Small Open Economy Extension (IRBC)

Macro II - Fluctuations - ENSAE, 2024-2025

Pablo Winant

2025-03-19

Introduction and Basic Facts

Why a small open economy?

What are the classical reasons to open economy to trade?

- trade integration

Why a small open economy?

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- trade integration
  - taste for variety
  - comparative advantage
- financial integration
  - smooth shock / insurance

From RBC to IRBC

RBC models have been very successful at matching Business Cycles

- (temporary) victory against keynesian view that short term fluctuations result from demand shocks
- so successful that facts at odd with theoretical predictions have been called "puzzles"

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Seminal Paper:

- *International Real Business Cycles*, Backus, Kehoe, Kydland (1992) (freshwater economists)

Very successful methodology:

- facts at odd with theoretical predictions have been called "puzzles"

IRBC Facts

Properties of Business Cycles in OECD Economies

	Std. Dev. (%)		Ratio of Standard Deviation to that of y					Autocorr. y	Correlation with Output					
Country	y	nx	c	x	g	n	z	y	c	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	−.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	−.24	−.46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	−.23	−.26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	−.30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	−.11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	−.68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	−.02	−.22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	−.68	.84	.93
U.K.	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	−.19	.47	.90
U.S.	1.92	.52	.75	3.27	.75	.61	.68	.86	.82	.94	.12	−.37	.88	.96
Europe	1.01	.50	.83	2.09	.47	.85	.98	.75	.81	.89	.10	−.25	.32	.85

Notes: Statistics are based on Hodrick-Prescott filtered data. Variables are: y, real output; c, real consumption; x, real fixed investment; g, real government purchases; nx, ratio of net exports to output, both at current prices; n, civilian employment; z, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:1 to 1990:2.

Figure 1: Moments

From Kehoe,Kydland (1995)

IRBC Facts

Properties of Business Cycles in OECD Economies

Country	Std. Dev. (%) y	nx	Ratio of Standard Deviation to that of y: c	x	g	n	z	Autocorr. y: y	Correlation with Output: c	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	−.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	−.24	−.46	.58	.65
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Figure 2: Moments

International Comovements in OECD Economies

Country	Correlation with Same U.S. Variable: y	c	x	g	n	z
Australia	.51	−.19	.16	.23	−.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	−.01	−.01	.53	.75
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United Kingdom	.55	.42	.40	−.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Notes: See Table 1.

Figure 3: Comoments

Stylized Facts

Properties of Business Cycles in OECD Economies

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Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical

International Comovements in OECD Economies

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Figure 5: Comoments

Domestically:

- output more variable than consumption
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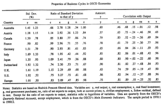
- productivity strongly procyclical
- trade balance strongly countercyclcal

Internationally:

Stylized Facts



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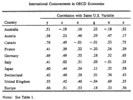

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- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output

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Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	−.23	−.26	.69	.84
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Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	−.02	−.22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	−.68	.84	.93
U.K.	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	−.19	.47	.90
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Europe	1.01	.50	.83	2.09	.47	.85	.98	.75	.81	.89	.10	−.25	.32	.85

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Figure 4: Moments

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Country	Correlation with Same U.S. Variable: y	c	x	g	n	z
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France	.41	.39	.22	−.20	.26	.39
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Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	−.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Notes: See Table 1.

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Can we replicate these moments with a BC model?

Modeling a Small Open Economy

Endowment model

Representative agents maximizes:

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + a_t \leq y_t + (1+r)a_{t-1}$$

Endowment economy:

- income $(y_t)_t$ is exogenously given
- for simplicity we assume it is deterministic

Small open economy:

- *open*: can save a_t which yields $a_{t+1}(1+r)$ in the next period
- *small*: country takes world interest rate r as given (no effect on world prices)

We solve this problem with the terminal conditions:

- a_{-1} given[1]
- $\lim \quad a_T \quad \geq 0$

Endowment model (3)

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t \left(y_t + (1+r)a_{t-1} - c_t - a_t \right)$$

First order conditions:

$$\begin{aligned} u'(c_t) &= \lambda_t && (1) \\ \lambda_t &= \beta(1+r)\lambda_{t+1} && (2) \end{aligned}$$

Under the *technical assumption* $\beta(1+r) = 1$ we get $c_t = c_{t+1}$ then

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

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Current Account

Reminders on Current Account

The **trade balance** is exports-imports (here $y_t - c_t$)
The **current account** is trade balance + net factor payments (here $y_t - c_t + ra_{t-1}$)
Positive **current account**: additional lending to the rest of the world.

