

# Small Open Economy Extension (IRBC)

Macro II - Fluctuations - ENSAE, 2024-2025

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## Introduction and Basic Facts

# Why a small open economy?

What are the classical reasons to open economy to trade?

- ▶ trade integration

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  - ▶ comparative advantage
- ▶ financial integration
  - ▶ smooth shock / insurance

## From RBC to IRBC

RBC models have been very successful at matching Business Cycles

- ▶ (temporary) victory against keynesian view that short term fluctuations result from demand shocks
- ▶ so successful that facts at odd with theoretical predictions have been called “puzzles”



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Seminal Paper:

- ▶ *International Real Business Cycles*, Backus, Kehoe, Kydland (1992) (freshwater economists)

Very successful methodology:

- ▶ facts at odd with theoretical predictions have been called “puzzles”

# IRBC Facts

Properties of Business Cycles in OECD Economies

Country	Std. Dev. (%)		Ratio of Standard Deviation to that of y					Autocorr. y	Correlation with Output					
	y	nx	c	x	g	n	z		c	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	-.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	-.24	-.46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	-.23	-.26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	-.30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	-.11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	-.68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	-.02	-.22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	-.68	.84	.93
U.K.	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	-.19	.47	.90
U.S.	1.92	.52	.75	3.27	.75	.61	.68	.86	.82	.94	.12	-.37	.88	.96
Europe	1.01	.50	.83	2.09	.47	.85	.98	.75	.81	.89	.10	-.25	.32	.85

Notes: Statistics are based on Hodrick-Prescott filtered data. Variables are: y, real output; c, real consumption; x, real fixed investment; g, real government purchases; nx, ratio of net exports to output, both at current prices; n, civilian employment; z, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:1 to 1990:2.

Figure 1: Moments

From Kehoe, Kydland (1995)

# IRBC Facts

Properties of Business Cycles in OECD Economies

Country	Year (1970-1990)	Business Cycles (1970-1990)					Properties with Japan				
		1	2	3	4	5	1	2	3	4	5
Australia	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40
Austria	1.18	1.11	1.8	1.82	2.0	1.12	29	37	40	39	40
Canada	1.20	1.00	2.0	1.84	2.0	1.8	31	37	40	39	40
France	1.20	1.00	2.0	1.84	2.0	1.8	31	37	40	39	40
Germany	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40
Italy	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40
Japan	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40
Switzerland	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40
United Kingdom	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40
U.S.	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40
Europe	1.12	1.28	2.00	1.75	2.8	1.9	40	40	40	40	40

## International Comovements in OECD Economies

Country	Correlation with Same U.S. Variable					
	y	c	x	g	n	z
Australia	.51	-.19	.16	.23	-.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	-.01	-.01	.53	.75
France	.41	.39	.22	-.20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	-.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	-.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Notes: See Table 1.

## Figure 3: Comoments

Figure 2:  
Moments

# Stylized Facts

Report of National Accounts to OECD Economies

Country	Est. Year	Ratio of Household Expenditure to GDP					Estimate	Expenditure on Other				
		1970	1975	1980	1985	1990		1970	1975	1980	1985	1990
Australia	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Canada	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
France	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Germany	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Italy	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Japan	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Netherlands	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U.K.	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U.S.	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Domestically:

► output more variable than consumption

Figure 4:  
Moments

International Comovements in OECD Economies

Country	Correlation with Euro U.S. Variable				
	1	2	3	4	5
Australia	21	-29	28	23	-18
Canada	18	23	20	20	17
France	16	40	-20	-20	20
Germany	41	29	22	-20	26
Italy	41	32	21	29	-16
Japan	46	44	36	21	28
Netherlands	42	49	38	31	26
United Kingdom	33	42	40	-26	40
U.S.	40	21	23	18	20

Internationally:

Figure 5:  
Comoments

## Stylized Facts

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Figure 4:  
Moments

Country	Correlation with Euro U.S. Variable				
	7	6	5	4	3
Australia	.31	-.19	.58	.23	-.18
Austria	.34	.33	.46	.29	.47
Canada	.38	.49	-.81	-.04	.20
France	.41	.39	.32	-.20	.16
Germany	.49	.49	.33	.24	.32
Italy	.41	.32	.31	.39	-.31
Japan	.60	.44	.04	.11	.32
Netherlands	.42	.48	.39	.31	.36
United Kingdom	.51	.42	.44	-.56	.49
Europe	.66	.31	.33	.19	.20

Source: See Table 1.

