

Image Segmentation II

DD2423 Image Analysis and Computer Vision

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November 23, 2020

Segmentation techniques

- Figure-ground segmentation: divide image in foreground and background regions. Methods:
 - Thresholding
 - Level-set methods
 - Energy minimization with graphs
- Image segmentation: divide image in regions with pixels of similar qualities. Methods:
 - K-means clustering
 - Split-and-merge
 - Watershedding
 - Mean-shift
 - Normalized cuts
 - Semantic segmentation

- Idea: Represent a shape by a curve

$$X(s) = (x(s), y(s)); s \in [0, 1]$$

- Closed contour (common in practice):

$$X(0) = X(1)$$

- Find the curve by minimizing some energy.

$$E = E_{internal} + E_{image}$$

Snakes – active contour models

- Internal energy (makes curve as short and straight as possible):

$$E_{internal} = \int \alpha(s) ||X_s(s)||^2 + \beta(s) ||X_{ss}(s)||^2 \, ds$$

$\alpha(s)$ and $\beta(s)$ can control shape for different points.

- Image energy (ex. maximize image gradient along curve):

$$E_{image} = E_{edge} = - \int ||\nabla I(X(s))||^2 \, ds$$

- Add whatever energy is suitable for the task. Examples:

E_{line} : fit curve to a line in the image

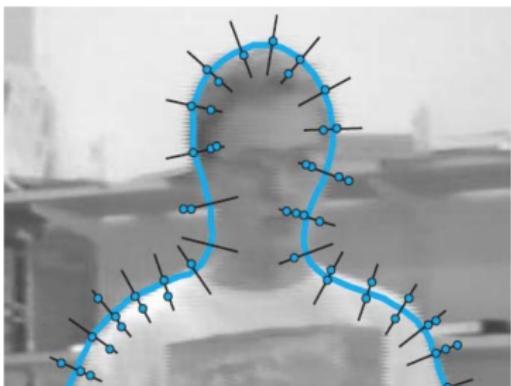
E_{spring} : make curve pass some given anchor points

Snakes – active contour models

- Discretization: Divide contour into pieces

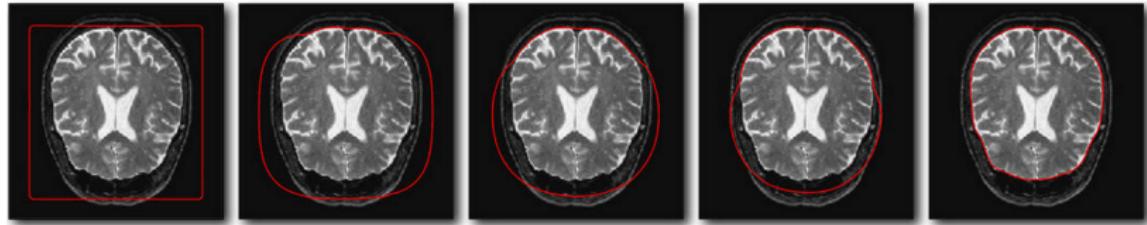
$$E = \sum_i \alpha_i \|X_i - X_{i+1}\|^2 + \beta_i \|X_{i-1} - 2X_i + X_{i+1}\|^2 - \|\nabla I(X_i)\|^2$$

- Iteratively, move along normals to gradually decrease energy.



Snakes – active contour models

- Strengths:
 - Easy to add custom energies to suit problem
 - Very good results on some problems (eg. medical)
 - Can be extended to particular shapes (eg. bodies)
- Weaknesses:
 - Can easily get stuck on wrong edges
 - Hard to prevent self-intersection
 - Cannot change topology



