What is Decidable about String Constraints with the ReplaceAll Function

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Anthony W. Lin (Oxford)
Zhilin Wu (Chinese Academy of Sciences)

String Data Type

Prevalent in today's software

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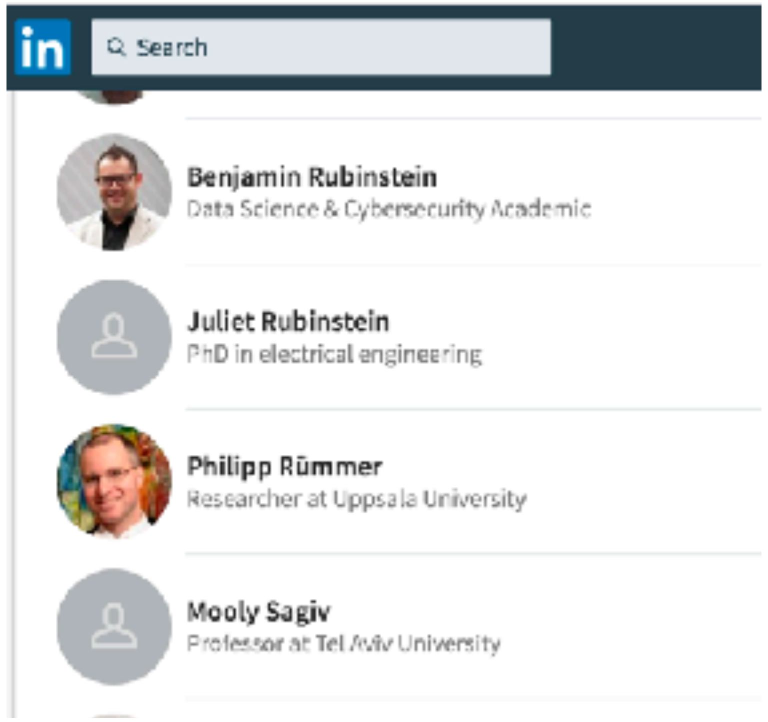

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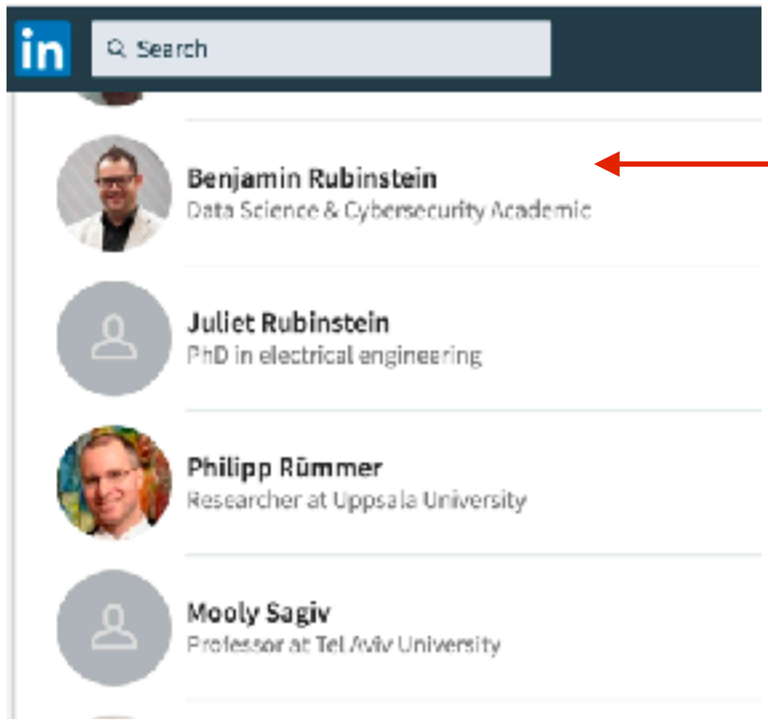
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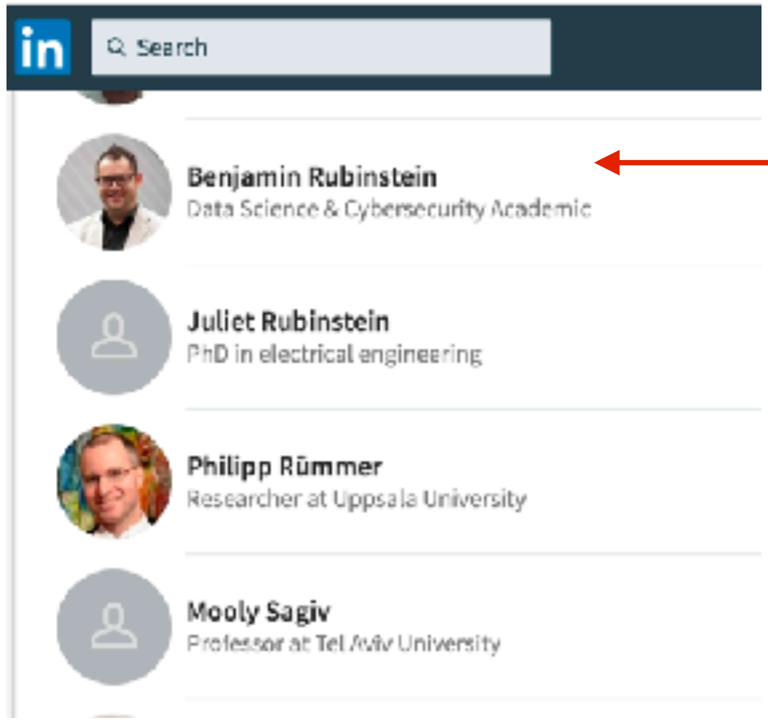

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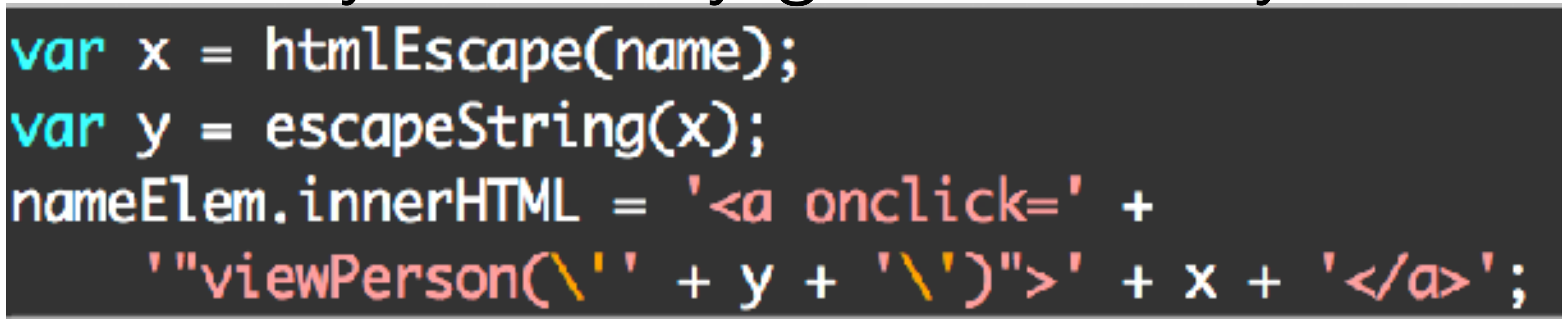

