

Small Open Economy Extension (IRBC)

Macro II - Fluctuations - ENSAE, 2023-2024

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Introduction and Basic Facts

Why a small open economy?

What are the classical reasons to open economy to trade

- ▶ trade integration

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What are the classical reasons to open economy to trade

- ▶ trade integration
 - ▶ taste for variety
 - ▶ comparative advantage
- ▶ financial integration
 - ▶ smooth shock / insurance

From RBC to IRBC

After the success of RBC models to match business cycles it didn't take long before the same methodology was applied to International Business Cycles

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Seminal Paper:

- ▶ *International Real Business Cycles*, Backus, Kehoe, Kydland (1992) (freshwater economists)

Very successful methodology:

- ▶ facts at odd with theoretical predictions have been called “puzzles”

IRBC Facts

Properties of Business Cycles in OECD Economies

Country	Std. Dev. (%)		Ratio of Standard Deviation to that of y					Autocorr. y	Correlation with Output					
	y	nx	c	x	g	n	z		c	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	-.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	-.24	-.46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	-.23	-.26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	-.30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	-.11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	-.68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	-.02	-.22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	-.68	.84	.93
U.K.	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	-.19	.47	.90
U.S.	1.92	.52	.75	3.27	.75	.61	.68	.86	.82	.94	.12	-.37	.88	.96
Europe	1.01	.50	.83	2.09	.47	.85	.98	.75	.81	.89	.10	-.25	.32	.85

Notes: Statistics are based on Hodrick-Prescott filtered data. Variables are: y, real output; c, real consumption; x, real fixed investment; g, real government purchases; nx, ratio of net exports to output, both at current prices; n, civilian employment; z, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:1 to 1990:2.

Figure 1: Moments

From Kehoe, Kydland (1995)

IRBC Facts

Properties of Business Cycles in OECD Economies

Country	Year (1970-1990)	Business Cycles (1970-1990)					Recessions with longer than average duration				
		1	2	3	4	5	6	7	8	9	10
Australia	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
Austria	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
Canada	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
France	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
Germany	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
Italy	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
Japan	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
Switzerland	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
United Kingdom	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
Europe	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18

International Comovements in OECD Economies

Country	Correlation with Same U.S. Variable					
	y	c	x	g	n	z
Australia	.51	-.19	.16	.23	-.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	-.01	-.01	.53	.75
France	.41	.39	.22	-.20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	-.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	-.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Notes: See Table 1.

Figure 3: Comoments

Figure 2:
Moments

Stylized Facts

Properties of Domestic Cycles in OECD Economies

Country	Year	Ratio of Domestic Investment to GDP				Correlation with Output
		1970-79	1980-89	1990-99	2000-09	
Australia	1970-79	1.28	1.15	1.04	1.00	0.10
Austria	1970-79	1.28	1.15	1.04	1.00	0.10
Canada	1970-79	1.28	1.15	1.04	1.00	0.10
France	1970-79	1.28	1.15	1.04	1.00	0.10
Germany	1970-79	1.28	1.15	1.04	1.00	0.10
Italy	1970-79	1.28	1.15	1.04	1.00	0.10
Japan	1970-79	1.28	1.15	1.04	1.00	0.10
Netherlands	1970-79	1.28	1.15	1.04	1.00	0.10
Spain	1970-79	1.28	1.15	1.04	1.00	0.10
Sweden	1970-79	1.28	1.15	1.04	1.00	0.10
U.K.	1970-79	1.28	1.15	1.04	1.00	0.10
U.S.	1970-79	1.28	1.15	1.04	1.00	0.10
Europe	1970-79	1.28	1.15	1.04	1.00	0.10

Domestically:

► output more variable than consumption

Figure 4:
Moments

International Comovements in OECD Economies

Country	Correlation with U.S. Output			
	1970-79	1980-89	1990-99	2000-09
Australia	0.11	-0.10	-0.10	-0.10
Austria	0.11	-0.10	-0.10	-0.10
Canada	0.11	-0.10	-0.10	-0.10
France	0.11	-0.10	-0.10	-0.10
Germany	0.11	-0.10	-0.10	-0.10
Italy	0.11	-0.10	-0.10	-0.10
Japan	0.11	-0.10	-0.10	-0.10
Netherlands	0.11	-0.10	-0.10	-0.10
Spain	0.11	-0.10	-0.10	-0.10
Sweden	0.11	-0.10	-0.10	-0.10
U.K.	0.11	-0.10	-0.10	-0.10
U.S.	0.11	-0.10	-0.10	-0.10
Europe	0.11	-0.10	-0.10	-0.10

