Assignment 6: Neural Networks using Keras and Tensorflow

DAT405 Introduction to Data Science and AI

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Hours spent on the assignment:

- Pauline Nässlander: 12 hours
- Albin Ekström: 12 hours

In [2]:
```python
# imports
from __future__ import print_function
import keras
from keras.datasets import mnist
from keras.models import Sequential
from keras.layers import Dense, Dropout, Flatten, Input
from keras.layers import Conv2D, MaxPooling2D
from keras import backend as K
import tensorflow as tf
from matplotlib import pyplot as plt
```

In [3]:
```python
# Hyper-parameters data-loading and formatting

batch_size = 128
num_classes = 10
epochs = 10

img_rows, img_cols = 28, 28

(x_train, lbl_train), (x_test, lbl_test) = mnist.load_data() # numbers dataset

if K.image_data_format() == 'channels_first':
    x_train = x_train.reshape(x_train.shape[0], 1, img_rows, img_cols)
    x_test = x_test.reshape(x_test.shape[0], 1, img_rows, img_cols)
    input_shape = (1, img_rows, img_cols)
else:
    x_train = x_train.reshape(x_train.shape[0], img_rows, img_cols, 1)
    x_test = x_test.reshape(x_test.shape[0], img_rows, img_cols, 1)
    input_shape = (img_rows, img_cols, 1)
```

```
Downloading data from https://storage.googleapis.com/tensorflow/tf-keras-datasets/mnist.npz
11493376/11490434 [==============================] - 0s 0us/step
11501568/11490434 [==============================] - 0s 0us/step
```

Preprocessing

In [4]:
```python
x_train = x_train.astype('float32')
x_test = x_test.astype('float32')

x_train /= 255
x_test /= 255

y_train = keras.utils.np_utils.to_categorical(lbl_train, num_classes)
y_test = keras.utils.np_utils.to_categorical(lbl_test, num_classes)
```

Question 1

First in the Preprocessing highlighted section the train and test data is converted to the data type float32. This is because the data will at some point be converted to a 32 bit float, since this is the most common training precision in a neural network, hence we do it immediately instead. Secondly, this number is divided with 255. Since the gray scale pixel values are numbers between 0 and 255 (1 byte) this will result in a float between 0.0 and 1.0. This is not a required step but it helps with the learning rate and other hyper-parameters. Lastly, the target data is one-hot encoded and will be represented in vector form. E.g. representing number 3 with one-hot encoding will get [0 0 0 1 0 0 0 0 0 0].

In [ ]:
```python
## Define model ##
model = Sequential()

model.add(Flatten())
model.add(Dense(64, activation = 'relu'))
model.add(Dense(64, activation = 'relu'))
model.add(Dense(num_classes, activation='softmax'))

model.compile(loss=keras.losses.categorical_crossentropy,
               optimizer = tf.keras.optimizers.SGD(lr = 0.1),
        metrics=['accuracy'],)
```

```
/usr/local/lib/python3.7/dist-packages/keras/optimizer_v2/gradient_descent.py:102: UserWarning: The `lr` argume
nt is deprecated, use `learning_rate` instead.
  super(SGD, self).__init__(name, **kwargs)
```

Question 2

Question 2 a)

There are 4 layers in the NN model. The layers are: a flatten layer (784 neurons) followed by two dense layers (64 neurons each) and one last dense layer (10 neurons). The flatten layer does not have an activation function but the first two dense layers have the activation function *relu* and the last dense layer has the activation function *softmax*. The relu function allows for very efficient computations since it simply outputs the input if it is larger than 0.0 and 0 if it is less than 0. Since the gradient computation is so simple it allows for speeded training. The softmax function is good when predicting a multinomial probability distribution, which is used in this network. This model requires class membership on more than two classes.

The total number of parameters in the model is given by the sum of the parameters on each layer. Total is 55 050 parameters. The input layer has the same amount of neurons as there are pixels in the image, each neuron will be given a value between 0 and 1 from the preprocessing. Each image is of size 28x28 = 784 pixel, hence 784 inputs/neurons. The output layer has the same amount of neurons as there are possible classes. There are 10 numbers between 0 and 9 that can be classified. Each class will get a probability of how likely the image contains that specific number, hence there are 10 outputs /neurons.

Question 2 b)

The loss-function used is *cross-entropy* and it's functional form is

$$H(P,Q) = -\sum P(x) * log(Q(x))$$

where P is the observed probabilities (is either 1 or 0) and Q is the approximation of the target distribution aka the output of the softmax function. Since the cross-entropy is logarithmic the derivative for bad predictions will be quite large which will allow for larger step size for backpropagation. This is better than the square residuals method which will result in smaller steps and longer training time.