Active contours example

Figure: Liptracking using snakes

Level set segmentation

- Write the curve a function of time t

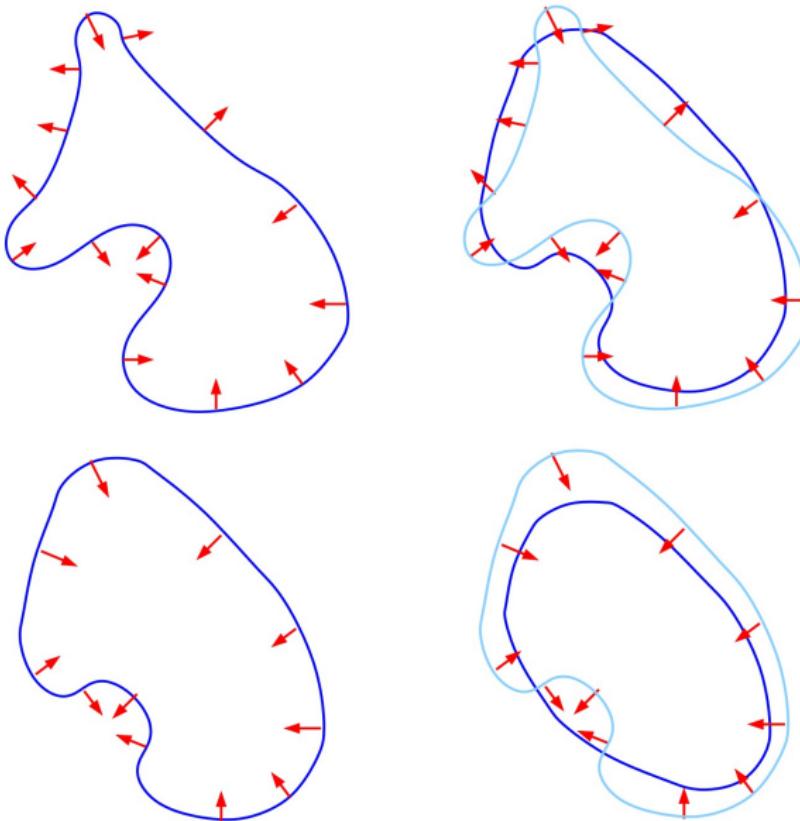
$$X(s, t) = (x(s, t), y(s, t))$$

- Let the curve move along normal N with speed given by the curvature κ .

$$\frac{\partial X}{\partial t} = \kappa N$$

- The effect of minimizing some internal energy.

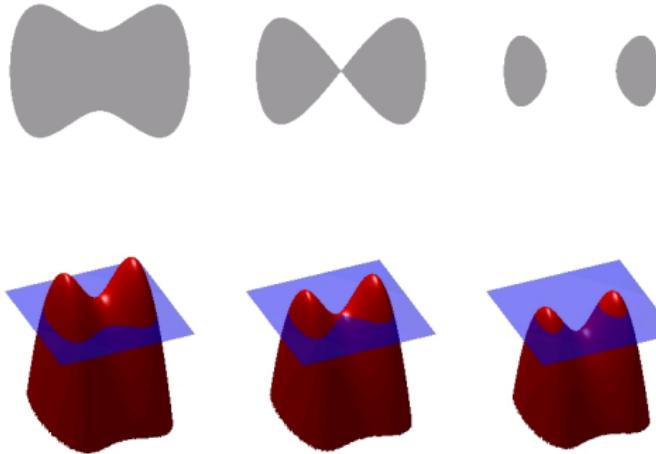
Level set segmentation



Level set segmentation

- Embed the curve $X(s, t)$ in a level set function ϕ :

$$\phi(X(s, t), t) = 0$$



- Idea: Update ϕ and let curve be given by level set $\phi = 0$.

Level set segmentation

- Apply the chain rule for derivation:

$$\frac{d\phi(X(s,t), t)}{dt} = \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial X} \frac{\partial X}{\partial t} = \frac{\partial \phi}{\partial t} + (\nabla \phi)^T \frac{\partial X}{\partial t},$$

where the gradient of the function is

$$\nabla \phi = \begin{pmatrix} \partial \phi / \partial x \\ \partial \phi / \partial y \end{pmatrix}$$

- The normal N can then be written as

$$N = \frac{\nabla \phi}{|\nabla \phi|}$$

Level set segmentation

- For the zero level set we have:

$$\frac{\partial \phi}{\partial t} + (\nabla \phi)^T \frac{\partial X}{\partial t} = 0$$

- With the curve moved as

$$\frac{\partial X}{\partial t} = \kappa N$$

the level set function is updated as

$$\frac{\partial \phi}{\partial t} = -(\nabla \phi)^T \frac{\partial X}{\partial t} = -\kappa (\nabla \phi)^T N = -\kappa |\nabla \phi|$$

- Here the curvature κ is given by

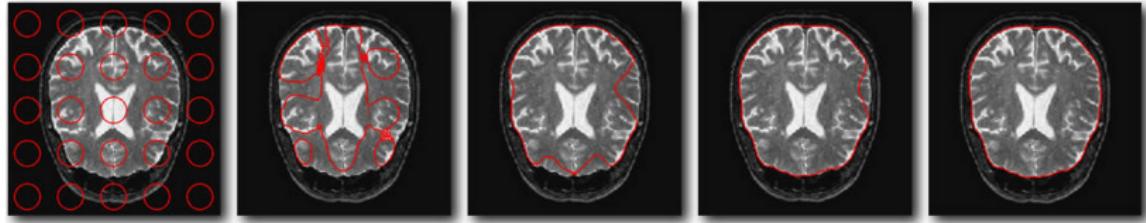
$$\kappa = \nabla \cdot N = \frac{\phi_{xx}\phi_y^2 - 2\phi_{xy}\phi_x\phi_y + \phi_{yy}\phi_x^2}{(\phi_x^2 + \phi_y^2)^{3/2}}$$

