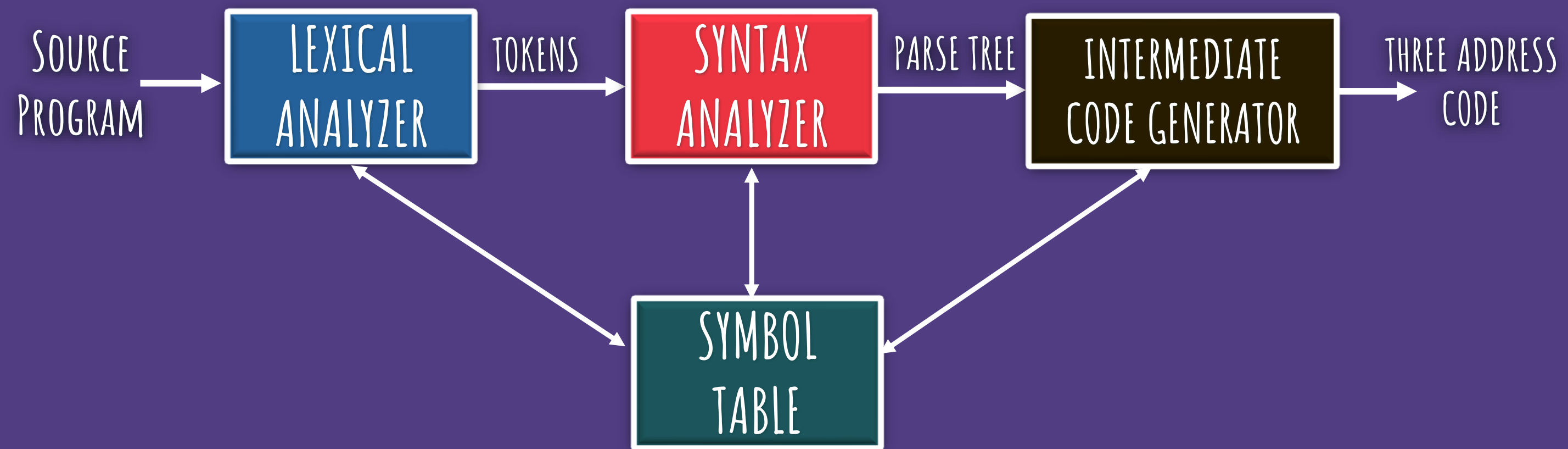




SYNTAX ANALYSIS

- ✿ Syntax analysis or parsing is the second phase of a compiler.
- ✿ A lexical analyzer can identify tokens with the help of regular expressions and pattern rules.
- ✿ A lexical analyzer cannot check the syntax of a given sentence due to the limitations of the regular expressions.
- ✿ Regular expressions cannot check balancing tokens, such as parenthesis.
- ✿ Syntax analysis phase uses **Context-Free Grammar (CFG)**, which is recognized by push-down automata.

MODEL OF A COMPILER FRONT END



REVIEW OF CFG

- ❁ A context-free grammar (grammar for short) consists of terminals, non-terminals, a start symbol and productions.
 - ❁ **Terminals** are the basic symbols from which strings are formed.
 - ❁ The term "token name" is a synonym for "terminal".

REVIEW OF CFG

- ❁ A context-free grammar (grammar for short) consists of terminals, non-terminals, a start symbol and productions.
 - ❁ **Non-terminals** are syntactic variables that denote sets of strings.
 - ❁ Non-terminals define sets of strings that help define the language generated by the grammar.
 - ❁ They also impose a hierarchical structure on the language that is useful for both syntax analysis and translation.

REVIEW OF CFG

- ✿ In a grammar, one nonterminal is distinguished as the **start symbol**, and the set of strings it denotes is the language generated by the grammar.
- ✿ Conventionally, the productions for the start symbol are listed first.

REVIEW OF CFG

- ❁ The **productions** of a grammar specify the manner in which the terminals and non-terminals can be combined to form strings. Each production consists of
 - ❁ A set of **production rules** which are the rules for replacing nonterminal symbols.
 - ❁ Production rules have the following form:
variable \rightarrow string of variables and terminals.

REVIEW OF CFG

- ✿ The grammar with the following productions defines simple arithmetic expression

$$\begin{aligned} \text{expr} &\rightarrow \text{expr} + \text{term} \\ \text{expr} &\rightarrow \text{expr} - \text{term} \\ \text{expr} &\rightarrow \text{term} \\ \text{term} &\rightarrow \text{term} * \text{factor} \\ \text{term} &\rightarrow \text{term} / \text{factor} \\ \text{term} &\rightarrow \text{factor} \\ \text{factor} &\rightarrow (\text{expr}) \\ \text{factor} &\rightarrow \mathbf{id} \end{aligned}$$

In this grammar, the **terminal symbols** are : $\text{id} + - * / ()$
The **nonterminal symbols** are : $\text{expr}, \text{term}, \text{factor}$
Start symbol : expr

REVIEW OF CFG

Notational Conventions

- ✿ These symbols are terminals:
 - a. Lowercase letters early in the alphabet, such as a, b, c.
 - b. Operator symbols such as +, *, and so on.
 - c. Punctuation symbols such as parentheses, comma, and so on.
 - d. The digits 0, 1, . . . , 9.
 - e. Boldface strings such as **id** or **if**, each of which represents a single terminal symbol.

REVIEW OF CFG

Notational Conventions

- ✿ These symbols are non-terminals
 - a. Uppercase letters early in the alphabet, such as A, B, C.
 - b. The letter S, which, when it appears, is usually the start symbol.
 - c. Lowercase, italic names such as *expr* or *stmt*.
 - d. When discussing programming constructs, uppercase letters may be used to represent non-terminals for the constructs. For example, non-terminals for expressions, terms, and factors are often represented by E, T, and F, respectively.