Internationally:

Figure 5:
Comovements

Stylized Facts

Properties of Domestic Cycles in OECD Economies

Country	Year	Ratio of Domestic Product to GDP				Correlation with Output			
		1	2	3	4	1	2	3	4
Australia	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Canada	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
France	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Germany	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Italy	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Japan	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Netherlands	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Sweden	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Switzerland	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
UK	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
USA	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Europe	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated

Figure 4:
Moments

International Comovements in OECD Economies

Country	Correlation with US GDP			
	1	2	3	4
Australia	1.0	1.0	1.0	1.0
Canada	1.0	1.0	1.0	1.0
France	1.0	1.0	1.0	1.0
Germany	1.0	1.0	1.0	1.0
Italy	1.0	1.0	1.0	1.0
Japan	1.0	1.0	1.0	1.0
Netherlands	1.0	1.0	1.0	1.0
Sweden	1.0	1.0	1.0	1.0
Switzerland	1.0	1.0	1.0	1.0
UK	1.0	1.0	1.0	1.0
USA	1.0	1.0	1.0	1.0
Europe	1.0	1.0	1.0	1.0

Internationally:

Figure 5:
Comoments

Stylized Facts

Properties of Business Cycles in OECD Economies

Country	Year	Ratio of Business Investment to GDP				Correlation with Output			
		1	2	3	4	1	2	3	4
Australia	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
Canada	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
France	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
Germany	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
Italy	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
Japan	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
Netherlands	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
Sweden	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
U.K.	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
U.S.	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8
Europe	1970-1979	1.28	1.15	1.18	1.08	0.8	0.8	0.8	0.8

Figure 4:
Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical

International Comovements in OECD Economies

Country	Correlation with U.S. Output			
	1	2	3	4
Australia	0.1	-0.1	-0.1	-0.1
Canada	0.1	-0.1	-0.1	-0.1
France	0.1	-0.1	-0.1	-0.1
Germany	0.1	-0.1	-0.1	-0.1
Italy	0.1	-0.1	-0.1	-0.1
Japan	0.1	-0.1	-0.1	-0.1
Netherlands	0.1	-0.1	-0.1	-0.1
Sweden	0.1	-0.1	-0.1	-0.1
U.K.	0.1	-0.1	-0.1	-0.1
U.S.	0.1	-0.1	-0.1	-0.1
Europe	0.1	-0.1	-0.1	-0.1

Figure 5:
Comovements

Internationally:

Stylized Facts

Properties of Economic Cycles in OECD Economies

Country	Year	Ratio of Domestic Product to GDP				Correlation with Output			
		1	2	3	4	1	2	3	4
Australia	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Canada	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
France	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Germany	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Italy	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Japan	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Netherlands	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Sweden	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Switzerland	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
UK	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
USA	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Europe	1970-1979	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Figure 4:
Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical

International Comovements in OECD Economies

Country	Year	Correlation with US GDP			
		1	2	3	4
Australia	1970-1979	0.1	0.1	0.1	0.1
Canada	1970-1979	0.1	0.1	0.1	0.1
France	1970-1979	0.1	0.1	0.1	0.1
Germany	1970-1979	0.1	0.1	0.1	0.1
Italy	1970-1979	0.1	0.1	0.1	0.1
Japan	1970-1979	0.1	0.1	0.1	0.1
Netherlands	1970-1979	0.1	0.1	0.1	0.1
Sweden	1970-1979	0.1	0.1	0.1	0.1
Switzerland	1970-1979	0.1	0.1	0.1	0.1
UK	1970-1979	0.1	0.1	0.1	0.1
USA	1970-1979	0.1	0.1	0.1	0.1
Europe	1970-1979	0.1	0.1	0.1	0.1

Figure 5:
Comovements

Internationally:

Stylized Facts

Properties of Business Cycles in OECD Economies

Country	1970-1980	Ratio of Business Cycles to GDP				Correlation with Output			
		1	2	3	4	1	2	3	4
Australia	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
Canada	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
France	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
Germany	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
Italy	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
Japan	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
UK	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
USA	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95
Europe	1.12	1.12	1.12	1.12	1.12	0.95	0.95	0.95	0.95

Figure 4:
Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

Internationally:

International Comovements in OECD Economies

Country	Correlation with US GDP			
	1	2	3	4
Australia	0.11	-0.10	-0.10	-0.10
Canada	0.11	-0.10	-0.10	-0.10
France	0.11	-0.10	-0.10	-0.10
Germany	0.11	-0.10	-0.10	-0.10
Italy	0.11	-0.10	-0.10	-0.10
Japan	0.11	-0.10	-0.10	-0.10
UK	0.11	-0.10	-0.10	-0.10
USA	0.11	-0.10	-0.10	-0.10
Europe	0.11	-0.10	-0.10	-0.10

Figure 5:
Comovements

Stylized Facts

	Est. Coef.		Adjusted R-squared		t-statistic		Correlation with Degree	
Country	1	2	1	2	1	2	1	2
Australia	1.64	1.73	0.19	0.19	1.40	1.40	-0.16	-0.16
Canada	1.10	1.10	0.19	0.19	1.40	1.40	-0.16	-0.16
France	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
Germany	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
Italy	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
Japan	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
Sweden	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
Switzerland	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
United Kingdom	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
United States	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
West Germany	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16
Yugoslavia	1.60	1.70	0.19	0.19	1.40	1.40	-0.16	-0.16

[illegible]

Figure 4:
Moments

Country	Correlation with Taxes U.S. Variable					
	γ	α	β	δ	λ	
Australia	.21	-.29	.28	.22	-.18	.22
Austria	.34	.20	.46	.29	.47	.17
Canada	.76	.49	-.81	-.51	.50	.73
France	.41	.39	.22	-.20	.16	.29
Germany	.49	.49	.30	.28	.32	.43
Italy	.41	.32	.31	.39	-.31	.33
Japan	.60	.44	.56	.11	.32	.58
Netherlands	.42	.48	.35	.35	.36	.43
United Kingdom	.53	-.42	.48	-.54	.49	.31
Sweden	.60	.31	.23	.18	.30	.56

Source: See Table 1.

Figure 5:
Comoments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

Internationally:

- ▶ smaller comovements in consumption

Stylized Facts

Country	No. of cycles (N)	National Business Indicator				Seasons	Correlation with Europe
		1	2	3	4		
Australia	1.04	1.12	1.08	1.00	1.00	1.00	0.93
Austria	1.08	1.13	1.18	1.00	1.00	1.00	0.93
Belgium	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Denmark	1.00	1.08	1.00	1.00	1.00	1.00	0.93
France	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Germany	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Greece	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Ireland	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Italy	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Japan	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Netherlands	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Portugal	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Spain	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Sweden	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Switzerland	1.00	1.08	1.00	1.00	1.00	1.00	0.93
UK	1.00	1.08	1.00	1.00	1.00	1.00	0.93
USA	1.00	1.08	1.00	1.00	1.00	1.00	0.93
Weighted average	1.00	1.08	1.00	1.00	1.00	1.00	0.93

Figure 4:
Moments

Country	Correlation with Δ Euro U.S. Variable				
	γ	α	β	δ	λ
Australia	.21	-.19	.18	-.18	.22
Austria	.34	.05	.46	.29	.47
Canada	.16	.49	-.81	-.51	.50
France	.41	.39	.52	-.20	.36
Germany	.46	.49	.33	.28	.52
Italy	.41	.82	.31	.39	.51
Japan	.60	.44	.56	.11	.50
Netherlands	.42	.48	.38	.36	.43
United Kingdom	.55	.42	.46	-.26	.49
Sweden	.46	.71	.73	.18	.55

Source: See Table 1.

Figure 5:
Comoments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

Internationally:

- ▶ smaller comovements in consumption
 - ▶ Backus-Kehoe-Kydland puzzle

Modeling a Small Open Economy

Endowment model

Take an endowment economy: income $(y_t)_t$ is exogenously given.
We assume it is deterministic

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + a_{t+1} \leq y_t + (1+r)a_t$$

Country takes world interest rate r as given

- ▶ a small open economy doesn't affect world prices

Endowment model (2)

We solve this problem with the terminal conditions:

▶ a_0 given

▶ $\lim_{T \rightarrow \infty} \frac{a_{T+1}}{(1+r)^T} \geq 0$

▶ *no-ponzi* condition

Endowment model (2)

We solve this problem with the terminal conditions:

▶ a_0 given

▶ $\lim_{T \rightarrow \infty} \frac{a_{T+1}}{(1+r)^T} \geq 0$

▶ *no-ponzi* condition

The no-ponzi condition will in effect eliminate diverging solutions. In a first order approximation, it selects the right eigenvalues.