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Figure 5:  
Comoments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated

Internationally:

## Stylized Facts

Country	Dependent Variables		Independent Variables		Control Variables	
	Mean	SD	Mean	SD	Mean	SD
USA	1.68	1.23	1.94	1.30	1.99	1.40
Canada	1.68	1.11	1.94	1.20	1.12	1.34
France	1.70	1.18	1.94	1.20	1.12	1.34
Germany	1.62	1.02	1.94	1.20	1.12	1.34
Italy	1.65	1.10	1.94	1.20	1.12	1.34
Japan	1.65	1.10	1.94	1.20	1.12	1.34
UK	1.65	1.10	1.94	1.20	1.12	1.34
Australia	1.65	1.10	1.94	1.20	1.12	1.34
Spain	1.65	1.10	1.94	1.20	1.12	1.34
Sweden	1.65	1.10	1.94	1.20	1.12	1.34
Norway	1.65	1.10	1.94	1.20	1.12	1.34
Finland	1.65	1.10	1.94	1.20	1.12	1.34
Denmark	1.65	1.10	1.94	1.20	1.12	1.34
Ireland	1.65	1.10	1.94	1.20	1.12	1.34
Portugal	1.65	1.10	1.94	1.20	1.12	1.34
Greece	1.65	1.10	1.94	1.20	1.12	1.34
Belgium	1.65	1.10	1.94	1.20	1.12	1.34
Austria	1.65	1.10	1.94	1.20	1.12	1.34
Switzerland	1.65	1.10	1.94	1.20	1.12	1.34
Luxembourg	1.65	1.10	1.94	1.20	1.12	1.34
Netherlands	1.65	1.10	1.94	1.20	1.12	1.34
Poland	1.65	1.10	1.94	1.20	1.12	1.34
Czech Republic	1.65	1.10	1.94	1.20	1.12	1.34
Slovak Republic	1.65	1.10	1.94	1.20	1.12	1.34
Slovenia	1.65	1.10	1.94	1.20	1.12	1.34
Hungary	1.65	1.10	1.94	1.20	1.12	1.34
Croatia	1.65	1.10	1.94	1.20	1.12	1.34
Serbia	1.65	1.10	1.94	1.20	1.12	1.34
Bulgaria	1.65	1.10	1.94	1.20	1.12	1.34
Romania	1.65	1.10	1.94	1.20	1.12	1.34
Latvia	1.65	1.10	1.94	1.20	1.12	1.34
Lithuania	1.65	1.10	1.94	1.20	1.12	1.34
Estonia	1.65	1.10	1.94	1.20	1.12	1.34
Malta	1.65	1.10	1.94	1.20	1.12	1.34
Cyprus	1.65	1.10	1.94	1.20	1.12	1.34
Montenegro	1.65	1.10	1.94	1.20	1.12	1.34
Albania	1.65	1.10	1.94	1.20	1.12	1.34
Bosnia and Herzegovina	1.65	1.10	1.94	1.20	1.12	1.34
Moldova	1.65	1.10	1.94	1.20	1.12	1.34
Ukraine	1.65	1.10	1.94	1.20	1.12	1.34
Belarus	1.65	1.10	1.94	1.20	1.12	1.34
Armenia	1.65	1.10	1.94	1.20	1.12	1.34
Georgia	1.65	1.10	1.94	1.20	1.12	1.34
Azerbaijan	1.65	1.10	1.94	1.20	1.12	1.34
Kazakhstan	1.65	1.10	1.94	1.20	1.12	1.34
Kyrgyzstan	1.65	1.10	1.94	1.20	1.12	1.34
Tajikistan	1.65	1.10	1.94	1.20	1.12	1.34
Uzbekistan	1.65	1.10	1.94	1.20	1.12	1.34
Turkmenistan	1.65	1.10	1.94	1.20	1.12	1.34
Yemen	1.65	1.10	1.94	1.20	1.12	1.34
Oman	1.65	1.10	1.94	1.20	1.12	1.34
Qatar	1.65	1.10	1.94	1.20	1.12	1.34
Bahrain	1.65	1.10	1.94	1.20	1.12	1.34</