REVIEW OF CFG

Notational Conventions

- ✿ Uppercase letters, such as X, Y, Z , represent grammar symbols; that is, either non-terminals or terminals.
- ✿ Lowercase letters late in the alphabet, chiefly u, v, \dots, z , represent (possibly empty) strings of terminals.
- ✿ Lowercase Greek letters α, β, γ for example, represent (possibly empty) strings of grammar symbols.

REVIEW OF CFG

Notational Conventions

- ✿ A set of productions $A \rightarrow \alpha_1, A \rightarrow \alpha_2, \dots, A \rightarrow \alpha_k$ with a common head A (call them A -productions), may be written $A \rightarrow \alpha_1 \mid \alpha_2 \mid \dots \mid \alpha_k$
- ✿ $\alpha_1, \alpha_2, \dots, \alpha_k$ are called the alternatives for A .
- ✿ Unless stated otherwise, the head of the first production is the start symbol.

REVIEW OF CFG

Notational Conventions

- ✿ Using these conventions, the grammar for arithmetic expression can be rewritten as

$$\begin{aligned} E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T / F \mid F \\ F &\rightarrow (E) \mid \text{id} \end{aligned}$$

DERIVATION

- ❁ The construction of a parse tree can be made precise by taking a derivational view, in which productions are treated as rewriting rules.
- ❁ Beginning with the start symbol, each rewriting step replaces a nonterminal by the body of one of its productions.

DERIVATION

- ✿ E.g. consider the following grammar , with a single non-terminal E

$$E \rightarrow E + E \mid E * E \mid - E \mid (E) \mid \text{id}$$

The production $E \rightarrow - E$ signifies that if E denotes an expression, then $- E$ must also denote an expression.

The replacement of a single E by $- E$ will be described by writing $E \Rightarrow -E$ which is read, "E derives - E."

DERIVATION

- ✿ The production $E \rightarrow (E)$ can be applied to replace any instance of E in any string of grammar symbols by (E) .
- ✿ e.g., $E * E \Rightarrow (E) * E$ or $E * E \Rightarrow E * (E)$
- ✿ We can take a single E and repeatedly apply productions in any order to get a sequence of replacements.
e.g., $E \Rightarrow - E \Rightarrow - (E) \Rightarrow - (id)$
- ✿ We call such a sequence of replacements a derivation of $-(id)$ from E .
- ✿ This derivation provides a proof that the string $-(id)$ is one particular instance of an expression.

Leftmost Derivation

- ✿ A leftmost derivation is obtained by applying production to the leftmost variable in each step.

$$s \rightarrow AB$$

$$A \rightarrow aaA \mid \varepsilon$$

$$B \rightarrow Bb \mid \varepsilon$$

- ✿ Leftmost Derivation

$$s \Rightarrow AB$$

$$\Rightarrow aaAB$$

$$\Rightarrow aaB$$

$$\Rightarrow aaBb$$

$$\Rightarrow aab$$

Rightmost Derivation

- ✿ A leftmost derivation is obtained by applying production to the rightmost variable in each step.

$$s \rightarrow AB$$

$$A \rightarrow aaA \mid \varepsilon$$

$$B \rightarrow Bb \mid \varepsilon$$

- ✿ Rightmost Derivation

$$s \Rightarrow AB$$

$$\Rightarrow ABb$$

$$\Rightarrow Ab$$

$$\Rightarrow aaAb$$

$$\Rightarrow aab$$

LEFTMOST & RIGHTMOST DERIVATION

- Let any set of production rules in a CFG be

$$X \rightarrow X+X \mid X^*X \mid X \mid a \quad \text{over an alphabet } \{a\}$$