Endowment model (3)

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t (y_t + (1+r)a_t - c_t - a_{t+1})$$

First order conditions:

$$u'(c_t) = \lambda_t \quad (1)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (2)$$

Under the *technical assumption* $\beta(1+r) = 1$ we get:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

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► problem isomorphic to consumption-savings decisions

Current Account

Reminders on Current Account

The **trade balance** is exports-imports (here $y_t - c_t$)

The **current account** is trade balance + net factor payments
(here $y_t - c_t + ra_t$)

Positive **current account**: additional lending to the rest of the world.

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Positive **current account**: additional lending to the rest of the world.

Using the formula from before

$$CA_0 = a_0 r + (1 - \frac{r}{1+r})y_0 - \frac{r}{1+r} \left\{ \sum_{t \geq 1}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

How does the current account reacts to income shocks?

- ▶ current account responds positively to *temporary* shock in income
- ▶ and to news about future income shocks:

Unit root

Still with the same formula:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

What is the effect of an increase in a_0 ?

- ▶ consumption rises permanently
 - ▶ by small amount r corresponding to interests paid forever on a_0
- ▶ this will correspond to a unit root in the solution

Exercise

From the first order conditions

$$u'(c_t) = \lambda_t \quad (3)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (4)$$

assuming $u(c_t) = \log(c_t)$, can you get the equation for the law of motion of a_t and show the presence of a unit root?

Adding capital

We add capital and production to our endowment economy:

$$y_t = z_t k_t^\alpha$$

$$k_t = (1 - \delta)k_{t-1} + i_{t-1}$$

The aggregate resource constraint becomes:

$$a_{t+1} + c_t + i_t = (1 + r)a_t + y_t$$

Now maximize $\sum_t \beta^t U(c_t)$

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We get first order conditions

$$\lambda_t = \beta \lambda_{t+1} (1 + r)$$

$$\lambda_t = \beta \lambda_{t+1} [(1 - \delta) + z_{t+1} f'(k_{t+1})]$$

where λ_t is lagrange multiplier associated to budget constraint.

Adding capital: optimality conditions

Since λ_t (constraint is always binding), we get:

$$(1 - \delta) + z_{t+1}f'(k_{t+1}) = 1 + r$$

$$k_{t+1} = \left(\frac{r + \delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha-1}}$$

and investment

$$i_t = \left(\frac{r + \delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha-1}} - (1 - \delta) \left(\frac{r + \delta}{\alpha z_t} \right)^{\frac{1}{\alpha-1}}$$

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Here investment is fully determined by productivity shocks

► too simple: no international dependence

Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$a_{t+1} + c_t + i_t + \frac{\omega (k_{t+1} - k_t)^2}{2 k_t} = (1 + r)a_t + z f(k_t)$$

$$k_{t+1} = (1 - \delta)k_t + i_t$$

where ω is an adjustment friction. Typically, ω is chosen so that the model replicates $\frac{Var(i_t)}{Var(y_t)}$ from the data.

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Cf tutorial.

A benchmark Small Open Economy Model

A benchmark Small Open Economy Model



Stephanie Schmitt-Grohe and Martin Uribe.

Figure 6: Stephanie Schmitt Grohe and Martin Uribe

Closing Small Economy Models,
Schmitt Grohe and Uribe
(2003), JIE

- ▶ small open economy model with production, consumption-leisure tradeoff and capital adjustment costs
 - ▶ = RBC+open+adj costs
- ▶ perform some moments matching
- ▶ compare different ways of stationarizing the model

The model

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + k_{t+1} + a_{t+1} = y_t + g_t - \frac{\omega}{2} (k_{t+1} - k_t)^2 + (1 - \delta)k_t + (1 + r^* + \pi(a_t))a_t$$

$$y_t = f(k_t, n_t, z_t)$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

$$\text{and } u(c, n) = \frac{1}{1-\sigma} \left(c^\psi (1-n)^{1-\psi} \right)^{1-\sigma}$$

How to make the distribution stationary?

The solution of the model exhibits a unit root:

$$a_t = a_{t-1} + \dots \text{other variables in } t-1 + \text{shocks in } t$$

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Problem:

- ▶ there isn't a unique deterministic steady-state
- ▶ the ergodic distribution of the model variables is not defined

This raises practical issues (notably for estimation) for the *linear* model.