Level set segmentation

- Thus, you can get the same effect by updating a level-set function $\phi(x, y)$, as you would for an active contour $X(s)$. [Internal energy]
- To make it useful for segmentation, we also need to add a component to $\phi(x, y)$ that depend on image $I(x, y)$. [Image energy]

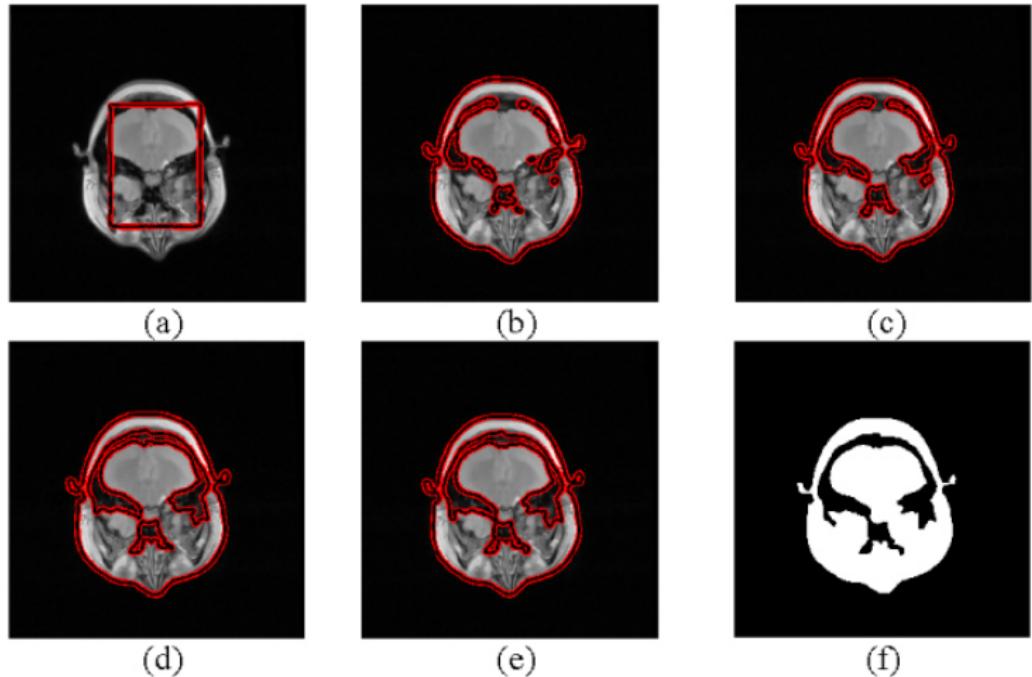
Level set segmentation

- Strengths:
 - Easy to add custom functions to suit problem
 - Can handle more complex shapes
 - Easily extended to high dimensions
 - Allows changes in topology (easier to initialize)
- Weaknesses:
 - Convergence speed can be slow
 - Can be hard to implement



Level-set based segmentation

Very common in medical imaging.



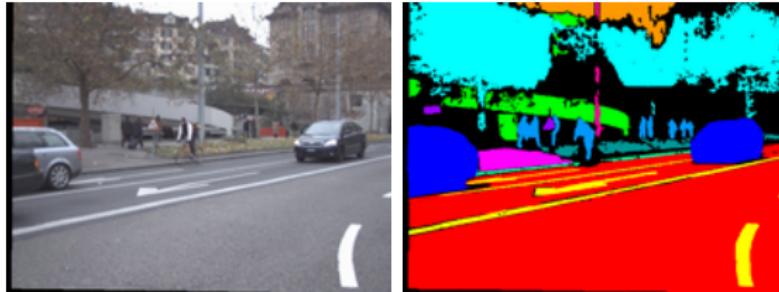
From zero (a) to 200 iterations (e) with final segmentation (f).

Level set example

Figure: MRI segmentation of lungs

Semantic segmentation with CRF

- Create a Conditional Random Field (CRF) with one node per pixel and links between all neighbouring nodes.
- Assume you have a set of models. Examples:
 - $L = \{foreground, background\}$
 - $L = \{sky, ground, vegetation, people, cars, houses\}$
- Each pixel x has a label $I_x \in L$ denoting which model it belongs to.
- Models are usually trained beforehand using a large database.
- A model can for example be represented by a color histogram.