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Positive **current account**: additional lending to the rest of the world.

Using the formula from before

$$CA_0 = a_{-1}r + (1 - \frac{r}{1+r})y_0 - \frac{r}{1+r}\left\{\sum_{t\geq 1}^{\infty} \frac{y_t}{(1+r)^t}\right\}$$

How does the current account reacts to income shocks?

- current account responds positively to *temporary* shock in income
- and to news about future income shocks:

Unit root

Still with the same formula:

$$c_0 = \frac{r}{1+r}\left\{(1+r)a_{-1} + \sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t}\right\}$$

What is the effect of an increase in a_{-1}?

```
- consumption rises permanently
- $a_t$ is constant, equal to $a_{-1}$
- agent consumes small amount $r$ corresponding to interest
```

- this will correspond to a unit root in the solution

Adding capital

We add capital and production to our endowment economy:

$$y_t = z_t k_{t-1}^{\alpha}$$

$$k_t = (1-\delta)k_{t-1} + i_t$$

The aggregate resource constraint becomes:

$$a_t + c_t + i_t = (1+r)a_{t-1} + y_t$$

Now maximize $\sum_t \beta^t U(c_t)$

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$$\lambda_t = \beta\lambda_{t+1}(1+r)$$

$$\lambda_t = \beta\lambda_{t+1}\left[(1-\delta) + z_{t+1}f'(k_t)\right]$$

where λ_t is lagrange multiplier associated to budget constraint.

Adding capital: optimality conditions

Since $\lambda_t > 0$ (constraint is always binding), we get:

$$(1-\delta) + z_{t+1} f'(k_t) = 1 + r$$

$$k_t = \left(\frac{r+\delta}{\alpha z_{t+1}}\right)^{\frac{1}{\alpha-1}}$$

and investment

$$i_t = \left(\frac{r+\delta}{\alpha z_{t+1}}\right)^{\frac{1}{\alpha-1}} - (1-\delta)\left(\frac{r+\delta}{\alpha z_t}\right)^{\frac{1}{\alpha-1}}$$

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Here investment is fully determined by productivity shocks

- too simple: no international dependence

Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$a_t + c_t + i_t + \frac{\omega}{2}\frac{(k_t - k_{t-1})^2}{k_t} = (1+r)a_{t-1} + zf(k_{t-1})$$

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Cf tutorial.

A benchmark Small Open Economy Model

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Figure 6: Stephanie Schmitt Grohe and Martin Uribe

Closing Small Economy Models, Schmitt Grohe and Uribe (2003), JIE

- small open economy model with production, consumption-leisure tradeoff and capital adjustment costs
  - = RBC+open+adj costs
- perform some moments matching
- compare different ways of stationarizing

The model

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

$$c_t + k_t + a_t = y_t + g_t - \frac{\omega}{2}(k_t - k_{t-1})^2 + (1-\delta)k_{t-1} + (1 + r^\star + \pi(a_{t-1}))a_{t-1}$$

$$y_t = f(k_{t-1}, n_t, z_t)$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

and

$$u(c, n) = \frac{1}{1-\sigma}\left(c^\psi (1-n)^{1-\psi}\right)^{1-\sigma}$$

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and

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The term π is there to make the model stationary.

How to make the distribution stationary?

The solution of the model exhibits a unit root:

$$a_t = a_{t-1} + \text{...other variables in t-1} + \text{shocks in t}$$

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Problem:

- there isn't a unique deterministic steady-state
- the ergodic distribution of the model variables is not defined

This raises practical issues (notably for estimation) for the *linear* model.

