\text{do } j = j - 1; \text{ while } (a[j] > n);$

Advances the right pointer step by step to the left skipping over large elements.

- $\text{do } (i >= j) \text{ break};$

The pointer meet or have crossed-return

$x = a(i); a(i) = a(j); a(j) = n ;$

swap $a(i)$ over $a(j)$

- $x = a(i); a(i) = a(n), a(n) = x;$

position the pivot

- $\text{quick sort } (m, j);$

Sort left partition

- $\text{quick sort } (I + 1, n);$

Sort partition.

Sample BNF

```

< code snippet>; := function definition >
< function definition > type> id (<param list>
{ <declaration list >< statement list> }

< param list > =param> | <param>, < param list >
<param> : <:type> id | < expression list > }

<type> : : + int / char /float/ void

<expression list > ::= =expression > |< expression >, < expression list >
<expression > ::= =expression > |< loop >, < expression >< loop>

<expression>
| -<expression>
|(expression >)
| id

```

<rop> ::= </>/<=/>==/!

<loop> ::= =+/-/*//

<declaration list> ::= declaration | <declaration>, <declaration list>

<statement list> ::= <statement> | <statement>, <statement list>

<statement> ::= var <expression>, (<expression list>) <statement>

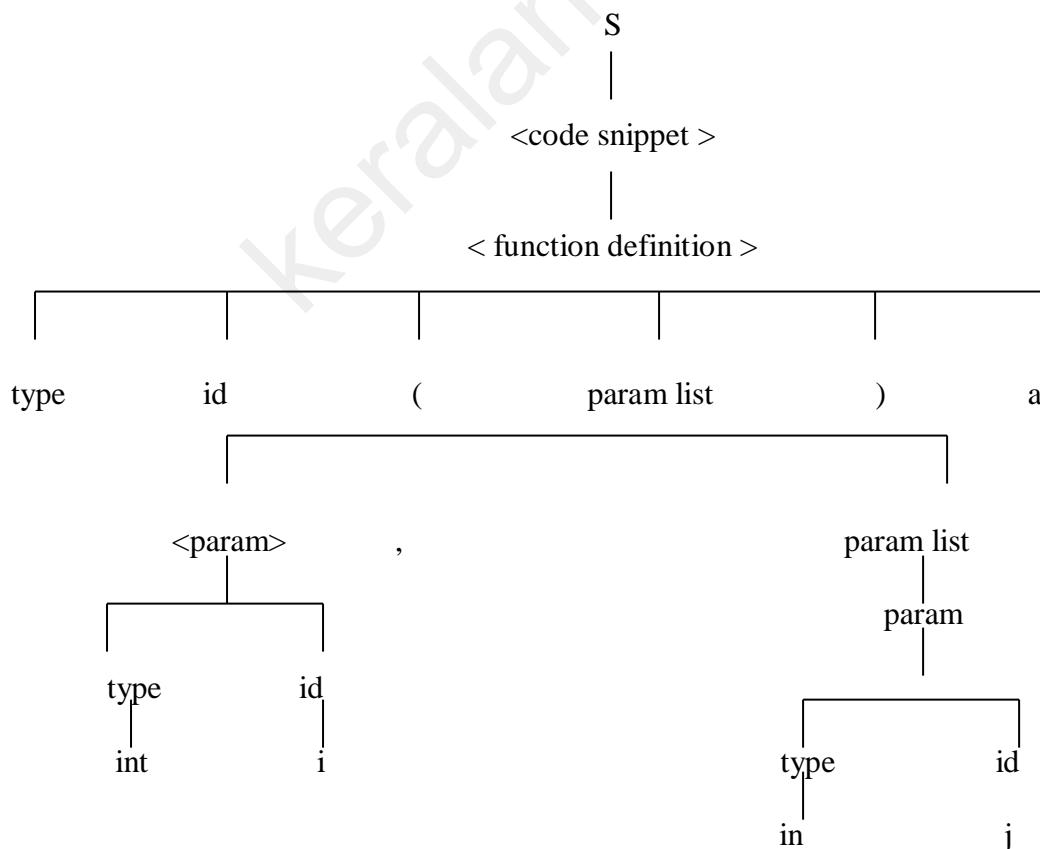
- | do <statement> while (<expression>)
- | while (<expression>) <statement>
- | for (<expression>; <expression>; <expression>)<statement>
- | null
- | continue
- | break
- | return
- | <function call>

Var ; id / id [<expression>]

<function call> ::= id (<expression list>)

<declaration list> ::= <declaration> / <declaration list>

<declaration> ::= <type> id:



Translating to the intermediate 3 – address code. We generate the intermediate code as we reduce the productions

1. $\langle \text{decl} \rangle :: \text{int I}$

Set up symbol table contain i as a integer type identifies of with 4 bytes

2. $\langle \text{decl} \rangle :: = \text{intj} :$

Set up symbol table to contain j as an integer type identifies of width 4 bytes.

3. $\langle \text{decl} \rangle :: = \text{intv} :$

Set up symbol table to contain variable v as an integer type of width bytes

var = $\langle \text{expression} \rangle ;$

$\langle \text{statement list} \rangle$

/ code generated is*

(14) $t_6 = 4 * i$

(15) $x = a[t_6] */$

while (1) { do $\langle \text{statement} \rangle$ while ($\langle \text{exp} \rangle$);

do $\langle \text{statement} \rangle$ while ($\langle \text{exp} \rangle$);

if ($\langle \text{expression} \rangle$ break ;

var = $\langle \text{expression} \rangle$

var = $\langle \text{expression} \rangle ;$

$\langle \text{statement list} \rangle$

/ code generated*

(16) $t_7 = 4 * i$

(17) $t_8 = 4 * j$

(18) $t_9 = a(t_8)$

(19) $a[t_7] = t_9 */$

⇒ while (1) {do $\langle \text{statement} \rangle$ while ($\langle \text{exp} \rangle$);

```

do < statement > while (<exp>);

if (exp) break;

var = <expression > ;

var = <expression >;

var = < expression >; }

/* code generated

```

$$(20) t_{10} = 4*j$$

$$(21) a[t_{10}] = x$$

$$(22) \text{ go to (8)}$$

$$(23) \dots \dots \ast /$$

11. <statement> \Rightarrow var = expression >
 $\Rightarrow X = a(i);$

```

/* code generated

```

$$(23) t_{11} = 4 * i$$

$$(24) x = a(t_{11}) \ast /$$

12. <statement > \Rightarrow var = <expression>
 a[i] = a(n)

```

/*code generated

```

$$(25) t_{12} = 4 * i$$

$$(26) t_{13} = 4 * n$$

$$(27) t_{14} = a(t_{13})$$

$$(28) a[t_{12}] = t_{14} \ast /$$

13. <statement > \Rightarrow var = <expression >
 $\Rightarrow a(n) = x$

$$(29) t_{15} = 4 * n$$

(30) $a[t_{15}] = x$

14. Putting the code together

- | | |
|-------------------------|-----------------------------|
| (1). $i=m-1$ | (2) $j=n$ |
| (3) $t_1=4*n$ | (4) $v=a(t_1)$ |
| (5) $i=i+1$ | (6) $t_2=4*I$ |
| (7) $t_3=a(t_2)$ | (8) if $t_3 < v$ go to (5) |
| (9) $j=j-1$ | (10) $t_4=4*j$ |
| (11) $t_5=a(t_4)$ | (12) if $t_5 > v$ go to (9) |
| (13) if $i \geq$ goto | (14) $t_6=4*i$ |
| (15) $x = a(t_8)$ | (16) $t_7=4*i$ |
| (17) $t_8=4 * j$ | (18) $t_9 = a(t_8)$ |
| (19) $a(t_7)=t_9$ | (20) $t_{10}=4*j$ |
| (21) $a(t_{10})=x$ | (22) go to (8) |
| (23) $t_{11}=4*i$ | (24) $x = a(t_{11})$ |
| (25) $t_{12} = 4*I;$ | (26) $t_{13}4*n$ |
| (27) $t_{14}=a(t_{13})$ | (28) $a[t_{12}] = t_{14}$ |
| (29) $t_{15}=4*n$ | (30) $a(t_{15})=x$ |

the flow graph

.....

.....

intermediate forms for control structure

1. $S \rightarrow \text{if } (B) S_1$
2. $S \rightarrow \text{if } (B) S_1 \text{ else } S_2$
3. $S \rightarrow \text{while } (B) S_1$
4. $S \rightarrow \text{do } S \text{ while } (B);$

5. $S \rightarrow \text{for } (E1; E2; E3)S$
 6. $S \rightarrow \text{switch } (E) \{ \text{case } ei : Si \} \text{default} : S_{n+1}$
01. The if statement

Construct : if (B) S_1

Code generated : B. code

S_1 . code

02. Modified construct : if $M_1(B) M_2 S_1 M_2$

Code generates $M_1 \rightarrow$ to B true

B. code

$(M_2) \rightarrow$ to B. false

B. true: S_1 code

B. false ; $(M_3) \setminus$

Ex: if ($a > b$) then $c = d + e$

$t = a > b$

IF J (t , false)

$c = d + e$

false : nop

03. The if – else statement

Construct : if (B) S_3 else S_4

Code generated : B. code

S_3 . code

S_4 . Code

04. Modified construct

If $M_4(B) \rightarrow$ to B. true

B. code

$M_5 \rightarrow$ to B false

B. true : S_3 - code

$M_6 \rightarrow$ to exit

B. false : S_4

Exit : NOP

05. Construct : while (B) S_5

Code generated : B. code

S_5 .Code

06. Modified construct

M_8 while $M_9(B)$ $M_{10}S_5M_{11}$

Start (M_8) \rightarrow B. true

(M_9)

B. code

$M_{10} \rightarrow$ B. false

S_5 . code

$M_{11} \rightarrow$ goto start

B. false : NOP

07. Construct : $S \rightarrow$ do S while (B);

Code generated : S code

B. Code

08. Modified construct

$S \rightarrow M_{12}$ do S while $M_{13}(B)M_{14}$

Code generated

Start : M_{12}

S. code

$M_{13} \rightarrow$ B. true

B. code

$M_{14} \rightarrow$ go to B. false

got to start

B. false : NOP

09. Construct $S \rightarrow$ for (E1;E2;E3;)S

- Code
 - E1. code
 - E2. code
 - E3. code
 - S. code
 - 10. Modified construct
 - $S \rightarrow \text{for } (M_1 E_1; M_2 E_3) M_4 S M_5$
 - Flow chart for
-
.....
.....

Code generated

```

Start : M1
    E1. code
    M2 → go to E2 true
    Start E2 : E2 Code
        M3 → E2 false
        go to start S;
    Start S : S4 code
        go to start E3

```

The SDTS for control structures

```

S → if (B1) M1 S1 M2
M1 → E{ gen(if j, B2(if j, B1.val,-); M1 Value, codeptr ) }
M2 → E(backpatch (M1.value, codeptr) )
S → if (B2) M3 S2 else M4 S3 M5
M3 → E (gen(if j, B2.val,-); M3.value = codeptr)
M4 → { gen(UJ,-); M4.value = codeptr ;
        backpatch (M3.value, codeptr) }
M5 → E { backpatch (M4.value, codeptr); }
S → M6 while (B3) M7 S4 M8
M6 → E { M6.value = codeptr ; }
M7 → E gen(if j, B3.val,-); M7.value = codeptr ;
M8 → E { gen(UJ, M6.val) backpatch (M7.val, codeptr); }

```

$S \rightarrow M_9 \text{ do } S \text{ while } M_{10}(B)M_{11}$

$M_9 \rightarrow E\{M_9\text{val} = \text{codeptr};\}$

$M_{10} \rightarrow E$

$M_{11} \rightarrow E\{ge = \text{codeptr};\}$

Syntax Directed Translation Scheme for evaluation of expressions.

$E_1 \rightarrow E\#$

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow \text{num}$

Sample expression

$4+3*6+4\#$

The derivation

$$\begin{array}{l}
 E_1 \xrightarrow{} E \# \\
 \quad \quad \quad \xrightarrow{} E+E \\
 \quad \quad \quad \xrightarrow{} E+E+4 \\
 \quad \quad \quad \xrightarrow{} E+E*E+4 \\
 \quad \quad \quad \xrightarrow{} E+E*6+4 \\
 \quad \quad \quad \xrightarrow{} E+3*6+4 \\
 \quad \quad \quad \xrightarrow{} 4+3*6+4
 \end{array}$$

The SDTS scheme

$$\begin{array}{l}
 E_1 \rightarrow E\#\{E_1 \text{ value}=E.\text{Value}\} \\
 E \rightarrow E_1+E_2\{E.\text{Value} = E_1\text{value} + E_2\text{ value}\} \\
 E \rightarrow E_1 * E_2 \{E \text{ value} = E_1\text{value} + E_2.\text{value}\} \\
 E \rightarrow \{E_1\} \{E.\text{value} = E_1\text{value}\} \\
 E \rightarrow \text{num} \{E.\text{value} = \text{num}\}
 \end{array}$$

The derivation tree

.....
.....
.....

The productions used in the reverse of a rightmost derivation.

$$\begin{aligned}
 E_1 &\rightarrow 4 \\
 E_2 &\rightarrow 3 \\
 E_3 &\rightarrow 6 \\
 E_4 &\rightarrow E_2 * E_3 = 18 \\
 E_5 &\rightarrow 4 \\
 E_6 &\rightarrow E_4 + E_5 = 22 \\
 E_7 &\rightarrow E_1 E_6 = 26
 \end{aligned}$$

The tree with reductions

Example 4.1:

Write SDT to build a syntax tree for expression ?