Question 2 c)

In [ ]:
```python
fit_info = model.fit(x_train, y_train,
           batch_size=batch_size,
           epochs=epochs,
           verbose=1,
           validation_data=(x_test, y_test))
score = model.evaluate(x_test, y_test, verbose=0)
print('Test loss: {}, Test accuracy {}'.format(score[0], score[1]))
```

```
Epoch 1/10
469/469 [==============================] - 2s 4ms/step - loss: 2.3006 - accuracy: 0.1133 - val_loss: 2.3002 - v
al_accuracy: 0.1135
Epoch 2/10
469/469 [==============================] - 2s 4ms/step - loss: 2.3000 - accuracy: 0.1144 - val_loss: 2.2991 - v
al_accuracy: 0.1135
Epoch 3/10
469/469 [==============================] - 2s 4ms/step - loss: 2.2994 - accuracy: 0.1145 - val_loss: 2.2989 - v
al_accuracy: 0.1135
Epoch 4/10
469/469 [==============================] - 2s 4ms/step - loss: 2.2986 - accuracy: 0.1154 - val_loss: 2.2982 - v
al_accuracy: 0.1512
Epoch 5/10
469/469 [==============================] - 2s 4ms/step - loss: 2.2977 - accuracy: 0.1188 - val_loss: 2.2972 - v
al_accuracy: 0.1135
Epoch 6/10
469/469 [==============================] - 2s 4ms/step - loss: 2.2962 - accuracy: 0.1190 - val_loss: 2.2952 - v
al_accuracy: 0.1135
Epoch 7/10
469/469 [==============================] - 2s 4ms/step - loss: 2.2945 - accuracy: 0.1177 - val_loss: 2.2947 - v
al_accuracy: 0.1028
Epoch 8/10
469/469 [==============================] - 2s 4ms/step - loss: 2.2918 - accuracy: 0.1309 - val_loss: 2.2893 - v
al_accuracy: 0.2052
Epoch 9/10
469/469 [==============================] - 2s 4ms/step - loss: 2.2876 - accuracy: 0.1382 - val_loss: 2.2836 - v
al_accuracy: 0.2027
Epoch 10/10
469/469 [==============================] - 2s 5ms/step - loss: 2.2808 - accuracy: 0.1609 - val_loss: 2.2744 - v
al_accuracy: 0.1135
Test loss: 2.2744319438934326, Test accuracy 0.11349999904632568
```

In [ ]:
```python
model.summary()
```

```
Model: "sequential_5"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 flatten_5 (Flatten)         (None, 784)               0

 dense_15 (Dense)            (None, 64)                50240

 dense_16 (Dense)            (None, 64)                4160

 dense_17 (Dense)            (None, 10)                650

=================================================================
Total params: 55,050
Trainable params: 55,050
Non-trainable params: 0
_________________________________________________________________
```

Question 2 d)

In [ ]:
```python
## Define model ##
model = Sequential()

model.add(Flatten())
model.add(Dense(500, activation = 'relu'))
model.add(Dense(300, activation = 'relu'))
model.add(Dense(num_classes, activation='softmax'))

model.compile(loss=keras.losses.categorical_crossentropy,
               optimizer = tf.keras.optimizers.SGD(lr = 0.1),
        metrics=['accuracy'],)
```

```
/usr/local/lib/python3.7/dist-packages/keras/optimizer_v2/gradient_descent.py:102: UserWarning: The `lr` argume
nt is deprecated, use `learning_rate` instead.
  super(SGD, self).__init__(name, **kwargs)
```

In [ ]:
```python
epochs = 40
fit_info = model.fit(x_train, y_train,
           batch_size=batch_size,
           epochs=epochs,
           verbose=1,
           validation_data=(x_test, y_test))
score = model.evaluate(x_test, y_test, verbose=0)
print('Test loss: {}, Test accuracy {}'.format(score[0], score[1]))
```