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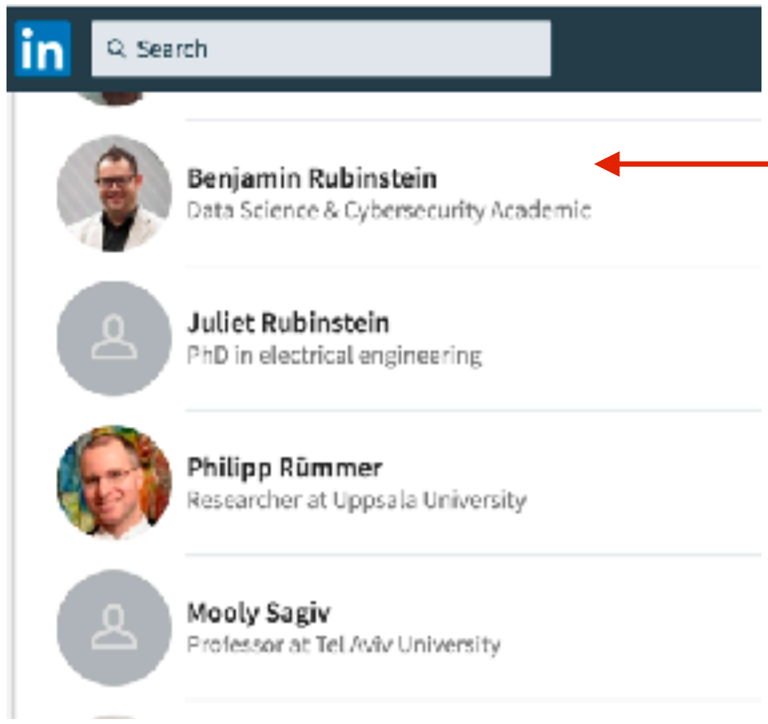
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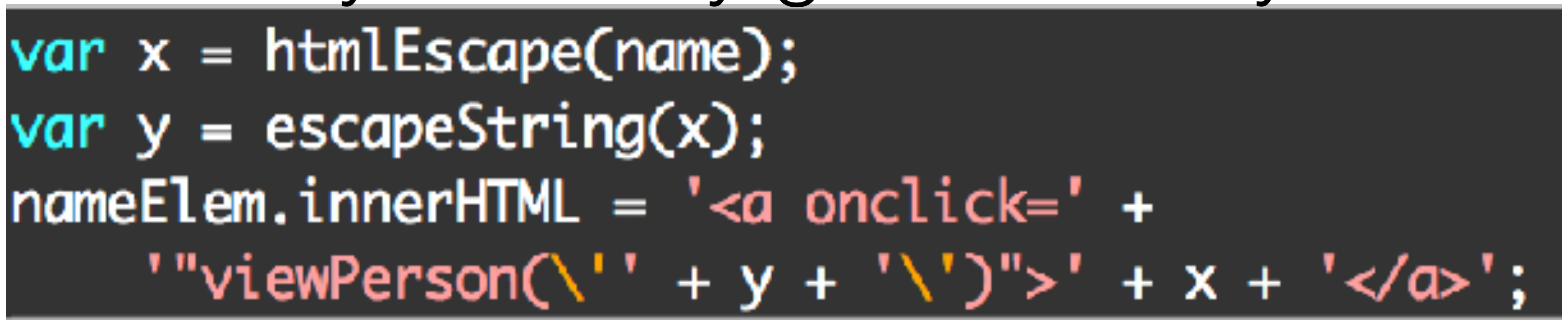

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Many string-related bugs — hard to find by random testing

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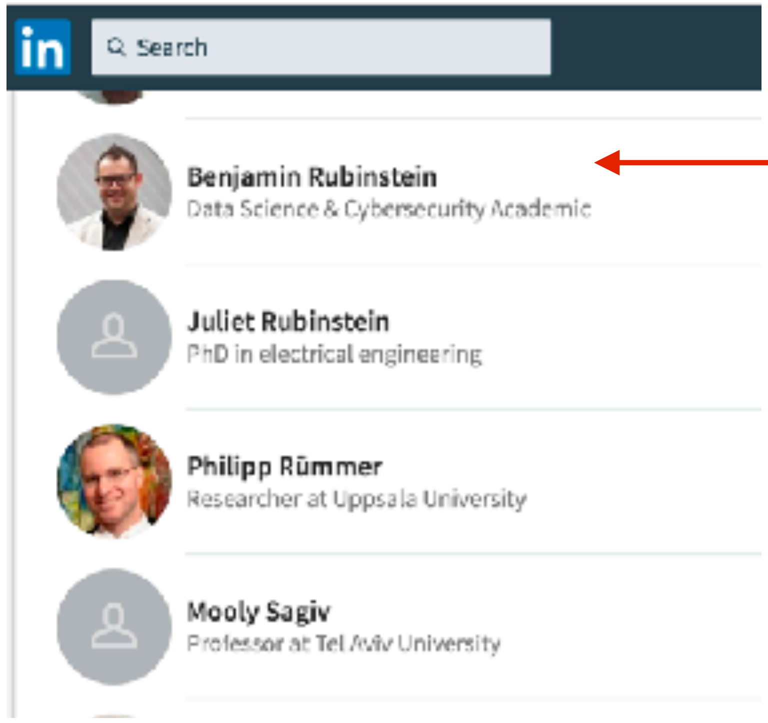
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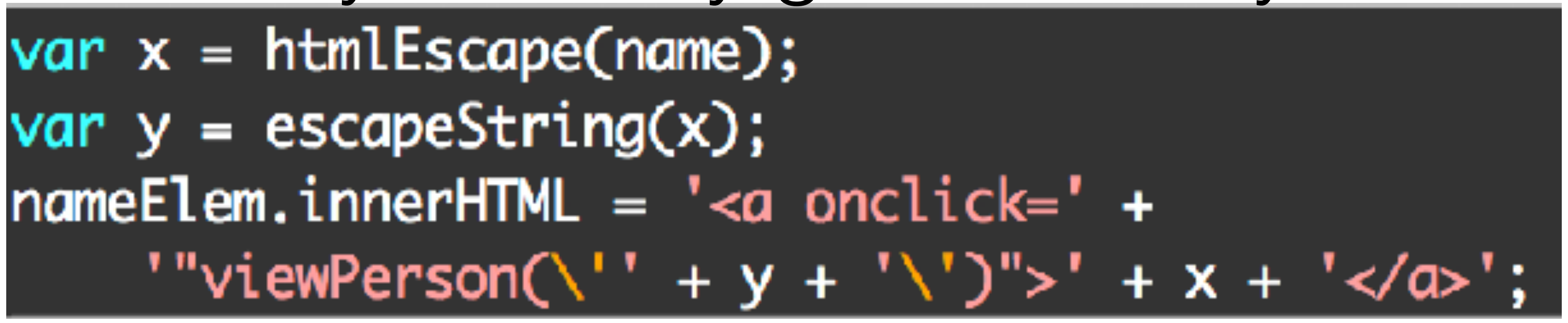