Sol : $E_1 \rightarrow E \#$

$$\begin{aligned}
 E &= E_1 * E_2 \\
 &= (E_1) \\
 &= \text{Id}
 \end{aligned}$$

The SDT

$$\begin{aligned}
 E &\rightarrow \text{id} \{ \text{create } E \text{ tree (1ptr = rptr=null, val=0)} \} \\
 E &\rightarrow E_1 + E_2 \{ E.\text{tree} = (1\text{ptr} = E_1.\text{tree}, \text{rptr} = E_2.\text{tree}, \text{val} = E_1.\text{val} + \text{val}) \} \\
 E &\rightarrow E_1 * E_2 \{ E.\text{tree} = 1\text{ptr} = E_1.\text{tree}, \text{rptr} = E_2.\text{tree}, \text{val} = E_1.\text{val} * E_2.\text{val} \} \\
 E &\rightarrow (E_1) \{ E.\text{tree} = E_1.\text{tree} \}
 \end{aligned}$$

Note : Intr is left pointer and rptr and is right pointer

Example : 4:1:

Write SDT to build a syntax tree a syntax tree for expression ?

Sol : $E_1 \rightarrow E \#$

$$\begin{aligned}
 E &\rightarrow E_1 + E_2 \\
 &\mid E_1 * E_2 \\
 &\mid (E_v) \\
 &\mid \text{id}
 \end{aligned}$$

The SDT

$$\begin{aligned}
 E &\rightarrow \text{id} \{ \text{create } E \text{ tree (1ptr = rptr=null, val=0)} \} \\
 E &\rightarrow E_1 + E_2 \{ E.\text{tree} = (1\text{ptr} = E_1.\text{tree}, \text{rptr} = E_2.\text{tree}, \text{val} = E_1.\text{val} + E_2.\text{val}) \} \\
 E &\rightarrow E_1 * E_2 \{ E.\text{tree} = 1\text{ptr} = E_1.\text{tree}, \text{rptr} = E_2.\text{tree}, \text{val} = E_1.\text{val} * E_2.\text{val} \} \\
 E &\rightarrow (E_1) \{ E.\text{tree} = E_1.\text{tree} \}
 \end{aligned}$$

Note : 1ptr is left pointer and rptr is right pointer

Example 4.2:

Write an SDT to create DAG ?

Sol : $E \rightarrow id \{ \text{If } id \text{ does not exist then}$

$\text{Get } E.\text{tree } (1\text{ptr} = \text{rptr} = \text{null}, \text{val}=0)$

$E \rightarrow E_1+E_2 \{ E.\text{tree} = (1\text{ptr} = E_1.\text{tree}, \text{val} = E_1.\text{val} + E_2.\text{val}) \}$

$E \rightarrow E_1*E_2 \{ E.\text{tree} = (1\text{ptr} = E_1.\text{tree}, \text{rptr} = E_2.\text{tree}, \text{val} * E_2.\text{val}) \}$

$E \rightarrow (E_1) \{ E.\text{tree} = E_1.\text{tree} \}$

Example 4.3:

Write SDT to store type information in the symbol table ?

Sol : $D \rightarrow T \{ L_n = T: \text{type} \}$
 L

$T \rightarrow \text{int} \{ T.\text{type} = \text{integers} \}$

$T \rightarrow \text{real} \{ T.\text{type} = \text{real} \}$

$L \rightarrow \{ L_1.\text{in} : L_{in} \}$

$L_1 \text{id} \{ \text{add type (id. Entry, Lin)} \}$

$L \quad id \{ \text{add type(id. Entry , L.in)} \}$

Note : id entry is the entry is the entry in the table

Write an SDT to generate three address code for any expression ?

Sol : $E_1 \rightarrow E\#$

$E \rightarrow E_1+T_1 \{ \text{new } t; t = \text{out}(+, E_1t, T.t); E.t=t \}$

$E \rightarrow T_1 \{ E.t = T_1t \}$

$T_1 \rightarrow T_1*F_1 \{ \text{new } t; t = \text{out}(*, T, t, F_1t); T.t = t \}$

$T \rightarrow F_1 \{ T.t = F_1.t \}$

$F \rightarrow id \{ F.t = id. \text{name} \}$

Note : new t creates a new temporary

Example 4.5:

Write an SDT to issue error messages ?

Sol : $E_1 \rightarrow E\#$

$E \rightarrow E_1+E_2$

$/E_1*E_2$

$/(E_1)$

$/id$

Both operands have to be of type float or int

$E_1 \rightarrow E\#$

$E \rightarrow E_1+E_2 \{ \text{if } (E_1.\text{type} = \text{int} \& E_2.\text{type} = \text{float}) \}$

Error();
 Else if (E₁. type = float & E₂.type = int)
 Error();

Similarly for E₁*E₂

Example 4.6:

Write an SDT to count the number of terminals.

Sol: E₁ → E {E₁.count = E. count + 1 }
 E → E₁+T {E. count = E₁.count + T. count }
 E → T {E. count = T. count }
 T → T₁*F {T. count = T₁.count + F. count + 1 }
 T → F {T. count = F. count }
 F → (E) {F. count = E. count }
 F → id {F. count = 1 }

Example : 4.7

Write an SDT to reverse an expression

Sol : E₁ → E# {E₁. val= # | E. val }
 E → E₁+T {E. val = || '+'|| E₁. val }
 E → T {E. val=T. Val}
 T → T₁ * F {E. val || '*'|| T₁. val }
 T → F {T. val=F. val }
 F → (E) {F. val = ')' | E. val || '(' }
 F → id {F. val = id name }

Example 4.8 :

Write an SDT for converting a binary number to its decimal equivalent.

Sol : N → ND/D

D → 0/1

The SDTS

D → 0 {(D. decval = 0)}
 D → 1 {(D. decval = 1)}
 N → D {N. decval = D. decval }
 N → N₁D {N. decval = N₁. decval * 2 + D. decval }

Note : ‘decval’ is decimal value

Example 4.9 :

Write an ADT for converting a integer string “1234” to its decimal equivalent

Sol : string → String chair string char /char
 Char → 0' – '9'
 SDTS
 Char → '0'- '9' { char . val = char = 0 to 9 }
 String → char {String. Val = char. Val }
 String → string₁ char { string. Val = string. Val*10+char. Val}

Example 4.10

Write postfix SDT to store type information in symbol table

Sol: $E_1 \rightarrow E\#$
 $E \rightarrow E_1 + T \{out('+)\}$
 $E \rightarrow T$
 $T \rightarrow T_1 * F \{out(*)\}$
 $T \rightarrow F$
 $F \rightarrow (E)$
 $F \rightarrow id \{enter symbol table id. Name = id. Type \}$

Classroom Practice Questions

$E \rightarrow E_1 + T \quad \{E \text{ val} = E_1 + T. \text{Val}\}$

$$E \rightarrow T \quad \{E \text{ val} = E_1 + T. \text{Val}\}$$

$T \rightarrow id$ { $T.$ Val= id }

- a) S-attributed
 - b) L-attributed
 - c) Both (a)and (b)
 - d) None

- ### 03. The following SDT checks

$S \rightarrow aSb \quad \{ S.\text{count} = S.\text{count} + 2 \}$

$S \rightarrow bSa \quad \{S.\text{count} = S.\text{count} + 2\}$

$S \xrightarrow{E} \{ S.\text{count} = 0 \}$

- (a) Equal number of a's and b's.

- (b) Number of a 's or number of b's in given string
 (c) Number of a's and b's in a given string
 (d) None
04. Consider the following Syntax Direct Translation scheme
- $$E \rightarrow E + T \{ E.\text{value} = '+' \parallel E.\text{Value} \parallel T.\text{value} \}$$
- $$E \rightarrow E - T \{ E.\text{value} = '-' \parallel E.\text{value} \parallel T.\text{value} \}$$
- $$E \rightarrow T \{ E.\text{value} = T.\text{value} \}$$
- $$T \rightarrow \text{id} \{ T.\text{value} = \text{id} \} \text{ (Assume } \parallel \text{ is string concatenation)}$$

The above SDTS specifies _____

- a) Expression Evaluation b) Infix to postfix conversion
 b) Infix to prefix conversion d) None of the Above

05. Consider the Syntax Directed Definition below

$$\begin{array}{ll} S \rightarrow S_1 S_2 c & \{S, t=S_1, t-S_2-t\} \\ S \rightarrow a & \{S, t=2\} \\ S \rightarrow b & \{S, t=6\} \end{array}$$

If the Top down parser carries out the translation for an input string "babcc" output is _____

- a) 0 b) 6
 c) 10 d) 5

06. Consider the SDT given below,

$$\begin{array}{ll} S \rightarrow S_1 S_2 c & \{S, \text{val}* S_2, \text{Val}=4\} \\ S \rightarrow a & \{S, \text{val}=6\} \\ S \rightarrow b & \{S, \text{val}=2\} \end{array}$$

If the SDT gives the input string abc, then the output is

- (a) 5 (b) 8
 (c) 0 (d) 2

07. Consider the SDTS (using synthesized attributes)

$$\begin{array}{l} E_1 \rightarrow E_2 * E_3 [E_1.\text{sem} = E_2.\text{sem} \times E_3.\text{sem}] \\ E_4 \rightarrow E_5 + E_6 [E_4.\text{sem} = E_5.\text{sem} + E_6.\text{sem}] \\ E \rightarrow \text{id} [E.\text{sem} = 1] \end{array}$$

The output for the input a+b+c+d+e is

- a) 3 (b) 4 (c) 5 (d) 6
 08. Consider the SDTS (using synthesized attributes)

$S \rightarrow aS \quad \text{out}(''1'')$
 $S \rightarrow a \quad \text{out}(''2'')$

The output for the input aaaa is

- a). “1111” b) “2222”
 c). “2111” d) “1112”

09. Consider the SDTS (using synthesized attributes)

$S \rightarrow aA \quad \text{out} (''1'')$
 $S \rightarrow aa \quad \text{out} (''2'')$
 $A \rightarrow a \quad \text{out} (''3'')$

The output for the input aa is

- a) 31 or 2 b) 2
 b) 31 d) None of the above

10. Consider the SDTS (using synthesized attributes)

$S \rightarrow aB \quad \{\text{out} "cat"\}$
 $S \rightarrow bA \quad \{\text{out} "dog"\}$
 $B \rightarrow aBB \quad \{\text{out} "pig"\}$
 $A \rightarrow bAA \quad \{\text{out} "cow"\}$
 $B \rightarrow b \quad \{\text{out} "deer"\}$
 $A \rightarrow a \quad (\text{out} "lion")$

The output for aabb is

- a) deer deer pig cat b) cat dog pig lion
 c) cat cow deer lion c) cat deer dog pig

11. Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals { S,A } and terminals { a, b }

$S \rightarrow aA\{\text{print 1}\}$
 $S \rightarrow a\{\text{print 2}\}$
 $A \rightarrow Sb\{\text{print 3}\}$

Using the above SDTS, The output printed by a bottom-up parser , for the input aab is ;

- (a) 1 3 2 b) 2 2 3

- 12 Here is a postfix SDT :

12. Here is a postfix SDT:

$S \rightarrow aS\{print\text{ "X"}\}$

$S \rightarrow bS \{ \text{print "y"} \}$

S → a {print “z”}

S → b {print “z”}

Suppose we execute this SDT in connection with a bottom-up parser. What will be printed in response to the input “ababb”

- a) zxyxxy
 - b) zyxyyy
 - c) zyxyx
 - d) zyyxx

13. The output for the SDT

A → A + A{print 1}

A → a { print 2}

For $a = a$ is

- a) 22121
 - b) 21221
 - c) 21212
 - d) 12122

14. Consider the SDTS given below

E1 → E print (“S”)

E → E+E printf(“+”)

E → E*E printf("*")

$$E \rightarrow (E)$$

E → id print(id. Value)

If we assume a bottom up shift reduce parser, the output for the input string $a+b^* c$ is

- (a) abc *+
 - (b) ab+ c *
 - (c) abc* or ab+c* depending on the compiler
 - (d) There will be an error and no parsing can take place

15. Consider the following translation scheme

$$S \longrightarrow AB$$

$B \rightarrow^* A \{ \text{print}^* \} B/E$

A → C+A { print+ } /C

C → S/id { print id }

Here id represents an integer For input ‘5* 6+7’ this translation prints _____

- (a) 3 7
- (b) 5*6+7
- (c) 56*7+
- (d) 5 6 7 +*

16. Consider the SDT given below

$$E \rightarrow E \uparrow E \{ \text{print } \}$$

$$E \rightarrow E^*E \{ \text{print}^* \}$$

$$E \rightarrow \text{id} \{ \text{print id} \}$$

If the shift reduce parser construct a parse tree for the input sentence a*b c, the ↑ above translation prints.

- (a) ab *c ↑
- (b) a b c ↑*
- © Both
- (d) None

5. Intermediated Code Generation

Introduction

In the analysis –synthesis model of a compiler, the front end analyzes a source program and creates an intermediate representation from which the back end generates target code. Ideally, details of the source language are confined to the front end, and details of the target machine to the back end. With a suitably defined intermediate representation, a compiler for language I and machine j can then be built by combining the front end for language I with the back end for machine j. We assume that a compiler front end is organized as in figure 5.1 where parsing, static checking and intermediate –code generation are done sequentially, sometimes they can be combined and folded into parsing.

.....
.....

Static checking includes type checking, which ensues that operators are applied to compatible operands. It also includes any syntactic checks that remain after parsing.

In the process of translating a program in a given source language into code for a given target machine, a compiler may construct a sequence of intermediate representation. High – level representations are close to the source language and low-level representations are close to the source language and low-level representations are close to the target machine.

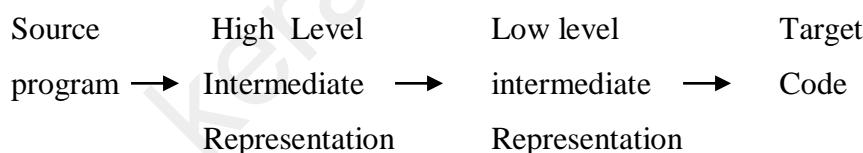


Fig 5.2 : A compiler might use a sequence of intermediate representations

Three –Address Code

In three –address Code, there is at the most one operator on the right side of an instruction : that is, no built –up arithmetic expressions are permitted. Thus a source-language expression like $x + y * z$ might be translated into the sequence of three-address instructions

$$T_1 = y * z$$

$$T_2 = x + t_1$$

Where t_1 and t_2 are compiler – generated temporary names.

