Introduction to DSGE Models

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Dynare

- Computes the solution of deterministic models
- Computes first and second order approximation to solution of stochastic models
- Estimates parameters of DSGE models (with maximum likelihood or Bayesian approach)
- Computes optimal policy Performs global sensitivity analysis

We'll only focus on point 1 and 2. For the remaining points, you can refer to the Dynare user guide.

Deterministic models

- These models are usually introduced to study the impact of a change in regime, as in the introduction of a new tax, for instance.
- Models assume full information, perfect foresight and no uncertainty around shocks
- Shocks can hit the economy today or at any time in the future, in which case they would be expected with perfect foresight. They can also last one or several periods.
- Models introduce a positive shock today and zero shocks thereafter (with certainty).
- The solution does not require linearization, in fact, it doesnít even really need a steady state. Instead, it involves numerical simulation to find the exact paths of endogenous variables that meet the model's first order conditions and shock structure.
- This solution method can therefore be useful when the economy is far away from steady state (when linearization o§ers a poor approximation).

Deterministic Models (see Juillard 1998)

- We are in a perfect foresight framework, agents know about future shocks.
- The solution method is based on work of Lafargue, Boucekkine and Juillard.
- The approximation imposes return to equilibrium in finite time (instead of asymptotically) !
  - Approximation of an infinite horizon model by a finite horizon one
- Computes the trajectory of the variables numerically
- The algorithm is a Newton-type method (Juillard 1996)

Stochastic Models

- In these models, shocks hit today (with a surprise), but thereafter their expected value is zero.
- Expected future shocks, or permanent changes in the exogenous variables cannot be handled due to the use of Taylor approximations around a steady state.
- When these models are linearized to the 1st order, agents behave as if future shocks where equal to zero (since their expectation is null), which is the certainty equivalence property.
- This is an often overlooked point in the literature which misleads readers in supposing their models may be deterministic.

Stochastic case: the general model

$$\begin{aligned} E_t \left\{ f\left(y_{t+1}, y_t, y_{t-1}, u_t\right) \right\} &= 0 \\ E\left(u_t\right) &= 0 \\ E\left(u_t u_t'\right) &= \Sigma_u \\ E\left(u_t u_s'\right) &= 0 \, \forall t \neq s \end{aligned}$$

where y_t is the vector of endogenous variables and u_t the one of exogenous stochastic shocks.

- shocks u_t are observed at the beginning of period t
- not all variables are necessarily present with a lead and a lag
- decisions affecting the current value of the variables y_t are function of the previous state of the system, y_{t-1} and the shocks.

A stochastic scale variable

At period t, the only stochastic variable is y_{t+1} and u_{t+1}.
We introduce the stochastic scale variable σ and auxiliary random variable ϵ_t such that:

$$\begin{aligned} u_{t+1} &= \sigma \epsilon_{t+1} \\ E\left(\epsilon_t\right) &= 0 \\ E\left(\epsilon_t \epsilon_t'\right) &= \Sigma_\epsilon \\ E\left(\epsilon_t \epsilon_t'\right) &= 0 \, \forall t \neq s \\ \Sigma_u &= \sigma^2 \Sigma_\epsilon \end{aligned}$$

Solution Function

$$y_t = g\left(y_{t-1}, u_t, \sigma\right)$$

If $\sigma = 0$, the model is deterministic. One can prove the existence of function g and the conditions (see Jin and Judd, "Solving Dynamic Stochastic Models"). Then:

$$y_{t+1} = g\left(y_t, u_{t+1}, \sigma\right)$$
$$y_{t+1} = g\left(g\left(y_{t-1}, u_t, \sigma\right), u_{t+1}, \sigma\right)$$

$$F\left(y_{t-1}, u_t, u_{t+1}, \sigma\right) = f\left(g\left(g\left(y_{t-1}, u_t, \sigma\right), u_{t+1}, \sigma\right), g\left(y_{t-1}, u_t, \sigma\right), y_{t-1},\right.$$

$$E_t\left\{F\left(y_{t-1}, u_t, \sigma\epsilon_{t+1}, \sigma\right)\right\} = 0$$

Perturbation method

- We need to obtain a Taylor expansion of the unknown solution function in the neighborhood of a problem we can solve, i.e., the steady state.
- The perturbations can be in the neighborhood of the ss OR increasing σ from zero.
- The Taylor approximation of the solution is taken with respect to y_{t-1}, u_t and σ

$$y_t = g\left(y_{t-1}, u_t, \sigma\right)$$

Deterministic Steady State

- The Taylor approximation is taken with respect to the deterministic steady-state, i.e. the one we obtain in absenc e of shocks. It satisfies:

$$
\begin{aligned}
f\left(\bar{y}, \bar{y}, \bar{y}, 0\right) = 0 \\
\bar{y} = g\left(\bar{y}, 0, 0\right)
\end{aligned}
$$

Clearly, there can be multiple ss. but the approximation is taken w.r.t. one only.