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XSS

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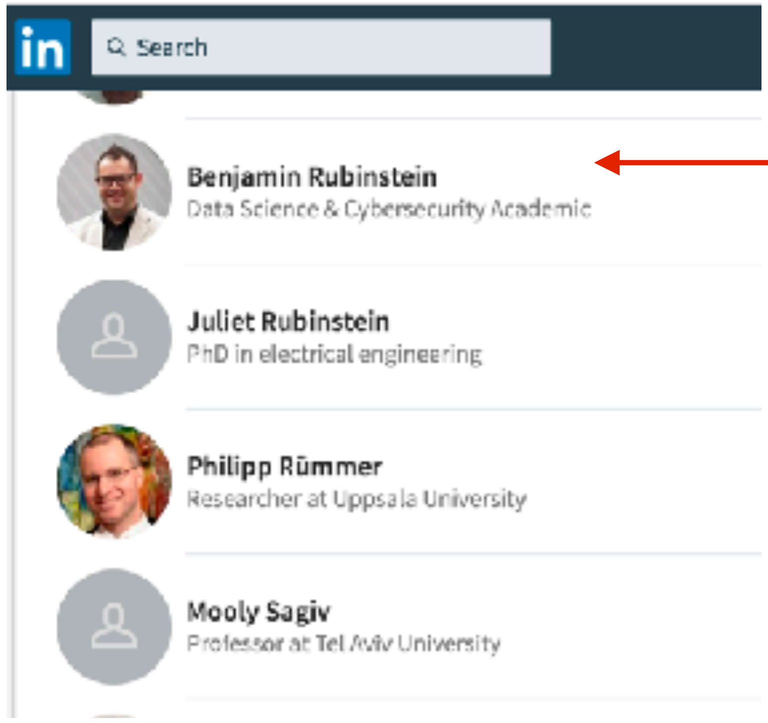
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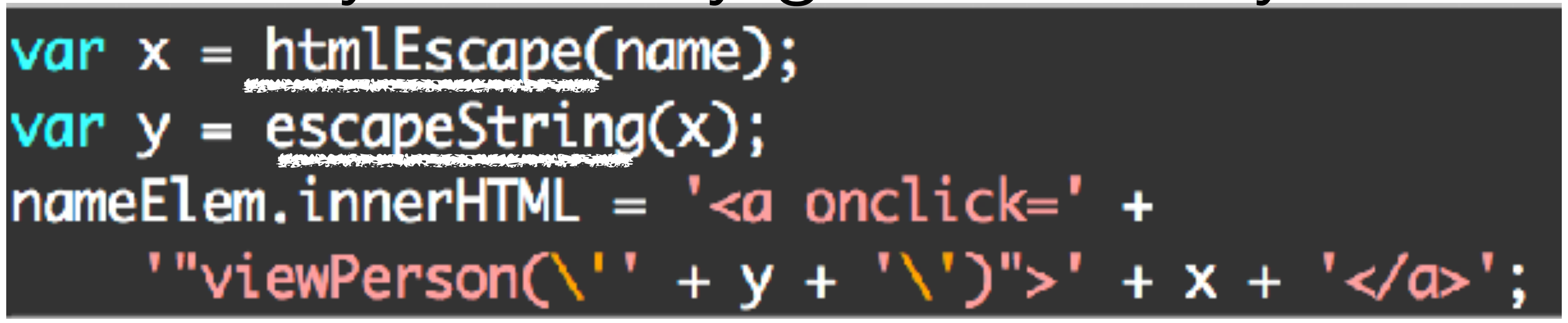

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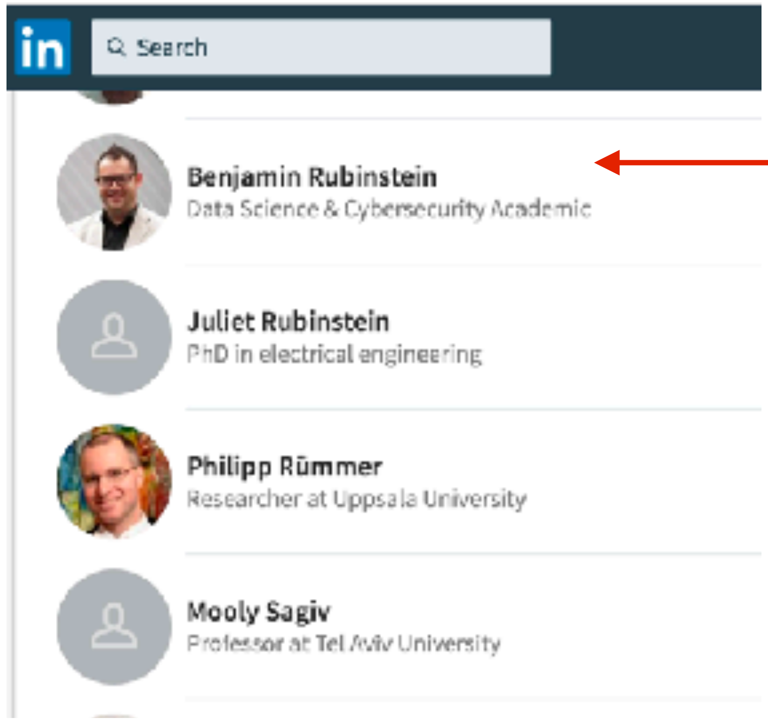
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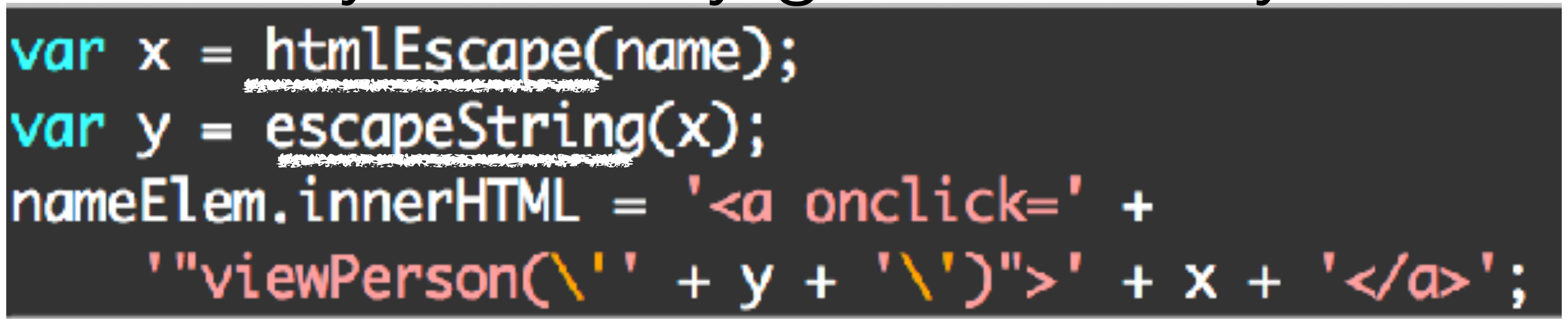

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XSS

Q: Does the sanitisation work?