Example : three address code is linearized representation of a syntax tree or a DAG, in which explicit names correspond to the interior nodes of the graph.

.....
.....
Fig : 5.3 DAG and its corresponding three-address code

Quadruples

A quadruple (or just “quad”) has four fields, which we call op, arg₁ arg₂ and result. The op field contains an internal code for the operator For instance, the three-address instruction $x = y + z$ is represented by placing +in op, y in arg₁ z in arg₂ and x in result.

The following are some exceptions to this rule :

1. Instructions with unary operator like $x = \text{minus } y$ or $x = y$ do not use arg₂. Note that for a copy statement like $x=y$, op is = while for most other operations, the assignment operator is implied.
2. Operators like param use neither arg₂nor result.
3. Conditional unconditional jumps put the target label in result.

Example : Three –address code for the assignment $a=b*-c+b*-c$:

$$T_1 = \text{minus } c$$

$$T_2 = b * T_1$$

$$T_3 = \text{minus } c$$

$$T_4 = b * T_3$$

$$T_5 = T_2 + T_4$$

$$a = T_5$$

(a) Three-address code

	OP	Arg ₁	Arg ₂	Result
	Minus	c		T_1
1	*	c	T_1	T_3
2	minus	c		T_3
3	*	b	T_3	t
4	+	T_2	T_4	T_5
5	=	T_5		a

(b) Quadruples

Fig . Three-address code and its quadruple representation

Triples

A triple has only three fields, which we call op, arg₁ and arg₂. Using triples, we refer to the result of an operation x op y by its position, rather than by an explicit temporary name.

The DAG and triple representations of expressions are equivalent.

0	minus	c	
1	*	b	(0)
2	minus	c	
3	*	b	(2)
4	+	(1)	(3)
5	=	a	(4)

Fig 5.4: Representations of $a = b * -c + b * c;$

Translation of Expressions

The translation of expressions into three –address code. An expression with more than one operator, like $a+b*c$, will translate into instructions with at the most one operator per instruction. an array reference $A(i) (j)$ will expand into a sequence of three-address instructions that calculate an address for reference

Production	Semantic Rules
$S \rightarrow id = E;$	$S.\text{code} = E.\text{code} $ $\text{gen}(\text{top.get(id.lexeme)}' = ' E.\text{addr})$
$E \rightarrow E_1 + E_2$	$E.\text{addr} = \text{new Temp}()$ $E.\text{code} = E_1.\text{code} E_2.\text{code} $ $\text{gen}(E.\text{addr} ' = ' E_1.\text{addr} + E_2.\text{addr})$
$1 - E_1$	$E.\text{addr} = \text{new Temp}()$ $E.\text{code} = E_1.\text{code} $ $\text{Gen}(E.\text{addr} ' = ' \text{minus}' E_1.\text{addr})$
$ (E_1)$	$E.\text{addr} = E_1.\text{addr}$ $E.\text{code} = E_1.\text{code}$
$ id$	$E.\text{addr} = \text{top.get(id.lexeme)}$ $E.\text{code} =$

Fig 5.5: three-address code for expressions

Control Flow

The translation of statements such as if –else- statements and while statements is tied to the translation of Boolean expressions. In Programming languages, Boolean expressions are often used to

1. Alter the flow of control. Boolean expressions are used as conditional expressions is implicit in a position reached in a program. For example, in if (E) s, the expression E must be true if statement S is reached
2. Compute logical values. A Boolean expression can represent true or false as values. Such Boolean expressions can be evaluated in analogy to arithmetic expressions using three-address instructions with logical operations. as values. Such Boolean expressions can be evaluated in analogy to arithmetic expressions using three-address instructions with logical operations.

Flow-of control statements

The translation of Boolean expression into three-address code in the context of statements such as those generated by the following grammar:

$$\begin{aligned} S &\rightarrow \text{if } (B)S_1 \\ S &\rightarrow \text{if } (B)S_1 \text{else } S_2 \\ S &\rightarrow \text{while } (B)S_1 \end{aligned}$$

In these productions, non-terminal B represents a Boolean expression and non-terminal S represents a Statement.

The translation of if (B) S₁ consist of B. code followed by S₁. code , as illustrated in fig. Within B. code are jumps based on the value of B. If B is true, control flows to the first instruction of S₁ Code , and if B is false, control flows to the instruction immediately following S₁ code.

.....
.....

The labels for the jumps in B. code and S.code are managed using inherited attributes. With a Boolean expression B, we associate two labels: B. true, the label to which control flows if B is true, B. false, the label to which control flows if B is false. With a statement S we

associated an inherited attribute S. next denoting a label for the instruction immediately after the code for S. In some cases, the instruction immediately following S. code is a jump to some label L. A jump to a jump to L from within S.code is avoided using S. next.

The syntax directed definition in fig. produces three-address code for Boolean expressions in the context of if, if else and while-statements

Prduction	Semantic Rules
P S	S. next= newlabel () P.code= S. code label(S. next)
S assign	S. code = assign. code
S if (B)S ₁	B.true=newlabel() B. false =S ₁ next = S. next S.code =B.code label (B.true) S ₁ code
S → if (B)S ₁ else S ₂	B. true =newlabel() B. false = newlabel() S ₁ next = S ₂ next = S. next S.code = B.code label(B.true) S ₁ code gen ('goto' S. next) label (B. false) S ₂ code
S While (B) S ₁	Begin = newlabel () B.true = newlabel () B. false = S. next S ₁ next = begin S. code = label (begin) B. code label(B.true) S ₁ .code gen ('goto' begin)
S → S ₁ S ₂	S ₁ next = newlabel () S ₂ next =S. next S. code= S ₁ . code label(S ₁ next) S ₂ . code

Fig 5.7: Syntax- directed definition for flow-of-control statements

Control-Flow Translation of Boolean Expressions

The semantic rules for Boolean expressions in Figure complement the semantic rules for statements in figure. A Boolean expression B is translated into three-address instructions that evaluate B using conditional and unconditional jumps to one of two labels: B true if B is true, and B. false if B is false

Production	Semantic Rules
$B \rightarrow b_1 b_2$	$B_1.\text{true} = B.\text{true}$ $B_1.\text{false} = \text{newlabel}()$ $B_2.\text{true} = B.\text{true}$ $B_2.\text{false} = B.\text{false}$ $B.\text{code} = B_1.\text{code} \text{label}(B_1.\text{false}) B_2.\text{code}$
$B \rightarrow B_1 \& \& B_2$	$B_1.\text{true} = \text{newlabel}()$ $B_1.\text{false} = B.\text{false}$ $B_2.\text{true} = B.\text{true}$ $B_2.\text{false} = B.\text{false}$ $B.\text{code} = B_1.\text{code} (B_1.\text{true}) B_2.\text{code}$
$B \rightarrow ! B_1$	$B_1.\text{true} = B.\text{false}$ $B_1.\text{false} = B.\text{true}$ $B.\text{code} = B_1.\text{code}$
$B \rightarrow E_1 \text{ rel } E_2$	$B.\text{code} = E_1.\text{code} E_2.\text{code}$ $\quad \text{gen}(\text{'if } E_1.\text{addr } \text{rel. op } E_2.\text{addr } \text{'goto' } B.\text{true})$ $\quad \text{gen}(\text{'goto' } B.\text{true})$
$B \rightarrow \text{true}$	$B.\text{code} = \text{gen}(\text{'goto' } B.\text{true})$
$B \rightarrow \text{false}$	$B.\text{code} = \text{gen}(\text{'goto' } B.\text{false})$

Fig 5.8 Generating three-address code for Booleans

Translate statement to 3 address statements

Example 5.1:

$$a = b + c * d / c$$

$$\text{Sol : (1) } (*, c, d)$$

$$(2) (/,(1),c)$$

$$(3) (+, b,(2))$$

(4) (MOV, (3),9,-1)

Example 5.2 :

for (j=0:j<10; j++) a= b+c

Sol: The control flow :

.....
.....

The translation

.....
.....

The code

- (1) (MOV, #0, j,-)
- (2) (<, j#, 10, T)
- (3) (IFJ, T, (8))
- (4) UJ, (6)
- (5) (+j, I, j)
- (6) (+, b, c,a)
- (7) UJ (2)
- (8) NOP

Example 5.3:

While (A<C) and (B.D) do if A = 1 then C=C+1

else while A < = D do A=A+3

Sol : (1) (<A,C,T1)

- (2) (>B,D, T2)
- (3) (and, T1, T2, T)
- (4) (IFJ, T, (15))
- (6) (=,A,1, T1)
- (7) (IFJ, T1,(10))
- (8) (+, I,C,C)
- (9) (UJ, (15))
- (10) (C=, A,D,T3)
- (11) (IFJ, T3, (15))
- (12) (+,#3, A A)
- (13) (UJ, (10)-)
- (14) (UJ, (1),-1)

(15) NOP

Example : 5.4

I f (a>7) or (b == 5) then x =7 else y=z

Sol : (1) (>, 9,7,T1)

(2) (==,b,5,T2)

(3) (or, T1, T2,T)

(4) (IF J, T,(7)

(5) (MOV, #7, X, -)

(6) (UJ, (8))

(7) (MOV, #2, y-

(8) NOP

Example 5.5:

Switch (i)

Case 1 : a=10: break ;

Case 2; a=20; break;

Default : a= 50: break

Sol : The flow chart

.....

.....

(1) (==,I,1,T1)

(2) (==,I, 2, T2)

(3) UJT1

(4) UJT2

(5) UJT

(6) (T1) (MOV, #,10,a)

(7) UJ(12)

(8) (T2)(MOV, # 20,a)

(9) UJ(12)

(10) (T3)MOV,#50,a)

(11) UJ(12)

(12) NOP

Example 5.6

-a+b/x ↑ d ↑ e * f/g

Sol (1) (-u, a, T)

(2) ↑, d,e,T2)

(3) ↑, x,T2,T3)

(4) (/,b, T3,T4)

(5) *, T4, 1, T5)

(6) /,T5,9,T6)

(7) (+,T1,T6,T7)

Note : ‘-u’ is unary minus

Example : 5.7:

a<b or c<d and c < f

Sol (1) (<,a,b,T1)

(2) (<,c,d,T2)

(3) (<,c,f.T3)

(4) (and, T2,T3,T4)

(5) (or, T1, T4, T5)

Directed Acyclic Graphs for Expressions

A directed acyclic graph (DAG) for an expression identifies the common sub expressions in the expression. Like a syntax tree, a DAG has a node for every sub expression of the expression; an interior node represents an operator and its children represent its operands. The difference is that a node in a DAG representing a common sub expression has more than one “parent,” in a syntax tree.

example : Construct a DAG for the following expression

$a+a^*(b-c+(b-c)*d)$

The leaf for a has two parents because a is common to the two sub expressions a and $a^*(b-c)$. Likewise, both occurrences of the common sub expression b-c are represented by the same node , which also has two parents.

.....
.....

Fig : 5.9: DAG for the expression $a+a^*(b-c)+(b-c)*d$.

Classroom Practice Questions

01. One of the purpose of using intermediate code in compilers is to
 a) make parsing and semantic analysis simpler

08. Consider the following code segment

```

x = u - t.,
y = x * v.,
x = y + w.,
y = t - z.,
y = x * y.,

```

The minimum number of total variables required to convert the above code segment to static single assignment from is _____

09. Consider the following intermediate program in three address code

```

p = a - b
q = p * c
p = u * v
q = p + q

```

Which one of the following corresponds to a static single assignment form of the above code ?

- | | |
|-------------------|-------------------|
| (a) $p_1 = a - b$ | (b) $p_3 = a - b$ |
| $q_1 = p_1 * c$ | $q_4 = p_3 * c$ |
| $p_1 = u * v$ | $p_4 = u * v$ |
| $q_1 = p_1 + q_1$ | $q_5 = p_4 + q_4$ |
| (c) $p_1 = a - b$ | (d) $p_1 = a - b$ |
| $q_1 = p_2 * c$ | $q_1 = p * c$ |
| $p_3 = u * v$ | $p_2 = u * v$ |
| $q_2 = p_4 + q_3$ | $q_2 = p + q$ |

10. Choose the incorrect statement

- (a) DAG representation of a basic block enables elimination of common sub expression.
- (b) DAG representation of a basic block enables dead code elimination.
- (c) DAG representation allows optical register usage by specifying & controlling the lifetime of variables in registers.
- (d) None

11. The DAG for $a*b*b$ is

.....
.....

12. The DAG for the expression

13. Consider the Three Address code below

$$d = a + b$$

$$e = c + d$$

$$f = d + e$$

$$b = c + e$$

$$e = b + f$$

The number of nodes in the DAG constructed for the above block of statements are _____

14. Consider the following block of three address statements

$$a = c + d$$

$$e = a + d$$

$$d = c + d$$

Then the number of nodes in the DAG constructed for the above block of statement is

15. For the code below

$$a = b + c$$

$$c = a + d$$

$$d = b + c$$

$$e = d - b$$

$$a = e + b$$

obtain the minimum number of registers to be allocated by the graph coloring algorithm .

- | | |
|-------|-------|
| (a) 2 | (b) 3 |
| (c) 4 | (d) 5 |

16. Consider the basic block given below

$$a = b + c$$

$$c = a + d$$

$$d = b + c$$

$$e = d - b$$

$$a = e + b$$

The minimum number of nodes and edges present in the DAG representation of the above basic block respectively.

- | | |
|--------------|--------------|
| (a) 6 and 6 | (b) 8 and 10 |
| (c) 9 and 12 | (d) 4 and 4 |

17. For a C program accessing X (i) (j) (k), the following intermediate code is generated by a compiler. Assume that the size of an integer is 32 bits and the size of character is 8 bits.