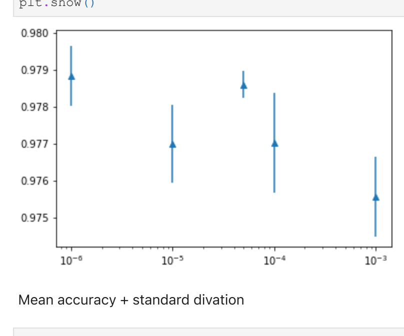
```
Epoch 1/40
469/469 [==============================] - 7s 13ms/step - loss: 0.4028 - accuracy: 0.8874 - val_loss: 0.2128 -
val_accuracy: 0.9400
Epoch 2/40
469/469 [==============================] - 6s 12ms/step - loss: 0.1892 - accuracy: 0.9451 - val_loss: 0.1638 -
val_accuracy: 0.9528
Epoch 3/40
469/469 [==============================] - 6s 13ms/step - loss: 0.1378 - accuracy: 0.9599 - val_loss: 0.1285 -
val_accuracy: 0.9615
Epoch 4/40
469/469 [==============================] - 6s 13ms/step - loss: 0.1090 - accuracy: 0.9687 - val_loss: 0.1084 -
val_accuracy: 0.9684
Epoch 5/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0880 - accuracy: 0.9743 - val_loss: 0.0968 -
val_accuracy: 0.9697
Epoch 6/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0746 - accuracy: 0.9784 - val_loss: 0.0852 -
val_accuracy: 0.9737
Epoch 7/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0626 - accuracy: 0.9821 - val_loss: 0.0850 -
val_accuracy: 0.9720
Epoch 8/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0536 - accuracy: 0.9846 - val_loss: 0.0771 -
val_accuracy: 0.9747
Epoch 9/40
469/469 [==============================] - 6s 13ms/step - loss: 0.0464 - accuracy: 0.9871 - val_loss: 0.0707 -
val_accuracy: 0.9783
Epoch 10/40
469/469 [==============================] - 6s 13ms/step - loss: 0.0402 - accuracy: 0.9890 - val_loss: 0.0689 -
val_accuracy: 0.9786
Epoch 11/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0346 - accuracy: 0.9909 - val_loss: 0.0669 -
val_accuracy: 0.9787
Epoch 12/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0303 - accuracy: 0.9923 - val_loss: 0.0685 -
val_accuracy: 0.9784
Epoch 13/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0263 - accuracy: 0.9936 - val_loss: 0.0644 -
val_accuracy: 0.9794
Epoch 14/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0229 - accuracy: 0.9944 - val_loss: 0.0622 -
val_accuracy: 0.9808
Epoch 15/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0200 - accuracy: 0.9958 - val_loss: 0.0625 -
val_accuracy: 0.9806
Epoch 16/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0174 - accuracy: 0.9965 - val_loss: 0.0683 -
val_accuracy: 0.9798
Epoch 17/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0150 - accuracy: 0.9973 - val_loss: 0.0620 -
val_accuracy: 0.9811
Epoch 18/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0129 - accuracy: 0.9979 - val_loss: 0.0630 -
val_accuracy: 0.9805
Epoch 19/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0116 - accuracy: 0.9984 - val_loss: 0.0616 -
val_accuracy: 0.9810
Epoch 20/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0101 - accuracy: 0.9988 - val_loss: 0.0705 -
val_accuracy: 0.9783
Epoch 21/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0088 - accuracy: 0.9991 - val_loss: 0.0605 -
val_accuracy: 0.9813
Epoch 22/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0079 - accuracy: 0.9992 - val_loss: 0.0609 -
val_accuracy: 0.9814
Epoch 23/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0069 - accuracy: 0.9995 - val_loss: 0.0630 -
val_accuracy: 0.9810
Epoch 24/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0063 - accuracy: 0.9995 - val_loss: 0.0770 -
val_accuracy: 0.9786
Epoch 25/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0058 - accuracy: 0.9996 - val_loss: 0.0635 -
val_accuracy: 0.9809
Epoch 26/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0050 - accuracy: 0.9998 - val_loss: 0.0629 -
val_accuracy: 0.9814
Epoch 27/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0045 - accuracy: 0.9998 - val_loss: 0.0645 -
val_accuracy: 0.9807
Epoch 28/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0040 - accuracy: 0.9999 - val_loss: 0.0644 -
val_accuracy: 0.9816
Epoch 29/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0037 - accuracy: 0.9999 - val_loss: 0.0642 -
val_accuracy: 0.9811
Epoch 30/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0034 - accuracy: 0.9999 - val_loss: 0.0629 -
val_accuracy: 0.9817
Epoch 31/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0032 - accuracy: 0.9999 - val_loss: 0.0635 -
val_accuracy: 0.9816
Epoch 32/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0029 - accuracy: 0.9999 - val_loss: 0.0655 -
val_accuracy: 0.9810
Epoch 33/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0028 - accuracy: 0.9999 - val_loss: 0.0644 -
val_accuracy: 0.9820
Epoch 34/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0025 - accuracy: 1.0000 - val_loss: 0.0653 -
val_accuracy: 0.9816
Epoch 35/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0024 - accuracy: 1.0000 - val_loss: 0.0650 -
val_accuracy: 0.9816
Epoch 36/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0022 - accuracy: 1.0000 - val_loss: 0.0661 -
val_accuracy: 0.9816
Epoch 37/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0021 - accuracy: 1.0000 - val_loss: 0.0658 -
val_accuracy: 0.9812
Epoch 38/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0020 - accuracy: 1.0000 - val_loss: 0.0657 -
val_accuracy: 0.9817
Epoch 39/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0019 - accuracy: 1.0000 - val_loss: 0.0664 -
val_accuracy: 0.9814
Epoch 40/40
469/469 [==============================] - 6s 12ms/step - loss: 0.0018 - accuracy: 1.0000 - val_loss: 0.0675 -
val_accuracy: 0.9814
Test loss: 0.06748238205909729, Test accuracy 0.9814000129699707
```