String theory (a la SMT)

Constants/Variables: over the string domain (over a finite alphabet)

String operations:

- equality (=)
- concatenation (+)
- regex matching
- length function (len)
- replaceAll
- …

Formulas: quantifier-free, first-order

Problem: satisfiability (existence of a solution)

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satisfiable: x -> ‘b’, y -> ‘’

String Solvers Everywhere

Kaluza	Z3	Z3-str
Kudzu	PISA	IBM AppScan
HAMPI	Saner	Sloth
S3	Stranger	STP
Norn	StrSolve	…
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CVC4 SUSHI **Focus on "heuristics"**

Decidable String Theories

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Word Equations

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Decidable [Makanin’77]

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Theory of Concatenation with Regular Constraints

s2 = s1+s1 /\ s3+s2 !=s1+s7+s8
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Long-standing classical open problem

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Many string operations are still missing

Problem: **replaceAll** is by and large missing

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Proposal: add **replaceAll** to string theories in a decidable way

The ReplaceAll Function

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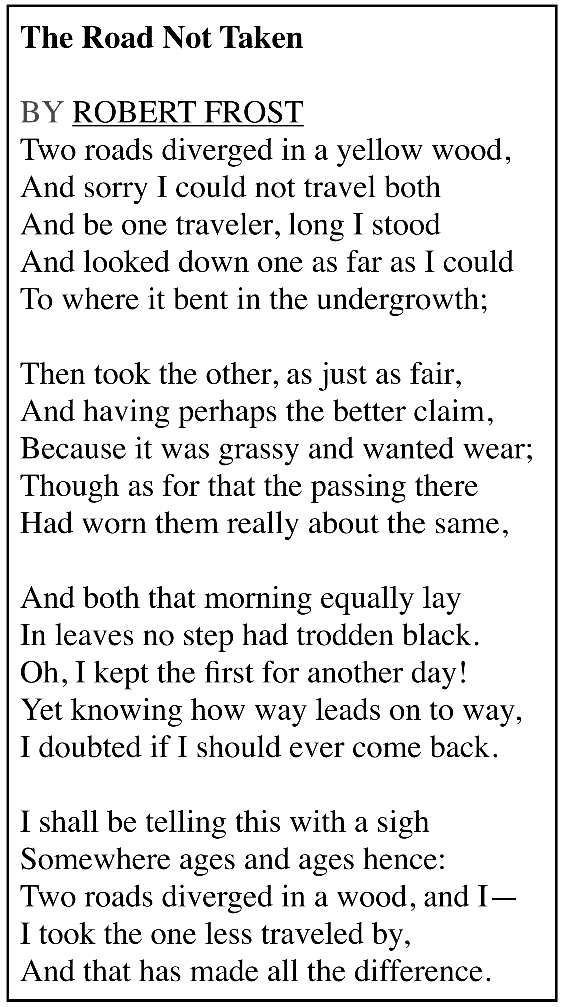

The Road Not Taken

BY ROBERT FROST
Two roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,

And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
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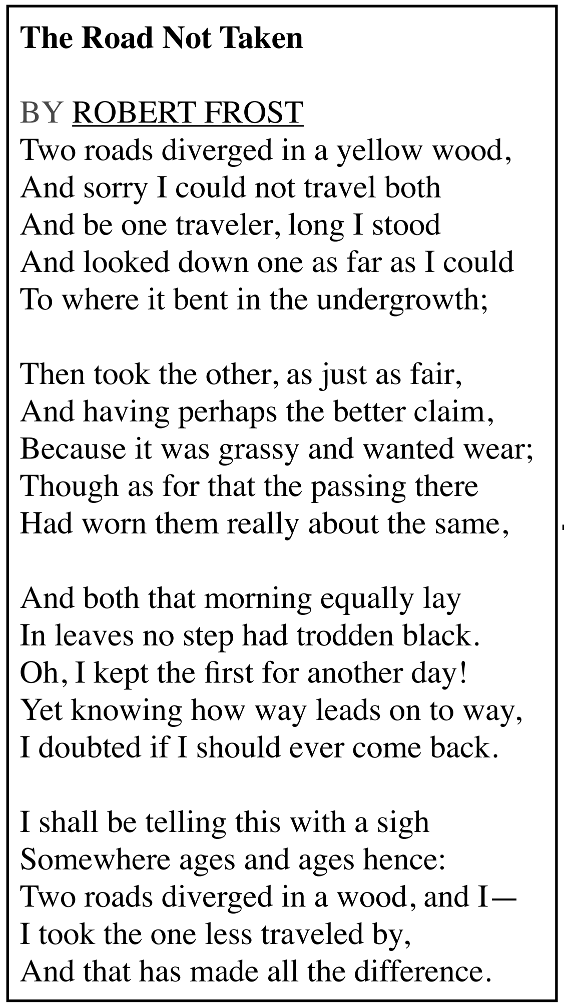

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$$\xrightarrow[\textit{rep} = \text{“Three”}]{\textit{pat} = \text{“Two”}}$$