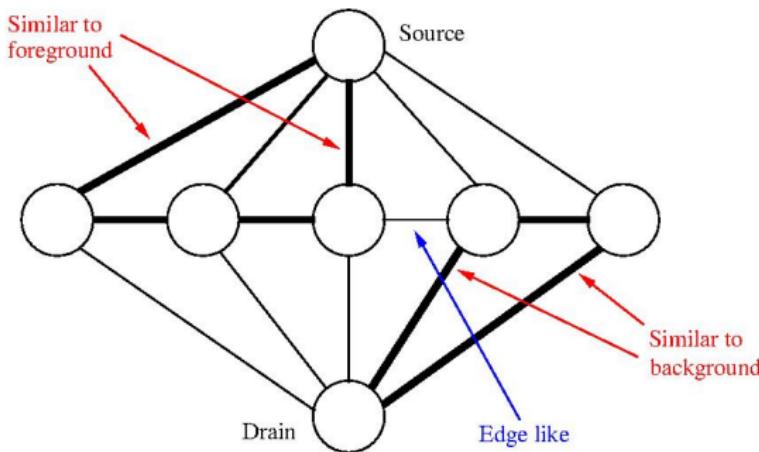
Semantic segmentation with CRF

- Idea: Set up the problem as a energy (cost) minimization problem.
- Find the combination of labels that minimizes the cost

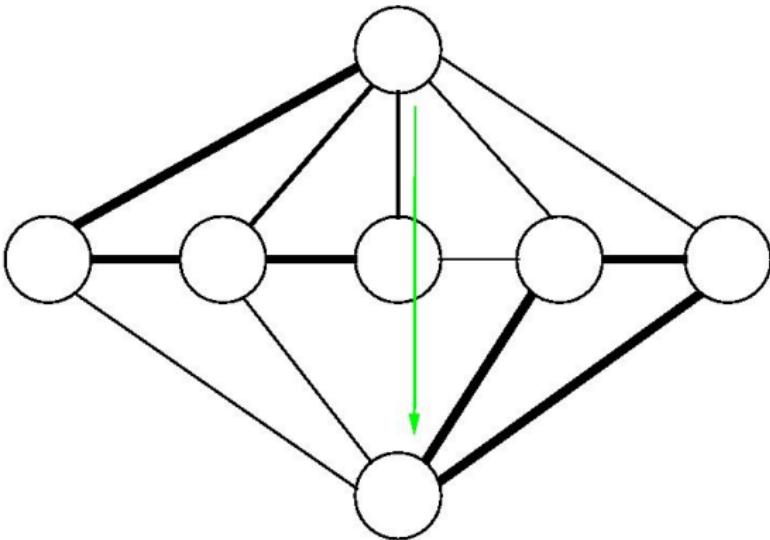
$$E = \sum_x \Psi_x(l_x) + \sum_x \sum_{y \in N_x} \Psi_{x,y}(l_x, l_y)$$

- Cost per node $\Psi_x(l_x)$
 - Low cost if pixel has a color similar to model l_x , high otherwise.
- Cost per link $\Psi_{x,y}(l_x, l_y)$
 - Normally $\Psi_{x,y}(l_x, l_y) = 0$, if $l_x = l_y$.
 - Low cost if pixels have different colors (edge)
 - High cost if pixels have similar colors (smooth surface)
- Result of energy minimization: assign pixels to most similar models, while aligning borders of segments to edges.