In [ ]:
```python
import statistics

r_factors = [1e-6, 1e-5, 0,5e-5, 1e-4, 1e-3] # 5 different regularization factors
```

In [ ]:
```python
std_accuracy = []
mean_accuracy = []

for r_factor in r_factors:
  std_list = []

  for i in range(0,3):
    model = Sequential()

    model.add(Flatten())
    model.add(Dense(500, activation='relu', kernel_regularizer=tf.keras.regularizers.l2(r_factor)))
    model.add(Dense(300, activation='relu', kernel_regularizer=tf.keras.regularizers.l2(r_factor)))
    model.add(Dense(num_classes, activation='softmax'))

    model.compile(loss=keras.losses.categorical_crossentropy,
                 optimizer = tf.keras.optimizers.SGD(lr = 0.1),
          metrics=['accuracy'],)

    fit_info = model.fit(x_train, y_train,
           batch_size=batch_size,
           epochs=epochs,
           verbose=0,
           validation_data=(x_test, y_test))
    score = model.evaluate(x_test, y_test, verbose=0)
    std_list.append(score[1]) # append accuracy

  std_accuracy.append(statistics.stdev(std_list))
  mean_accuracy.append(statistics.mean(std_list))
```

```
/usr/local/lib/python3.7/dist-packages/keras/optimizer_v2/gradient_descent.py:102: UserWarning: The `lr` argume
nt is deprecated, use `learning_rate` instead.
  super(SGD, self).__init__(name, **kwargs)
```

In [ ]:
```python
plt.errorbar(r_factors, mean_accuracy, std_accuracy, linestyle='None', marker='^')
ax = plt.gca()
ax.set_xscale('log')
plt.show()
```



Mean accuracy + standard divation

In [ ]:
```python
print(mean_accuracy[0] + std_accuracy[0])
```

```
0.9796354324492814
```

The closest result to Hintons result (0.9814 accuracy) is achieved when the regularization factor is 1e-6 which has mean accuracy plus standard deviation = 0.9796. This is a worse result than the model without regularization but it is important to keep in mind that these models are only trained for 10 epochs while the previous one was trained for 40 epochs.

Question 3

In [95]:
```python
model = Sequential()

model.add(Conv2D(32, kernel_size= (3,3), activation='relu', input_shape = (28,28,1)))
model.add(Conv2D(64, kernel_size= (3,3), activation='relu'))
model.add(MaxPooling2D(pool_size= (3,3)))
model.add(Dropout(0.5))
model.add(Flatten())
model.add(Dense(300, activation='relu'))
model.add(Dense(num_classes, activation='softmax'))

model.compile(loss=keras.losses.categorical_crossentropy,
              optimizer = tf.keras.optimizers.SGD(lr = 0.1),
        metrics=['accuracy'],)
```

```
/usr/local/lib/python3.7/dist-packages/keras/optimizer_v2/gradient_descent.py:102: UserWarning: The `lr` argume
nt is deprecated, use `learning_rate` instead.
  super(SGD, self).__init__(name, **kwargs)
```