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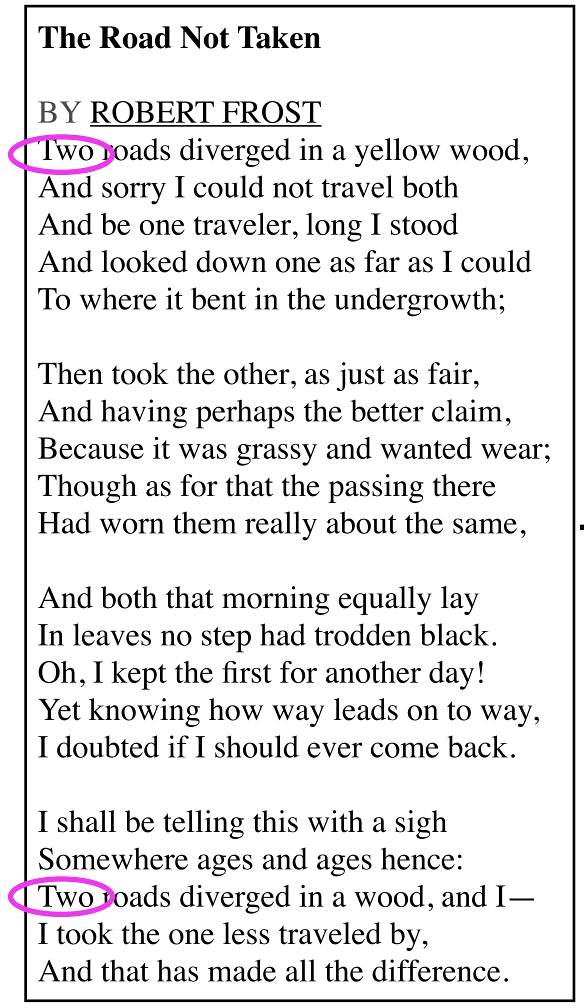

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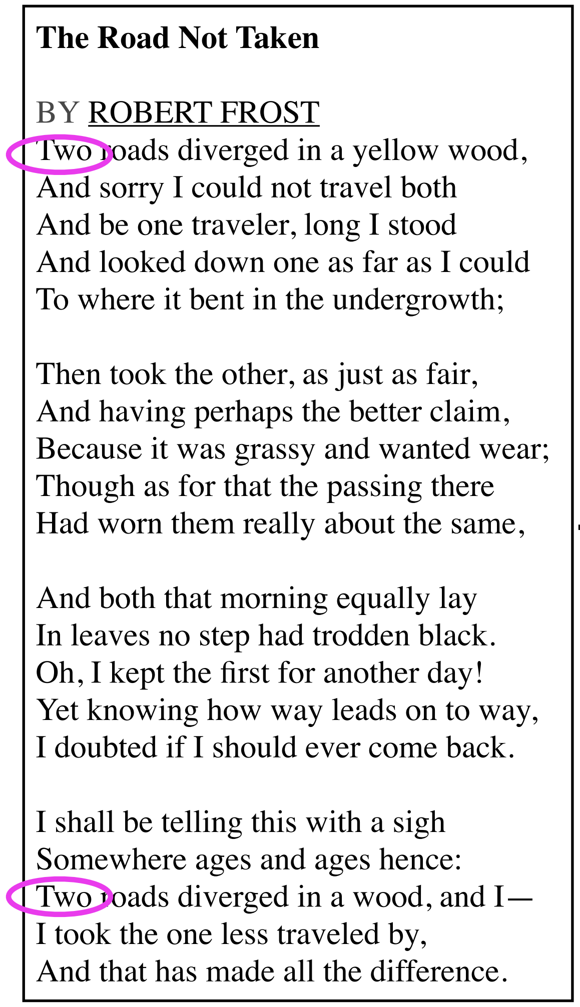

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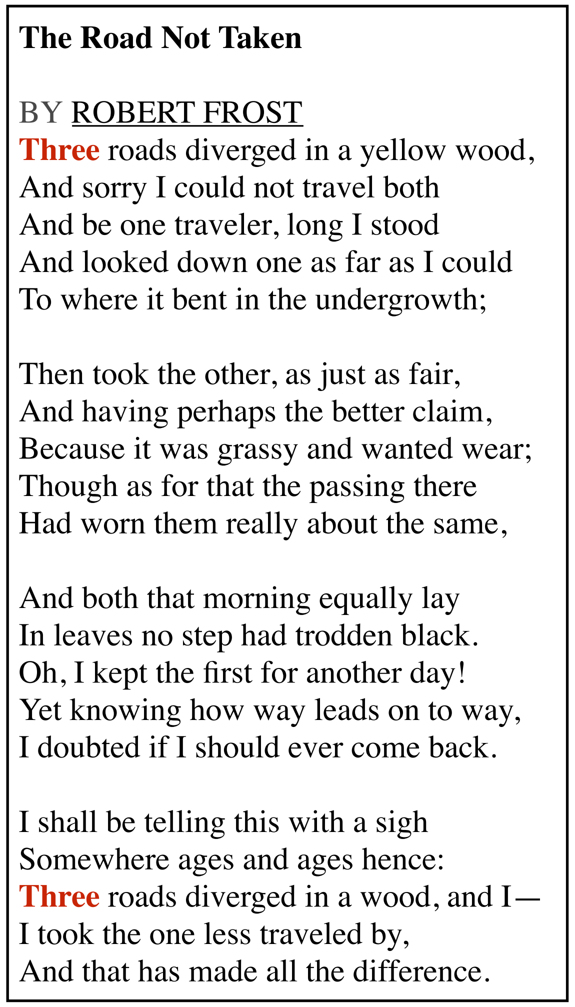

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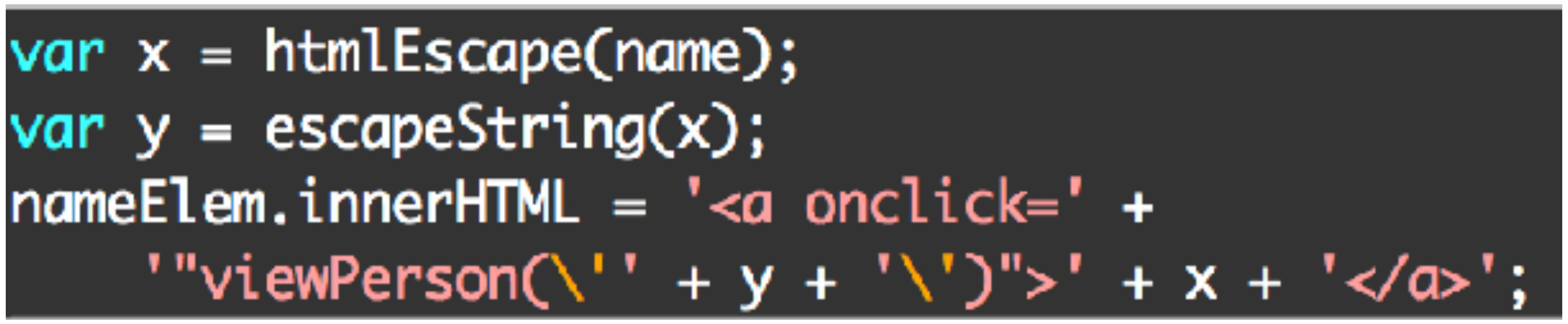

