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Utbildningskod: *Education code:* TNA003	Modul: *Module:* TEN1	

1. $\sqrt{x} - \ln x = h$

$f(x) = \sqrt{x} - \ln x \qquad D_f \quad x > 0$ R

$\lim_{x \to 0^+} \sqrt{x} - \ln x = 0 - (-\infty) = \infty$ R   Ändpunkter

$\lim_{x \to \infty} \sqrt{x} - \ln x = \infty$ R   Enligt hastighetstabellen

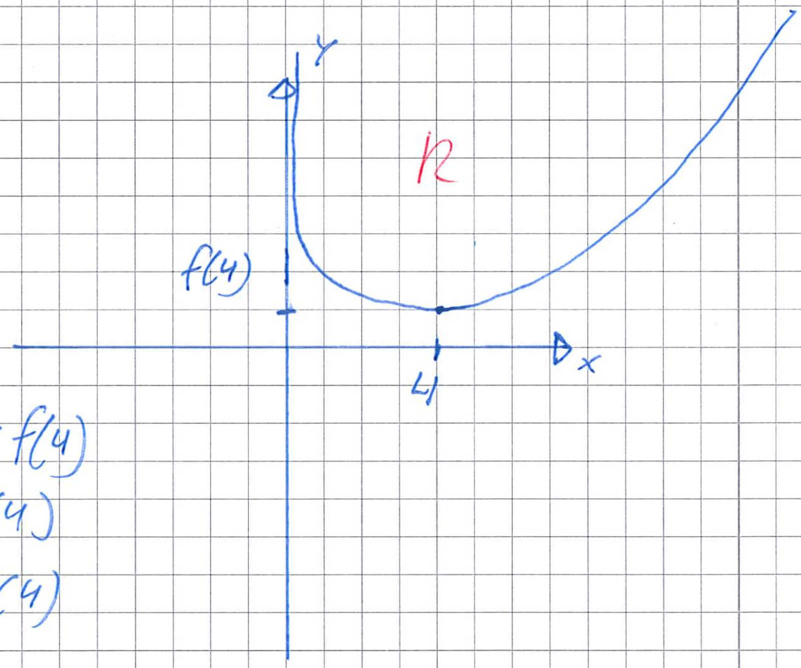
$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x} = \frac{x - 2\sqrt{x}}{x 2\sqrt{x}}$ R $x = 4$ Derivata

$f(4) = 2 - \ln 4 > 0$ Stationära punkter

R

Teckentabell

	0		4		x
$x - 2\sqrt{x}$		$-$	0	$+$	
$x 2\sqrt{x}$		$+$	$+$	$+$	R
f'		$-$	0	$+$	
f		$\searrow$	Lok Min	$\nearrow$	



K:
- 0 rötter om $h < f(4)$
- 1 rot om $h = f(4)$
- 2 rötter om $h > f(4)$

R

Svar: ↑

6p

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2. a) $\lim_{x\to 1} \frac{e^{x-1}-1}{\sin(x-1)}$ $\left\{ t = x-1 \atop x\to 1,\, t\to 0 \right\}$

$= \lim_{t\to 0} \frac{e^t-1}{\sin t} = \lim_{t\to 0} \underbrace{\frac{e^t-1}{t}}_{\to 1} \cdot \underbrace{\frac{t}{\sin t}}_{\to 1} = 1\cdot 1 = 1$ R

R

Enligt SGU , $t\to 0$

b) $\lim_{x\to 0} \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{x\ln(1+4x)} = \lim_{x\to 0} \frac{(\sqrt{2+x^2}-\sqrt{2-x^2})(\sqrt{2+x^2}+\sqrt{2-x^2})}{(x\ln(1+4x))(\sqrt{2+x^2}+\sqrt{2-x^2})}$

$= \lim_{x\to 0} \frac{2x}{\ln(1+4x)(\sqrt{2+x^2}+\sqrt{2-x^2})} = \lim_{x\to 0} \underbrace{\frac{4x}{\ln(1+4x)}}_{\to 1} \cdot \frac{1}{2(\sqrt{2+x^2}+\sqrt{2-x^2})}$ R

Enligt SGU

vilket ?