In [96]:
```python
fit_info = model.fit(x_train, y_train,
           batch_size=batch_size,
           epochs=40,
           verbose=1,
           validation_data=(x_test, y_test))
score = model.evaluate(x_test, y_test, verbose=0)
print('Test loss: {}, Test accuracy {}'.format(score[0], score[1]))
```

```
Epoch 1/40
469/469 [==============================] - 4s 9ms/step - loss: 0.3951 - accuracy: 0.8767 - val_loss: 0.0949 - v
al_accuracy: 0.9703
Epoch 2/40
469/469 [==============================] - 4s 9ms/step - loss: 0.1089 - accuracy: 0.9666 - val_loss: 0.0568 - v
al_accuracy: 0.9821
Epoch 3/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0777 - accuracy: 0.9759 - val_loss: 0.0393 - v
al_accuracy: 0.9869
Epoch 4/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0619 - accuracy: 0.9807 - val_loss: 0.0358 - v
al_accuracy: 0.9883
Epoch 5/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0530 - accuracy: 0.9835 - val_loss: 0.0314 - v
al_accuracy: 0.9896
Epoch 6/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0454 - accuracy: 0.9857 - val_loss: 0.0319 - v
al_accuracy: 0.9901
Epoch 7/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0411 - accuracy: 0.9870 - val_loss: 0.0273 - v
al_accuracy: 0.9898
Epoch 8/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0377 - accuracy: 0.9880 - val_loss: 0.0271 - v
al_accuracy: 0.9898
Epoch 9/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0335 - accuracy: 0.9889 - val_loss: 0.0261 - v
al_accuracy: 0.9908
Epoch 10/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0300 - accuracy: 0.9903 - val_loss: 0.0259 - v
al_accuracy: 0.9913
Epoch 11/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0283 - accuracy: 0.9908 - val_loss: 0.0254 - v
al_accuracy: 0.9915
Epoch 12/40
469/469 [==============================] - 4s 9ms/step - loss: 0.0249 - accuracy: 0.9922 - val_loss: 0.0258 - v
al_accuracy: 0.9914
Epoch 13/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0234 - accuracy: 0.9922 - val_loss: 0.0229 - v
al_accuracy: 0.9917
Epoch 14/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0239 - accuracy: 0.9927 - val_loss: 0.0230 - v
al_accuracy: 0.9920
Epoch 15/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0208 - accuracy: 0.9931 - val_loss: 0.0222 - v
al_accuracy: 0.9918
Epoch 16/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0192 - accuracy: 0.9937 - val_loss: 0.0232 - v
al_accuracy: 0.9919
Epoch 17/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0193 - accuracy: 0.9937 - val_loss: 0.0233 - v
al_accuracy: 0.9920
Epoch 18/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0175 - accuracy: 0.9940 - val_loss: 0.0234 - v
al_accuracy: 0.9924
Epoch 19/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0159 - accuracy: 0.9945 - val_loss: 0.0212 - v
al_accuracy: 0.9932
Epoch 20/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0148 - accuracy: 0.9952 - val_loss: 0.0221 - v
al_accuracy: 0.9930
Epoch 21/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0153 - accuracy: 0.9950 - val_loss: 0.0224 - v
al_accuracy: 0.9924
Epoch 22/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0137 - accuracy: 0.9955 - val_loss: 0.0260 - v
al_accuracy: 0.9916
Epoch 23/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0136 - accuracy: 0.9955 - val_loss: 0.0219 - v
al_accuracy: 0.9924
Epoch 24/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0124 - accuracy: 0.9959 - val_loss: 0.0207 - v
al_accuracy: 0.9926
Epoch 25/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0126 - accuracy: 0.9959 - val_loss: 0.0219 - v
al_accuracy: 0.9932
Epoch 26/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0121 - accuracy: 0.9961 - val_loss: 0.0211 - v
al_accuracy: 0.9936
Epoch 27/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0113 - accuracy: 0.9962 - val_loss: 0.0227 - v
al_accuracy: 0.9930
Epoch 28/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0112 - accuracy: 0.9964 - val_loss: 0.0214 - v
al_accuracy: 0.9926
Epoch 29/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0097 - accuracy: 0.9970 - val_loss: 0.0218 - v
al_accuracy: 0.9928
Epoch 30/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0094 - accuracy: 0.9968 - val_loss: 0.0253 - v
al_accuracy: 0.9921
Epoch 31/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0094 - accuracy: 0.9970 - val_loss: 0.0242 - v
al_accuracy: 0.9928
Epoch 32/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0087 - accuracy: 0.9969 - val_loss: 0.0236 - v
al_accuracy: 0.9929
Epoch 33/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0084 - accuracy: 0.9973 - val_loss: 0.0236 - v
al_accuracy: 0.9930
Epoch 34/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0083 - accuracy: 0.9972 - val_loss: 0.0232 - v
al_accuracy: 0.9927
Epoch 35/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0078 - accuracy: 0.9974 - val_loss: 0.0215 - v
al_accuracy: 0.9935
Epoch 36/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0079 - accuracy: 0.9974 - val_loss: 0.0217 - v
al_accuracy: 0.9934
Epoch 37/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0073 - accuracy: 0.9973 - val_loss: 0.0235 - v
al_accuracy: 0.9931
Epoch 38/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0064 - accuracy: 0.9979 - val_loss: 0.0242 - v
al_accuracy: 0.9930
Epoch 39/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0071 - accuracy: 0.9975 - val_loss: 0.0249 - v
al_accuracy: 0.9930
Epoch 40/40
469/469 [==============================] - 4s 8ms/step - loss: 0.0065 - accuracy: 0.9979 - val_loss: 0.0212 - v
al_accuracy: 0.9939
Test loss: 0.021202296018600464, Test accuracy 0.9939000010490417
```

Question 3 a)

Using two convolutional layers, one max pooling layer and onelayer of dropout will produce a model with accuracy 0.991. This solution was inspired by https://www.geeksforgeeks.org/applying-convolutional-neural-network-on-mnist-dataset/ . The max pooling layer takes the maximum value from a kernel in this case 3 by 3 pixels large and outputs this into a smaller image with the maximum values. This is good since it reduces the dimensionality. The dropout layer reduces the risk of overfitting the model by stochastically removing units with a probability of 0.5. The hidden neurons can not co-adapt to others which makes the model more general.