$t0 = i * 1024$

$t1 = j * 32$

$t2 = k * 4$

$t3 = t1 + t0$

$t4 = t3 + t2$

$t5 = X(t4)$

Which one of the following statements about the source code for the C program is CORRECT ?

- a) X is declared as “int X[32] [32] [8]”
- b) X is declared as “ int X [4] [1024] [32]”
- c) X is declared as “char X [4] [32] [8]”
- d) X is declared as “char X[32] [16] [2]

18. The following syntax tree :

.....
.....

Represent the assignment

$a = (b - (c + d)) + e * f;$

Convert this tree to (op, arg1, arg2)triples, following these rules :

1. Evaluate the right sub tree of anode before the left subtree.
2. Number the instruction (1), (2) .

Then identify the list below , the one triple, with its instruction number, that would appear in your translation

- | | |
|---------------------|-----------------|
| (a) (1) [+ , e,f] | b) (3)[*,e,f] |
| (c) (3) [- , b, (2) | d) (4) [+,b(1) |

Key

01.(c) 02(d) 03(d) 04(d) 04(d) 05(b) 06(c) 07(c)

09 (b) 10 (d) 11 (d) 12(b) 13.8 14.4 15.(b) 16(b)

17 (a) 18(b)

Chapter – 1

Finite Automata and Regular Sets

Level – 1 Questions

01. The class of regular sets are
 - (a) closed under finite union
 - (b) sometimes closed under infinite union
 - (c) closed under finite intersection
 - (d) all of the above
02. When an NFA is converted to an equivalent DFA accepting the same language, the number of states
 - (a) necessarily decreases
 - (b) necessarily increases
 - (c) always remains the same
 - (d) sometimes remains the same
03. Which of the following is most approximate in the case of Finite language
 - (a) It can be regular language
 - (b) It can be CFL
 - (c) It can be CSL
 - (d) It can be REL
04. Choose the correct answer
 - (a) The class of finite automata can be put into a one to one correspondence with the integers.
 - (b) The class of finite automata can be put into a one to one correspondence with the formal languages.
 - (c) The class of push down automata is larger than the class of finite automata and can be put into a one to one correspondence.
 - (d) None of the above
05. The minimal finite automata accepting the set of all strings in $(0 + 1)^*$ that has the last two symbols same has
 - (a) 40 states
 - (b) 5 states
 - (c) 1000 states
 - (d) 1000000 states

06. The set of all strings over {0, 1} where the 10th symbol from the right end is a 1 and the 10th symbol from the left end is a 0 and the 20th symbol from the right end is a 1 is best described as
 (a) regular set (b) CFL
 (c) CSL (d) R.E. set
07. The minimal DFA that acts as a mod 3 counter and outputs a 1 whenever the input in binary treated as binary integer is divisible by 3 has
 (a) 3 states (b) 4 states
 (c) 5 states (d) none of the above
08. The minimal finite automata accepting the set of all strings over {0, 1} where the fourth symbol from the right end is a 1 has
 (a) 4 states (b) 8 states
 (c) 10 states (d) 16 states
09. The regular expression denoting the set of all strings not containing two consecutive 1's given by
 (a) $(0+10)^*(\varepsilon+0)$ (b) $(0+10)^*(\varepsilon+1)$
 (c) $(1+01)^*$ (d) $(\varepsilon+0)(101)^*(\varepsilon+0)$
10. Let L = {Set of all binary strings whose integer equivalent is divisible by 4}. Then the number of states in Minimal FA accepting 'L' is
 (a) 2 (b) 3
 (c) 4 (d) 5
11. The regular expression with will strings 0's and 1's with at least two consecutive 0's is
 (a) $(1+10)^*$ (b) $(0+1)^*00(0+1)$
 (c) $(0+1)^*011$ (d) $0^*1^*2^*$
12. Give the regular expression that derives all strings of a's and b's where each string contain even occurrences of substring ab.
 (a) $(abab)^*$ (b) $(b+abab)^*$
 (c) $(b+aa^*bb^*aa^*b)^*$ (d) $(b.aa^*bb^*aa^*b)^*$
13. Which of the following strings will the transition diagram given below recognize?
 (a) Ending with 10 b) Not ending with 10
 (c) Not ending with 01 d) None

14. The regular expression denoting the set of all binary strings not containing 000 as a substring is
- $(0+01+001)^*(\varepsilon + 0 + 00)$
 - $(\varepsilon + 0 + 00) (1+10+100)^*$
 - $(0+1)^* 000(0+1)^*$
 - none of the above
15. The minimum state automaton to recognize the set denoted by $(a+b)^* abb$ has
- 3 states
 - 4 states
 - 6 states
 - none of the above
16. What is the language expressed by the regular expression $(0^*011^*100^*)^* \cup (1^*100^*011^*)^*$
- The set of all strings, which contain equal number of 01's and 10's
 - The set of all strings, which contain 01 or 10 as substring.
 - The set of all strings that have same number of disjoint appearances of 10 and 01
 - None of the above
17. Let $\Sigma = \{a, b\}$. Which of the following regular expression define the language :
- $L = \{ w \in \Sigma^* / w \text{ has atleast one substring } ab \}$
- $b^*a^*abb^*a^*$
 - $(a \cup b)^* (ab)^*(b \cup a)^*$
 - $(a^*b^*)^*ab(a^*b^*)^*$
 - $b^*a^*(ab \cup \phi^*)b^*a^*$
18. The minimal finite automata accepting the set denoted by $(0+1)^* (00+11)$ has
- 3 states
 - 4 states
 - 5 states
 - 6 states
19. The regular expression

$$(aa)^* + a(aa)^* + aaaaa^*a^*$$

is the same as

- (a) $(a+aa+aaa)^*$
 (b) $aaa^* + aaaaa^* + aaaaaaa^*$
 (c) $(aaa)^*a^{***}(a^*+aa^*)a^*$
 (d) none of the above
20. The complement of the language
 $L = \{a^n b^n / n \neq 100\}$ is _____
 (a) Regular b) context free
 (b) Context sensitive d) None
21. If P is a prime Number and a is positive integer such that $a^{p-1} = r \pmod{p}$, then r is
 (a) a (b) $1/a$
 (c) 1 (d) a^2
22. The set of all strings over {0, 1} starting with 00 and ending in 11 is
 (a) 0011 (b) $00(0+1)^*+11$
 (c) $(00)^*(11)^*$ (d) 0^*1^*
23. The language $\{ww^R | w \text{ is in } (a+b)^*\}$ is accepted by
 a) DFA b) NFA
 c) 2DFA d) multi tape, multithreaded turing machine

Level – 2 Questions

01. Choose the true statement :
 (a) The minimal finite automata accepting the set of all strings over {0, 1} containing an even number of 0's and an even number of 1's has 4 states.
 (b) The minimal finite automata accepting the set of all strings not containing three consecutive 0's has 5 states.

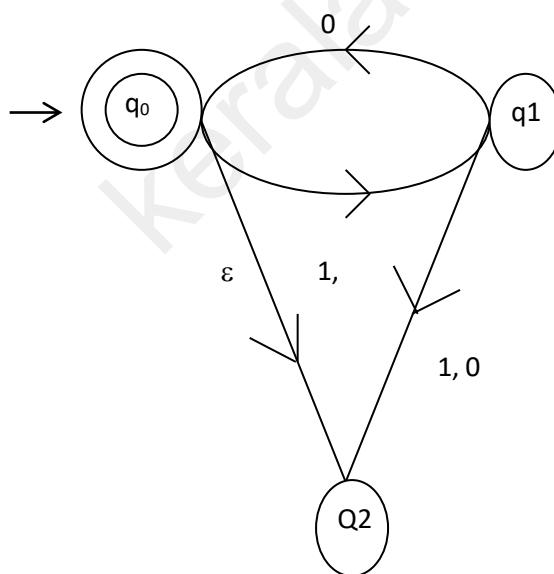
- (c) The minimal finite automata accepting the set of all strings in $(0+1)^*$ that start with a 0 or start with a 1 has 3 states.
- (d) The minimal finite automata is not always unique.
02. Choose the true statement :
- (a) The minimal finite automata accepting the set of all strings over $\{0, 1\}$ where the 1000 symbol from the right end is a 0 has over a 10^{100} no. of states.
- (b) The minimal finite automata accepting the set of all even length strings in $(0+1+2+3)^*$ has 4 states.
- (c) The minimal finite automata accepting the set of all odd numbered strings over a finite alphabet which has between 100 to 200 symbols has 5 states.
- (d) The minimal finite automata accepting the empty set is not unique.
03. The Mealy machine M which recognizes all strings over $\{a, b\}$ where the last two symbols are the same
- (a) no equivalent moore machine
- (b) has a moore machine equivalent of 1000 states (minimal)
- (c) such a Mealy machine does not exist
- (d) has an equivalent Moore machine
04. Which of the following statements is false?
- (a) the finite automaton in which the output is associated with state only is called a Moore machine.
- (b) the finite automaton in which the output is associated with state and input is called a Mealy machine.
- (c) the finite automaton in which the movement of read head is in either direction (left or right) is called a two way finite automaton

- (d) None of the above
06. When an NFA is converted to an equivalent DFA, the number of states
- (a) is the same
 - (b) is always more
 - (c) is always exponential to the number of states of the NFA
 - (d) is bounded by 2^n , where n is the number of states of the DFA
06. Choose the true answer.
- (a) Every formal language can be described using a one symbol alphabet.
 - (b) Every formal language can be described using a two symbol alphabet.
 - (c) Every formal language can be described without using ϵ .
 - (d) Every formal language does not have a finite description.
07. Consider the set of all strings over {0, 1} not containing two consecutive 0's. The regular expression for the same is
- (a) $(1+01)^*(\epsilon+0)$
 - (b) $(1+01)^*(\epsilon+0)(1+01)$
 - (c) $(1+01)^*(\epsilon+0) (1+01)^*(\epsilon+0)$
 - (d) $(1+01)^*(\epsilon+0) (1+01)^*(\epsilon+0)(1+01)^*(\epsilon+0)$
08. The set of all strings over {0,1} where every block of 7 consecutive symbols contains at least 2 0's and 3 1's is
- (a) a finite set.
 - (b) a regular set that is not finite
 - (c) a context free language that is not finite

- (d) a.r.e. set that is not finite.
09. The minimal finite automata accepting the set of all strings over $\{0,1,2\}$ that interpreted as a base 3 number is congruent to 0 modulo 7 and also congruent to 0 modulo 5 has
- (a) 35 states
 - (b) 12 states
 - (c) 36 states
 - (d) none of the above
10. The minimal finite automata accepting the set of all strings over $(0 + 1)^*$ that do not have the sub string 0001 has
- (a) 5 states b) 6 states
 - c) 7 states d) none of these

Common Data for Q11 & Q12 is given below.

Consider the transition Diagram.



11. Language accepted by FA is

- (a) $L = \{(101)^n : n \geq 0\}$
- (b) $L = \{(10)^n : n \geq 0\}$

- (c) $L = \{10^n : n \geq 0\}$
- (d) $L = \{(1^n 0) : n \geq 0\}$
12. By referring to the above result, what happens when this automaton is presented with the string 111?
- (a) It will be accepted
- (b) It will be rejected by moving towards q_1
- (c) It will be rejected by moving towards dead state
- (d) Both (b) & (c)
13. The minimal finite automata accepting the set of all strings over $\{0,1\}$ starting with a 1 that interpreted as the binary representation of an integer are congruent to zero modulo 29 has
- (a) 25 states (b) 26 states
- (c) 29 states (d) 31 states
14. One of the following regular expressions is not the same as others. Which one?
- (a) $(a^* + b^*a^*)^*$
- (b) $(a^*b^* + b^*a^*)^* (a^*b^*)^*$
- (c) $((ab)^* + a^*)^*$
- (d) $(a+b)^*a^*b^*a^*b^*$
15. The language $\{ww | w \text{ in } (0+1)^* \text{ and } |w| < 100000\}$ is
- A. a finite set B. regular set
- C. CFL D. recursive set
- (a) D>C>B>A (b) A>B>C>D
- (c) B>D>C>A (d) C>B>A>D

16. Choose the non regular set

- (a) $L = \{a^i b^j c^k \mid i < j < k < 10^{100}\}$
- (b) $L = \{a^i b^j c^k \mid i < j < k < 10^{100}\}$
- (c) $L = \{a^i b^j \mid j = i * l \text{ and } j < 10^{100}\}$
- (d) $L = \{0^m 1^n 0^{m+n} \mid m, n = 1 \text{ and } m, n < 10^{100}\}$

17. Consider the languages $L_1 L_2$ and L_3 as given below.

$$L_1 = \{0^p 1^q \mid p, q \in \mathbb{N}\}$$

$$L_2 = \{0^p 1^q \mid p, q \in \mathbb{N} \text{ and } p = q\} \text{ and}$$

$$L_3 = \{0^p 1^q 0^r \mid p, q, r \in \mathbb{N} \text{ and } p = q = r\}.$$

Which of the following statements is not

TRUE?

- (a) Pushdown automata (PDA) can be used to recognize L_1 and L_2
- (b) L_1 is a regular language
- (c) All the three languages are context free
- (d) Turing machines can be used to recognize all the languages

18. Consider the languages

$$L_1 : \{www \mid |w| < 3000, w \in (a+b)^*\}$$

$$L_2 : \{a^n b^m \mid m \geq 2000^n\}$$

Choose the correct statement

- (a) L_1 and L_2 are both regular
- (b) L_1 is regular and L_2 is context free but not regular
- (c) L_2 is regular and L_1 is context free
- (d) None of the above

19. Consider the regular expression identities

- i. $r^* + s^* + t^* = (r + s + t)^*$
 - ii. $(rrss)^*rr = rr(ssrr)^*$
 - iii. $(r + s + t)^* = (r^*s^*t^* + \epsilon)^*$
- (a) all are valid
 - (b) (i) and (ii) are valid but (iii) is not
 - (c) (ii) and (iii) are valid but (i) is not
 - (d) (i) and (iii) are valid but (ii) is not

20. The minimal DFA accepting the set $(a+\epsilon^*) (a+\epsilon)$ has ____ states

- (a) 1
- (b) 2
- (c) 3
- (d) 4

21. The minimal finite automata accepting the set of all strings over $\{0, 1\}$ where the number of 0's is divisible by three hundred and the number of 1's is divisible by two hundred has, choose the true statement

- (a) 3000 states is the best case
- (b) 40000 states or more
- (c) 60000 states in the minimal machine
- (d) 8000 states only in the minimal machine

22. Choose the correct statement

- (a) The set of all formal languages is countable
- (b) The set of all languages is not countable
- (c) The regular sets are not countably infinite.
- (d) The family of all finite automata is not countably infinite.