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Figure 4:  
Moments

Country	Correlation with Base U.S. Variable				
	y	z	g	s	i
Australia	.71	-.19	.58	.25	-.18
Austria	.54	.23	.46	.29	.47
Canada	.78	.49	-.85	.54	.50
France	.41	.59	.33	-.30	.26
Germany	.49	.49	.33	.28	.52
Italy	.41	.82	.31	.39	-.35
Japan	.60	.44	.56	.11	.32
Netherlands	.42	.48	.39	.31	.36
United Kingdom	.53	.42	.46	-.56	.48
Sweden	.66	.31	.53	.18	.20

Name: See Table 1

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical

Internationally:

Figure 5:  
Comoments

## Stylized Facts

	Std. Dev. (%)		Relative Standard Deviation (in %)		Asymmetry		Persistence (Lag Length)	
Country	1	2	3	4	5	6	7	8
Australia	1.04	1.23	0.80	3.70	1.39	1.00	1	1
Canada	1.04	1.23	0.80	3.70	1.39	1.00	1	1
France	1.06	1.16	0.80	3.60	1.30	1.00	1	1
Germany	1.05	1.18	0.80	3.60	1.30	1.00	1	1
Italy	1.02	1.08	0.80	3.60	1.30	1.00	1	1
Japan	1.04	1.23	0.80	3.70	1.39	1.00	1	1
Spain	1.04	1.23	0.80	3.70	1.39	1.00	1	1
Sweden	1.04	1.23	0.80	3.70	1.39	1.00	1	1
Switzerland	1.04	1.23	0.80	3.70	1.39	1.00	1	1
United Kingdom	1.04	1.23	0.80	3.70	1.39	1.00	1	1
United States	1.04	1.23	0.80	3.70	1.39	1.00	1	1
West Germany	1.04	1.23	0.80	3.70	1.39	1.00	1	1
Yugoslavia	1.04	1.23	0.80	3.70	1.39	1.00	1	1

[illegible]

Figure 4:  
Moments

Country	Correlation with Same U.S. Variable					
	y	v	s	g	v	i
Australia	.21	-.19	.18	.20	-.18	.22
Austria	.34	.20	.48	.29	.47	.17
Canada	.58	.49	-.81	-.51	.50	.73
France	.41	.39	.52	-.20	.38	.29
Germany	.40	.49	.51	.28	.32	.43
Italy	.41	.32	.31	.39	-.31	.23
Japan	.80	.44	.56	.11	.30	.18
Netherlands	.42	.49	.39	.35	.36	.43
United Kingdom	.33	.42	.40	-.56	.40	.32
Europe	.86	.21	.23	.19	.20	.56

Source: See Table 1.

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical

Internationally:

Figure 5:  
Comoments



## Stylized Facts

	Ref. Exp.	Ratio of Observed to Expected	Intercept	Correlation with Slope
Chlorine	1	1	1	1
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13
Asbestos	1.04	1.23	0.00	0.13
Lead	1.04	1.23	0.00	0.13
Chlorine	1.04	1.23	0.00	0.13

[illegible]

Figure 4:  
Moments

Country	Correlation with Same U.S. Variable					
	y	a	b	c	d	e
Australia	.21	-.19	.18	.20	-.18	.21
Austria	.38	.33	.40	.20	.47	.17
Canada	.58	.49	-.81	-.51	.50	.73
France	.41	.39	.52	-.10	.36	.39
Germany	.40	.40	.51	.28	.32	.43
Italy	.51	.32	.31	.39	-.31	.33
Japan	.80	.84	.54	.11	.30	.38
Netherlands	.42	.48	.38	.31	.36	.41
United Kingdom	.33	.42	.44	-.26	.40	.31
Europe	.60	.21	.23	.18	.20	.26

Source: See Table 1.