- The leftmost derivation for the string "a+a*a" may be

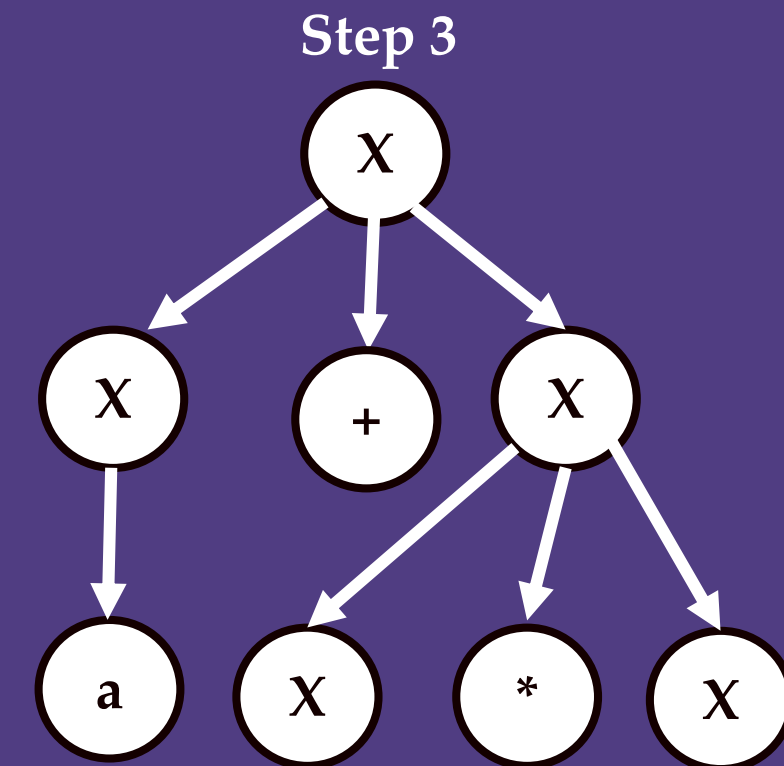
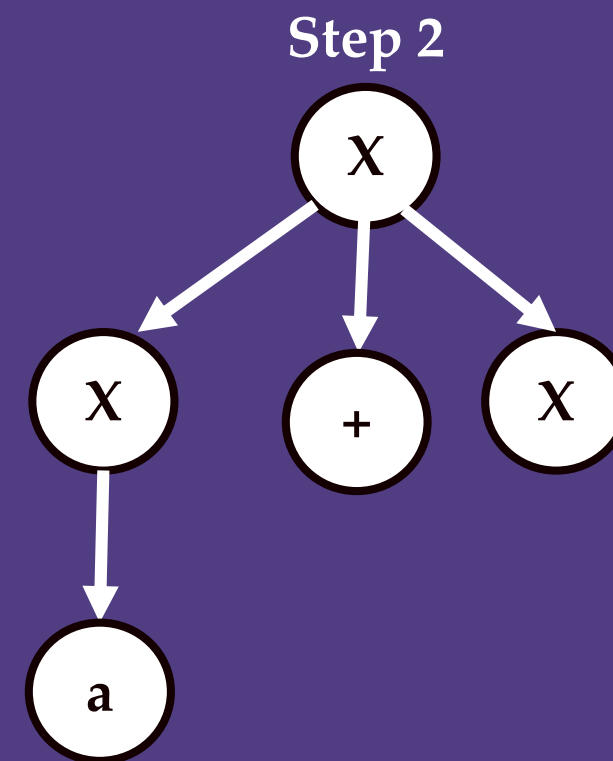
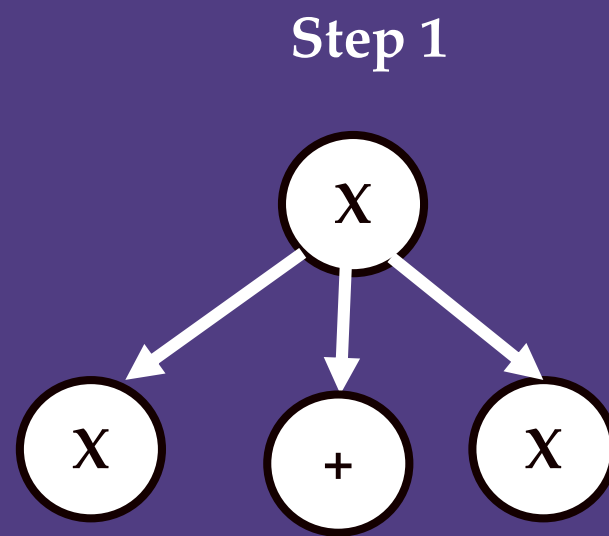
$$X \Rightarrow X+X \Rightarrow a+X \Rightarrow a + X^*X \Rightarrow a+a^*X \Rightarrow a+a^*a$$

- The rightmost derivation for the string may be

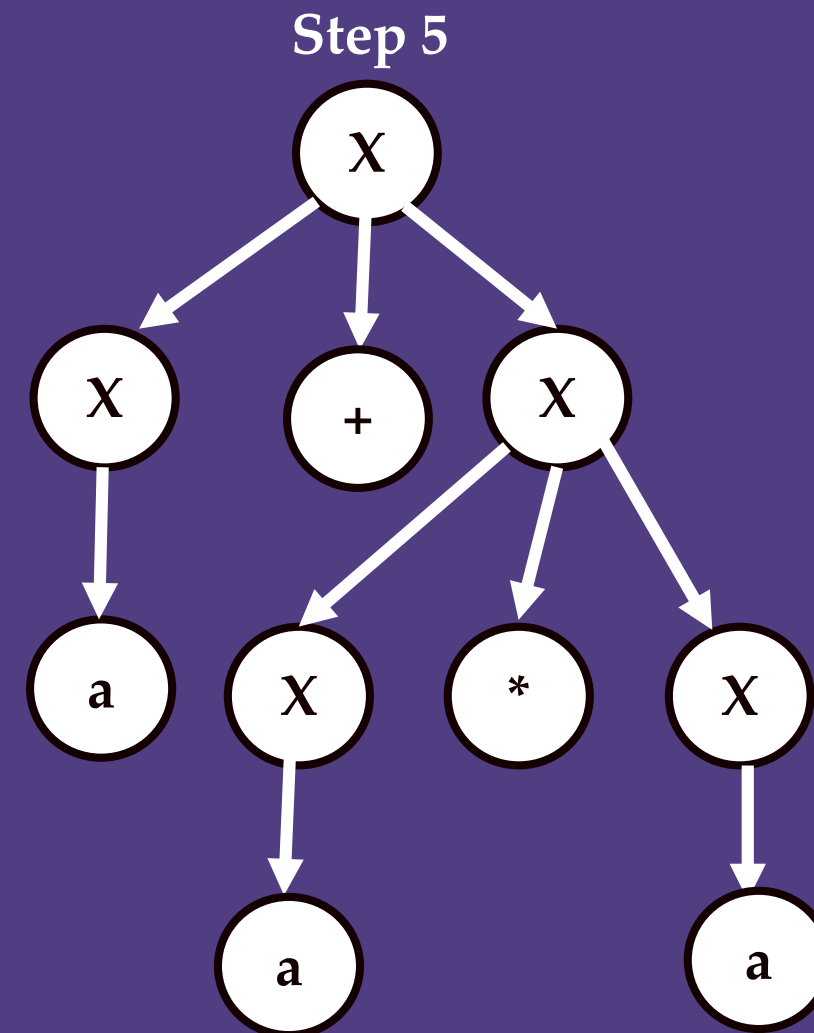
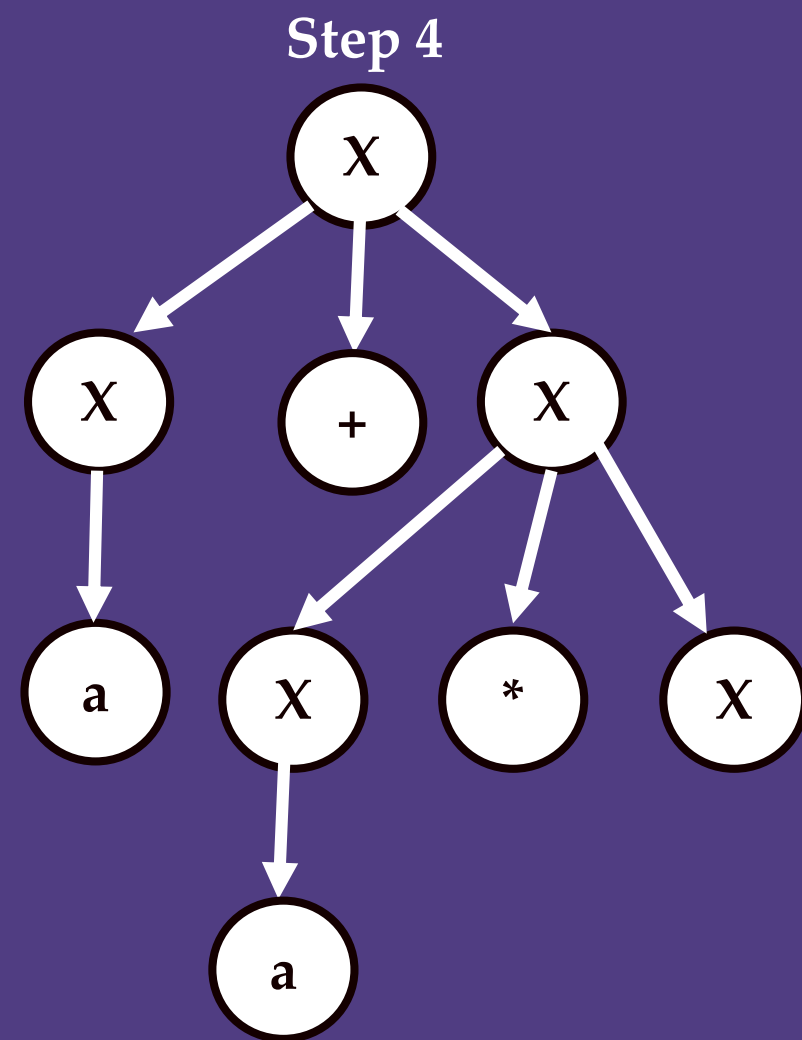
$$X \Rightarrow X^*X \Rightarrow X^*a \Rightarrow X+X^*a \Rightarrow X+a^*a \Rightarrow a+a^*a$$