Solution

- Dynare takes the first and second order Taylor approximation of the solution function
- It creates a structural state space representation (matrix)
- It computes the Schur decomposition.
- It checks the Blanchard-Kahn conditions.
- It selects the stable trajectory.
- More details (see Juillard's material).

Writing a ".mod" file

- preamble: lists variables and parameters
- model: spells out the model
- steady state or initial value: gives indications to find the steady-state of a model or the starting point for simulations or impulse response functions based on the model's solution
- shocks: defines the shocks to the system
- computation: instructs Dynare to undertake specific operations (e.e. forecasting estimating impulse response functions)

Writing a ".mod" file (2)

- Each instruction of the model must be terminated by a semi-colon (;), although a single instructions can span two lines, if you need extra space (just don't put a semi-colon)
- You can comment a line with forward slashes (//) or an entire section with /*, */

Example (from Collard 2001): stochastic case

- Firms are producing a homogeneous final product that can be either consumed or invested by means of capital and labor services. Firms own their capital stock and hire labor supplied by the households. Households own the firms. In each and every period three perfectly competitive markets open 0 the markets for consumption goods, labor services, and financial capital in the form of firm's shares.
- Utility function:

$$E_t \sum \beta \left(\ln c_t - \theta \frac{h_t^{1+\psi}}{1+\psi} \right)$$

where $0 < \beta < 1$ is a constant discont factor, c_t is consumption in period t, h_t is the fraction of total available time dvoted to productive activity in period t, $\theta > 0$ and $\psi > 0$.

An example (cont'd)

There exists a central planner that maximizes the household's utility functino subject to the following budget constraint:

$$c_t + i_t = y_t$$

Investment is used to form physical capital, which accumulates in the standard form as:

$$k_{t+1} = e^{b_t} i_t + (1-\delta)k_t,\ 0 < \delta < 1$$

where b_t is a shock affecting incorporated technological progress. Output is produced by means of capital and labour services, relying on a constant returns to scale technology, represented by the following Cobb-Douglas production function ($0 < \alpha < 1$):

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$
$$A_t = e^{a_t}$$

Dynare

- We assume that the shocks to technology are distributed with zero mean but display both persistence across time and correlation in the current period. Let us consider the joint process (a_t, b_t) defined as:

$$\begin{pmatrix} a_t \\ b_t \end{pmatrix} = \begin{pmatrix} \rho & \tau \\ \tau & \rho \end{pmatrix} \begin{pmatrix} a_{t-1} \\ b_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t \\ v_t \end{pmatrix}$$

- where:

$$\begin{aligned} E(\epsilon_t) = E(v_t) &= 0 \\ E(\epsilon_t \epsilon_s) &= 0, \; t \neq s \\ E(v_t v_s) &= 0, \; t \neq s \\ E(\epsilon_t v_s) &= 0, \; t \neq s \end{aligned}$$

Equilibrium equations

- Arbitrage consuption/labour effort:
- $$c_t \theta h_t^{1+\psi} = (1-\alpha) y_t$$
- Euler:

$$\beta E_t \left[\left(\frac{e^{b_t} c_t}{e^{b_{t+1}} c_{t+1}} \right) \left(e^{b_{t+1}} \right) \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right] = 1$$

$$y_t = e^{a_t} k_t^{\alpha} h_t^{1-\alpha}$$

$$k_{t+1} = e^{b_t} (y_t - c_t) + (1-\delta) k_t$$

$$a_t = \rho a_{t-1} + \tau b_{t-1} + \epsilon_t$$
$$b_t = \tau a_{t-1} + \rho b_{t-1} + \nu_t$$

Preamble

- endogenous variables: "var y, c, k, a, h, b;"
- exogenous variables: "varexo e, u;"
- the list of parameters: "parameters beta, rho, alpha, delta, theta, psi, tau;"
- Parameters values:

```
"alpha = 0.36;" (capital elasticity in the production fu
"rho = 0.95;" (shock persistence)
"tau = 0.025;"(cross persistence)
"beta = 0.99;"(discount factor)
"delta = 0.025;"(capital depreciation rate)
"psi = 0;"(labor supply elasticity)
"theta = 2.95;" (disutility of labor)
"phi = 0.1;"(not in the model but useful to express the
```

The model

- Model declaration: It starts with the instruction model; and ends with end;, in between all equilibrium conditions are written exactly the way we write it by hand.
- There need to be as many equations as you declared endogenous variables.
- Equations are entered one after the other; no matrix representation is necessary. Variable and parameter names used in the model block must be the same as those declared in the preamble; variable and parameter names are case sensitive.