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

Internationally:

Figure 5:  
Comoments

## Stylized Facts

Country	Real GDP (constant 1990 prices)		Population		Fertilizer use (kg/ha)	
	1990	1999	1990	1999	1990	1999
Algeria	1.00	1.23	1.00	1.10	1.00	1.10
Argentina	1.00	1.10	1.00	1.05	1.00	1.05
Australia	1.00	1.10	1.00	1.05	1.00	1.05
Brazil	1.00	1.10	1.00	1.05	1.00	1.05
Canada	1.00	1.10	1.00	1.05	1.00	1.05
France	1.00	1.10	1.00	1.05	1.00	1.05
Germany	1.00	1.10	1.00	1.05	1.00	1.05
India	1.00	1.10	1.00	1.05	1.00	1.05
Japan	1.00	1.10	1.00	1.05	1.00	1.05
South Korea	1.00	1.10	1.00	1.05	1.00	1.05
Spain	1.00	1.10	1.00	1.05	1.00	1.05
Sweden	1.00	1.10	1.00	1.05	1.00	1.05
Switzerland	1.00	1.10	1.00	1.05	1.00	1.05
Taiwan	1.00	1.10	1.00	1.05	1.00	1.05
United Kingdom	1.00	1.10	1.00	1.05	1.00	1.05
United States	1.00	1.10	1.00	1.05	1.00	1.05
West Germany	1.00	1.10	1.00	1.05	1.00	1.05

[illegible]

Figure 4:  
Moments

Country	Correlation with Euro U.S. Variable					
	y	a	b	c	d	e
Australia	.21	-.19	.28	.25	-.18	.22
Austria	.34	.33	.46	.20	.47	.17
Canada	.58	.49	-.81	-.54	.50	.73
France	.41	.39	.52	-.10	.36	.39
Germany	.46	.48	.50	.28	.32	.43
Italy	.41	.32	.31	.39	-.35	.33
Japan	.80	.44	.56	.11	.32	.38
Netherlands	.42	.48	.38	.31	.36	.41
United Kingdom	.55	.42	.44	-.56	.40	.31
Europe	.60	.21	.23	.18	.20	.26

Source: See Table 1.

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

Internationally:

- ▶ smaller comovements in consumption

Figure 5:  
Comoments

# Stylized Facts

Properties of National Accounts & GDP Estimates

Country	Est. Year	Index of Domestic Production					Index	Correlation with Output				
		1970	1975	1980	1985	1990		1970	1975	1980	1985	1990
Australia	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Canada	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
France	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Germany	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Italy	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Japan	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Netherlands	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U.K.	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U.S.	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Sweden	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Notes: Statistics are based on the latest available data. Statistics are for 1970-1990. A full description of the data sources and the methods used to calculate the statistics is available in the appendix. The correlation coefficients are calculated for the period 1970-1990. The correlation coefficients are calculated for the period 1970-1990. The correlation coefficients are calculated for the period 1970-1990.

Figure 4:  
Moments

Domestically:

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- ▶ output autocorrelated
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Internationally:

- ▶ smaller comovements in consumption
- ▶ Backus-Kehoe-Kydland puzzle

International Comovements in OECD Economies

Country	Correlation with Euro G.D.P. (1970-1990)				
	1	2	3	4	5
Australia	21	-29	28	22	-18
Canada	18	21	20	20	17
France	10	-40	-20	20	17
Germany	41	29	22	-22	29
Italy	41	32	21	29	-15
Japan	40	44	20	21	22
Netherlands	42	40	20	21	26
United Kingdom	33	42	40	-26	40
Sweden	40	21	21	18	20

Notes: See Table 1.