LEFTMOST DERIVATION

✿ Step-wise derivation of the string is



LEFTMOST DERIVATION

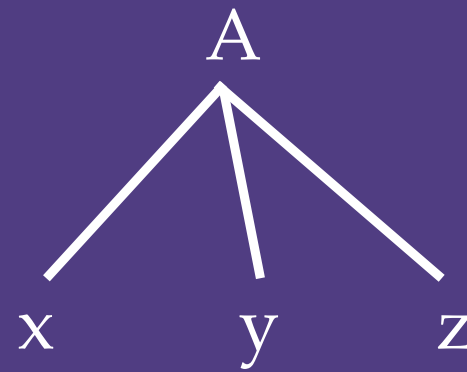


PARSE TREES

- ❁ Parse tree is a hierarchical structure which represents the derivation of the grammar to yield input strings.
- ❁ Simply it is the graphical representation of derivations.
- ❁ Root node of parse tree has the start symbol of the given grammar from where the derivation proceeds.
- ❁ Leaves of parse tree are labeled by non-terminals or terminals.
- ❁ Each interior node is labeled by some non terminals.

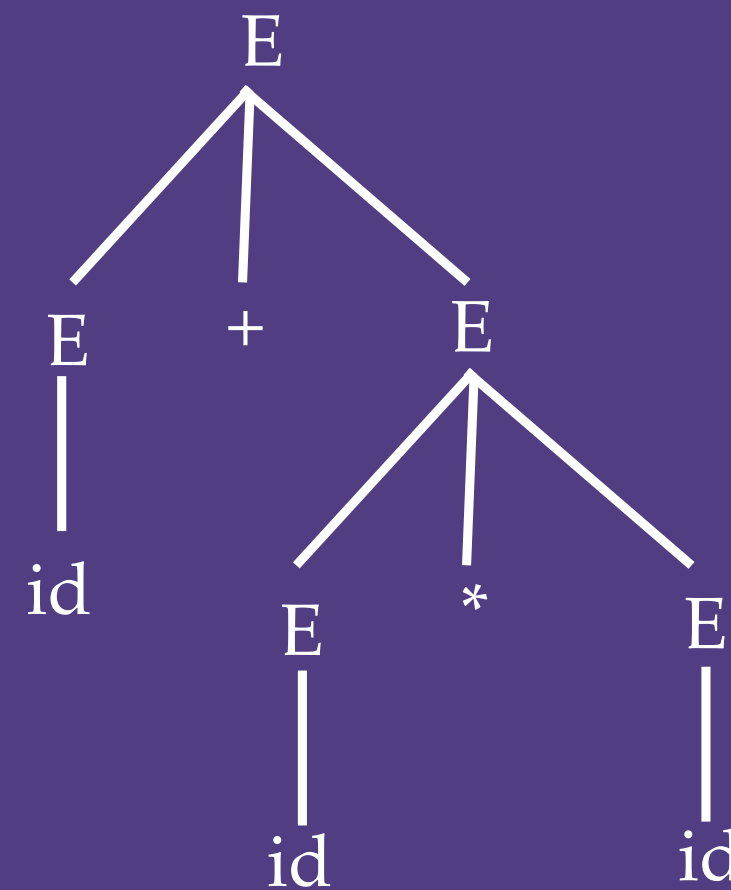
PARSE TREES

- ✿ If $A \rightarrow xyz$ is a production, then the parse tree will have A as interior node whose children are x , y and z from its left to right.



PARSE TREES

- ✿ Construct the parse tree for $E \rightarrow E + E \mid E * E \mid E \mid \text{id}$



YIELD OF A PARSE TREE

- ❁ The leaves of the parse tree are labeled by non-terminals or terminals and read from left to right, they constitute a sentential form, called the yield or frontier of the tree.
- ❁ The string `id + id * id`, is the yield of the parse tree.