$= 1 \cdot \frac{1}{2(\sqrt{2+0}+\sqrt{2-0})}$

$= \frac{1}{4\sqrt{2}}$ R

5p

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3. a) $\int \cos(x^{1/3})\,dx = 3\sin(x^{1/3})$

0

b) $\int \frac{1}{x+2\sqrt{x}+2}\,dx = \left\{ \begin{matrix} t=\sqrt{x} & dx = 2t\,dt \\ x = t^2 \end{matrix} \right\}$

$= \int \frac{2t}{t^2+2t+2}\,dt = \int \frac{2t}{(t+1)^2+1}\,dt \quad \left\{ \begin{matrix} u=(t+1) \quad dt = du \\ t = u-1 \end{matrix} \right.$

$= \int \frac{2u-2}{u^2+1}\,du = \int \frac{2u}{u^2+1}\,dt - 2\int \frac{1}{u^2+1}\,dt$

$= \ln(u^2+1) - 2\arctan u + C$

$= \ln(t^2+2t+2) - 2\arctan(t+1) + C$

$= \ln(x+2\sqrt{x}+2) - 2\arctan(\sqrt{x}+1) + C$

Svar: a) $3\sin(x^{1/3})$

b) $\ln(x+\sqrt{x}+2) - 2\arctan(\sqrt{x}+1) + C$

3

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4. $y = \arctan x - \arctan\left(\frac{1}{x}\right) + \ln(1+x^2)$ $\quad D_f = \mathbb{R} \setminus \{0\}$

$$\lim_{x\to 0^+} \arctan x - \arctan\left(\frac{1}{x}\right) + \ln(1+x^2) =$$

$$= 0 - \frac{\pi}{2} + 0 = -\frac{\pi}{2}$$

$$\lim_{x\to 0^-} \arctan x - \arctan\left(\frac{1}{x}\right) + \ln(1+x^2) =$$

$$= 0 + \frac{\pi}{2} + 0 = \frac{\pi}{2}$$

Ingen lodrät asymptot.

$$\lim_{x\to \pm\infty} \arctan x - \arctan\left(\frac{1}{x}\right) + \ln(1+x^2) =$$

$$= \pm\frac{\pi}{2} + \infty = \infty$$

Ingen vågrät asymptot.

$$\lim_{x\to \pm\infty} \frac{\arctan x - \arctan\left(\frac{1}{x}\right) + \ln(1+x^2)}{x} = 0$$

Enligt hastighetstabellen går x snabbare än $\ln(1+x^2)$ och arctan är betydelselös. ?

Ingen sned asymptot

$$y' = \frac{1}{x^2+1} + \frac{1}{x^2+1} + \frac{2x}{1+x^2} = \frac{2(x+1)}{x^2+1} \qquad x = -1$$

Stationär punkt

$$y(-1) = \cancel{-\frac{\pi}{4} + \frac{\pi}{4}} + \ln 2$$

Teckentabell

x		-1		0	
$2x+2$	$-$	0	$+$		$+$
x^2+1	$+$	$+$	$+$		$+$
f'	$-$	0	$+$		$+$
f	$\searrow$	lok min	$\nearrow$		$\nearrow$