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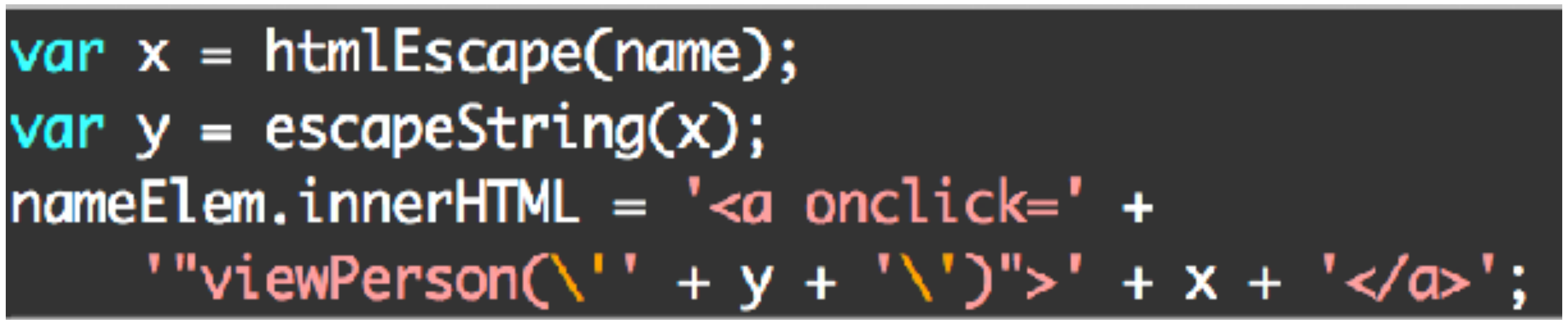

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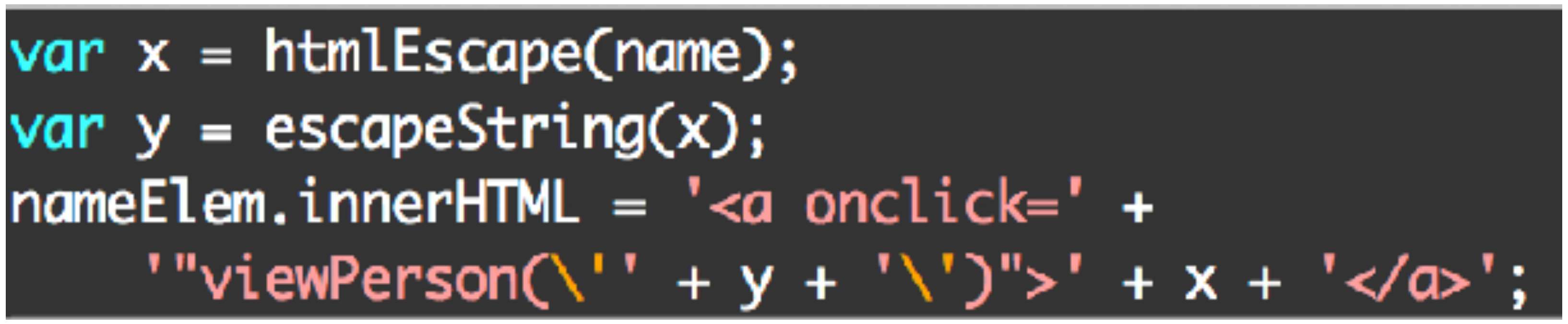

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<u>HTML template (with Mustache)</u>

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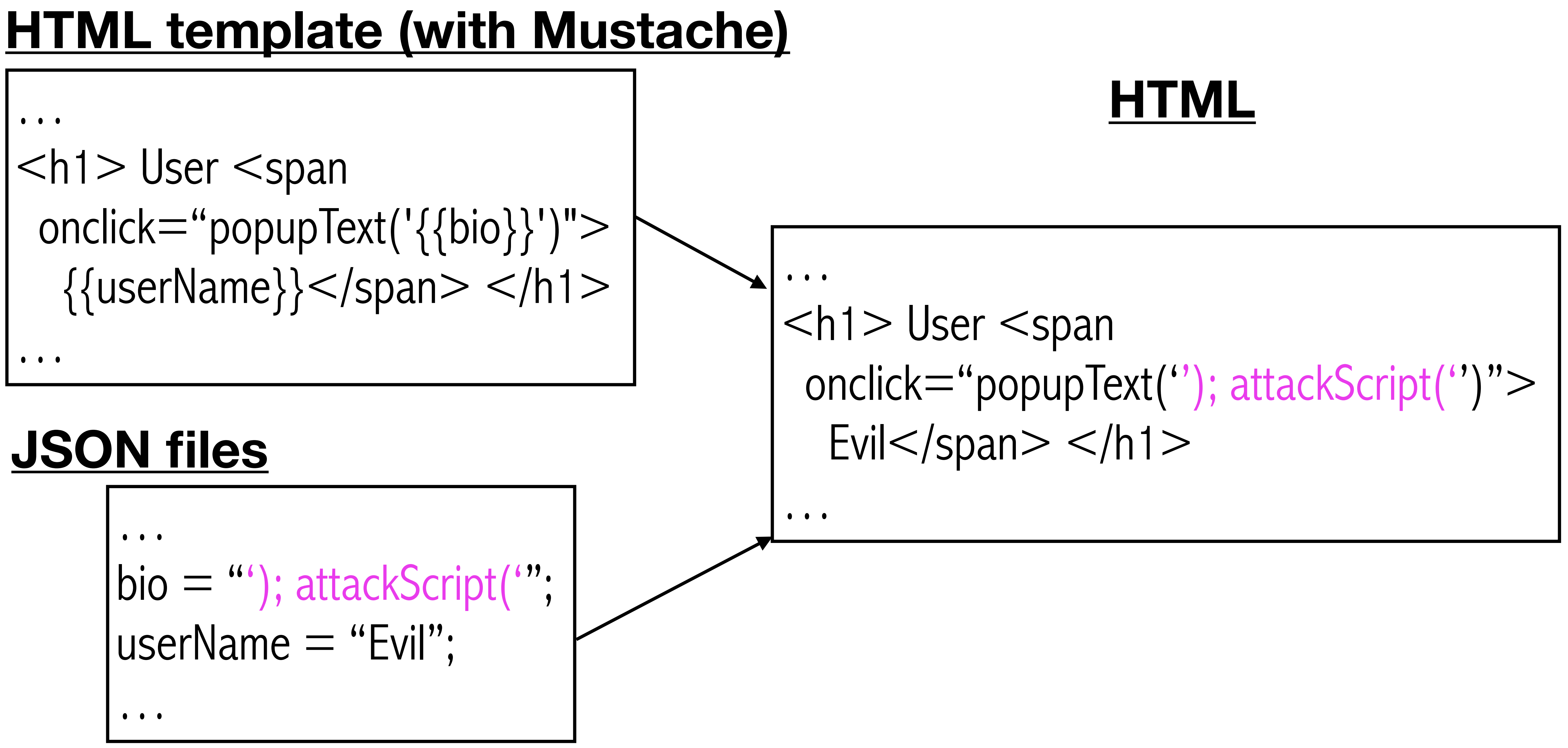