AMBIGUITY

- ❁ An ambiguous grammar is one that produces more than one leftmost or more than one rightmost derivation for the same sentence.
- ❁ For most parsers, it is desirable that the grammar be made unambiguous, for if it is not, we cannot uniquely determine which parse tree to select for a sentence.
- ❁ In other cases, it is convenient to use carefully chosen ambiguous grammars, together with disambiguating rules that "throw away" undesirable parse trees, leaving only one tree for each sentence.

AMBIGUITY

✿ Consider the input string $\text{id}+\text{id}*\text{id}$

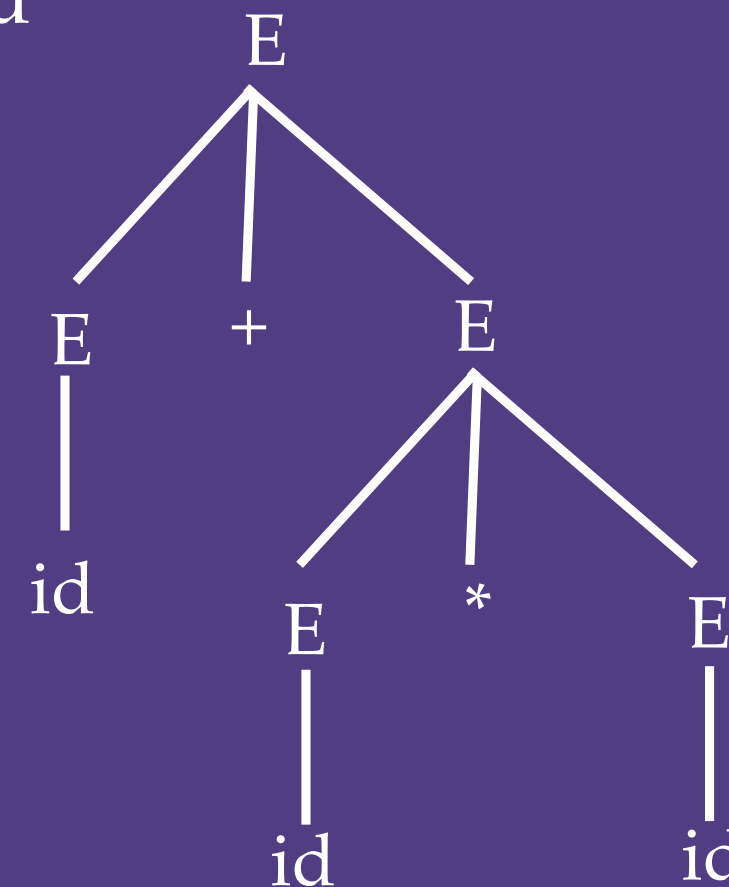
$E \Rightarrow E + E$

$\Rightarrow \text{id} + E$

$\Rightarrow \text{id} + E * E$

$\Rightarrow \text{id} + \text{id} * E$

$\Rightarrow \text{id} + \text{id} * \text{id}$



AMBIGUITY

✿ Consider the input string $\text{id}+\text{id}*\text{id}$

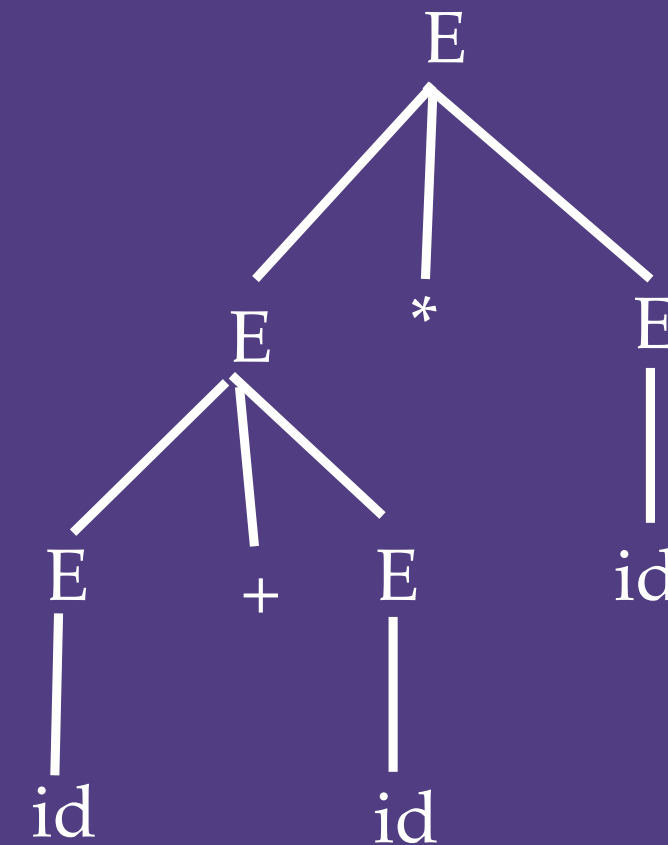
$E \Rightarrow E * E$

$\Rightarrow E + E * E$

$\Rightarrow \text{id} + E * E$

$\Rightarrow \text{id} + \text{id} * E$

$\Rightarrow \text{id} + \text{id} * \text{id}$



WRITING A GRAMMAR

- ❁ Grammars are describing most, but not all, of the syntax of the programming languages.
- ❁ E.g. Identifiers need to be declared before they are used cannot be described by a context-free grammar.
- ❁ The sequences of tokens accepted by a parser form a superset of the programming language.
- ❁ Subsequent phases of the compiler must analyze the output of the parser to ensure compliance with rules that are not checked by the parser.

WRITING A GRAMMAR

Lexical Versus Syntactic Analysis

- ❁ Everything that can be described by regular expression can also be described by a grammar.
- ❁ Then, why regular expression is used to describe the lexical syntax of a language?

