Small Open Economy Extension (IRBC)

Macro II - Fluctuations - ENSAE, 2023-2024

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Introduction and Basic Facts

Why a small open economy?

What are the classical reasons to open economy to trade

- trade integration

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  - taste for variety

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What are the classical reasons to open economy to trade

- trade integration
  - taste for variety
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- financial integration
  - smooth shock / insurance

From RBC to IRBC

After the success of RBC models to match business cycles it didn't take long before the same methodology was applied to International Business Cycles

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Seminal Paper:

- *International Real Business Cycles*, Backus, Kehoe, Kydland (1992) (freshwater economists)

Very successful methodology:

- facts at odd with theoretical predictions have been called "puzzles"

IRBC Facts

Properties of Business Cycles in OECD Economies

	Std. Dev. (%)		Ratio of Standard Deviation to that of y					Autocorr. y	Correlation with Output					
Country	y	nx	c	x	g	n	z	y	c	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	−.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	−.24	−.46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	−.23	−.26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	−.30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	−.11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	−.68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	−.02	−.22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	−.68	.84	.93
U.K.	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	−.19	.47	.90
U.S.	1.92	.52	.75	3.27	.75	.61	.68	.86	.82	.94	.12	−.37	.88	.96
Europe	1.01	.50	.83	2.09	.47	.85	.98	.75	.81	.89	.10	−.25	.32	.85

Notes: Statistics are based on Hodrick-Prescott filtered data. Variables are: y, real output; c, real consumption; x, real fixed investment; g, real government purchases; nx, ratio of net exports to output, both at current prices; n, civilian employment; z, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:1 to 1990:2.

Figure 1: Moments

From Kehoe,Kydland (1995)

IRBC Facts

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Figure 2: Moments

International Comovements in OECD Economies

	Correlation with Same U.S. Variable					
Country	y	c	x	g	n	z
Australia	.51	−.19	.16	.23	−.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	−.01	−.01	.53	.75
France	.41	.39	.22	−.20	.26	.39
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Italy	.41	.02	.31	.09	−.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	−.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Notes: See Table 1.

Figure 3: Comoments

Stylized Facts

Properties of Business Cycles in OECD Economies

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Internationally:

Stylized Facts

Properties of Business Cycles in OECD Economies

Country	Std. Dev. (%) y	Std. Dev. (%) nx	Ratio of Standard Deviation to that of y: c	x	g	n	z	Autocorr. y: y	Correlation with Output: c	x	g	nx	n	z
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Domestically:

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Figure 4: Moments

Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical

International Comovements in OECD Economies

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Figure 5: Comoments

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- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output

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Europe	1.01	.50	.83	2.09	.47	.85	.98	.75	.81	.89	.10	−.25	.32	.85

Notes: Statistics are based on Hodrick-Prescott filtered data. Variables are: y, real output; c, real consumption; x, real fixed investment; g, real government purchases; nx, ratio of net exports to output, both at current prices; n, civilian employment; z, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:1 to 1990:2.

Figure 4: Moments

International Comovements in OECD Economies

Country	Correlation with Same U.S. Variable					
	y	c	x	g	n	z
Australia	.51	−.19	.16	.23	−.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	−.01	−.01	.53	.75
France	.41	.39	.22	−.20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	−.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	−.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Notes: See Table 1.

Figure 5: Comoments

Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output

Internationally:

- smaller comovements in consumption

Stylized Facts

Properties of Business Cycles in OECD Economies

Country	Std. Dev. (%) y	nx	Ratio of Standard Deviation to that of y c	x	g	n	z	Autocorr. y	Correlation with Output c	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	−.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.56	1.23	.84	.57	.65	.75	−.24	−.46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	−.23	−.26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	−.30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	−.11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	−.68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	−.02	−.22	.60	.98
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Internationally:

- smaller comovements in consumption
    - Backus-Kehoe-Kydland puzzle

Modeling a Small Open Econmomy

Endowment model

Take an endowment economy: income $(y_t)_t$ is exogenously given. We assume it is deterministic

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + a_{t+1} \leq y_t + (1+r)a_t$$

Country takes world interest rate r as given

- a small open economy doesn't affect world prices

Endowment model (2)

We solve this problem with the terminal conditions:

- a_0 given
- $\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} \geq 0$
  - *no-ponzi* condition

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- $\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} \geq 0$
  - *no-ponzi* condition

The no-ponzi condition will in effect eliminate diverging solutions.
In a first order approximation, it selects the right eigenvalues.