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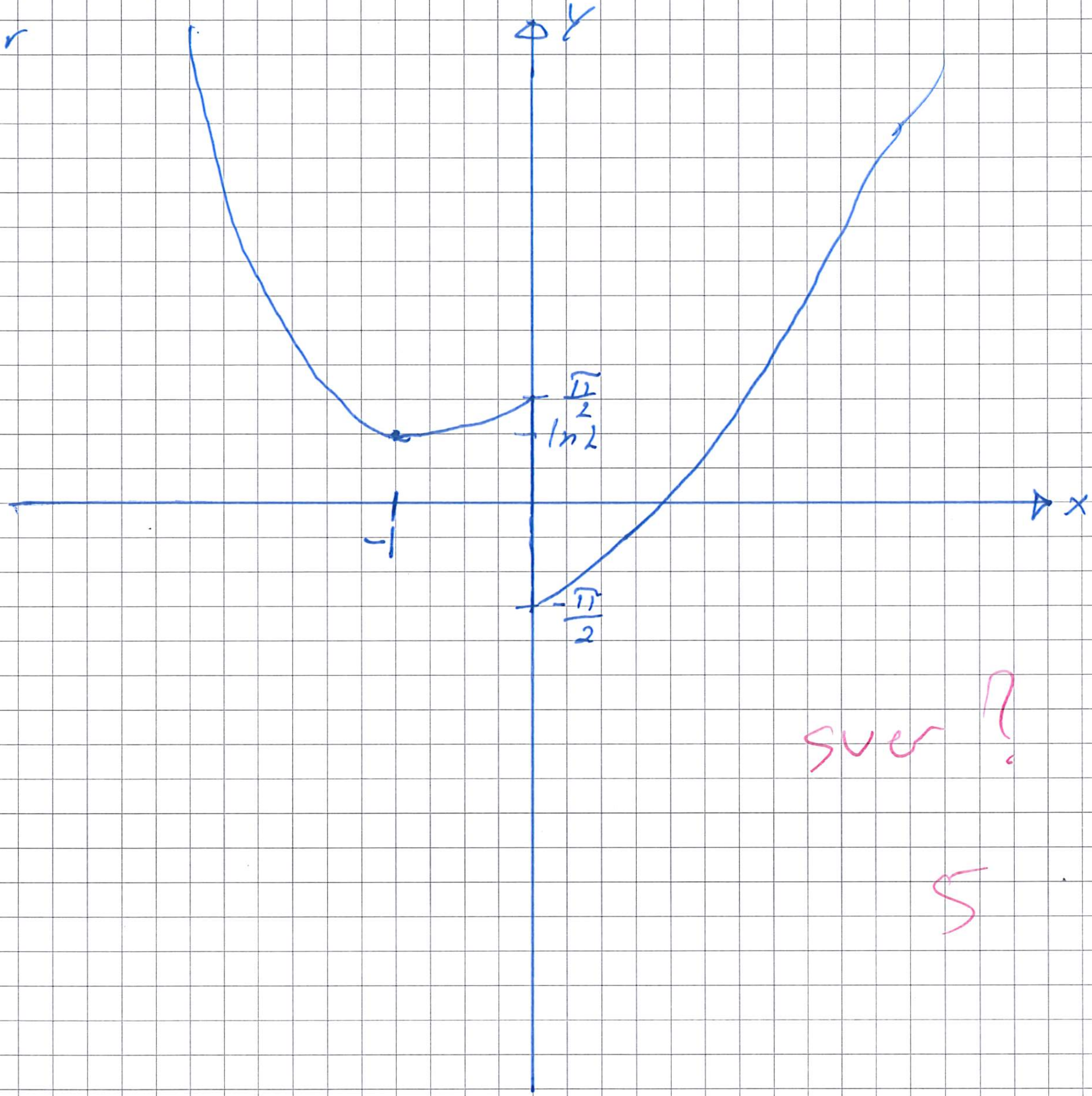
4. Figur



svar?

5

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5. a) $f(x) = x^2 e^{3x}$

$$f'(x) = \lim_{h \to 0} \frac{f(h+x) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(h+x)^2 e^{3(h+x)} - x^2 e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{h^2 e^{3(h+x)} + 2hx e^{3(h+x)} + x^2 e^{3(h+x)} - x^2 e^{3x}}{h}$$

$$= \lim_{h \to 0} \underbrace{h e^{3(h+x)}}_{\to 0} + 2x e^{3(h+x)} + \frac{x^2 e^{3(h+x)} - x^2 e^{3x}}{h}$$

$$= \lim_{h \to 0} 2x e^{3(h+x)} + 3 \frac{x^2 e^{3x} e^{3h} - x^2 e^{3x}}{3h}$$

$$= \lim_{h \to 0} 2x e^{3(h+x)} + 3x^2 e^{3x} \left(\underbrace{\frac{e^{3h} - 1}{3h}}_{\to 1} \right)$$

enligt SGV $\frac{e^t - 1}{t}$

$$= 2x e^{3x} + 3x^2 e^{3x}$$

3

Skriv ej på baksidan! Do not write on the other side!

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5 b)

$$f(x) = \begin{cases} 4+\sqrt{x}\ln x & , x>0 \\ A & , x=0 \\ B+\dfrac{1-\sqrt{1-x}}{x} & , x<0 \end{cases} \qquad A, B\,?$$

$\lim_{x\to 0^+} 4+\sqrt{x}\ln x = 4$ enligt hastighetstabellen är $\sqrt{x}$ $(=0)$ snabbare än $\ln x$ $(-\infty)$.

$$\lim_{x\to 0^-} B+\frac{1-\sqrt{1-x}}{x} = \lim_{x\to 0^-} B+\frac{(1-\sqrt{1-x})(1+\sqrt{1-x})}{x(1+\sqrt{1-x})} =$$

$$\lim_{x\to 0^-} B+\frac{1-1+x}{x(1+\sqrt{1-x})} = \lim_{x\to 0^-} B+\frac{1}{1+\sqrt{1-x}} = B+\frac{1}{2}$$

$A=4$ $\quad B=3{,}5$ Gör att funktionen blir kontinuerlig!