WRITING A GRAMMAR

Lexical Versus Syntactic Analysis

1. Separating the syntactic structure of a language into lexical and non-lexical parts provide a convenient way of modularizing the front end of a compiler into two manageable-sized components.
2. The lexical rules of a language are frequently quite simple, and to describe them we do not need a notation as powerful as grammars.

WRITING A GRAMMAR

Lexical Versus Syntactic Analysis

3. Regular expressions generally provide a more concise and easier-to-understand notation for tokens than grammars.
4. More difficult lexical analyzers can be constructed automatically from regular expressions than arbitrary grammars.

WRITING A GRAMMAR

Lexical Versus Syntactic Analysis

- ❁ Regular expressions are most useful for describing the structure of constructs such as identifiers, constants, keywords and white space.
- ❁ Grammars are most useful for describing nested structures such as balanced parentheses, matching begin-end's, corresponding if-then-else's and so on. These cannot be described by regular expressions.

WRITING A GRAMMAR

Eliminating Ambiguity

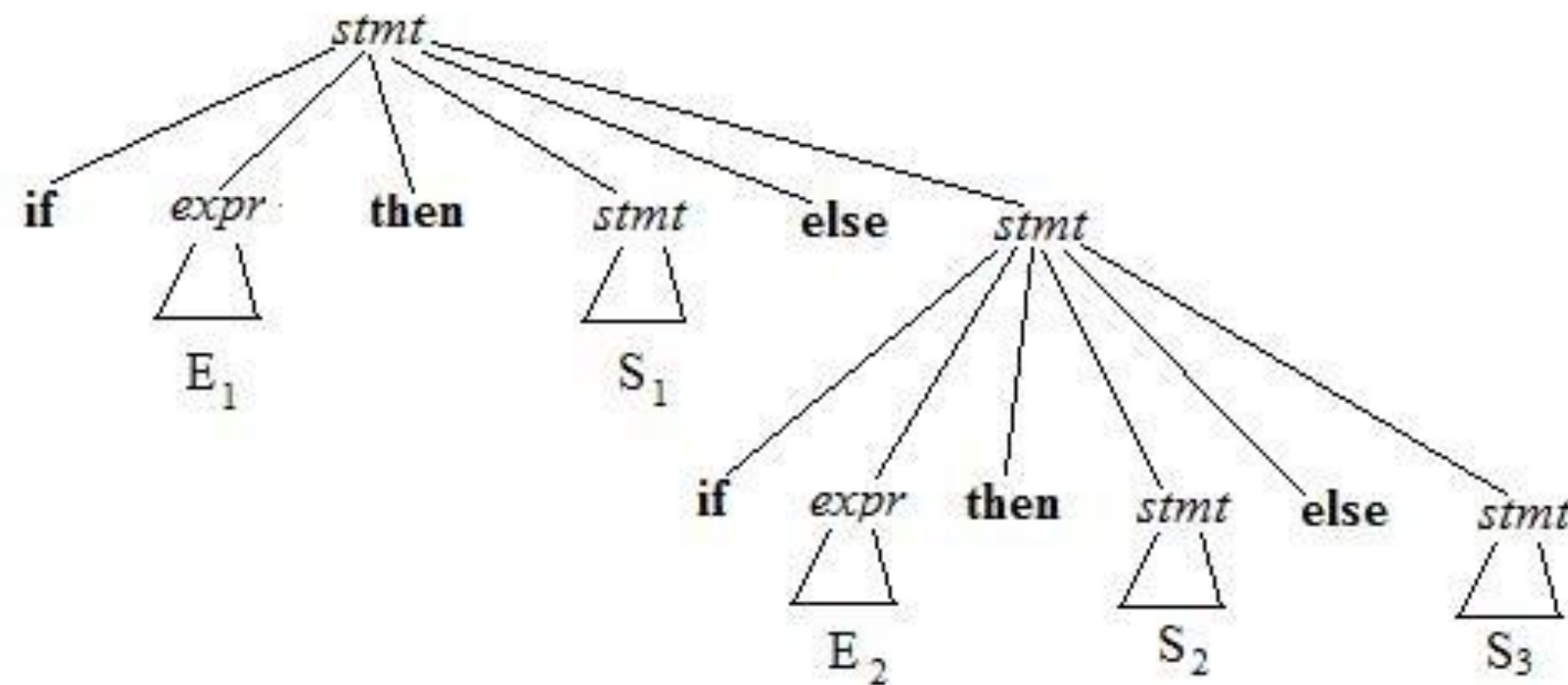
- ✿ Sometimes an ambiguous grammar can be rewritten to eliminate the ambiguity.
- ✿ Consider the dangling else grammar

$$\begin{aligned} \text{stmt} \rightarrow & \textit{if expr then stmt} \\ & | \textit{if expr then stmt else stmt} \\ & | \textit{other} \end{aligned}$$

Here, **other** stands for any other statement

Eliminating Ambiguity

The compound conditional statement,
if E_1 then S_1 else if E_2 then S_2 else S_3
 has the parse tree

