EL2450: Hybrid and Embedded Control Systems: Homework 1

[To be handed in **January 29**]

Introduction

The objective of this homework is to understand the basics of digital control including modeling, controller design, and implementation. The process to be controlled is a double tank system, shown in Figure 1. The water is pumped into the upper tank by an electric pump. The water flows through a hole in the bottom of the upper tank to the lower tank. In the lower tank a similar hole is located letting the water flow back to the main reservoir. Let A be the cross-sectional areas of the tanks and a_1 and a_2 the effective outlet areas. Assume that the outgoing flow q from the pump is directly proportional to the voltage u applied to the motor. Let β be the proportionally coefficient. The water level of the upper and lower tanks can be described by the following differential equations:

$$
\begin{aligned}
\frac{dh_1(t)}{dt} &= -\alpha_1\sqrt{2gh_1(t)} + \beta u(t), \\
\frac{dh_2(t)}{dt} &= \quad \alpha_1\sqrt{2gh_1(t)} - \alpha_2\sqrt{2gh_2(t)}, \\
y(t) &= h_2(t),
\end{aligned}
$$

where h_1 is the level of the upper tank, h_2 the level of the lower tank, $\alpha_1 = a_1/A$, and $\alpha_2 = a_2/A$. In this homework the water level of the lower tank is to be controlled.

Exercises

Download the package `H1_src.zip` from the Canvas course page. You will edit the files during this homework. After you change a file you have to execute it for the changes to take effect.

Coupled tank model

1. Verify the model by opening the Simulink model `tanks.mdl`. Open the Two tanks block by double-clicking on it. Open the Tank 1 block. What is modeled by the gain Tap that has the value zero? **[1p]**

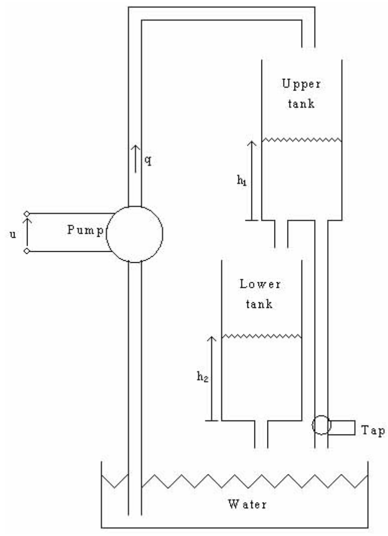

Figure 1: Coupled tanks with pump.

Continuous control design

The scaled and linearized system is described by the transfer function

$$
\begin{aligned}
\Delta X_1(s) &= \frac{k}{1+\tau s}\Delta U(s),\\
\Delta X_2(s) &= \frac{\gamma}{1+\gamma\tau s}\Delta X_1(s),\\
\Delta Y(s) &= \Delta X_2(s),\\
G(s) &= \frac{\Delta Y(s)}{\Delta U(s)}.
\end{aligned} \tag{1}
$$

2. Edit the file `control_design.m` and fill in the transfer functions for the upper and lower tank. State the MATLAB code in your report.**[1p]**

We should now try to control the system. The design method that we will use is pole-placement. We have the following step response **requirements** on the closed loop system:

- rise time (T_r) less than 6 s
- overshoot (M) less than 35 %
- settling-time (T_{set}) with a 2 % error band less than 30 s

The PID-controller to be used is given by

$$F(s) = K\left[1 + \frac{1}{T_I s} + \frac{T_D N s}{s+N}\right].$$

The closed loop system will have 4 poles. The poles will be placed using the closed-loop characteristic equation

$$(s+\chi)^2(s^2 + 2\zeta\omega_0 s + \omega_0^2) = 0.$$

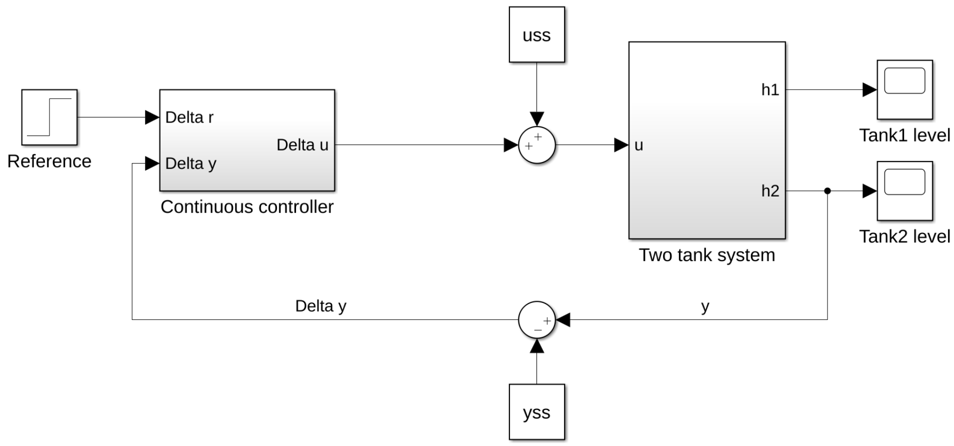

Figure 2: Simulink model of closed loop system.