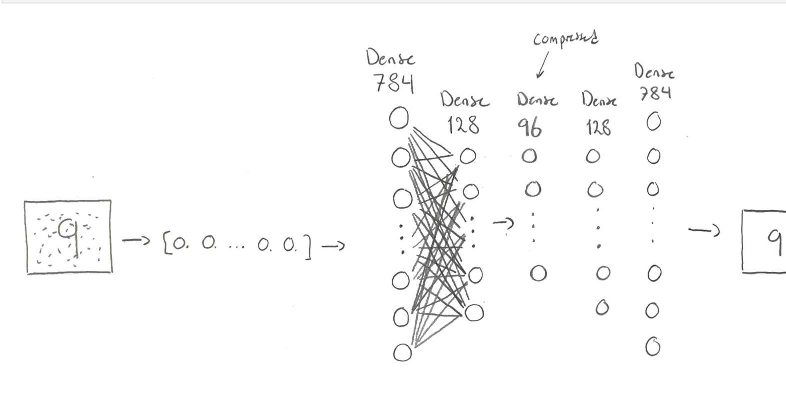
Question 3 b)

In this application it is suitable to use convolutional layers because of their ability to represent an image internally (in the model). Since the MNIST dataset contains two dimensional images this becomes an useful ability.

Question 4

In [5]:
```python
import numpy as np
def salt_and_pepper(input, noise_level=0.5):
    """
    This applies salt and pepper noise to the input tensor - randomly setting bits to 1 or 0.
    Parameters
    ----------
    input : tensor
        The tensor to apply salt and pepper noise to.
    noise_level : float
        The amount of salt and pepper noise to add.
    Returns
    -------
    tensor
        Tensor with salt and pepper noise applied.
    """
    # salt and pepper noise
    a = np.random.binomial(size=input.shape, n=1, p=(1 - noise_level))
    b = np.random.binomial(size=input.shape, n=1, p=0.5)
    c = (a==0) * b
    return input * a + c

#data preparation
flattened_x_train = x_train.reshape(-1,784)
flattened_x_train_seasoned = salt_and_pepper(flattened_x_train, noise_level=0.4)

flattened_x_test = x_test.reshape(-1,784)
flattened_x_test_seasoneed = salt_and_pepper(flattened_x_test, noise_level=0.4)
```

In [6]:
```python
latent_dim = 96  # Dimensions of compressed data

input_image = keras.Input(shape=(784,)) # Shape: [0. 0. ... 0. 1.], size 784
encoded = Dense(128, activation='relu')(input_image)
encoded = Dense(latent_dim, activation='relu')(encoded)
decoded = Dense(128, activation='relu')(encoded)
decoded = Dense(784, activation='sigmoid')(decoded)

autoencoder = keras.Model(input_image, decoded) # Autoencodern, encoder -> decoder
encoder_only = keras.Model(input_image, encoded) # Encodern

encoded_input = keras.Input(shape=(latent_dim,))
decoder_layer = Sequential(autoencoder.layers[-2:])
decoder = keras.Model(encoded_input, decoder_layer(encoded_input))

autoencoder.compile(optimizer='adam', loss='binary_crossentropy')
```



In [42]:
```python
autoencoder.summary()
```

```
Model: "model_30"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 input_21 (InputLayer)       [(None, 784)]             0

 dense_40 (Dense)            (None, 128)               100480

 dense_41 (Dense)            (None, 96)                12384

 dense_42 (Dense)            (None, 128)               12416

 dense_43 (Dense)            (None, 784)               101136

=================================================================
Total params: 226,416
Trainable params: 226,416
Non-trainable params: 0
_________________________________________________________________
```