The model: timing conventions

- If x is *decided* in period t then we simply write x.
- When the variable is decided in t-1, such as the capital stock in our simple model, we write x(-1).
- When a variable is decided in the next period, $t + 1$, such as consumption in the Euler equation, we write $x(+1)$.

The model

```
model;
c*theta*h^(1+psi)=(1-alpha)*y;
k = beta*(((exp(b)*c)/(exp(b(+1))*c(+1)))
*(exp(b(+1))*alpha*y(+1)+(1-delta)*k));
y = exp(a)*(k(-1)^alpha)*(h^(1-alpha));
k = exp(b)*(y-c)+(1-delta)*k(-1);
a = rho*a(-1)+tau*b(-1) + e;
b = tau*a(-1)+rho*b(-1) + u;
end;
```

The model (for stochastic models)

- initval;
- y = 1.08068253095672; c = 0.80359242014163; h = 0.29175631001732; k = 11.08360443260358; a = 0; b = 0; e = 0; u = 0; end;
- Adding steady just after your initval block will instruct Dynare to consider your initial values as mere approximations and start simulations or impulse response functions from the exact steady state.
- You need to enter a "good" steady state. Dynare can help in finding your model's steady state by calling the appropriate Matlab functions.

The model: the check command

after the initval or endval block (following the steady command if you decide to add one): "check" command. This computes and displays the eigenvalues of your system which are used in the solution method. As mentioned earlier, a necessary condition for the uniqueness of a stable equilibrium in the neighborhood of the steady state is that there are as many eigenvalues larger than one in modulus as there are forward looking variables in the system.

Shocks (stochastic)

- Can only be temporary and hit the system today (expectation of future shocks must be zero).
- We can make the efect of the shock propagate slowly throughout the economy by introducing a latent shock variable ϵ_t and ν_t that affects the model's true exogenous variable, a_t and b_t ,AR(1)
- In that case, though, we would declare a_t and b_t as endogenous variables and ϵ_t and ν_t as exogenous variables

```
shocks;
var e; stderr 0.009;
var u; stderr 0.009;
var e, u = phi*0.009*0.009;
end;
```

Computation (stochastic)

- stoch_simul is usually the appropriate command
- it computes a Taylor approximation of the decision and transition functions for the model (the equations listing current values of the endogenous variables of the model as a function of the previous states of the model and current shocks), impulse response functions and various descriptive statistics (moments, variance decomposition, correlation and autocorrelation coefficients)

Computation (stochastic)

Main options (see the Manual)

- irf=INTEGER: number of periods on which to compute the IRFS (default=40). Setting IRF=0, suppresses the plotting of IRF's
- order=1 or 2: order of Taylor approximation (default=2), unless you're working with a linear model, in which case order is automatically set to 1.

Output (stochastic)

- Model summary: a count of the various variable types in your model (endogenous, jumpers, etc...)
- If you use the check command: eigenvalues and a confirmation of the Blanchard-Kahn conditions
- Matrix of covariance of exogenous shocks (consistent with the input on the shock)
- Policy and transition functions
- Moments of simulated variables (up to fourth moments)
- Correlation of simulated variables (contemporaneous)
- Autocorrelation of simulated variables: up to the f-th lag as specified in the options of stoch simul.

Deterministic model, shock

- $A_t \neq e^{a_t}$, $\epsilon_t = 0$, $v_t = 0$

  - The economy is at the steady state (A=1)

  - There is an unexpected drop in TFP of 10% at the beginning of period 1

  - Deterministic shocks are described in shocks block

Deterministic model, shock

```
initval;
A=1;
y =to be computed;
c = to be computed;
h = to be computed;
k = to be computed;
a = 0;
b = 0;
end;
steady;
shocks;
var A;
periods 1;
values 0.9;
end;
```

Deterministic model, shock

- the economy is at the steady-state

- TFP jumps by 4% in period 5 and grows by 1% during the 4 following periods

```
shocks;
var A;
periods 4, 5, 6, 7, 8;
values 1.04, 1.05, 1.06, 1.07, 1.08;
end;
```

Deterministic model, shock

- The economy is at the initial steady state (A=1)

- In period 1, TFP jumps to 1.05, permanently

Deterministic model, shock

```
model..remember that the exo variable is here A.
end;
initval;
A=1; ...initial steady state..
end;
steady;
endval;
A=1.05; ..new steady state, to be computed.. end;
```

Deterministic model, output

- Not very detailed
- Steady state values, if command steady
- Eigenvalues, if check
- Some intermediate output: the errors at each iteration of the Newton solver used to estimate the solution to your model
- Sou need custom codeif you want richer statistics