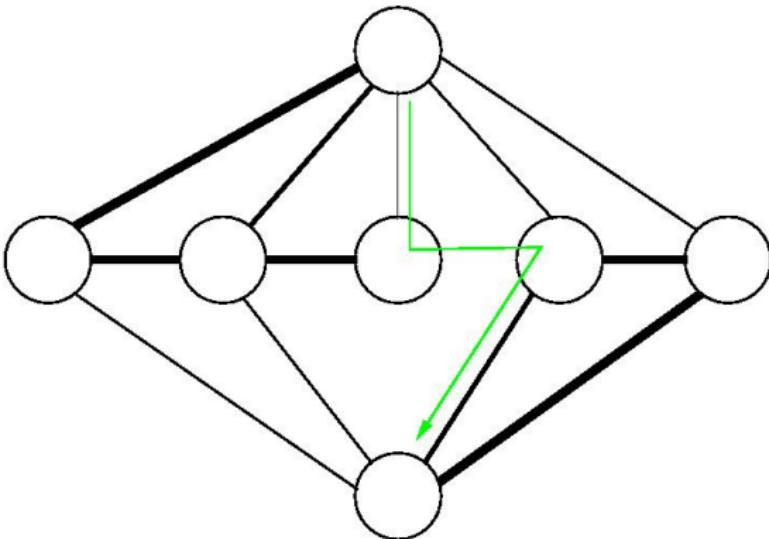
Graph cuts using 5 pixels



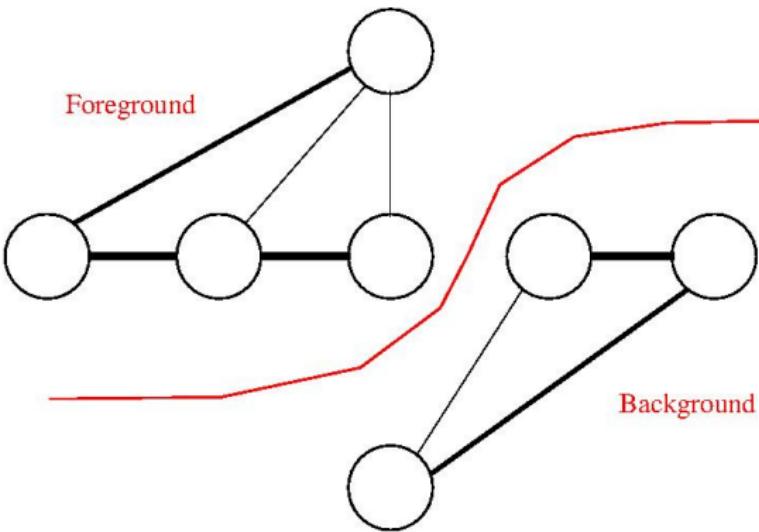
- Graph cuts can be used to minimize the energy E for $|L| = 2$.
- Add links to Source and Drain with costs given by $\Psi_x(I_x)$.
Source = foreground, Drain = background.
- Goal: Find the lowest cost split of the graph into two pieces.



- Push flow from Source to Drain until links saturate.
- Imagine links are pipes and you push flow (water).
- Width of links illustrate how much more flow can be pushed.



- Try all possible paths from Source to Drain.



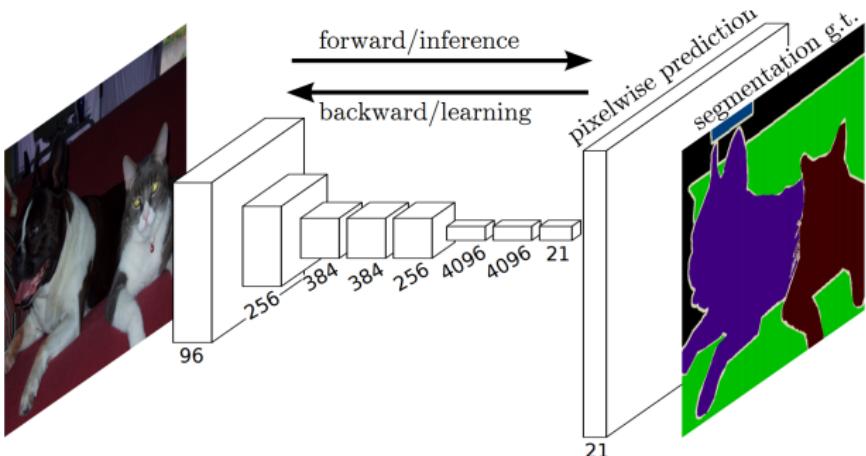
- In the end no more flow can be pushed from Source to Drain.
- Nodes have been divided into two segments.
- The saturated links correspond to the non-zero terms in E .

GrabCut: example of energy based segmentation



- Left: A couple of strokes are applied to create colour models of background and foreground.
- Right: Afterwards background can be changed to something else.

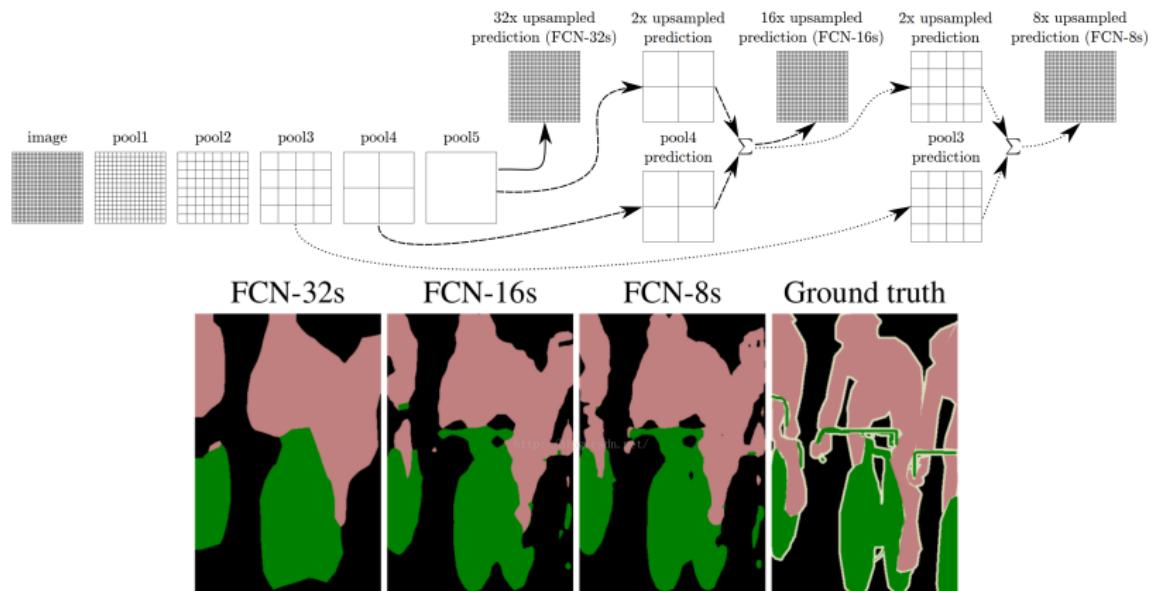
Fully convolutional network (FCN)