In [7]:
```python
fit_info_AE = autoencoder.fit(flattened_x_train_seasoned, flattened_x_train,
                epochs=32,
                batch_size=64,
                shuffle=True,
                validation_data=(flattened_x_test_seasoneed, flattened_x_test))
```

```
Epoch 1/32
938/938 [==============================] - 9s 6ms/step - loss: 0.1925 - val_loss: 0.1538
Epoch 2/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1476 - val_loss: 0.1410
Epoch 3/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1376 - val_loss: 0.1336
Epoch 4/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1324 - val_loss: 0.1310
Epoch 5/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1290 - val_loss: 0.1281
Epoch 6/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1267 - val_loss: 0.1272
Epoch 7/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1249 - val_loss: 0.1253
Epoch 8/32
938/938 [==============================] - 5s 5ms/step - loss: 0.1237 - val_loss: 0.1245
Epoch 9/32
938/938 [==============================] - 7s 7ms/step - loss: 0.1227 - val_loss: 0.1234
Epoch 10/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1219 - val_loss: 0.1232
Epoch 11/32
938/938 [==============================] - 6s 6ms/step - loss: 0.1211 - val_loss: 0.1224
Epoch 12/32
938/938 [==============================] - 8s 8ms/step - loss: 0.1206 - val_loss: 0.1223
Epoch 13/32
938/938 [==============================] - 7s 7ms/step - loss: 0.1200 - val_loss: 0.1218
Epoch 14/32
938/938 [==============================] - 7s 8ms/step - loss: 0.1196 - val_loss: 0.1218
Epoch 15/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1191 - val_loss: 0.1213
Epoch 16/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1187 - val_loss: 0.1212
Epoch 17/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1184 - val_loss: 0.1210
Epoch 18/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1181 - val_loss: 0.1214
Epoch 19/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1178 - val_loss: 0.1208
Epoch 20/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1175 - val_loss: 0.1204
Epoch 21/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1173 - val_loss: 0.1204
Epoch 22/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1171 - val_loss: 0.1204
Epoch 23/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1169 - val_loss: 0.1202
Epoch 24/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1167 - val_loss: 0.1203
Epoch 25/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1165 - val_loss: 0.1200
Epoch 26/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1163 - val_loss: 0.1198
Epoch 27/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1161 - val_loss: 0.1197
Epoch 28/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1160 - val_loss: 0.1205
Epoch 29/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1158 - val_loss: 0.1206
Epoch 30/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1157 - val_loss: 0.1196
Epoch 31/32
938/938 [==============================] - 4s 4ms/step - loss: 0.1156 - val_loss: 0.1198
Epoch 32/32
938/938 [==============================] - 4s 5ms/step - loss: 0.1155 - val_loss: 0.1196
```

Question 4 a)

The denoising deep autoencoder model purposely adds noise to each image. Then it compresses this image and outputs a clean image without noise. Why would we do this? This is because it helps with overfitting, the hidden layers become more robust and it prevents the encoder from learning a simple identity function.

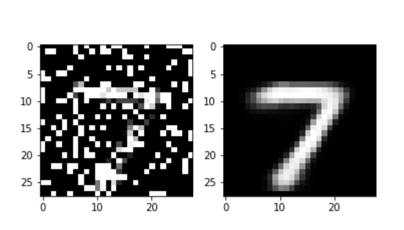
The data preparation code transforms the representation of the train and test images from four dimensions to two dimensions by reshaping it so that each image is represented as one row in the new 2 dimensional matrix. The difference to the previous representation where each image was represented as a 2 dimensional array with one datapoint in each row.

In the model definition the goal is reached from taking in the original input size of the reshaped training data 784 data points. Then adding a hidden Dense layer and outputting the dimensions of the compressed image (96 data points). Then it adds a hidden Dense layer after the compression (which takes in 96 data points) and outputs this to a last Dense layer. This layer will output 784 data points with values between 0 and 1 (as the activation function of the last layer is sigmoid) to be classified as the denoised image.

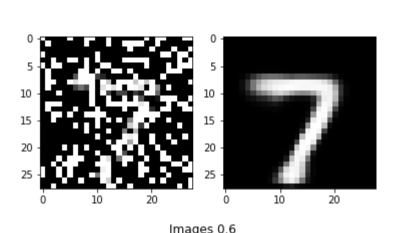
The loss function used is called binary crossentropy which calculates the loss somewhat similar to the crossentropy loss function but is more suitable for binary decisions. It is very suitable for solving many classification problems at the same time as long as each classification can be described as a binary decision (in this case 0 or 1). In short the binary crossentropy loss function calculates how far away from the target value the prediction is for each class. As in the case of the autoencoder the target value is the unseasoned image and each class is one pixel, this loss function will describe how well the image was reconstructed.