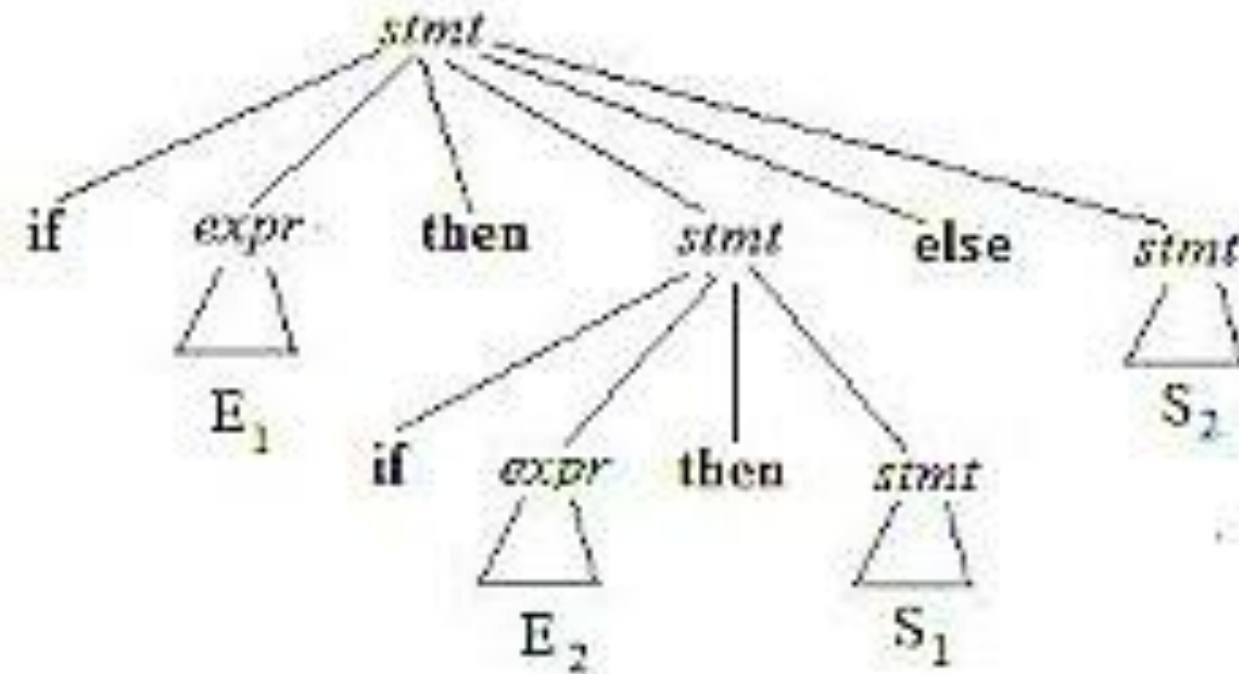
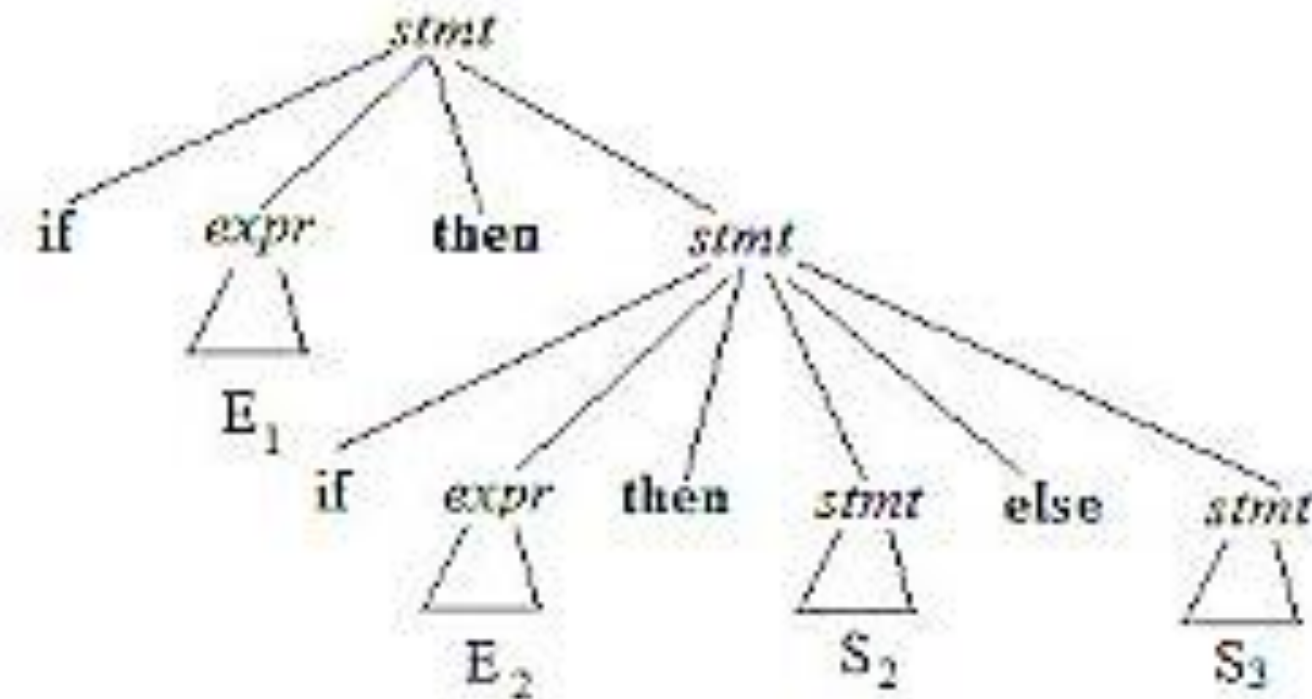


Eliminating Ambiguity

The grammar is ambiguous since the string

if E_1 then if E_2 then S_1 else S_2

has two parse trees



WRITING A GRAMMAR

Eliminating Ambiguity

- ❁ In all programming languages with conditional statements of this form, the first parse tree is preferred.
- ❁ The general rule is, “Match each else with the closest unmatched then”.

WRITING A GRAMMAR

Eliminating Ambiguity

- ❁ Unambiguous grammar for if-then-else statements,

```
stmt → matched_stmt  
      | open_stmt  
matched_stmt → if expr then matched_stmt else matched_stmt  
              | other  
open_stmt → if expr then stmt  
            | if expr then matched_stmt else open_stmt
```

WRITING A GRAMMAR

Elimination of Left Recursion

- ❁ A grammar is left recursive if it has a nonterminal A such that there is a derivation $A \xRightarrow{*} A\alpha$ for some string α .
- ❁ Top-down parsing methods cannot handle left recursive grammars, so a transformation is needed to eliminate left recursion.