Endowment model (3)

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t \left(y_t + (1+r)a_t - c_t + a_{t+1} \right)$$

First order conditions:

$$u'(c_t) = \lambda_t \quad (1)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (2)$$

Under the *technical assumption* $\beta(1+r) = 1$ we get:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

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- problem isomorphic to consumption-savings decisions

Current Account

Reminders on Current Account

The **trade balance** is exports-imports (here $y_t - c_t$)
The **current account** is trade balance + net factor payments (here $y_t - c_t + ra_t$)
Positive **current account**: additional lending to the rest of the world.

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Positive **current account**: additional lending to the rest of the world.

Using the formula from before

$$CA_0 = a_0 r + (1 - \frac{r}{1+r}) y_0 + \frac{r}{1+r} \left\{ \sum_{t \geq 1}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

How does the current account reacts to income shocks?

- current account responds positively to *temporary* shock in income
- and to news about future income shocks:

Unit root

Still with the same formula:

$$c_0 = \frac{r}{1+r}\left\{(1+r)a_0 + \sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t}\right\}$$

What is the effect of an increase in a_0?

- consumption rises permanently
  - by small amount r corresponding to interests paid forever on a_0
- this will correspond to a unit root in the solution

Exercise

From the first order conditions

$$u'(c_t) = \lambda_t \quad (3)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (4)$$

assuming $u(c_t) = \log(c_t)$, can you get the equation for the law of motion of a_t and show the presence of a unit root?

Adding capital

We add capital and production to our endowment economy:

$$y_t = z_t k_t^{\alpha}$$

$$k_t = (1-\delta)k_{t-1} + i_{t-1}$$

The aggregate resource constraint becomes:

$$a_{t+1} + c_t + i_t = (1+r)a_t + y_t$$

Now maximize $\sum_t \beta^t U(c_t)$

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Now maximize $\sum_t \beta^t U(c_t)$

We get first order conditions

$$\lambda_t = \beta\lambda_{t+1}(1+r)$$

$$\lambda_t = \beta\lambda_{t+1}\left[(1-\delta) + z_{t+1}f'(k_{t+1})\right]$$

where λ_t is lagrange multiplier associated to budget constraint.

Adding capital: optimality conditions

Since λ_t (constraint is always binding), we get:

$$(1-\delta) + z_{t+1} f'(k_{t+1}) = 1 + r$$

$$k_{t+1} = \left(\frac{r+\delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha - 1}}$$

and investment

$$i_t = \left(\frac{r+\delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha - 1}} - (1-\delta) \left(\frac{r+\delta}{\alpha z_t} \right)^{\frac{1}{\alpha - 1}}$$

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Here investment is fully determined by productivity shocks

- too simple: no international dependence

Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$a_{t+1} + c_t + i_t + \frac{\omega}{2}\frac{(k_{t+1} - k_t)^2 - \delta k^\star)^2}{k_t} = (1+r)a_t + zf(k_t)$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

where ω is an adjustment friction. Typically, ω is chosen so that the model replicates $\frac{Var(i_t)}{Var(y_t)}$ from the data.

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Cf tutorial.

A benchmark Small Open Economy Model

A benchmark Small Open Economy Model

Figure 6: Stephanie Schmitt Grohe and Martin Uribe

Closing Small Economy Models, Schmitt Grohe and Uribe (2003), JIE

- small open economy model with production, consumption-leisure tradeoff and capital adjustment costs
  - $=$ RBC$+$open$+$adj costs
- perform some moments matching
- compare different ways of stationarizing the model

The model

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + k_{t+1} + a_{t+1} = y_t + g_t - \frac{\omega}{2}(k_{t+1} - k_t)^2 + (1-\delta)k_t + (1 + r^{\star} + \pi(a_t))a_t$$

$$y_t = f(k_t, n_t, z_t)$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

and $u(c,n) = \frac{1}{1-\sigma}\left(c^{\psi}(1-n)^{1-\psi}\right)^{1-\sigma}$

How to make the distribution stationary?