- Train a network for classification with many different classes
- Apply it to many different overlapping windows in the image
- Segment image based on most likely class in each window

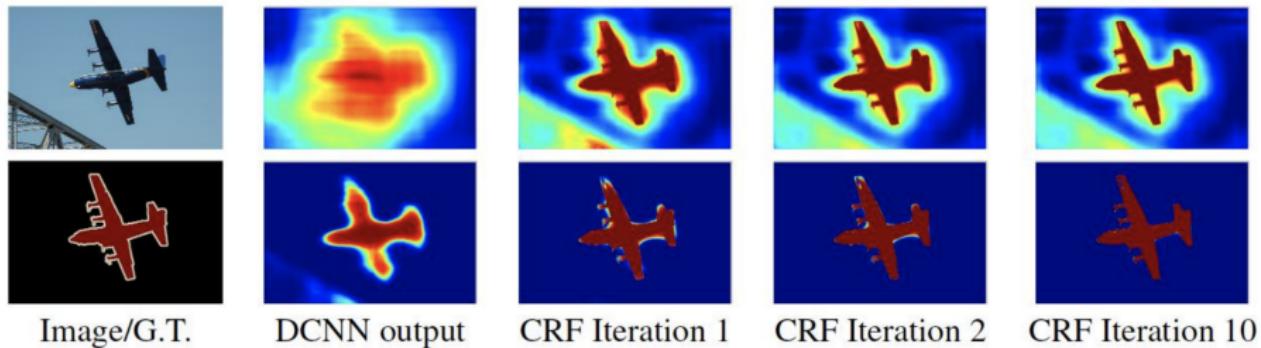
Long et al., “Fully Convolutional Networks for Semantic Segmentation”, CVPR 2015.

Fully convolutional network (FCN)



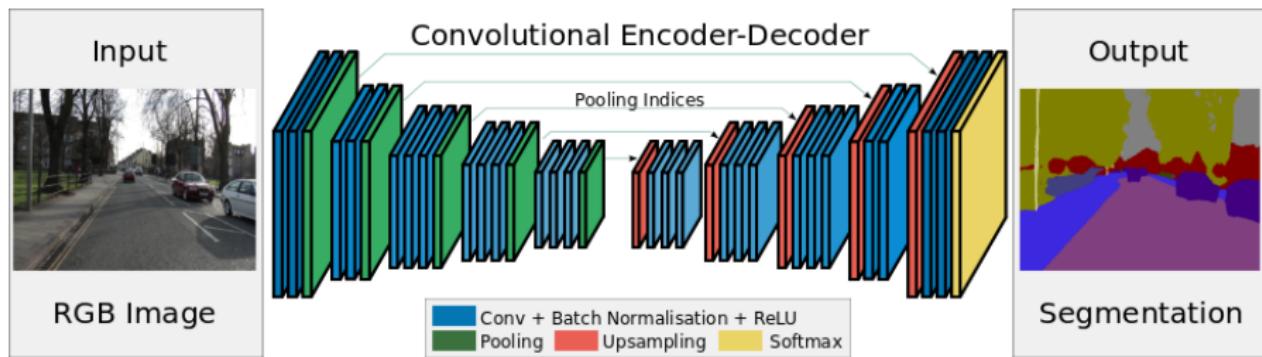
- For better segmentation, combine different layers of the network

Fully convolutional network (FCN)



- Use the deep neural network simply for feature learning
- But apply a Conditional Random Field (CRF) at the end