The calculation of the PID parameters is implemented in the function `polePlacePID.m`. Open the file and make sure you understand how to use it.

3. Open the Simulink model by typing `tanks`. Also open the Simulink model `controllers`. Move the continuous controller to the tanks model and connect them as shown in the Figure 2. Check the Reference block and describe what the reference signal will look like. Explain why the `uss` and `yss` blocks are needed. **[2p]**

4. Edit the file `control_design.m`. Use the function `[K_pid,Ti,Td,N]=polePlacePID(chi,omega0,zeta,Tau,Gamma,K)` to derive the PID parameters. Also fill in the transfer function for the controller F. State the MATLAB code in your report. Note that the parameters will be calculated in the next task. **[2p]**

5. Simulate the system for the different values of the parameters as specified in the table below. Which one of the parameters settings gives the best control performance, i.e. fulfills the requirements stated on page 2 best? Please motivate your answer by the simulation results. **[2p]**

χ	ζ	ω_0	T_r	M	T_{set}
0.5	0.7	0.1			
0.5	0.7	0.2			
0.5	0.8	0.2			

6. What is the cross over frequency of the open loop system? Please motivate how you derive it. **[1p]**

Digital control design

The controller you just designed will be implemented digitally. This system can be described as in the Figure 3. In this part of the homework, the A/D and D/A converters can be neglected, and will be covered in the last part

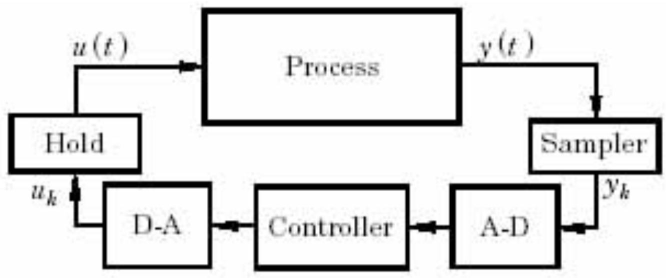

Figure 3: Model of digital control system.

of the homework. In Simulink, the sampling block is included at the input connector of all discrete blocks. Hence no explicit sampler is needed when simulating the system with a discrete controller.

7. Open the Simulink model. Connect a zero-order hold block after the continuous controller and before the double-tank process. Try to change the sampling time T_s of the zero-order hold. What are the differences in the control performance, compared to the continuous case? For which sampling times are the requirements on page 2 not fulfilled anymore? **[5p]**

8. Discretize the continuous controller into state space form (using MATLAB function `c2d(F,`T_s`,'ZOH')`, where `F` is the model you want to discretize) under a certain sampling time T_s. Then move the discretized controller (from `controller.mdl`) into the loop, replace the continuous controller and remove the zero-order hold. Compare the simulation results with the previous case in Question 7 (under different sampling times). Are there any differences in control performance? Please motivate your answer by simulation results. **[5p]**

9. When implementing a continuous controller digitally, in which interval should the sampling time be to keep the control performance? (*Hint*: Use the cross over frequency of the open loop system, see Lecture 4 Slide 41). **[2p]**

10. What is the maximum possible sampling time without affecting control performance in Question 8? Compare with the calculated sampling time in Question 9 and comment on possible differences. **[3p]**

Discrete control design

11. Simulate the closed-loop system in Question 8 when the sampling time $T_s = 4s$. How is the control performance? **[1p]**

We will now try a different approach. We will derive a sampled model of the process for which we design a discrete controller.