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bio = “‘); attackScript(‘”;
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…
```

HTML

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replaceAll in String Theory

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pat can be a <u>regular expression (over string constants)</u>

(semantics: leftmost/longest match)

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Most common usage: pat/rep are constants

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Not so uncommon usage: rep is a variable, pat is a constant

mustache(x,z,bio,userName) := y = replaceAll(x,{{bio}},bio) /\
z = replaceAll(y,{{userName}},userName)

Bad News

Proposition (Folklore): String constraints with equality, regex, and replaceAll (pat/rep constants) is **undecidable**

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1	2	3
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x in (1+2+3)* /\
y = replaceAll(x,1,a) /\ y’ = replaceAll(y,2,ab) /\ y’’ = replaceAll(y’,3,bba) /\
z = replaceAll(x,1,baa) /\ z’ = replaceAll(z,2,aa) /\ z’’ = replaceAll(z,3,bb) /\
y’’ = z’’

“Straight-Line” Framework

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Consider a symbolic execution in a program

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$$S ::= \quad y := f(x_1, \ldots, x_n) \mid \mathbf{assert}(g(x_1, \ldots, x_n)) \mid S_1; S_2$$

where

$$f : (\Sigma^*)^n \to \Sigma^* \qquad\qquad g : (\Sigma^*)^n \to \{0, 1\}$$

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Path Feasibility Problem: decide if there exist input strings that satisfy all the assertions

“Straight-Line” Framework

[Lin&Barcelo,POPL’16]

Consider a symbolic execution in a program

$$S ::= \quad y := f(x_1, \ldots, x_n) \mid \mathbf{assert}(g(x_1, \ldots, x_n)) \mid S_1; S_2$$

where

$$f : (\Sigma^*)^n \to \Sigma^* \qquad\qquad g : (\Sigma^*)^n \to \{0, 1\}$$

Path Feasibility Problem: decide if there exist input strings that satisfy all the assertions

Theorem: Path feasibility is decidable whenever:

1. assign.: concatenation/finite transducer
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Key Idea: NO general string equality in conditionals!

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```
x1 := x + ‘aba’ + y ∧
y1 := replaceall(x1,’a’,’c’) ∧
assert( y1 in (‘b’)* )
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Formula (use SSA form)

<u>Path Feasibility = Satisfiability (in disguise)</u>

Why Straight-Line

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- Prohibits bad formulas

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x in (1+2+3)* /\
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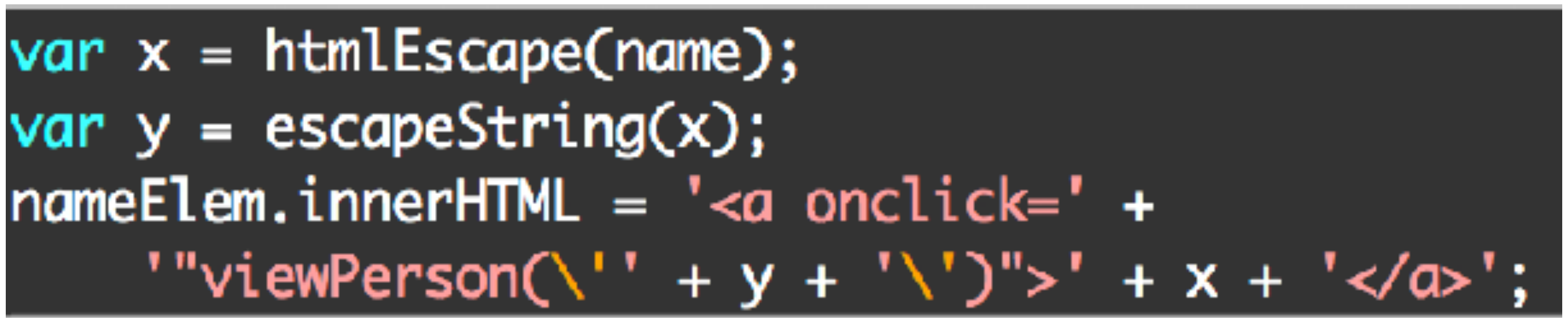

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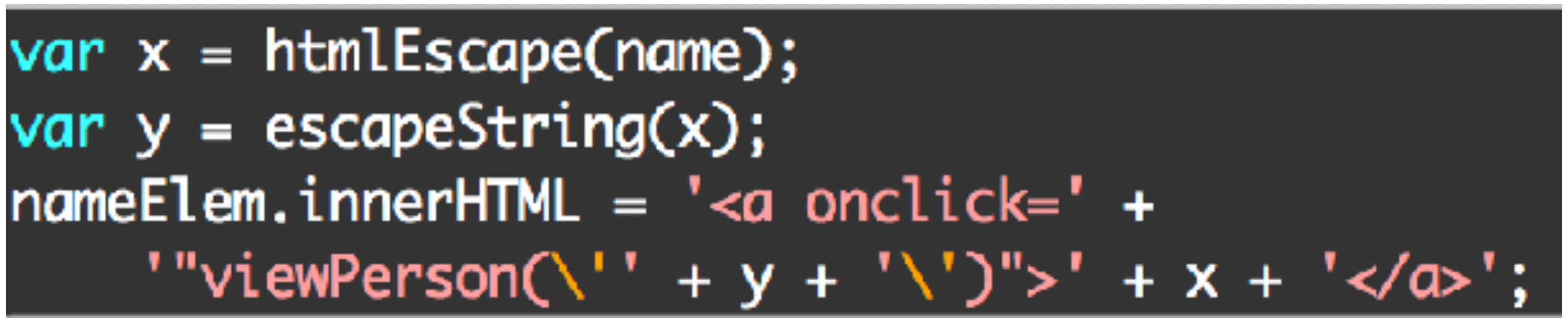

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LOTS of existing benchmarks are in SL

Limitation of SL [LB'16]

HTML template (with Mustache)

```
…
<h1> User <span
 onclick="popupText('{{bio}}')">
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…
```

JSON files

```
…
bio = "'); attackScript('";
userName = "Evil";
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Requires more general replaceall!

x = replaceAll(text,’{{bio}}’,bio)

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A more expressive decidable straight-line fragment with **replaceAll**!

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5 ———> 'aaaaa'

z = x*y ———> z = **replaceAll**(x,'a',y)

z = x+y ———> z = x+y   (concatenation)

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Lemma: Concatenation in SL can be expressed as **replaceAll**(VAR,regex,VAR).

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$$X = Y + \text{'aba'} + Y + Z \longrightarrow \begin{array}{l} X0 = \text{replaceAll}(\text{'yabayz'},y,Y) \land \\ X\ = \text{replaceAll}(X',z,Z) \end{array}$$

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<u>Definition</u>: The <u>pre-image</u> of a language *L* under **replaceAll**$_{\text{pat}}$ with pattern pat is:

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There are many other combinations!