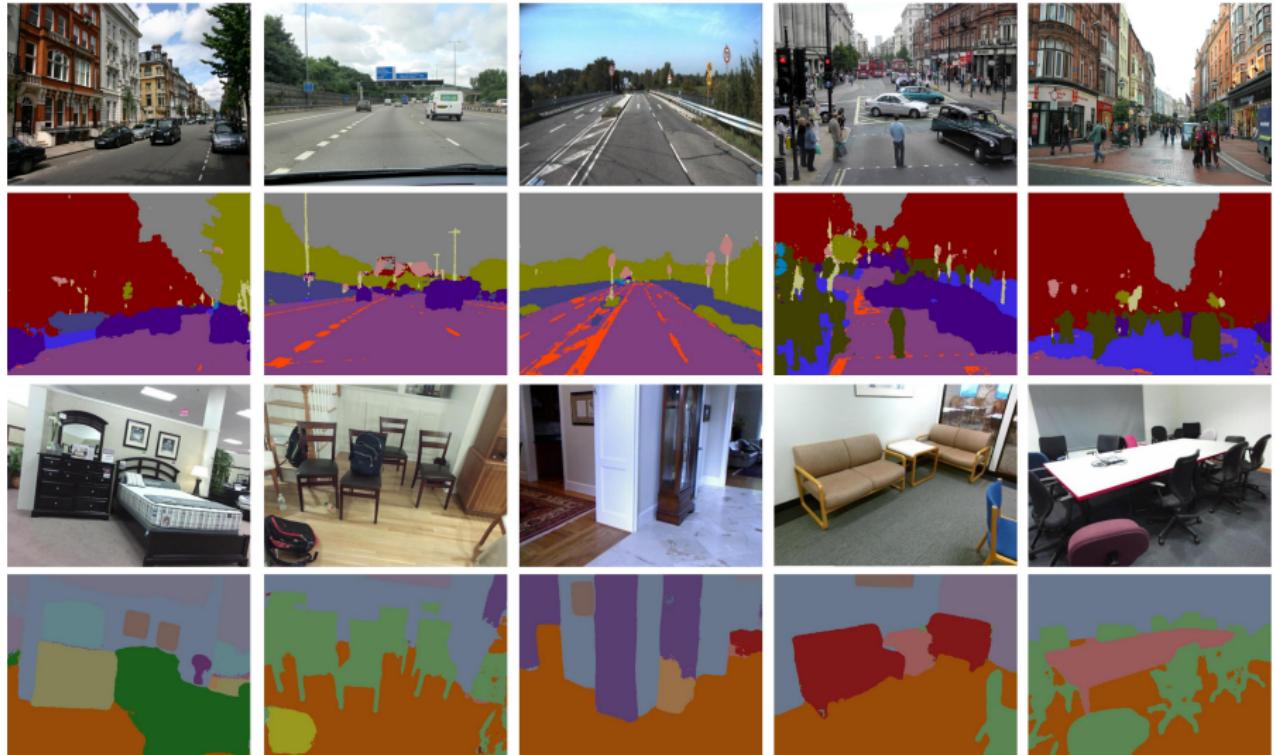
SegNet: Segmentation with encoder-decoder network



- FCN applied to multiple windows can be slow
- A faster alternative is to use an encoder-decoder network
- In SegNet, images upsampled by propagating pooling indices

Badrinarayanan et al., "SegNet: A Deep Convolutional Encoder-Decoder Architecture for Image Segmentation." arXiv preprint, 2015.

SegNet: Segmentation with encoder-decoder network

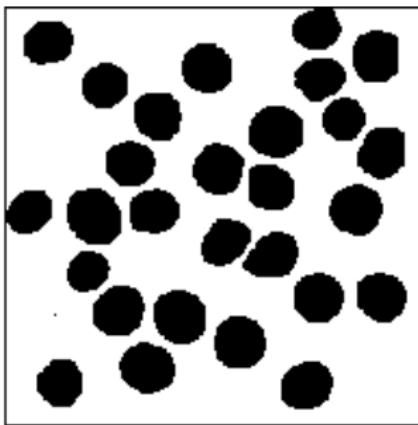


[Figure](#): SeqNet video

- Often there is a need to 'clean up' the segmentation.
 - Some segments are simply too small and only due to noise.
 - Others have holes, because colours look like background.
- Solution: Apply (non-linear) morphological operations.



A



B

- A morphological operator is defined by a structuring element (or kernel) of size $n \times n$ and a set operator.
- Kernel is shifted over the image and its elements are compared with the underlying pixels.
- If the two sets of elements match the condition defined by the set operator, the pixel underneath the center of the structuring element is set to a pre-defined value (0 or 1 for binary images).

A	the (usually binary) image
A^C	the complement of the image (inverse)
$A \cup B$	the union of images A and B
$A \cap B$	the intersection of images A and B
$A - B = A \cap B^C$	the difference between A and B (pixels in A not in B)
$\#A$	the cardinality of A (area of the object)

- The basic effect of the operator on a binary image is to gradually enlarge the boundaries of regions of foreground pixels.
- Areas of foreground pixels grow in size, while holes in regions become smaller.
- Let A and B denote sets in \mathbb{R}^2 with elements a and b .

