Audio Filters

Woohyun Rim

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Contents

1 Analog Filter Units

1.1 First-order Filters

1.1.1 Shelving Filters

First-order shelving filters can be constructed by simply adding a bottom constant function and a low- or high-pass filter. [1]

$$H(s) = A + B\frac{1}{s+\omega_0} \to C\frac{s+a}{s+b}, \tag{1}$$

where a, b, and C are positive. We want to express this transfer function into these three variables:

- gain at low frequency,
- gain at high frequency, and
- the center frequency (where the gain is the geometric average in amplitude sense or the arithmetic average in decibel sense of the lowest and the highest gains).

The gain at the low frequency is,

$$H(s)\big|_{s=2\pi if, f\to 0} = H(s)\big|_{s=0} = C\,\frac{a}{b}. \tag{2}$$

The gain at the low frequency is,

$$H(s)\big|_{s=2\pi if, f\to\infty} = C. \tag{3}$$

The center frequency f_c satisfies

$$|H(s = 2\pi i f_c)| = C\sqrt{\frac{a}{b}}. \tag{4}$$

The left side expands:

$$|H(s = 2\pi i f_c)| = C\sqrt{\frac{a^2 + (2\pi f_c)^2}{b^2 + (2\pi f_c)^2}}. \tag{5}$$

Thus, the condition for f_c simplifies:

$$\begin{aligned} & \sqrt{\frac{a}{b}} = \sqrt{\frac{a^2 + (2\pi f_c)^2}{b^2 + (2\pi f_c)^2}} && (6)\\ \Rightarrow \quad & a(b^2 + (2\pi f_c)^2) = b(a^2 + (2\pi f_c)^2) && (7)\\ \Rightarrow \quad & (b-a)(2\pi f_c)^2 + ab(a-b) = 0 && (8)\\ \overset{\text{unless } a=b}{\Rightarrow} \quad & (2\pi f_c)^2 = ab && (9) \end{aligned}$$

To find the expression of the transfer function in terms of $H(0)$, $H(\infty)$, and f_c, let us find the expression of a, b, and C in terms of those variables.

$$\begin{aligned} & \frac{a}{b} = \frac{H(0)}{H(\infty)}, \quad (2\pi f_c)^2 = ab && (10)\\ \Rightarrow \quad & a = (2\pi f_c)\sqrt{\frac{H(0)}{H(\infty)}}, \quad b = (2\pi f_c)\sqrt{\frac{H(\infty)}{H(0)}}; && (11)\\ & C = H(\infty). && (12) \end{aligned}$$

Thus, the transfer function in terms of meaningful parameters is written by

$$\begin{aligned} H(s) = C\,\frac{s+a}{s+b} &= H(\infty)\frac{\frac{s}{2\pi f_c} + \sqrt{\frac{H(0)}{H(\infty)}}}{\frac{s}{2\pi f_c} + \sqrt{\frac{H(\infty)}{H(0)}}} && (13)\\ &= \frac{s\sqrt{H(\infty)} + \omega_c\sqrt{H(0)}}{\frac{s}{\sqrt{H(\infty)}} + \frac{\omega_c}{\sqrt{H(0)}}}, && (14) \end{aligned}$$

where $\omega_c = 2\pi f_c$.

References

[1] *Low and High Shelf Filters,* `https://ccrma.stanford.edu/~jos/fp/Low_High_Shelf_Filters.html`

[2] *Shelving filter,* `https://www.recordingblogs.com/wiki/shelving-filter`

[3] Jonathan S. Abel and David P. Berners, *Filter Design Using Second-Order Peaking and Shelving Sections, Proceedings ICMC 2004* (2004) `http://hdl.handle.net/2027/spo.bbp2372.2004.152`