In [59]:
```python
levels = [0.4, 0.5, 0.6, 0.7, 0.8, 0.9]

for noise_level in levels:
  flattened_x_train_seasoned = salt_and_pepper(flattened_x_train, noise_level)
  flattened_x_test_seasoned = salt_and_pepper(flattened_x_test, noise_level)

  fig, (ax1, ax2) = plt.subplots(1, 2)
  fig.suptitle('Images {0}'.format(noise_level))

  predict = autoencoder.predict(flattened_x_test_seasoned, verbose=0)
  predict = predict[0].reshape((28, 28))
  original = flattened_x_test_seasoned[0].reshape((28, 28))

  ax1.imshow(original, cmap= 'gray')
  ax2.imshow(predict, cmap= 'gray')
```

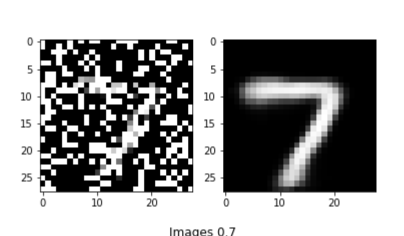
Images 0.4



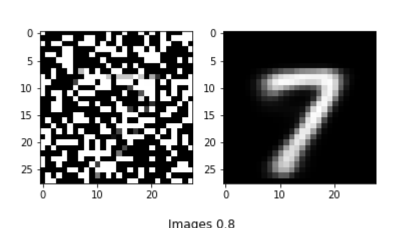
Images 0.5



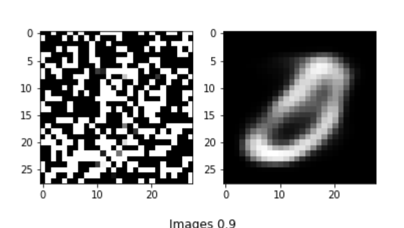
Images 0.6



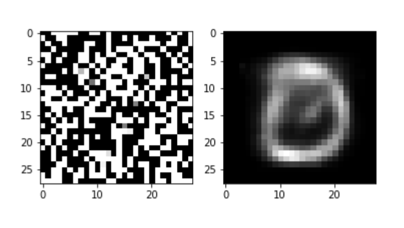
Images 0.7



Images 0.8



Images 0.9



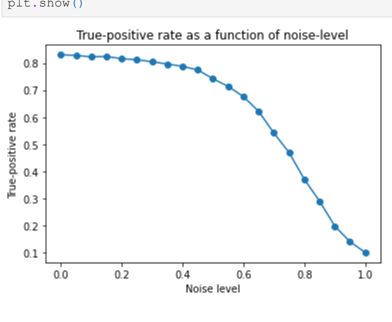
Question 4 b)

At a noise level of 0.5 it becomes difficult to identify the digits by using the human eye and for noise level 0.8 the autoencoder stops working.

In [84]:
```python
from sklearn.metrics import confusion_matrix
import numpy as np
```

In [97]:
```python
noise_levels = np.linspace(0,1,21)
all_TPR = []

for noise_level in noise_levels:
  flattened_x_train_seasoned = salt_and_pepper(flattened_x_train, noise_level)
  flattened_x_test_seasoned = salt_and_pepper(flattened_x_test, noise_level)

  predict = autoencoder.predict(flattened_x_test_seasoned, verbose=0).reshape((-1, 28, 28))
  y_predict = model.predict(predict)

  matrix = confusion_matrix(y_test.argmax(axis=1), y_predict.argmax(axis=1))

  FN = matrix.sum(axis=1) - np.diag(matrix)
  TP = np.diag(matrix)

  # Sensitivity, hit rate, recall, or true positive rate
  TPR = TP/(TP+FN)
  all_TPR.append(np.mean(TPR))
```

In [98]:
```python
plt.plot(noise_levels, all_TPR, '-o')
plt.title("True-positive rate as a function of noise-level")
plt.xlabel("Noise level")
plt.ylabel("True-positive rate")
plt.show()
```



Question 4 c)

As shown in the graph above the true positive rate decreases as the noise level increases for this combination of autoencoder and model. Note that for low values of noise (under 0.4) the TPR (True positive rate) does not decrease noticeably. Before compiling, some increment in performance was expected for noise levels larger than 0. However, this was not the case for this particular model as can be confirmed by looking at the graph above . This might be due to the fact that the best model created in assignment 3 already was robust and since it used a max pooling and dropout layer it did not have a problem with overfitting resulting in the autoencoder being redundant.

Question 4 d)

Trying to first add random noice to image, but it seems to only output garbage. We didn't find any other solution to this problem.

Maybe you could input an image of a number into the encoder and then stop it at the compression. Add random noise to that image and it will generate synthetic "hand-written" numbers.

In [29]:
```python
matrix = np.zeros((3,96))
test = salt_and_pepper(matrix, noise_level=1)

for i in range(0,3):
  predict = decoder.predict(test)
  predict = predict[i].reshape((28, 28))
  fig, ax = plt.subplots()
  ax.imshow(predict, cmap= 'gray')
```