23. The transition function for the transition graph is

- (a) $\delta : Q \times \Sigma^* \rightarrow Q$
- (b) $\delta : Q \times \Sigma^* \rightarrow Q$
- (c) $\delta : Q \times \Sigma^* \rightarrow 2^{Q \times \Sigma}$
- (d) All of the above

Solutions

Level – 1

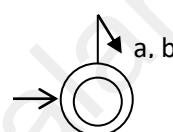
01. Ans : (d)

Sol : The operation of finite union & finite intersection preserve the regular sets.

The set $\{\epsilon\} \cup \{aa\} \cup \{aaa\} \dots = a^*$ is the example of the intersection and union of regular sets being regular.

02. Ans : (d)

Sol : consider the NFA



03. Ans : (a)

Sol : All finite languages are regular.

04. Ans : (a)

Sol : The finite automata can be recursively enumerated.

05. Ans : (b)

Sol : It has 5 states

06. Ans : (d)

Sol : The set can be described by a regular expression.

07. Ans : (a)

Sol : A mod 3 counter needs 3 states in the minimal DFA.

08. Ans : (d)

Sol : This is the example of a DFA where the NFA to DFA requires an exponential number of states.

09. Ans : (b)

Sol : (a), (c) & (d) contain ϵ . Which is a negative string that does not contain two consecutive 1's.

10. Ans : (b)

Sol : The decimal 4 of L needs three bits to represent it. So a 3 state modulo machine is needed.

11. Ans : (b)

Sol :00.....: 2 0's are needed

$$\text{So } (0+1)^* 00 (0+1)^*$$

12. Ans : (c)

Sol : we need

$$[(a+b)^* a (a+b)^* b (a+b)^* a (a+b)^* b (a+b)^*]$$

13. Ans : (b)

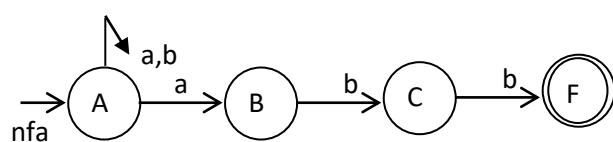
Sol : 10 is not recognized & 01 is recognized

14. Ans (a) & (b)

Sol : After ϵ , 0, or 00 after 1 appears. (c) contains 000.

15. Ans (b)

Sol :



16. Ans (c)

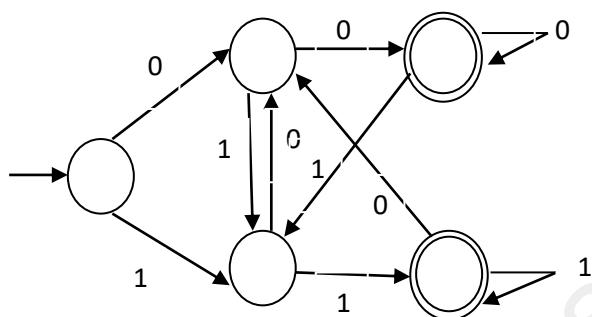
Sol : Use the method of elimination with the strings 0 & 1

17. Ans : All of the choices

Sol : The sub string ab is in all the choices (a) – (d)

18. Ans (c)

Sol :



19. Ans : (a)

Sol : (a) is a^* which is the given R.E. (b) does not allow a & (c) does not allow a.

Sol : Use the method of elimination with the strings 0 & 1

20. Ans : (c)

Sol : The complement is a DCFL that is not regular. DCFLs are closed under complement.

Sol : Use the method of elimination with the strings 0 & 1

21. Ans : (c)

Sol : This is a statement of Fermat's Theorem

22. Ans : (b)

Sol : 00 _ _ start with 00

— 11 end with 11

00 (0+1)* 11 start with 00 & end with 11

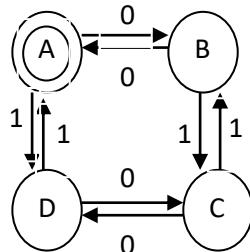
23. Ans : (d)

Sol : The language of palindrome is not regular or a DCFL. It is a CFL & hence R.E. So (d) is the answer.

Level – 2

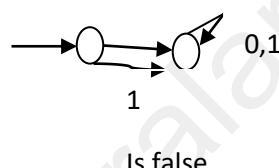
01. Ans : (a)

Sol : a)



(b) is false as a minimal FA is unique up to a homomorphism.

(c) $0(0+1)^* + 1(0+1)^*$



Is false

02. Ans : (a)

Sol : The DFA has 2^{1000} states & hence has more than a 10^{1000} no. of states.

03. Ans : (d)

Sol : Every moore machine has an equivalent mealy m/c & vice versa.

04. Ans : (d)

Sol : These are the standard definition of the Moore & Mealy machines & the two way finite automata

05. Ans : (d)

Sol : If NFA contains n states, then DFA contains atmost 2^n states.

06. Ans : (c)

Sol : ϵ is just symbol so some other symbol like λ can be used for the empty string.

07. Ans : (a)

Sol : After one or no 0's, a '1' compulsorily occurs.

08. Ans : (b)

Sol : Set = $(00)^+ + (111)^+ + \text{other}$

So it is infinite. It can be expressed as a regular expression so it is regular.

09. Ans : (d)

Sol : The answer is a modulo 7 m/c multiplied by a modulo 5 m/c.

10. Ans : (a)

Sol : FA has 5 states

11. Ans : (b)

Sol : Evidently $(10)^*$ is accepted by the FA.

12. Ans : (c)

Sol : 111 will go to dead state

13. Ans : (d)

Sol : A modulo 29 machine has 31 states as it starts with 1.

14. Ans : (c)

Sol : (a), (b) & (d) are $(a+b)^*$

Where (c) does not contain 'b'

15. Ans : (a)

Sol : The given set is finite. Every finite set is regular. Every regular set is DCFL. Every DCFL is a CFL. Every CFL is DSL. Every CSL is recursive. Every recursive set is R.E.

16. Ans : (b)

Sol : By the Pumping Lemma for regular sets, (b) is not regular. The remaining choice are all finite sets.

17. Ans : (d)

Sol : L_1 requires no stack

L_2 requires one stack

L_3 requires 2 stack

So L_1 is Regular language, L_2 is CFL, and

L_3 is CSL and every RL, CFL, CSL are RES, accepted by TM

18. Ans : (b)

Sol : L_1 is a finite set & hence is regular. L_2 is a CFL that is not regular.

19. Ans : (c)

Sol : $(ra)^* r = r(ar)^*$

So (ii) is true

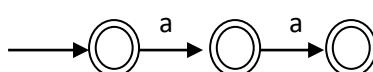
$$(r + s + t)^* = (r^*s^*t^* + \varepsilon)^*$$

$$= (r^*s^*t^*)^*$$

$$= (r + s + t)^*$$

20. Ans : 3

Sol : The minimal DFA is



21. **Ans : (c)**

Sol : $M_1 0'$ need 300 states

$M_2 1'$ need 200 states

$$M_3 = M_1 \times M_2 = 60000 \text{ states}$$

22. **Ans : (b)**

Sol : Set of all formal languages is uncountable, since there is no one – to – one relation between set of formal languages and N.

23. **Ans : (c)**

Sol : The Question deals with the conversion of a transition diagram having ϵ -moves. The non-determinism requires all possibilities to be considered.

Chapter – 2

Context – free Languages & Push down Automata

Level – 1 Questions

01. The intersection of two CFL's

- (a) is always a CFL
- (b) may be a CFL
- (c) is never a CFL
- (d) is always a CSL

02. Which of the following statements is true?

- (a) if a language is context free it can always be accepted by a deterministic push down automaton
- (b) The union of two CFLs is a CFL
- (c) The context free languages are closed under intersection
- (d) The complement of a CFL is always a CSL

03. Consider the following two languages

$$L_1 = \{1^n 0^n 1^n 0^n / n \geq 0\}$$

$$L_2 = \{a^n b^k / n \leq k \leq 2n\}$$

Which of the following statement is true.

- (a) both L_1 and L_2 are context free
- (b) L_1 is context free but not L_2
- (c) L_2 is context free but not L_1
- (d) neither L_1 nor L_2 is context free

04. The language generated by the following context free grammar is,

$$S \rightarrow AB, A \rightarrow aA/a, B \rightarrow bBc/\epsilon$$

- (a) $\{a^m b^n c^n / m \geq 0, n \geq 0\}$
- (b) $\{a^n b^m c^n / m \geq 0, n \geq 0\}$
- (c) $\{a^n b^n c^m / m \geq 0, n \geq 0\}$
- (d) None of the above

05. The recursive sets, CFLs and R.E. sets are closed under

- (a) Union and complement
- (b) Intersection and complement
- (c) Union and intersection
- (d) Union

06. CFL is not closed under

- (a) Intersection
- (b) complement
- (c) difference
- (d) all the above

07. Which of the following language cannot be accepted by any deterministic push down automaton

- (a) The set of all strings over {a, b} consisting of equal number of a's and b's.
- (b) A language of palindromes
 $\{x \in \{a, b\}^* / x = \text{rev}(x)\}$
- (c) $\{wcw^R / w \in \{a, b\}^*\}$
- (d) all of the above
08. The language which is accepted by LBA is called as
- (a) Regular
- (b) Context free
- (c) Context dependant
- (d) None of the above
09. Choose the correct statement
- (a) The nondeterministic & deterministic finite automata are not equivalent.
- (b) The nondeterministic & deterministic PDA are not equivalent.
- (c) The nondeterministic & deterministic TMs are not equivalent.
- (d) The nondeterministic & deterministic halting TMs are not equivalent.
10. In the case of non – determinism choose the incorrect statement
- (a) Deterministic and nondeterministic finite automata are equivalent in power.
- (b) Deterministic and nondeterministic push down automata are equivalent in power.
- (c) Deterministic and non deterministic two push down tape machines are equivalent in power.
- (d) Deterministic and nondeterministic three counter machines are equivalent in power.
11. The language $0^* \cup \{0^n 1^n / n \geq 1\} \cup \{0^n 1^{2n} / n \geq 1\}$ is accepted by
- (a) 2 DFA
- (b) DPDA
- (c) PDA
- (d) LBA but not by a PDA

12. Consider a PDA M accepting a CFL L. Choose the correct statement.
- A PDA can never loop on its input and makes as many moves as the size of the input.
 - In a PDA the maximum size of the stack for an input of size n, is a polynomial p (n).
 - In a PDA the maximum size of the stack for an input of size n, is an exponential of $O(2^n)$
 - A PDA can be defined in two equivalent models having acceptance by final state or acceptance by empty store.
13. Consider the CFG G generating the CFL $L = \{ww^R \mid w \in \{a, b\}^*\}$. Choose the correct statement.
- As L is a CFL, g must be ambiguous.
 - L is CFL that is also a DCFL.
 - If L is defined over a single alphabet {a} then L is accepted by non deterministic finite automata.
 - the exists an LR (k) that generates L.
14. Let L be a language over the alphabet {a} & be generated by some CFG G. Choose the correct statement.
- L is a CSL that is not a CFL
 - L is a CFL that is not a DCFL
 - L is a DCFL that is not regular
 - L can be denoted by some regular expression
15. Consider the language $L = \{ww \mid w \in \Sigma^*\}$.
- L is a CSL that is not a CFL as down by the pumping Lemma for CFLs.
 - L is a CSL as shown by the pumping Lemma for CFLs.
 - L does not model the definition before use requirements in some programming languages.
 - L can be described by extended regular expression which arranges the standard definition of regular expression with the intersection & complement operators.
16. Consider the language

$$L = \{a^i b^j a^i b^j \mid i, j \geq 1\}.$$

Choose the correct statement.

- (a) L can be accepted by some PDA
- (b) L can be described by the intersection of regular expression.
- (c) L models the requirement that the number of parameters in procedure definition & use be the same.
- (d) The pumping Lemma for regular sets shows that L is a CSL.

17. Consider the languages

- i. $L_1 = \{a^n b^{2n} | n \geq 1\}$
- ii. $L_2 = \{a^n b^{2n} | n \geq 1\}$

Choose the false statement

- (a) L_1 is a DCFL
- (b) L_2 is a DCFL
- (c) $L_1 \cup L_2$ is accepted by some PDA
- (d) $L_1 \cup L_2$ is a DCFL

18. Consider the languages

$$L_1 = \{a^n b^n c^i \mid n, i \geq 1\} \text{ &} \\ L_2 = \{a^m b^j c^j \mid m, j \geq 1\} \text{ &}$$

Choose the false statement.

- (a) $L_1 \& L_2$ are CFLs
- (b) L_1 may have an ambiguous CSG generating it
- (c) L_2 may have an ambiguous CFG generating it.
- (d) $L_1 \cup L_2$ is not inherently ambiguous.

19. Define a linear grammar G having rules of the form

$$A \rightarrow a, \quad A \rightarrow aB$$

$$A \rightarrow Ba, \quad A \rightarrow B,$$

$$A \rightarrow ab.$$

Where A, B are non-terminals. & a, b are terminals, Let G generates a CFL L.

Choose the correct statement

- (a) Every CFL does not have a linear grammar.