Figure 5:  
Comovements

# Stylized Facts

Properties of National Accounts & GDP Estimates

Country	Est. Year	Index of Domestic Production					Index	Correlation with Output				
		1970	1975	1980	1985	1990		1970	1975	1980	1985	1990
Australia	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Canada	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
France	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Germany	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Italy	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Japan	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Netherlands	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U.K.	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U.S.	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Sweden	1970	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Notes: Statistics are based on the latest available data. Statistics are for 1970-1990. A full description of the data sources and the methods used to calculate the statistics is available in the appendix. The correlation coefficients are calculated for the period 1970-1990. The correlation coefficients are calculated for the period 1970-1990. The correlation coefficients are calculated for the period 1970-1990.

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International Comovements in OECD Economies

Country	Correlation with Euro G.D.P. (1970-1990)				
	1	2	3	4	5
Australia	21	-29	28	22	-18
Canada	18	21	20	20	17
France	16	48	-25	20	17
Germany	41	29	22	-22	26
Italy	41	32	21	29	-15
Japan	46	44	26	21	22
Netherlands	42	48	28	21	26
United Kingdom	33	42	48	-26	26
Sweden	40	21	21	18	22

Notes: See Table 1.

Figure 5:  
Comovements

# Stylized Facts

Properties of Stylized Facts in 1980 Economy

Country	GDP	Index of Domestic Production				Imports	Productivity with Output			
		1	2	3	4	5	6	7	8	9
Canada	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
France	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Germany	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Italy	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
Japan	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4
UK	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
USA	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
Sweden	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7
Denmark	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
Netherlands	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9
Belgium	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
Australia	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1
New Zealand	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2
Spain	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3
Portugal	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4
Greece	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
Ireland	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
Iceland	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Luxembourg	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8
Austria	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9
Finland	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
Switzerland	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1
South Korea	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2
Singapore	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3
Hong Kong	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
Taiwan	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
Malaysia	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
Thailand	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7
Philippines	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8
Indonesia	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
Brazil	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
Argentina	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1
Chile	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2
Colombia	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3
Venezuela	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
Ecuador	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
Peru	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6
Bolivia	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7
Paraguay	4.8	4.8	4.8	4.8	4.8	4.8	4.8	4.8	4.8	4.8
Uruguay	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9
Costa Rica	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
Panama	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1
Jamaica	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2
Haiti	5.3	5.3	5.3	5.3	5.3	5.3	5.3	5.3	5.3	5.3
Dominican Republic	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4
Cuba	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5
Yugoslavia	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6
Czech Republic	5.7	5.7	5.7	5.7	5.7	5.7	5.7	5.7	5.7	5.7
Slovak Republic	5.8	5.8	5.8	5.8	5.8	5.8	5.8	5.8	5.8	5.8
Hungary	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9
Poland	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0
Romania	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1
Bulgaria	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2
Greece	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3
Turkey	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4
Israel	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5
South Africa	6.6	6.6	6.6	6.6	6.6	6.6	6.6	6.6	6.6	6.6
Nigeria	6.7	6.7	6.7	6.7	6.7	6.7	6.7	6.7	6.7	6.7
Kenya	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8
Uganda	6.9	6.9	6.9	6.9	6.9	6.9	6.9	6.9	6.9	6.9
Rwanda	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0
Burundi	7.1	7.1	7.1	7.1	7.1	7.1	7.1	7.1	7.1	7.1
Tanzania	7.2	7.2	7.2	7.2	7.2	7.2	7.2	7.2	7.2	7.2
Zambia	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3
Malawi	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4
Mozambique	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5
Botswana	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6
Lesotho	7.7	7.7	7.7	7.7	7.7	7.7	7.7	7.7	7.7	7.7
Swaziland	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8
Sierra Leone	7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9
Liberia	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
Ivory Coast	8.1	8.1	8.1	8.1	8.1	8.1	8.1	8.1	8.1	8.1
Ghana	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.2
Senegal	8.3	8.3	8.3	8.3	8.3	8.3	8.3	8.3	8.3	8.3
Mali	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4
Niger	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5
Chad	8.6	8.6	8.6	8.6	8.6	8.6	8.6	8.6	8.6	8.6
Cameroon	8.7	8.7	8.7	8.7	8.7	8.7	8.7	8.7	8.7	8.7
Cote d'Ivoire	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8
Upper Volta	8.9	8.9	8.9	8.9	8.9	8.9	8.9	8.9	8.9	8.9
Benin	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0
Nigeria	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1
Kenya	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2
Uganda	9.3	9.3	9.3	9.3	9.3	9.3	9.3	9.3	9.3	9.3
Rwanda	9.4	9.4	9.4	9.4	9.4	9.4	9.4	9.4	9.4	9.4
Burundi	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5
Tanzania	9.6	9.6	9.6	9.6	9.6	9.6	9.6	9.6	9.6	9.6
Zambia	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7
Malawi	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8
Mozambique	9.9	9.9	9.9	9.9	9.9	9.9	9.9	9.9	9.9	9.9
Botswana	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0