WRITING A GRAMMAR

Elimination of Left Recursion

- ✿ $A \rightarrow A\alpha \mid \beta$ is left recursive
- ✿ This can be made non-left recursive by

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \varepsilon$$

WRITING A GRAMMAR

Elimination of Left Recursion

Eliminate left recursion from the grammar

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid \text{id} \end{aligned}$$

After eliminating left recursion,

$$\begin{aligned} E &\rightarrow TE' \\ E' &\rightarrow +TE' \mid \varepsilon \\ T &\rightarrow FT' \\ T' &\rightarrow *FT' \mid \varepsilon \\ F &\rightarrow (E) \mid \text{id} \end{aligned}$$

Elimination of Left Recursion

Eliminate left recursion from the grammar

$$\begin{aligned} S &\rightarrow ABC \\ A &\rightarrow Aa | Ad | b \\ B &\rightarrow Bb | e \\ C &\rightarrow Cc | g \end{aligned}$$

After eliminating left recursion,

$$\begin{aligned} S &\rightarrow ABC \\ A &\rightarrow bA' \\ A' &\rightarrow aA' | \varepsilon | dA' \\ B &\rightarrow eB' \\ B' &\rightarrow bB' | \varepsilon \\ C &\rightarrow gC' \\ C' &\rightarrow cC' | \varepsilon \end{aligned}$$

WRITING A GRAMMAR

Elimination of Left Recursion

- ✿ Immediate left recursion can be eliminated by the following technique, which works for any number of A-productions.
- ✿ First group the productions as

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \cdots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n$$

where no β_i begins with an A.

WRITING A GRAMMAR

Elimination of Left Recursion

- ✿ Then replace the A-productions by

$$\begin{array}{l} A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \cdots \mid \beta_n A' \\ A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \cdots \mid \alpha_m A' \mid \epsilon \end{array}$$

- ✿ The non terminal A generates the same strings as before but it is no longer left recursive.

WRITING A GRAMMAR

Elimination of Left Recursion

- ✿ The procedure eliminates all left recursion from the A and A' productions (provided no α_i is ϵ), but it does not eliminate left recursion involving derivations of two or more steps.
- ✿ E.g.

$$\begin{array}{l} S \rightarrow A a \mid b \\ A \rightarrow A c \mid S d \mid \epsilon \end{array}$$

Elimination of Left Recursion

INPUT : Grammar G with no cycles or ε -productions.

OUTPUT: An equivalent grammar with no left recursion.

- 1) arrange the nonterminals in some order A_1, A_2, \dots, A_n .
- 2) **for** (each i from 1 to n) {
- 3) **for** (each j from 1 to $i - 1$) {
- 4) replace each production of the form $A_i \rightarrow A_j \gamma$ by the
 productions $A_i \rightarrow \delta_1 \gamma \mid \delta_2 \gamma \mid \dots \mid \delta_k \gamma$, where
 $A_j \rightarrow \delta_1 \mid \delta_2 \mid \dots \mid \delta_k$ are all current A_j -productions
- 5) }
- 6) eliminate the immediate left recursion among the A_i -productions
- 7) }

Elimination of Left Recursion

Eliminate left recursion from the grammar

$$\begin{aligned} S &\rightarrow Aa \mid b \\ A &\rightarrow Ac \mid Sd \mid \varepsilon \end{aligned}$$

Substitute S in $A \rightarrow Sd$ to obtain the following A -productions,
 $A \rightarrow Ac \mid Aad \mid bd \mid \varepsilon$

Eliminating left recursion yields the following grammar

$$\begin{aligned} S &\rightarrow Aa \mid b \\ A &\rightarrow bdA' \mid A' \\ A' &\rightarrow cA' \mid adA' \mid \varepsilon \end{aligned}$$

WRITING A GRAMMAR

Left Factoring

- ❁ Left factoring is a grammar transformation that is useful for producing a grammar suitable for predictive or top-down parsing.
- ❁ When the choice between two alternative A-productions is not clear, we may be able to rewrite the productions to defer the decision until enough of the input has been seen to make the right choice.

WRITING A GRAMMAR

Left Factoring

- ✿ For e.g., if we have two productions

$$\begin{aligned} \text{stmt} &\rightarrow \textit{if expr then stmt else stmt} \\ &| \textit{if expr then stmt} \end{aligned}$$

- ✿ On seeing the input *if*, we cannot immediately tell which production to choose to expand *stmt*.

WRITING A GRAMMAR

Left Factoring

- ✿ In general, if $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$ are two productions, and the input begins with a non empty string derived from α , we do not know whether to expand A to $\alpha\beta_1$ or $\alpha\beta_2$
- ✿ We can defer the decision by expanding A to $\alpha A'$
- ✿ After seeing the input derived from α we can expand A' to β_1 or β_2

WRITING A GRAMMAR

Left Factoring

- ✿ Left factored the original productions become,

$$\begin{aligned} A &\rightarrow \alpha A' \\ A' &\rightarrow \beta_1 \mid \beta_2 \end{aligned}$$

Left Factoring

INPUT : Grammar G.

OUTPUT : An equivalent left-factored grammar.

METHOD: For each nonterminal A , find the longest prefix α common to two or more of its alternatives. If $\alpha \neq \epsilon$ — i.e., there is a nontrivial common prefix — replace all of the A -productions $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \cdots \mid \alpha\beta_n \mid \gamma$, where γ represents all alternatives that do not begin with α , by

$$\begin{aligned} A &\rightarrow \alpha A' \mid \gamma \\ A' &\rightarrow \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n \end{aligned}$$

Here A' is a new nonterminal. Repeatedly apply this transformation until no two alternatives for a nonterminal have a common prefix. \square

WRITING A GRAMMAR

Left Factoring

- ✿ Apply left factoring to the “dangling-else” problem

$$S \rightarrow i E t S \mid i E t S e S \mid a$$

$$E \rightarrow b$$

$$S \rightarrow i E t S S' \mid a$$

$$S' \rightarrow e S \mid \epsilon$$

$$E \rightarrow b$$