- (b) Every regular set does not have a linear grammar.
- (c) A linear grammar does not describe all the languages that can be denoted by regular expressions.
- (d) A linear grammar can generate the set $\{a^n b c^n \mid n \geq 1\}$

Level – 2 Questions

01. The language $\{ww^R w w w w w w w^R w w w w w^R \mid w \text{ in } (a+b)^*\}$ is accepted by
- (a) PDA
 - (b) NFA
 - (C) 2DFA
 - (d) multitape, multiheaded turing machine
02. **Statement (I)** : The language $L = \text{set of all strings not containing } 101 \text{ as a substring}$ is regular set.
- Statement (II)** : L satisfies the pumping Lemma for regular sets.
- (a) I is true, II is false
 - (b) I is false II is true
 - (c) I & II are true
 - (d) I & II are false
03. A is given a CFG, G_1 that generates language F is not a DCFL L_1 & a LR(k) grammar G_2 that generates a languages $L_2\$$. Choose the correct statement
- (a) For every G_1 there exists a grammar G_2 .
 - (b) Every G_1 & G_2 are unambiguous.
 - (c) $L_1 L_2$ is always a DCFL.
 - (d) L_2 is always a DCFL if $\$$ is a symbol not in the vocabulary
04. A DPDA's that is accepted by empty store. Choose the correct statement.

- (a) For every DCFL, there exists a DPDA that accepted by empty store.
- (b) For every DCFL that satisfies the prefix property, there does not exist, a DPDA that is accepted by empty store.
- (c) If L is a DCFL then $L\$$ can be generated by some LR(k) grammar.
- (d) Every DCFL can be defined to be accepted by an empty store or final state.
05. Consider the machines
- M_1 = a NPDA accepting L_1
- M_2 = a 2 way PDA accepting L_2
- M_3 = a PDA with two stacks accepting L_3 .
- Choose the false statement
- (a) For any CFL L , there exists some $M_1, M_2, & M_3$.
- (b) For some CSL L , there exits some $M_1, M_2, & M_3$.
- (c) For every recursive set, there exits some $M_2, & M_3$.
- (d) The power of the machines are not $M_1 \leq M_2 < M_3$.
06. Consider the properties of a context – free grammar G in the chomsky normal form (CNF), the Greibach Normal Form (GNF) & an operator grammar (OG). Choose the false statement.
- (a) In CNF, the r.h.s of a rule is either a terminal or two adjacent non – terminals.
- (b) In GNF, the r.h.s of a rule is either a terminal or a terminal followed by a string of non – terminals
- (c) In OG, the r.h.s of every rule does not have two adjacent non – terminals.
- (d) No grammar cannot be in CNF, GNF & OG at the same time

07. We are given a type 0 grammar G_0 , a type 1 grammar G_1 , a type 1 grammar G_1 , a type 2 grammar G_2 , & a type 3 grammar G_3 , generating the formal language L_0, L_1, L_2 & L_3 respectively.

Choose the true statement,

- (a) G_0 cannot generate all possible L_1
- (b) G_1 cannot generate all possible L_2
- (c) G_2 cannot generate all possible L_3
- (d) G_3 cannot generate all the finite sets

08. We are given a PDA M_1 accepting L_1 & a PDA M_2 accepting L_2 .

Choose the false statement.

- (a) We can construct a PDA M_3 that accepts all the strings M_1 & M_2 accept.
- (b) We cannot construct a PDA that accepts some strings accepted by both M_1 & M_2 .
- (c) We cannot construct a PDA that accepts string $\overline{L}_1, \overline{L}_2$.
- (d) We can construct a PDA that accepts all the strings not in L_1 .

09. Consider the languages

i. $L_1 = \{a^n b^{2n} \mid n \geq 1\}$

ii. $L_2 = \{ca^n b^n \mid n \geq 1\}$

Choose the false statement.

- (a) L_1 is accepted by some DPDA
- (b) L_2 is accepted by some DPDA
- (c) $L_1 \cup L_2$ is not accepted by any DPDA
- (d) $L_1 \cup L_2$ is not accepted by some PDA

10. Consider the languages

- i. $L_1 = \{ww^R \mid w \in \Sigma^*\}$
- ii. $L_2 = \{w \neq w^R \mid w \in \Sigma^*\}$
- iii. $L_3 = \{wxw^R \mid w, x \in \Sigma^*\}$

When $\Sigma = \{a, b\}$.

Choose the false statement.

- (a) L_1 cannot be accepted by any DPDA & needs a PDA or LBA to accept it.
 - (b) L_2 can be accepted by a DPDA but not a finite automata.
 - (c) L_3 is a sample regular language $\{a, b\}^* - \{a, b\} \{a, b\}^*$ & can be accepted by some finite automata.
 - (d) All L_1, L_2 & L_3 are inherently ambiguous languages i.e. any CFG constructed for them will be necessarily ambiguous.
11. We are given the regular sets $R_1 = \epsilon^*$ & R_2 = set of all strings over $\{a, b\}$ where the 11th symbol from the right end is a. We are also given an arbitrary CFL L_1 & an arbitrary DCFL L_2 .

Choose the false statements.

- (a) $L_1 = R_1$ or $L_1 = R_2$ are both undecidable
- (b) $L_2 = R_1$ or $L_2 = R_2$ are both decidable
- (c) $L_1 \cap L_2 \cap R_1$ has an emptiness problem that is undecidable.
- (d) $R_1 = R_2$ is undecidable

12. Consider the following machine

- (i) A PDA M_1 where the stack size cannot be more than a function f of the input size n .
- (ii) A DPDA M_2 where the stack size bounded by some prime number.

- (iii) A ODA M_3 where the maximum stack size should be $m!$ for some integer m .

Choose the false statement

- (a) M_1 accepts only regular sets
 - (b) M_2 accepts only regular sets
 - (c) M_3 can accept CFLs
 - (d) None of the above
13. Consider the language $L_1 = \{a^n b^{2n} \mid n \geq 1\}$, the homomorphism $h([p]) = a$, $h([q]) = aa$, Let $L_2 = h^{-1}(L_1)$ & $R = p^* q^*$.

Choose the false statement

- (a) $L_1 \cap R = \{a^m b^m \mid m \geq 1\}$
- (b) L_2 & L_1 are both CFLs that are not regular.
- (c) L_2 is not a CFL but is a CSL.
- (d) None of the above.

Solutions

Level -1

- 01. Ans : (b) & (d)**

Sol : 1. $a^* \cap b^* = \emptyset$ a CFL

2. The CFLs are not closed under intersection.

- 02. Ans : (b) & (d)**

Sol : The DCFLs are a smaller class than the CFLs. The CFLs are closed under union but not under intersection. Every CFL is a CSL. Complement of CFL is recursive

- 03. Ans : (a)**

Sol : (b) L_1 is a standard CSL

(c) L_2 is CFL

04. Ans : (d)

Sol : $S \rightarrow AB$

$A \xrightarrow{*} a^+s$

$B \xrightarrow{*} \{b^n c^n / n \geq 0\}$

Note that ϵ is not generated by the grammar

05. Ans : (d)

Sol : The CFLs are not closed under intersection or complement.

06. Ans : (d)

Sol : A CFL is not closed under intersection, or complement & hence difference

07. Ans : (b)

Sol : The language $L = \{ww^R / w \in \Sigma^*\}$ is a CFL but not a DCFL.

08. Ans : (c)

09. Ans : (b)

Sol : The PDA are not the same for non determinism.

10. Ans : (b)

Sol : The DPDA & PDA are not equivalent. Non-determinism does not add any power to the finite automata or the TM.

11. Ans : (c)

Sol : The language is a CFL but not a DCFL.

12. Ans : (d)

Sol : Any CFL can be accepted by a PDA that can be accepted by final state. This has an equivalent model of acceptance by final state.

For (a), (b), (c) we note that a PDA can make an infinite number of moves on ϵ input, & for each one of these moves, the store can grow in size. So in general the PDA needs an infinite push down store.

13. Ans : (c)

- Sol :** (c) if the vocabulary is $\{a\}$ then $L = (aa)^*$ which is a regular set
(b) L is a standard CFL, the language of all palindromes is not a DCFL.
(d) G is unambiguous. G is a partially CFG. Ambiguity problem is for all CFGs.
(d) A L is not a DCFL, no LR (k) can generate it.

14. Ans : (d)

Sol : CFLs over a single alphabet are regular.

15. Ans : (a)

Sol : L is a standard CSL that is not a CFL (& hence not regular). This can be shown using the pumping Lemma for CFLs. Also it models the definition before use requirement in programming languages.

16. Ans : (c)

Sol : L is a CSL that is not a CFL. It cannot be shown to be a CFL by using pumping Lemma for CFLs. A LBA can be constructed by accepting L & so it is a CSL. L models the requirement of the same procedure to have the same number of parameters.

17. Ans : (d)

Sol : We can construct DPDA's to accept L_1 & L_2 . We can construct a nondeterministic PDA to accept $L_1 \cup L_2$. However $L_1 \cup L_2$ will require non-determinism & cannot be a DCFL.

18. Ans : (d)

Sol: It can be shown that L_1 & L_2 are CFLs by constructing CFG's generating them. Ambiguity can be artificially added to them to make them generated by ambiguous CSG's.

It runs out that $L_1 \cup L_2$ is inherently ambiguous. As strings $\{a^n b^n c^n\}$ will have two derivations.

19. Ans : (a)

Sol: The languages generated by linear grammars are a proper subset of the CFLs. The linear grammar $S \rightarrow aSc/b$ generates $\{a^n b c^n \mid n \geq 1\}$

All right linear grammars & all left linear grammars are trivially linear grammar. So all regular sets can be generated by linear grammars. So all regular expressions can be denoted by linear grammar.

Level – 1

01. Ans : (d)

Sol: The language is a CSL & hence a R.E. set accepted by a TM or its Vice versa.

02. Ans : (b)

Sol: satisfying the pumping Lemma give no conclusion.

$\therefore L$ is a regular set that satisfies the Pumping Lemma.

03. Ans : (d)

Sol : (a) not true as G_1 every CFL is not a DCFL. So every CFG L_1 does not have an equivalent LR(k) grammar G_1 .

(b) LR(k) grammars are unambiguous, but G_1 being an arbitrary CFG it may be ambiguous.

(c) The concatenation of two CFLs is a CFL that need not be a DCFL.

(d) If L is a DCFL the $L\$$ is LR(k) for some k .

04. Ans : (c)

Sol : The DPDA that is accepted by final state accepts DCFLs. If the prefix property is satisfied we can use DPDA's that is accepted by empty store.

(c) Standard theorem. If L is a DCFL then $L\$$ can be generated by some LR (k) grammar.

05. Ans : (d)

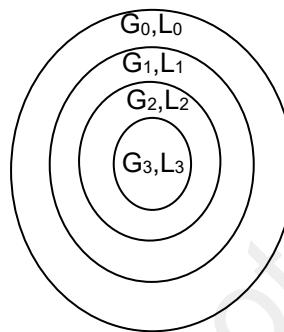
Sol. M_1 accepts all CFLs. M_2 accepts some CSLs. M_3 is a variation of the Turing machines.

06. Ans : (d)

Sol : Consider the grammar $S \rightarrow a$. It is in CNF, GNF & OG at the same time.

07. Ans : (d)

Sol :



08. Ans : (d)

Sol : (a) The union of two CFLs is a CFL.

(b) The intersection of two CFLs need not be a CFL.

(c) The complement of a CFL need not be a CFL & the CSLs are closed under concatenation.

(d) The CFLs are not closed under complement.

09. Ans : (c)

Sol: L_1 is a DCFL as a DPDA can be constructed to accept it. L_2 also can be accepted by an easily constructed DPDA. Since L_1 & L_2 are CFLs. $L_1 \cup L_2$ is a CFL & can therefore be accepted by some PDA.

$L_1 \cup L_2$ is a DCFL as DPDA can use the presence of the first symbol 'c' to table the non determinism.

10. Ans : (d)

Sol : As (c) is a regular set it will have unambiguous left linear or right linear grammar generating it.

As L_2 is a DCFL it can be accepted by DPDA. L_1 is a standard CFL that is not a CSL.

11. Ans : (d)

Sol : It is decidable a DCFL L_2 is equivalent to a given regular set R. This is undecidable for DCFLs.

The intersection of two DCFLs can give all the valid computations of a turing machine & so is the same as the halting problem.

It is decidable if two regular sets are equivalent.

12. Ans (c)

Sol : For $M_1 \cup M_2$ the stack size is bounded. So the stack can be stored in the finite control. So $M_1 \cup M_2$ accept only regular sets.

For M_3 the stack is not bounded. At the most we have to add the stack with dummy symbols so that the stack is $m!$ for some m. So this is the general definition of a PDA and accept all the CFLs.

So for M_3 also the stack size is bounded by $m!$. Hence the memory is finite and can be stored in finite. Hence M_2 accepts only regular set.

13. Ans : (c)

Sol: L is a CFL. The CFLs are closed under homomorphism & inverse homomorphism, so $h^{-1}(L_1) = L_2$ is a CFL. L_2 has p's & q's jumbled up intersection with R arrange the p's before the q's. As a's, (bb)'s are the same in L_1 they are the same number of p's and q's in any string in L_2 .

Chapter – 3

Turing Machines, Modifications, CSLs, Recursive & R.E. sets

Level – 1 Questions

01. To evaluate expressions in C without function calls, which of the following is mandatory?
 - (a) one stack is enough
 - (b) two stacks are must
 - (c) unlimited number of stacks are required
 - (d) a turing machine is required and it must be non deterministic in the general case
02. Some how I proved that $P = NP$ then which of the following statements is true?
 - (a) NP is closed under complement
 - (b) NP – CoNP (complement of NP)
 - (c) Both a and b are true
 - (d) None of the above is true
03. Recursive sets are closed under
 - (a) Kleene closure
 - (b) Substitution
 - (c) Homomorphism
 - (d) Inverse homomorphism
04. Consider the following languages

$$L_1 = \{<M> : L(M) \text{ contains a word of length } < 1000\}$$

$L_2 = \{<M> : M \text{ accepts some word within 1000 steps of computation}\}$

(R → Recursive, RE → recursively enumerable)

Which of the following statements is true?