Then

$$A \oplus B = \{c \in \mathbb{R}^2 : c = a + b\}$$

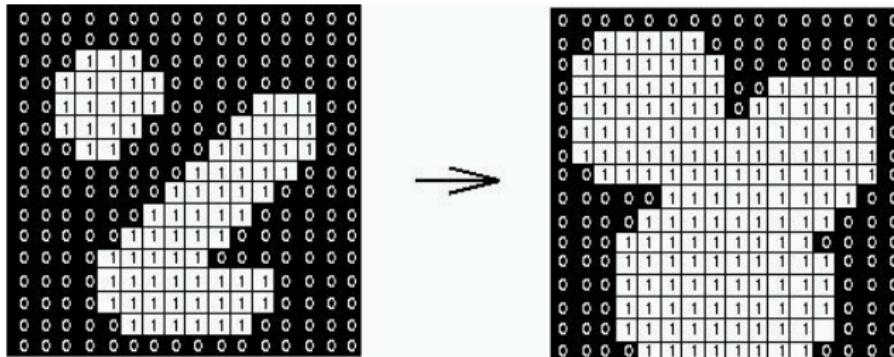
- Typically: A = binary image, B = mask (structuring element)
Example: 3×3 structuring element

1	1	1
1	1	1
1	1	1

$$B = \{(-1, -1), (0, -1), (1, -1), (-1, 0), (0, 0), (1, 0), (-1, 1), (0, 1), (1, 1)\}$$

Dilation

- If at least one pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value.
- If all the corresponding pixels in the image are background, however, the input pixel is left at the background value.

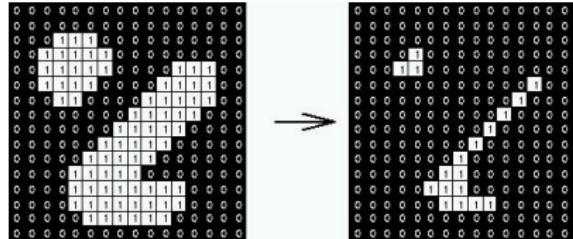


Erosion

- The basic effect of the operator on a binary image is to erode away the boundaries of regions of foreground pixels.
- Thus areas of foreground pixels shrink in size, and holes within those areas become larger.

$$A \ominus B = \{c \in \mathbb{R}^2 : c + b \in A, \forall b \in B\}$$

- If for every pixel in the structuring element, the corresponding pixel in the image underneath is a foreground pixel, then the input pixel is left as it is.
- If any of the corresponding pixels in the image are background, however, the input pixel is also set to background value.



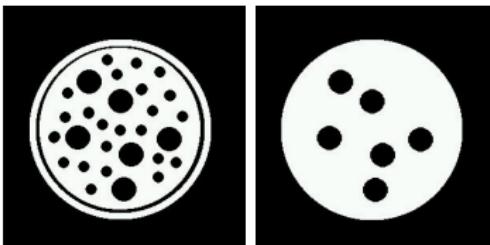
Opening and closing

- Opening: an opening is defined as an erosion followed by a dilation using the same structuring element for both operations

$$A \circ B = (A \underbrace{\ominus}_{\text{erosion}} B) \underbrace{\oplus}_{\text{dilation}} B$$

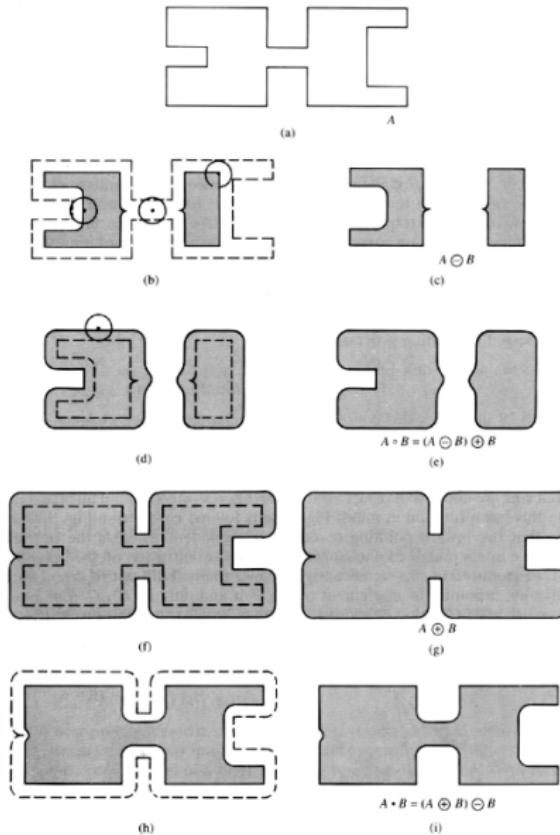
- Closing:

$$A \bullet B = (A \oplus B) \ominus B$$



- Opening is the dual of closing: opening the foreground pixels with a particular structuring element is equivalent to closing the background pixels with the same element.

Opening and closing



Summary of good questions

- What is an active contour?
- How do level set methods represent segmentations?
- What are the differences and similarities between active contours and level set methods?
- What kinds of cost functions does an energy formulation for segmentation often include?
- What is the purpose of graph cuts for segmentation?
- How does a graph cut work?
- What does a morphological opening and closing operation do?

- Gonzalez and Woods: Chapters 9.1-9.4
- Szeliski: Chapter 5