12. The continuous-time linearized system can be written as

$$\Delta\dot{x}(t) = A\Delta x(t) + B\Delta u(t),$$
$$\Delta y(t) = C\Delta x(t) = [0\ 1]\Delta x(t),$$

where $\Delta x = [\Delta x_1 \ \Delta x_2]^\top$. Determine the matrices A and B based on Equation (1). Discretize this system with $T_s = 4s$ using `c2d`, and determine the discrete-time system of the form

$$\begin{aligned}\Delta x(k+1) &= \Phi \Delta x(k) + \Gamma \Delta u(k),\\ \Delta y(k) &= C\Delta x(k) = [0\ 1]\Delta x(k).\end{aligned}$$

Save the parameters Φ, Γ, C in the file `control_design.m`. Please specify the values of Φ and Γ in your report. **[2p]**

13. Analyze the observability and reachability of the discrete-time plant model. **[1p]**

Next, we want to design a state-feedback controller such that the closed-loop system has the same poles as the continuous-time closed-loop system. The discrete-time system poles are located at $z_i = e^{T_s p_i}$, where p_i are the poles of the continuous-time system. In order to do this we use a control law of the form

$$\Delta u(k) = -L\Delta \hat{x}(k) + l_r r(k),$$

where $\hat{x}(k)$ is the estimated state obtained by a dynamic observer, $r(k)$ is the reference signal, and $l_r = \frac{1}{C(I-\Phi+\Gamma L)^{-1}\Gamma}$ is the reference gain.

14. Why is the reference gain l_r necessary? **[1p]**

15. Write down the state space equations for the dynamic observer. Now we can define an augmented system as, $x_a(k+1) = A_a x_a(k) + B_a r(k)$, where $x_a(k) = [\Delta x(k)^\top \Delta \hat{x}(k)^\top]^\top$ is the augmented state vector. This augmented system represents the closed-loop dynamics of the water-tank system. Derive the matrices A_a and B_a depending on Φ, Γ, C, K, L, and l_r. **[3p]**

16. An important property of observer-based controllers is the separation principle, which states that L and K can be designed independently when placing the poles of the closed-loop system. In order to show this we introduce the state vector $z(k) = [\Delta x(k)^\top \Delta \tilde{x}(k)^\top]^\top$, where $\Delta \tilde{x}(k) = \Delta x(k) - \Delta \hat{x}(k)$ is the reconstruction error. Verify that the closed-loop system can then be written as

$$z(k+1) = \begin{bmatrix} \Phi - \Gamma L & \Gamma L \\ 0 & \Phi - KC \end{bmatrix} z(k) + \begin{bmatrix} \Gamma l_r \\ 0 \end{bmatrix} r(k). \tag{2}$$

Motivate why this result shows that the separation principle holds. **[3p]**

17. Convert the poles for the continuous-time closed-loop system to discrete-time. Please specify the continuous-time and discrete-time poles in your report. Using the MATLAB command `acker`, design the controller gain L and the observer gain K, such that the closed-loop system has the desired discrete-time poles that you obtained in the beginning of this task. How would you design K? Explain the reasoning

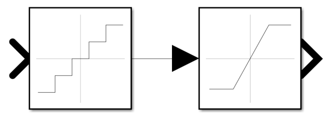

Figure 4: Quantizer with saturation.

behind this choice. Verify that A_a has the desired poles. (Hint: When computing the poles of a closed-loop system the MATLAB function `minreal` can be used to ensure a minimal realization.) **[3p]**

18. Open the Simulink model and change to the discrete designed controller (from `controller.mdl`). Simulate the system again and compare the outcome with the previous results in Question 11. Conclusions? Does the output reach the desired steady-state value? If not, motivate why. Please motivate your answer by simulation results. **[2p]**

Quantization

We will next investigate the effect of quantization. In a digital control system quantization appears in three different parts. When the signal is converted from analog to digital (A/D) the signal is quantized. The quantization level depends on the number of bits of the converter. In the control algorithm the output signal is computed. Here the size of the memory, used to store the signal value, contributes to the quantization. The less memory used the greater the quantization. The conversion of the signal back to analog (D/A) gives the same effect.

19. Suppose you want to construct a A/D converter. The signal to be converted vary between 0 and 100. What will the quantization level be if the the number of bits used is 10? **[3p]**

20. In Simulink, the quantization block does not have any upper or lower limits. Try to construct a quantization block by connecting a quantizer with a saturation as shown in Figure 4. Why is the saturation necessary? **[2p]**

21. Open the Simulink model with the discrete designed controller. Add your own quantization block (with the same quantization level) according to Figure 3 and ensure that you quantize $y(t)$ and $u(t)$ and not Δy and Δu. This corresponds to an A/D and a D/A converter. Set the saturation for both blocks to 0 for the lower and 100 for the upper limit. Simulate the system for different values of the quantization level. For which quantization level will the control performance start to degrade? How many bits does this correspond to? **[5p]**

Questions to the TAs

For this homework the responsible TAs are:

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