- (a) $L_1 \in R$ and $L_2 \in RE$
 - (b) $L_1 \in R$ and $L_2 \in RE$
 - (c) $L_1 \in RE$ and $L_2 \notin RE$
 - (d) $L_1 \in RE$ and $L_2 \in RE$
05. The language generated by the grammar $S \rightarrow aSa|bSb|d$ is accepted by a
- | | |
|-------------|-------------------|
| A. DPDA | B. PDA |
| C. LBA | D. Turing machine |
| (a) D>C>B>A | (b) A>B>C>D |
| (c) B>C>D>A | (d) C>D>B>A |
06. The language consisting of an equal number of a's and b's can be accepted by a
- | | |
|-------------------|-------------------|
| A. DPDA | B. PDA |
| C. turing machine | D. LBA |
| (a) C > D > B > A | (b) A > B > C > D |
| (c) B > C > D > A | (d) D > C > B > A |
07. If a 2DFA we supply an unlimited amount of ink so that it can write on its input tape, it become a
- | | |
|----------|--------------------|
| (a) DPDA | (b) PDA |
| (c) LBA | (d) Turing machine |
08. A finite set that is accepted by a finite automata is always
- | |
|---------------------------|
| (a) type 0 but not type 1 |
|---------------------------|

- (b) type 1 but not type 2

(c) type 2 but not type 3

(d) always type 3

09. Which of the following languages are closed under complement?

(a) CFL

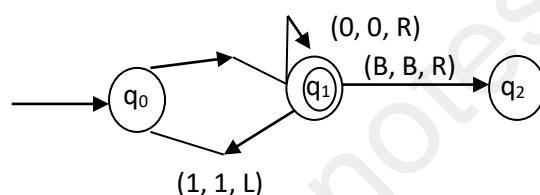
(b) CSL

(c) Recursive sets

(d) Recursively enumerated sets

Statement for linked answer Q 10 to Q 11 is given below :

Let M be the Turing Machine given below :



10. The languages accepted by M is

 - (a) $\{00^n \mid n \geq 0\}$
 - (b) $\{0^n 1 0^n 1 \mid n \geq 1\}$
 - (c) $\{0^n 1 \ 0^n 1 \mid n \geq 1\}$
 - (d) $\{0^n 1 0^n 1 \mid n \geq 1\}$

11. The above language is

 - (a) Regular language
 - (b) DCFL
 - (c) CFL
 - (d) CSL

12. Choose the correct statement

 - (a) The recursive sets can be effectively enumerated

- (b) The formal language can be effectively enumerated.
- (c) All real numbers can be effectively enumerated
- (d) The finite automata can be effectively enumerated
13. Choose the correct statement
- (a) If any problem in P is closed under complement, then $P = NP$.
- (b) If every problem in NP is closed under complement, then $P = NP$.
- (c) If every problem in NP is NP – complete, then $P = NP$.
- (d) If any problem in NP is not NP – complete, then $P \neq NP$.
14. Choose the undecidable problem.
- (a) Equivalence of finite automata
- (b) Equivalence of regular expression
- (c) Emptiness problem of CSLs
- (d) Emptiness problem of R.E sets
15. The following are not countable
- (a) the subsets of all regular sets
- (b) the subsets of all CFLs
- (c) The subsets of all CSLs
- (d) All of the above
16. The following problems are not decidable
- (a) Whether a CFL is infinite or finite
- (b) The membership problem of CSL's.
- (c) The membership problem of recursive sets
- (d) Whether a CFL consists of all strings over the terminal vocabulary.

17. The union of the classes of regular sets and context sensitive languages is
- (a) not countably infinite
 - (b) countably infinite
 - (c) sometimes countably infinite
 - (d) none of the above

18. Let M_1 be a deterministic finite automata accepting a language L_1 & let M_2 be a non – deterministic finite automata accepting a language L_2 .

Choose the correct statement :

- (a) L_1 is equal to L_2
 - (b) It is decidable if $L_1 = L_2$
 - (c) There is no algorithm to decide if L_1 is empty
 - (d) The intersection of L_1 & L_2 is always a finite set
19. Let M_1 be a deterministic push down automata accepting a DCFL L_1 . Let G_2 be an LR(k) grammar generating the language L_2 . Let M_2 be a two way deterministic finite automata (2DFA) accepting the language L_3 .

Choose the correct statement.

- (a) There is no algorithm to decide if $L_1 \cap L_2$ is empty.
 - (b) $L_1 = L_2$
 - (c) $L_2 = L_3$
 - (d) The problem of whether $L_1 = L_2$ is undecidable
20. Let M_1 be a push down automata accepting a language L_1 .

Choose the correct statement :

- (a) $L_1 = \emptyset$ is decidable

- (b) $L_1 = \Sigma^*$ is decidable
- (c) L_1 is a finite set is not decidable
- (d) L_1 is an infinite set is not decidable
21. Given an arbitrary C program C_1 and a JAVA program, J_2 . Choose the correct statement.
- (a) C_1 may not be an algorithm but J_2 is always an algorithm.
- (b) C_1 may not be an algorithm but J_2 may not be an algorithm.
- (c) Both C_1 & J_2 are algorithms always.
- (d) neither C_1 nor J_2 necessarily an algorithm.
22. Consider an arbitrary C program which has some assignment statement
a. Choose the correct statement.
- (a) It is decidable whether a is executed.
- (b) It is undecidable whether a is live or dead always
- (c) It is decidable if a is repeatedly executed in some iteration.
- (d) None of the above
23. Choose the correct statement
- (a) The graph coloring problem is undecidable.
- (b) The register optimization problem in compilers can be reduced to
graph coloring problem is undecidable.
- (c) The graph coloring problem with less than 10 nodes is intractable.
- (d) The graph coloring problem is decidable and seemingly intractable.
24. A is given a recursive set L accepted by some LBA M & a homomorphism
h. Choose the correct statement.

- (a) It may not be possible to decide if $L = \emptyset$ but it is always possible to decide if $h(L) = \emptyset$
- (b) It is undecidable if $h(L)$ is a R.E. set
- (c) The emptiness problem of $h(L)$ is the same as the halting problem of turing machine
27. A considered the sets of Turing machines describing all the Hamiltonian cycle problems. Choose the correct statement.
- (a) The membership problem of S is decidable
- (b) It is decidable if the complement of S is empty
- (c) It is decidable if S is empty, finite or infinite
- (d) It is decidable if $S = \Sigma^*$

Level - 2 Questions

01. A single tape turning machine M has two states q_0 and q_1 , of which q_0 is the starting state. The tape alphabet of M is $\{0, 1, B\}$ and its input alphabet is $\{0, 1\}$. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table.

	0	1	B
q_0	$q_0, 1, R$	$q_0, 1, R$	q_1, B, R
q_1	$q_0, 1, R$	$q_0, 0, R$	Halt

Which of the following statements is true about M ?

- (a) M accepts all strings that end with 0 only
- (b) M accepts all strings that end with 1 only
- (c) M accepts all strings in $(0+1)^*$ only
- (d) M accepts all strings in $(0+1)^* 01 (0+1)^* + 1^* 0^*$

02. Match the following

Grammar	Classification
P.S→ aS bS a, S→ A, A→ a	1. Type 3 and right linear
Q.S→ aAb ab, C→ c/cC S→ AC/abc	2. Type 2 and not generating a regular set
R.S→ LWaR Wa → Waaaa WR→ W1 R aW1→ W1a LW1→ LW WR → W2R aW2R→ W2Raaaa LW2R→ ϵ	3. Type 0 and not generating a CFL
S.S→ aSBC aBC CB → BC aB → ab bB → bb bC → bc cC → cc	4. Type 1 and not generating a CFL
	5. LR (k)
	6. Type 3 and left linear

Codes :

- (a) P-1, Q=5, R-3, S-4
- (b) P=6, Q-4, R-3, S-5
- (c) P-1, Q-3, R-6, S-2
- (d) P-1, Q-2, R-3, S-6

03. Choose the true statement.

- (a) The regular sets, R.E. sets, CSLs, DCFLs and CFLs are closed under intersection
- (b) The regular sets, recursive sets, DCFLs and CFLs are preserved under the operation of complement
- (c) The regular sets are closed under intersection but the CFLs are not
- (d) The CFLs are closed under complement but the regular sets and R.E. sets are not

Statement for Linked answer Q.04 and Q.05 is given below.

Ram obtains a turing machine that can accept the empty set L_n . Shyam obtains a turing machine that can accept the set of all strings over the input alphabet L_m . Thomas studies L_n to classify it in the Chomsky hierarchy. Rahim construct a turing machine to accept L_1 the complement of L_n .

04. Choose the correct statement :

- (a) Thomas will not be able to determine if L_n is a regular set.
- (b) Thomas will not be able to determine all strings in L_n as L_n is a recursive set.
- (c) Shyam will not be able to determine if L_m is the same as L_n as the equivalence of turing machines is undecidable.
- (d) Shyam will not be able to determine if L_m is a subset of L_n as the equivalence problem of turing machines and the containment problem is undecidable.

05. Choose the correct statement :

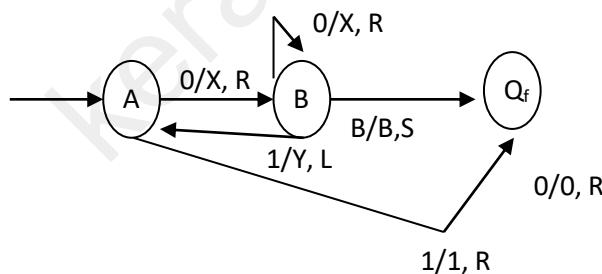
- (a) Rahim will not be able to determine L_1 as L_1 is not R.E.
- (b) Rahim will not be able to determine L_1 as L_1 is recursive.
- (c) Rahim will not be able to determine L_1 as L_1 is R.E.
- (d) Rahim will not be able to determine L_1 as L_n is a finite.

06. Consider the complexity classes P, NP, PSPACE, EXPTIME, then which of the following relationship is true?

- (a) $P \subset NP \subset PSPACE \subset EXPTIME$
- (b) $P \subset NP \subset EXPTIME \subset PSPACE$
- (c) $P \subset PSPACE \subset NP \subset EXPTIME$
- (d) $NP \subset EXPTIME \subset P \subset PSPACE$

07. Which of the following statement is false
- A Turing machine is more powerful than a finite state machine because it has halt state.
 - A finite state machine can be assumed to be a Turing machine of finite tape length, with rewinding capability and unidirectional tape movement.
 - both (a) and (b)
 - none of the above
08. Choose the false statement
- Every CSL can be accepted by a finite automata if we allow an unbounded number of states.
 - The complement of the language $L = \{ww \mid w \in \{a, b\}^*\}$ is a CFL.
 - The complement of the language $L = (a^n b^n c^n \mid n \geq 1)$ is a CFL
 - Every regular expression does not denote a regular set

Common Data for Q09 and Q10 is given below.



09. The above TM accepts
- 0^*
 - 1^*
 - $(0+1)^+$
 - $(0+1)^*$
10. The language accepted by the above TM is
- finite

- (b) recursive set
- (c) CSL but not CFL
- (d) CFL but not regular

Common Data for Q11 & Q12 is given below.

Consider a language L_1 . $L_1 = \{0, 1\}^*$ if another language L_2 is a CFL.

L_1 = empty set if another language L_2 is not a CFL.

L_2 = a language that is R.E. and its complement is R.E.

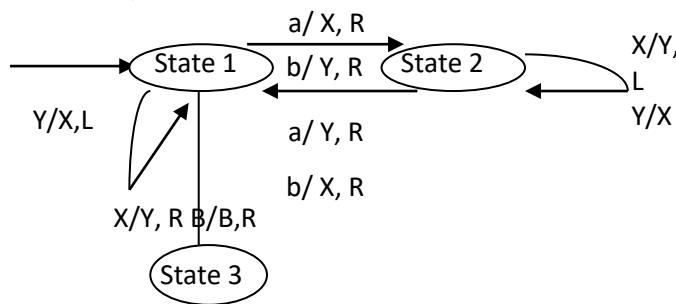
11. L_1 is

- (a) regular and not finite
- (b) CFL and not regular
- (c) CSL and not a CFL
- (d) r.e and not recursive

12. L_2 is

- | | |
|-----------|-------------|
| (a) empty | (b) regular |
| (c) CFL | (d) r.e |

13. Consider the following Turing machine M :



Assume that initially tape contains the strings over the alphabet {a, b} and can end of the string is identified with the help of continuous blank symbols after the given string.

The above Turing machine M accepts

- (a) All strings of a's and b's including ϵ .
 - (b) All strings of a's and b's including ϵ .
 - (c) All odd length strings over {a, b}.
 - (d) All even length strings over {a, b}.
14. Choose the correct statement in relation to a given instance P of a post corresponding problem.
- (a) P can be simulated by the intersection of two deterministic push down automata.
 - (b) P can be simulated by the intersection of two regular sets.
 - (c) P cannot be reduced to the emptiness problem of context sensitive languages (CSLs)
 - (d) P can be simulated by the intersection of two languages L_1 & L_2 , where L_1 is generated by an LR(k) grammar & L_2 is a language for which a description as a regular expression R exists.
15. Consider a language L_1 generated by a CSG G_1 & a language L_2 accepted by a non-deterministic LBA M_2 . Choose the correct statement.
- (a) It is undecidable whether L_1 is empty, finite, infinite or the same as Σ^* .
 - (b) It is undecidable if a given string w is generated by G_1 or accepted by G_1 or accepted by M_2 .
 - (c) It is decidable if $L_1 = L_2$ or $L_1 \leq L_2$.
 - (d) It is decidable if L_1 or L_2 is accepted by some two way nondeterministic finite automata (2NFA) M_3 .
16. Consider a language L_1 accepted by deterministic Turing Machine M_1 & a language L_2 accepted by a two push down tape machine M_2 . Choose the correct statement.