Figure 4:  
Moments

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Comoments

International Comovements in OECD Economies

Country	Correlation with Euro G.D.P. (1980-1990)				
	1	2	3	4	5
Australia	0.1	0.1	0.1	0.1	0.1
Austria	0.2	0.2	0.2	0.2	0.2
Canada	0.3	0.3	0.3	0.3	0.3
France	0.4	0.4	0.4	0.4	0.4
Germany	0.5	0.5	0.5	0.5	0.5
Italy	0.6	0.6	0.6	0.6	0.6
Japan	0.7	0.7	0.7	0.7	0.7
Netherlands	0.8	0.8	0.8	0.8	0.8
Sweden	0.9	0.9	0.9	0.9	0.9
Switzerland	1.0	1.0	1.0	1.0	1.0
United Kingdom	1.1	1.1	1.1	1.1	1.1
USA	1.2	1.2	1.2	1.2	1.2

Source: See Table 1.

Can we replicate these moments with a BC model?

## Modeling a Small Open Economy

# Endowment model

Representative agents maximizes:

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + a_t \leq y_t + (1 + r)a_{t-1}$$

**Endowment economy:**

- ▶ income  $(y_t)_t$  is exogenously given
- ▶ for simplicity we assume it is deterministic

**Small open economy:**

- ▶ *open*: can save  $a_t$  which yields  $a_{t+1}(1 + r)$  in the next period
- ▶ *small*: country takes world interest rate  $r$  as given (no effect on world prices)

We solve this problem with the terminal conditions:

- ▶  $a_{-1}$  given<sup>1</sup>

- ▶  $\lim_{T \rightarrow \infty} a_T \geq 0$

## Endowment model (3)

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t (y_t + (1+r)a_{t-1} - c_t - a_t)$$

First order conditions:

$$u'(c_t) = \lambda_t \quad (1)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (2)$$

Under the *technical assumption*  $\beta(1+r) = 1$  we get  $c_t = c_{t+1}$  then

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$



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$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

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$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t (y_t + (1+r)a_{t-1} - c_t - a_t)$$

First order conditions:

$$u'(c_t) = \lambda_t \quad (1)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (2)$$

Under the *technical assumption*  $\beta(1+r) = 1$  we get  $c_t = c_{t+1}$  then

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

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We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t (y_t + (1+r)a_{t-1} - c_t - a_t)$$

First order conditions:

$$u'(c_t) = \lambda_t \quad (1)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (2)$$

Under the *technical assumption*  $\beta(1+r) = 1$  we get  $c_t = c_{t+1}$  then

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

# Current Account

## Reminders on Current Account

The **trade balance** is exports-imports (here  $y_t - c_t$ )

The **current account** is trade balance + net factor payments  
(here  $y_t - c_t + ra_{t-1}$ )

Positive **current account**: additional lending to the rest of the world.

# Current Account

## 💡 Reminders on Current Account

The **trade balance** is exports-imports (here  $y_t - c_t$ )

The **current account** is trade balance + net factor payments (here  $y_t - c_t + ra_{t-1}$ )

Positive **current account**: additional lending to the rest of the world.