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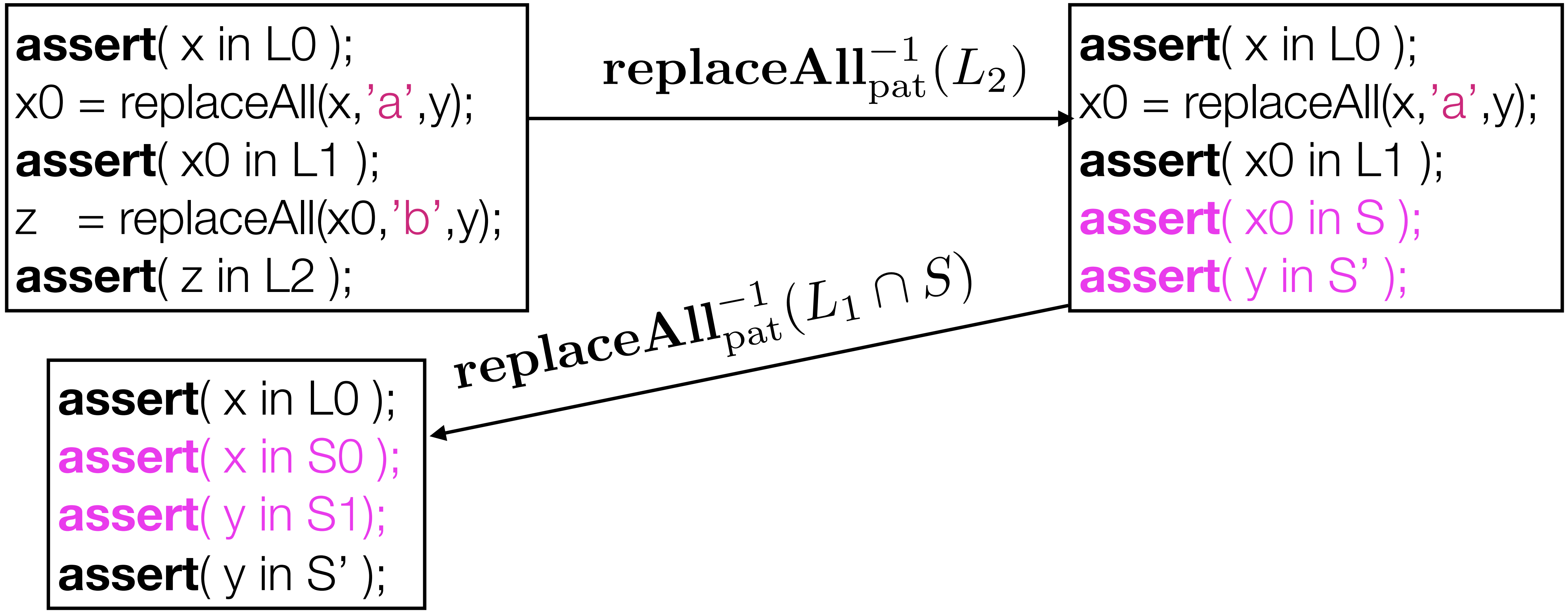
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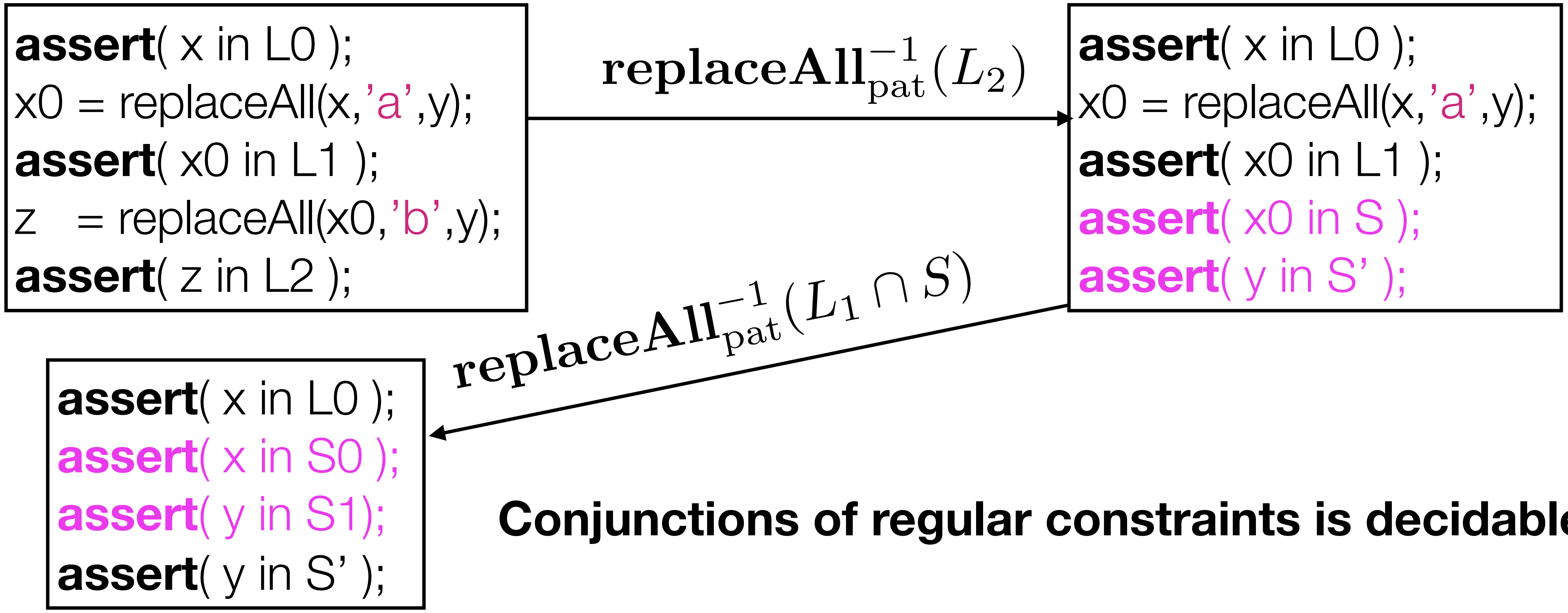
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Conjunctions of regular constraints is decidable!

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prohibits this crazy use

Extensions

Length Constraints

```
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Note:

1. Some length constraints are regular
   len(x) < 7 len(x) mod 7 = 3
2. [LB'16] decidable for **replaceAll**(VAR,regex,const)

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Proof is by a reduction from PCP

Final Words

Summary:

- Decidability boundary of string solving with **replaceAll**
- Reason to be positive!

Ongoing work:

- Computational complexity issues
- Unify transducers [LB'16] with **replaceAll**(VAR,regex,VAR)
- String solver based on our constraint language

https://github.com/TinyYan/z3-replaceAll