- no unconditional moments

How to get rid of the unit root?

General idea:

- introduce a force that pulls the level of foreign assets towards equilibrium

Schmitt Grohe and Uribe (2003) consider many options:

- debt-elastic interest rate:

$$1 + r = 1 + r^{\star} + \pi(a_d)$$

  - with $\pi(0) = 0$ and $\pi'(0) > 0$
  - π can be understood as a risk premium on rising debt

- endogenous time-discount (aka Usawa preferences)

$$\beta(c_t) = (1 + c_t)^{-\chi}$$

- costs of adjustment for international portfolios

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SGU show that the choice of the stationarization device has little effect for the dynamics (moments) of most variables

Calibration

Parameters	Values
σ	2
ψ	1.45
α	0.32
ω	0.028
r	0.04

Parameters	Values
δ	0.1
ρ	0.42
σ^2	0.0129
$A^\star$	-0.7442
χ	0.000742

Results

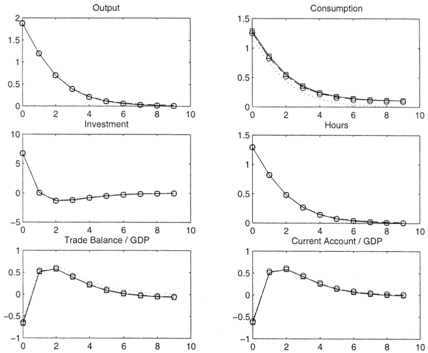

Fig. 1. Impulse response to a unit technology shock in Models 1–5. Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashed line: Debt-elastic interest rate model; Dash–dotted line: Portfolio adjustment cost model; Dotted line: complete asset markets model; Circles: Model without stationarity inducing elements.

Figure 7: Impulse Response Function

Table 3
Observed and implied second moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
Volatilities:						
std(y_t)	2.8	3.1	3.1	3.1	3.1	3.1
std(c_t)	2.5	2.3	2.3	2.7	2.7	1.9
std(i_t)	9.8	9.1	9.1	9	9	9.1
std(h_t)	2	2.1	2.1	2.1	2.1	2.1
std$\left(\frac{tb_t}{y_t}\right)$	1.9	1.5	1.5	1.8	1.8	1.6
std$\left(\frac{ca_t}{y_t}\right)$		1.5	1.5	1.5	1.5	
Serial correlations:						
corr(y_t, y_{t-1})	0.61	0.61	0.61	0.62	0.62	0.61
corr(c_t, c_{t-1})	0.7	0.7	0.7	0.78	0.78	0.61
corr(i_t, i_{t-1})	0.31	0.07	0.07	0.069	0.069	0.07
corr(h_t, h_{t-1})	0.54	0.61	0.61	0.62	0.62	0.61
corr$\left(\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}}\right)$	0.66	0.33	0.32	0.51	0.5	0.39
corr$\left(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}\right)$		0.3	0.3	0.32	0.32	
Correlations with output:						
corr(c_t, y_t)	0.59	0.94	0.94	0.84	0.85	1
corr(i_t, y_t)	0.64	0.66	0.66	0.67	0.67	0.66
corr(h_t, y_t)	0.8	1	1	1	1	1
corr$\left(\frac{tb_t}{y_t}, y_t\right)$	−0.13	−0.012	−0.013	−0.044	−0.043	0.13
corr$\left(\frac{ca_t}{y_t}, y_t\right)$		0.026	0.025	0.05	0.051	

Note. The first column was taken from Mendoza (1991). Standard deviations are measured in percent per year.

Figure 8: Moments (from SGU)

Conclusions

- The model matches unconditional correlations fairly well
  - The stationarization device has little effect on the moments
- Unconditional correlations are not that great
  - a limitation of the moment matching method?
- Correlation of consumption with output is too high
  - and probably cross-correlation of consumption too low
  - still the Backus-Kehoe-Kydland puzzle...