Using the formula from before

$$CA_0 = a_{-1}r + (1 - \frac{r}{1+r})y_0 - \frac{r}{1+r} \left\{ \sum_{t \geq 1}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

How does the current account reacts to income shocks?

- ▶ current account responds positively to *temporary* shock in income
- ▶ and to news about future income shocks:

## Unit root

Still with the same formula:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

What is the effect of an increase in  $a_{-1}$ ?

- consumption rises permanently
  - $a_t$  is constant, equal to  $a_{-1}$
  - agent consumes small amount  $r$  corresponding to interest
- this will correspond to a unit root in the solution

## Adding capital

We add capital and production to our endowment economy:

$$y_t = z_t k_{t-1}^\alpha$$

$$k_t = (1 - \delta)k_{t-1} + i_t$$

The aggregate resource constraint becomes:

$$a_t + c_t + i_t = (1 + r)a_{t-1} + y_t$$

Now maximize  $\sum_t \beta^t U(c_t)$

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We get first order conditions

$$\lambda_t = \beta \lambda_{t+1} (1 + r)$$

$$\lambda_t = \beta \lambda_{t+1} [(1 - \delta) + z_{t+1} f'(k_t)]$$

where  $\lambda_t$  is lagrange multiplier associated to budget constraint.



## Adding capital: optimality conditions

Since  $\lambda_t > 0$  (constraint is always binding), we get:

$$(1 - \delta) + z_{t+1}f'(k_t) = 1 + r$$

$$k_t = \left( \frac{r + \delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha-1}}$$

and investment

$$i_t = \left( \frac{r + \delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha-1}} - (1 - \delta) \left( \frac{r + \delta}{\alpha z_t} \right)^{\frac{1}{\alpha-1}}$$

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Here investment is fully determined by productivity shocks

► too simple: no international dependence

## Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$a_t + c_t + i_t + \frac{\omega (k_t - k_{t-1})^2}{2 k_t} = (1 + r)a_{t-1} + z f(k_{t-1})$$

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Cf tutorial.

## A benchmark Small Open Economy Model

# A benchmark Small Open Economy Model



Stephanie Schmitt-Grohe and Martin Uribe.

Figure 6: Stephanie Schmitt Grohe and Martin Uribe

*Closing Small Economy Models,*  
Schmitt Grohe and Uribe  
(2003), JIE

- ▶ small open economy model with production, consumption-leisure tradeoff and capital adjustment costs
  - ▶ = RBC+open+adj costs
- ▶ perform some moments matching
- ▶ compare different ways of stationarizing

# The model

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

$$c_t + k_t + a_t = y_t + g_t - \frac{\omega}{2} (k_t - k_{t-1})^2 + (1 - \delta) k_{t-1} + (1 + r^* + \pi(a_{t-1})) a_{t-1}$$

$$y_t = f(k_{t-1}, n_t, z_t)$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

and

$$u(c, n) = \frac{1}{1 - \sigma} (c^\psi (1 - n)^{1 - \psi})^{1 - \sigma}$$



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The term  $\pi$  is there to make the model stationary.

## How to make the distribution stationary?

The solution of the model exhibits a unit root:

$$a_t = a_{t-1} + \dots \text{other variables in } t-1 + \text{shocks in } t$$

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Problem:

- ▶ there isn't a unique deterministic steady-state
- ▶ the ergodic distribution of the model variables is not defined

This raises practical issues (notably for estimation) for the *linear* model.