Svar $A=4$ $\quad B=3{,}5$

3

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6. a) $f(x) = \int_1^{x^{1/3}} \frac{\ln t - 1}{\arctan t} \, dt, \; x \geq 1$

Max, Min?

Max $\ln 3 + \frac{\pi}{2}$

$/ \, 0$

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6b) $x^4+y^4-4xy=0$ $\quad P(\sqrt{2}, \sqrt{2})$

$f(x)=x^4+y^4-4xy$

$f'(x)=4x^3+4y'^3-4y' = 4x^3+4y'(y'^2-1)$

$y=-2x+2$

0

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7. $I(h) = \int_0^\infty \left(\frac{\ln(x+h+1)}{\sqrt{x+h}} - \frac{\ln x}{\sqrt{x}} \right) dx$ för $h>0$

$$\int \overbrace{\frac{\ln(x+h+1)}{\sqrt{x+h}}}^{A} dx - \int \overbrace{\frac{\ln x}{\sqrt{x}}}^{B} dx$$

A: $\left\{ \begin{matrix} t = \sqrt{x+h} & dx = 2t\,dt \\ x = t^2 - h & \end{matrix} \right\} = \int \frac{\ln(t^2+1)\cdot 2t}{t} dt =$ P.I

$= \int 2\ln(t^2+1)dt = 2t\ln(t^2+1) - \int \frac{4t^2}{t^2+1} dt$

$-4\int \frac{t^2}{t^2+1} dt = -4\int \left(1 - \frac{1}{t^2+1}\right) dt$ Polynomdivision

$= -4t + 4\arctan t$

$A = 2t\ln(t^2+1) - 4t + 4\arctan t =$

$= 2\sqrt{x+h}\ln(x+h+1) - 4\sqrt{x+h} + 4\arctan\sqrt{x+h} + C$

B: $\left\{ \begin{matrix} t = \sqrt{x} & dx = 2t\,dt \\ t^2 = x & \end{matrix} \right\} = \int \frac{\ln t^2}{t} \cdot 2t\,dt = -\int 2\ln t^2 \, dt =$

P.I

$= -2t\ln t^2 + \int \frac{4t^2}{t^2} dt = -2t\ln t^2 + 4t$

$= -2\sqrt{x}\ln x + 4\sqrt{x} + C$

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7. Forts. A+B ger

$$2\sqrt{x+h}\ln(x+h+1)-4\sqrt{x+h}+4\arctan\sqrt{x+h}-2\sqrt{x}\ln x+4\sqrt{x}+C$$

Vi vill beräkna $x\to\infty$ - $x\to 0$ för varje värde $h>0$ och sedan $h\to 0^+$ i det resultatet. $\underline{C}$ tar ut varann

$$\lim_{R\to\infty} 2\sqrt{R+h}\ln(R+h+1)-4\sqrt{R+h}+4\arctan\sqrt{R+h}-2\sqrt{R}\ln R+4\sqrt{R}$$

$$\lim_{a\to 0} 2\sqrt{a+h}\ln(a+h+1)-4\sqrt{a+h}+4\arctan\sqrt{a+h}-2\sqrt{a}\ln a+4\sqrt{a}$$
$$=2\sqrt{h}\ln(h+1)-4\sqrt{h}+4\arctan\sqrt{h}$$

X: $$\lim_{R\to\infty} 2\sqrt{R+h}\ln(R+h+1)-4\sqrt{R+h}+4\arctan\sqrt{R+h}-2\sqrt{R}\ln R+4\sqrt{R}$$
$$-2\sqrt{h}\ln(h+1)+4\sqrt{h}-4\arctan\sqrt{h}$$

$$\lim_{R\to\infty}\lim_{h\to 0^+}=2\sqrt{R}\ln(R+1)-4\sqrt{R}+4\arctan\sqrt{R}-2\sqrt{R}\ln R+4\sqrt{R}$$
$$=2\pi$$

Svar: Det närmsta jag kom till ett svar för $I(h)$ är X.

$$\lim_{h\to 0^+}(I(h))=2\pi$$

1/6