- ▶ no unconditional moments

How to get rid of the unit root?

General idea:

- ▶ introduce a force that pulls the level of foreign assets towards equilibrium

Schmitt Grohe and Uribe (2003) consider many options:

- ▶ debt-elastic interest rate:

$$1 + r = 1 + r^* + \pi(a_d)$$

- ▶ with $\pi(0) = 0$ and $\pi'(0) > 0$
- ▶ π can be understood as a risk premium on rising debt
- ▶ endogenous time-discount (aka Usawa preferences)

$$\beta(c_t) = (1 + c_t)^{-\chi}$$

- ▶ costs of adjustment for international portfolios

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- ▶ costs of adjustment for international portfolios

SGU show that the choice of the stationarization device has little effect for the dynamics (moments) of most variables

Calibration

Parameters	Values
σ	2
ψ	1.45
α	0.32
ω	0.028
r	0.04

Parameters	Values
δ	0.1
ρ	0.42
σ^2	0.0129
A^*	-0.7442
χ	0.000742

Results

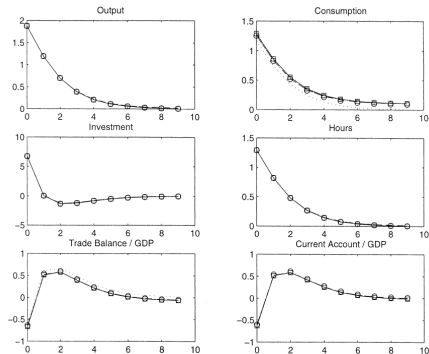


Fig. 1. Impulse response to a unit technology shock in Models 1–5. Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashed line: Debt-elastic interest rate model; Dash-dotted line: Portfolio adjustment cost model; Dotted line: complete asset markets model; Circles: Model without stationarity inducing elements.

Figure 7: Impulse Response Function

Table 3
Observed and implied second moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
<i>Volatilities:</i>						
$\text{std}(y_t)$	2.8	3.1	3.1	3.1	3.1	3.1
$\text{std}(c_t)$	2.5	2.3	2.3	2.7	2.7	1.9
$\text{std}(i_t)$	9.8	9.1	9.1	9	9	9.1
$\text{std}(h_t)$	2	2.1	2.1	2.1	2.1	2.1
$\text{std}\left(\frac{ib_t}{y_t}\right)$	1.9	1.5	1.5	1.8	1.8	1.6
$\text{std}\left(\frac{ca_t}{y_t}\right)$		1.5	1.5	1.5	1.5	
<i>Serial correlations:</i>						
$\text{corr}(y_t, y_{t-1})$	0.61	0.61	0.61	0.62	0.62	0.61
$\text{corr}(c_t, c_{t-1})$	0.7	0.7	0.7	0.78	0.78	0.61
$\text{corr}(i_t, i_{t-1})$	0.31	0.07	0.07	0.069	0.069	0.07
$\text{corr}(h_t, h_{t-1})$	0.54	0.61	0.61	0.62	0.62	0.61
$\text{corr}\left(\frac{ib_t}{y_t}, \frac{ib_{t-1}}{y_{t-1}}\right)$	0.66	0.33	0.32	0.51	0.5	0.39
$\text{corr}\left(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}\right)$		0.3	0.3	0.32	0.32	
<i>Correlations with output:</i>						
$\text{corr}(c_t, y_t)$	0.59	0.94	0.94	0.84	0.85	1
$\text{corr}(i_t, y_t)$	0.64	0.66	0.66	0.67	0.67	0.66
$\text{corr}(h_t, y_t)$	0.8	1	1	1	1	1
$\text{corr}\left(\frac{ib_t}{y_t}, y_t\right)$	-0.13	-0.012	-0.013	-0.044	-0.043	0.13
$\text{corr}\left(\frac{ca_t}{y_t}, y_t\right)$		0.026	0.025	0.05	0.051	

Note: The first column was taken from Mendoza (1991). Standard deviations are measured in percent per year.

Figure 8: Moments (from SGU)

Conclusions

- ▶ The model matches unconditional correlations fairly well
 - ▶ The stationarization device has little effect on the moments
- ▶ Unconditional correlations are not that great
 - ▶ a limitation of the moment matching method?
- ▶ Correlation of consumption with output is too high
 - ▶ and probably cross-correlation of consumption too low
 - ▶ still the Backus-Kehoe-Kydland puzzle...