- ▶ no unconditional moments

# How to get rid of the unit root?

*General idea:*

- ▶ introduce a force that pulls the level of foreign assets towards equilibrium

Schmitt Grohe and Uribe (2003) consider many options:

- ▶ debt-elastic interest rate:

$$1 + r = 1 + r^* + \pi(a_d)$$

- ▶ with  $\pi(0) = 0$  and  $\pi'(0) > 0$
- ▶  $\pi$  can be understood as a risk premium on rising debt
- ▶ endogenous time-discount (aka Usawa preferences)

$$\beta(c_t) = (1 + c_t)^{-\chi}$$

- ▶ costs of adjustment for international portfolios

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- ▶ costs of adjustment for international portfolios

SGU show that the choice of the stationarization device has little effect for the dynamics (moments) of most variables

# Calibration

Parameters	Values
$\sigma$	2
$\psi$	1.45
$\alpha$	0.32
$\omega$	0.028
$r$	0.04

Parameters	Values
$\delta$	0.1
$\rho$	0.42
$\sigma^2$	0.0129
$A^*$	-0.7442
$\chi$	0.000742

# Results

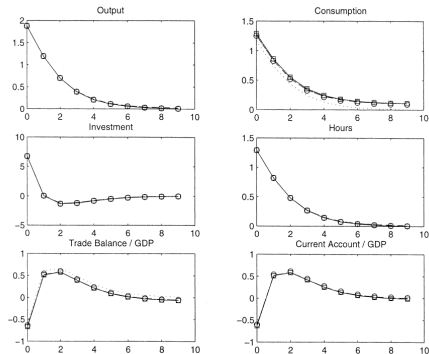


Fig. 1. Impulse response to a unit technology shock in Models 1–5. Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashed line: Debt-elastic interest rate model; Dash-dotted line: Portfolio adjustment cost model; Dotted line: complete asset markets model; Circles: Model without stationarity inducing elements.

Figure 7: Impulse Response Function

Table 3  
Observed and implied second moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
<i>Volatilities:</i>						
$\text{std}(y_t)$	2.8	3.1	3.1	3.1	3.1	3.1
$\text{std}(c_t)$	2.5	2.3	2.3	2.7	2.7	1.9
$\text{std}(i_t)$	9.8	9.1	9.1	9	9	9.1
$\text{std}(h_t)$	2	2.1	2.1	2.1	2.1	2.1
$\text{std}\left(\frac{ib_t}{y_t}\right)$	1.9	1.5	1.5	1.8	1.8	1.6
$\text{std}\left(\frac{ca_t}{y_t}\right)$		1.5	1.5	1.5	1.5	
<i>Serial correlations:</i>						
$\text{corr}(y_t, y_{t-1})$	0.61	0.61	0.61	0.62	0.62	0.61
$\text{corr}(c_t, c_{t-1})$	0.7	0.7	0.7	0.78	0.78	0.61
$\text{corr}(i_t, i_{t-1})$	0.31	0.07	0.07	0.069	0.069	0.07
$\text{corr}(h_t, h_{t-1})$	0.54	0.61	0.61	0.62	0.62	0.61
$\text{corr}\left(\frac{ib_t}{y_t}, \frac{ib_{t-1}}{y_{t-1}}\right)$	0.66	0.33	0.32	0.51	0.5	0.39
$\text{corr}\left(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}\right)$		0.3	0.3	0.32	0.32	
<i>Correlations with output:</i>						
$\text{corr}(c_t, y_t)$	0.59	0.94	0.94	0.84	0.85	1
$\text{corr}(i_t, y_t)$	0.64	0.66	0.66	0.67	0.67	0.66
$\text{corr}(h_t, y_t)$	0.8	1	1	1	1	1
$\text{corr}\left(\frac{ib_t}{y_t}, y_t\right)$	-0.13	-0.012	-0.013	-0.044	-0.043	0.13
$\text{corr}\left(\frac{ca_t}{y_t}, y_t\right)$		0.026	0.025	0.05	0.051	

Note: The first column was taken from Mendoza (1991). Standard deviations are measured in percent per year.

Figure 8: Moments (from SGU)



# Conclusions

- ▶ The model matches unconditional correlations fairly well
  - ▶ The stationarization device has little effect on the moments
- ▶ Unconditional correlations are not that great
  - ▶ a limitation of the moment matching method?
- ▶ Correlation of consumption with output is too high
  - ▶ and probably cross-correlation of consumption too low
  - ▶ still the Backus-Kehoe-Kydland puzzle...