The solution of the model exhibits a unit root:

$$a_t = a_{t-1} + \text{...other variables in t-1} + \text{shocks in t}$$

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Problem:

- there isn't a unique deterministic steady-state
- the ergodic distribution of the model variables is not defined

This raises practical issues (notably for estimation) for the *linear* model.

- no unconditional moments

How to get rid of the unit root?

General idea:

- introduce a force that pulls the level of foreign assets towards equilibrium

Schmitt Grohe and Uribe (2003) consider many options:

- debt-elastic interest rate:

$$1 + r = 1 + r^{\star} + \pi(a_d)$$

  - with $\pi(0) = 0$ and $\pi'(0) > 0$
  - π can be understood as a risk premium on rising debt

- endogenous time-discount (aka Usawa preferences)

$$\beta(c_t) = (1 + c_t)^{-\chi}$$

- costs of adjustment for international portfolios

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- costs of adjustment for international portfolios

SGU show that the choice of the stationarization device has little effect for the dynamics (moments) of most variables

Calibration

Parameters	Values
σ	2
ψ	1.45
α	0.32
ω	0.028
r	0.04

Parameters	Values
δ	0.1
ρ	0.42
σ^2	0.0129
$A^\star$	-0.7442
χ	0.000742

Results

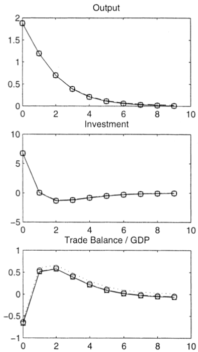

Fig. 1. Impulse response to a unit technology shock in Models 1–5. Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashed line: Debt-elastic interest rate model; Dash–dotted line: Portfolio adjustment cost model; Dotted line: complete asset markets model; Circles: Model without stationarity inducing elements.

Figure 7: Impulse Response Function

Table 3
Observed and implied second moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
Volatilities:						
$\mathrm{std}(y_t)$	2.8	3.1	3.1	3.1	3.1	3.1
$\mathrm{std}(c_t)$	2.5	2.3	2.3	2.7	2.7	1.9
$\mathrm{std}(i_t)$	9.8	9.1	9.1	9	9	9.1
$\mathrm{std}(h_t)$	2	2.1	2.1	2.1	2.1	2.1
$\mathrm{std}\left(\frac{tb_t}{y_t}\right)$	1.9	1.5	1.5	1.8	1.8	1.6
$\mathrm{std}\left(\frac{ca_t}{y_t}\right)$		1.5	1.5	1.5	1.5	
Serial correlations:						
$\mathrm{corr}(y_t, y_{t-1})$	0.61	0.61	0.61	0.62	0.62	0.61
$\mathrm{corr}(c_t, c_{t-1})$	0.7	0.7	0.7	0.78	0.78	0.61
$\mathrm{corr}(i_t, i_{t-1})$	0.31	0.07	0.07	0.069	0.069	0.07
$\mathrm{corr}(h_t, h_{t-1})$	0.54	0.61	0.61	0.62	0.62	0.61
$\mathrm{corr}\left(\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}}\right)$	0.66	0.33	0.32	0.51	0.5	0.39
$\mathrm{corr}\left(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}\right)$		0.3	0.3	0.32	0.32	
Correlations with output:						
$\mathrm{corr}(c_t, y_t)$	0.59	0.94	0.94	0.84	0.85	1
$\mathrm{corr}(i_t, y_t)$	0.64	0.66	0.66	0.67	0.67	0.66
$\mathrm{corr}(h_t, y_t)$	0.8	1	1	1	1	1
$\mathrm{corr}\left(\frac{tb_t}{y_t}, y_t\right)$	−0.13	−0.012	−0.013	−0.044	−0.043	0.13
$\mathrm{corr}\left(\frac{ca_t}{y_t}, y_t\right)$		0.026	0.025	0.05	0.051	

Note. The first column was taken from Mendoza (1991). Standard deviations are measured in percent per year.

Figure 8: Moments (from SGU)

Conclusions

- The model matches unconditional correlations fairly well
  - The stationarization device has little effect on the moments
- Unconditional correlations are not that great
  - a limitation of the moment matching method?
- Correlation of consumption with output is too high
  - and probably cross-correlation of consumption too low
  - still the Backus-Kehoe-Kydland puzzle...