- (a) The membership problem of $L_2 \& L_1 \cap L_2$ is decidable.
- (b) The equivalence problem of $L_1 \& L_2$, viz. L_1 viz. $L_1 = L_2$ can be resolved by some algorithm.
- (c) It is possible that $L_1 = L_2 = \Sigma^*$.
- (d) It is decidable if $L_1 \& L_2$ are regular sets.
17. Consider a CFG C_1 generating syntactically correct C programs & a context free grammar J_1 generating syntactically correct JAVA programs.
- Choose the correct statement.
- (a) It is decidable if $C_1 \& J_1$ decidable all and only algorithms
- (b) It is undecidable if C_1 is ambiguous
- (c) It is decidable if J_1 is ambiguous
- (d) Both $C_1 \& J_1$ have equivalent LR(k) grammars.
18. A takes two arbitrary DPDA $M_1 \& M_2$. He wants to determine if $M_1 \& M_2$ accept some strings in common.
- (a) A has to check for all strings of length ≤ 10
- (b) A has to check for all strings of length $\leq 2^{10}$
- (c) A has to check for all strings of length $\leq 2^{10}$
- (d) A can never determine if $M_1 \& M_2$ accept some string in common.
19. A is given a software package S_1 . To test it exhaustively he generates test data using a form counter machine. Choose the correct statement.
- (a) By generating enough test data, S_1 can be exhaustively tested, so it is a decidable problem.
- (b) Exhaustive testing is an undecidable problem.
- (c) Though exhaustive testing is undecidable we can use enough test data to see if that S_1 contains < 10 bugs.

- (d) none of the above.
20. A argues that as the membership problem of regular sets is decidable, if we form the infinite union of regular sets we should decide the membership problem. Choose the correct statement.
- (a) A's statement is always true
- (b) A's statement is always false
- (c) A's statement is never true for any particular choice of regular sets.
- (d) none of the above
21. A is given a R.E. set L_1 accepted by some Turing Machine M_1 . He is also given that L_1 is a R.E. set that is not recursive.
- (a) $\overline{L_1}$ is always decidable
- (b) $\overline{L_1}$ is partially decidable
- (c) $\overline{L_1}$ is undecidable
- (d) $\overline{L_1}$ can be accepted by the finite union of 1000 counter machines
22. A is given a non-deterministic Turing machine M_1 that accepts a language L_1 & a deterministic Turing machine accepting a language L_2 . Choose the correct statement.
- (a) As any nondeterministic Turing machine has an equivalent deterministic Turing machine $L_1 = L_2$
- (b) If M_2 is obtained by replacing the non determinism on M_1 by determinism then the time complexity of L_2 is polynomial in the size of the input always.
- (c) $L_1 = \emptyset$ may be undecidable, but $L_2 = \emptyset$ is always decidable if $h(L_1) = L_2$.
- (d) Both L_1 & L_2 suffer from the halting problem.

23. A is given languages L_1, L_2, L_3, L_4 & L_5 ; which are a R.E. set, a CSL, a CFL and a regular set. He is also given an ϵ -free homomorphism h . Choose the correct answer.
- (a) $h(L_1), h(L_2), h(L_3), h(L_4)$ & $h(L_5)$, are regular sets & so their emptiness problem is decidable.
 - (b) $h(L_1), h(L_2), h^{-1}(L_3), h^{-1}(L_4)$, & $h^{-1}(L_5)$, all are CFLs and so their equivalence problem is decidable for any pair of languages.
 - (c) $h^{-1}(h(L_1)), h^{-1}(h(L_2)), h^{-1}(h(L_3)), h^{-1}(h(L_4))$, & $h^{-1}(h(L_5))$, are all CSLs or recursive sets and so the problem of whether they are pairwise disjoint is decidable.
 - (d) The membership problem of L_1 is undecidable but the membership problem of L_2, L_3, L_4 & L_5 are undecidable.

Solutions

Level -1

01. **Ans (b)**

Sol : To evaluate we need a turing machine. A stack can only parse it.

02. **Ans (c)**

Sol : P is closed under complement, If NP is closed under complement then $P = NP$. If complement of $P = NP$ then $P = NP$

03. **Ans (d)**

Sol : ϵ cannot be in any recursive set of (a), (b) & (c) are excluded.

04. **Ans (d)**

Sol : L_1 cannot be decided in a finite time, so it is R.E. L_2 is simple we run the TM for 1000 steps. So L_2 is recursive.

05. **Ans (a)**

Sol : The given grammar is a DCFL & hence DPDA can accept it. It is the language of palindrome with 'd' in the center. Every DCFL is accepted by a DPDA. A DPDA can be simulated by a PDA. A PDA can be simulated by an LBA. An LBA can be simulated by a TM.

06. Ans (a)

Sol : The language is a DCFL & hence can be accepted by a DPDA. Hence by a PDA. Hence by a LBA. Hence by a TM

07. Ans (c)

Sol : A LBA is nothing, but a 2 DFA with an unlimited amount of ink.

08. Ans (d)

Sol : A finite set is any way accepted by a FA.

09. Ans (b) & (c)

Sol : The class of CSLs & recursive sets are closed under complement.

10. Ans (c)

Sol : The string must end with a 0. The 1 takes it back to q_0 & hence we take $0^n 1 0^n$.

11. Ans (a)

Sol : The language is a regular set & all the answers in a way are correct. (a) is the tightest & closest answer.

12. Ans (d)

Sol : The class of finite automata can be put into a one to one correspondence with the integer.

13. Ans (b)

Sol : P is closed under complement. If the class NP is closed under complement than $P = NP$.

14. **Ans (d)**

15. **Ans (d)**

Sol : The subsets of Σ^* are CSLs, CFLs, regular sets etc and all are the formal languages & hence are uncountable.

16. **Ans (d)**

Sol : $L = \Sigma^*$ is not decidable for CFLs.

17. **Ans (b)**

Sol : The union of two countable infinite sets is countable infinite.

18. **Ans (b)**

Sol : (a) Since L_1 & L_2 may be accepted by two different finite automata, they may be different finite automata, they may be different regular sets. So it is not true that L_1 is always the same as L_2 .

(b) The equivalence problem of regular sets is decidable. So this is a true statement.

(c) The emptiness problem of regular sets is decidable. For the algorithm just check strings of length 0 to number of states of the minimal DFA accepting the language. If any one of them is accepted then the regular set is not empty. So this is a false statement.

(d) For a counter example consider $L_1 = L_2 = \Sigma^*$. Then $L_1 \cap L_2 = \Sigma^*$ is not a finite set. So this is a false statement.

19. **Ans (a)**

Sol : (a) The intersection of two DCFLs is empty is the same as the halting problem of Turing machines and so is undecidable. So this is a true statement.

(b) L_1 & L_2 may be different DCFLs. So this is a false statement.

(c) L_2 need not be a regular set always. So $L_2 = L_3$ is false.

- (d) It is decidable if a DCFL L_1 is equal to a given regular set L_3 . So this is false.

20. **Ans (a)**

Sol : Two CFLs & the invalid computation of Turing machine can be described by a regular set. So this is undecidable. So (b) is a false statement.

- (c) It is decidable if a CFL is finite. Construct the graph of the (CFG) grammar generating L_1 . If it has no cycles the L_1 is finite. So (c) is a false statement.
- (d) It is decidable if a CFL is infinite. If the graph of the reduced CFG generating L_1 .

- (a) The emptiness problem of CFG's is decidable. Construct the reduced grammar generating L_1 ; if it vanishes then L_1 is empty. So (a) is a true statement.
- (b) It is undecidable for a CFL L_1 , $L_1 = \Sigma^*$. This follows from the fact that the valid computations of a Turing machine can be given by the intersection of a cycle then L_1 is infinite. So (b) is a false statement.

21. **Ans (d)**

Sol : Both C_1 & J_2 are procedure which can realise all the R.E. Sets. So neither C_1 nor J_2 is necessarily an algorithm.

22. **Ans (b)**

Sol : (a) Whether 'a' is ever executed reduces the halting problem of Turing machines & so is undecidable.

- (b) Whether 'a' is live or dead reduces the halting problem of turing machine
- (c) It is undecidable if C contains a repeated iteration.

23. **Ans (d)**

Sol : (a) The graph coloring problem is NP – complete problem which is

Perhaps intractable but always decidable as there exists an algorithm of exponential complexity for the problem.

- (b) Register optimization can be resolved by the graph coloring problem.
- (c) If we have a finite number of nodes then a simple polynomial algorithm suffices for graph coloring & so it is intractable
- (d) The graph coloring problem has an algorithm of less than or equal to exponential complexity.

24. Ans : (c)

Sol: If L is recursive set $h(L)$ will be in general R.E set. So for context sensitive language & recursive sets we have to use ϵ - free homomorphism in closure properties.

25. Ans : (d)

- Sol:** (a) M_1 M_2 and M_3 describe all the R.E. sets and so the halting problem is undecidable.
- (b) The equivalence problem of R.E. sets is undecidable
 - (c) It is undecidable if a turing machine accepts all the strings over the terminal alphabet.
 - (d) M_3 can realise a universal turing machine & so can simulate any turing machine.

26. Ans : (d)

Sol: M_1 & M_2 are just variations of the standard model of the turing machine & accept all the R.E. sets & nothing else.

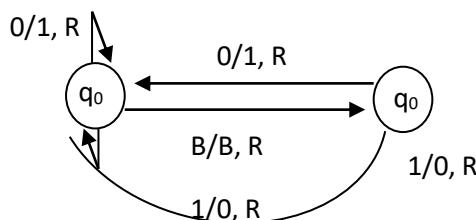
27. Ans : (c)

Sol: By Rice's Theorem all non-trivial properties of R.E. sets are undecidable. The (c) is a trivial property as there are an infinite number of graphs which have the Hamiltonian cycle.

Level – 2

01. Ans : (a)

Sol :



1) ϵ as input – m/c gets into an infinite loop. So (d) is not a correct choice.

2) 0 as input – accept

02. Ans : (d)

Sol : S generates $\{a^n b^n c^n / n, m \geq 1\}$

03. Ans : (c)

Sol : The DCFLs are not closed under intersection. The CFLs are not closed under intersection. The regular sets are closed under intersection.

04. Ans : (a)

Sol : (a) Regularity is decidable

(c) Membership of R.E. is decidable

05. Ans : (d)

Sol : $L_1 = \overline{L_n} = \Sigma^*$ & $L_n = \emptyset$

06. Ans : (a)

Sol : It is generally accepted that (a) is the true case

07. Ans : (c)

Sol : Any automata has a halt state. A TM is not a FA.

08. Ans : (d)

Sol : The regular expression denotes regular sets.

09. Ans : (c)

Sol : ϵ is not accepted by the TM

10. Ans : (b)

Sol : The TM halts so it accepts a recursive set.

11. Ans : (a)

Sol : $L_1 = (0+1)^*$ is a regular & infinite set by itself.

12. Ans : (d)

Sol : L_2 can be any language.

13. Ans : (d)

Sol : ϵ is accepted & so in 'aa'. Use the methods of elimination

14. Ans : (a)

Sol : (a) The intersection of two DCFLs gives the valid computations of a Turing Machine which corresponds to solutions to the PCP problem P. So this is true.

(b) The intersection of two regular sets is regular. For a regular set we have a FA M accepting it. The halting problem of M is decidable. So this is a false statement.

15. Ans : (a)

Sol : (a) A CSL can be obtained from the intersection of two CFLs, as the Boolean closure of two CFLs are the CSLs. Now the intersection of two CFLs give the valid computations of a Turing Machine & so the given choice reduces to the halting problem which is undecidable. So (a) is true statement.

(b) The CSLs are automatically recursive sets, so their membership problem is decidable. So this is a false statement.

(c) The equivalence problem of two CFLs is undecidable, so this gives that the equivalence problem of CFLs is undecidable. So this is a false statement.

(d) It is undecidable if a CFL is regular, so this is undecidable.

16. Ans : (c)

Sol : (a) L_1 is recursive set & L_2 is R.E. set. The membership problem of L_2 is the halting problem of TMs & so undecidable. So this is a false statement.

(b) As L_1 is recursive set for which the halting problem is decidable & L_2 is a R.E. set for which the halting problem is undecidable, this is a false statement.

(c) It is undecidable if $L_1 = \emptyset$. So it is undecidable if $L_1 = \Sigma^*$. L_1 is a R.E. set. However some L_1 & L_2 may be Σ^* . So this is true.

(d) It is undecidable if recursive sets and R.E. sets are regular. So this is a false statement.

17. Ans : (b)

Sol : (a) C_1 & J_1 generate all C & JAVA programs which are procedures & not algorithms.

(b) It is undecidable if a CFG C_1 is ambiguous

(c) It is undecidable if a cfg J_2 is ambiguous

(d) Both C_1 & J_1 deal with CFG's. All CFG's do not have LR(k) equivalents.

18. Ans : (d)

Sol : The intersection of two DCFLs can give all the valid computations of a turing machine & so is undecidable.

19. Ans : (b)

Sol : Exhaustive testing reduce to the halting problem of turing machine. So we can never decide if S_1 will ever halt.

20. Ans : (a)

Sol : (a) The infinite union of regular sets can yield any formal languages including undecidable once.

(b) If we choose all regular sets as $\{\epsilon\}$ then Lavanya's statement is true.

(c) If we choose all regular sets as if then Lavanya's statement is true.

21. Ans : (c)

Sol : As the R.E. sets are not closed under complement, \overline{L}_1 cannot be accepted by a Turing machine. A multi counter Turing machine is just a variation of the standard model of a Turing machine.

22. Ans : (d)

Sol : (a) M_1 & M_2 may be machine generating different R.E. sets

(b) Normally removing non-determinism leads to an exponential blown-up in the time complexity.

(c) The emptiness problem of L_1 and L_2 are undecidable.

(d) Both M_2 & M_1 can be corrected to the standard model of turing machines.

23. Ans : (d)

Sol : By definition recursive sets have a decidable membership problem. The membership of R.E. sets is undecidable are closed under ϵ - free homomorphism & so we use the standard results

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