Numeric

Using Numpy

Selection from w3resources and rougier/numpy100

```
import numpy as np
```

Exercise 1

- Write a NumPy program to generate five random numbers from the normal distribution.

```
np.random.rand(5)
```

```
array([0.21510737, 0.21120555, 0.34300793, 0.90026871, 0.71797054])
```

Exercise 2 Write a NumPy program to generate six random integers between 10 and 30.

```
np.random.randint(10, 31, size=6)
```

```
array([16, 20, 20, 14, 20, 19])
```

Exercise 3 Create a 3x3 matrix with values ranging from 0 to 8

```
np.random.randint(0, 9, size=(3,3))
```

```
array([[7, 0, 8],
       [1, 7, 0],
       [5, 4, 6]])
```

Exercise 4 Create 2d array M such of size 3*3 such that $M_{ij} = i \times j$

```
u = np.arange(3)+1 # creates vector with 1,2,3
u[:,None]*u[None,:]
```

```
array([[1, 2, 3],
       [2, 4, 6],
       [3, 6, 9]])
```

Exercise 5 Create 3 vectors of length 5 and create a matrix where each column is one of the vector.

```
a = np.arange(0,5)
b = np.arange(5,10)
c = np.arange(10,15)
np.column_stack([a,b,c])
```

```
array([[ 0,  5, 10],
       [ 1,  6, 11],
       [ 2,  7, 12],
       [ 3,  8, 13],
       [ 4,  9, 14]])
```

Exercise 6 Create 3 vectors of length 5 and create a matrix where each row is one of the vector.

```
a = np.arange(0,5)
b = np.arange(5,10)
c = np.arange(10,15)
np.row_stack([a,b,c])
```

```
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14]])
```

Exercise 7 Find indices of non-zero elements from `np.array([1,2,0,0,4,0])`. Replace them with -1.0

```
v = np.array([1,2,0,0,4,0])
indices = ~np.isclose(v, 0.0)
#better than `v !=0.0` in case values are close but not equal to 0
v[indices]=-1.0
v
```

```
array([-1, -1,  0,  0, -1,  0])
```

Exercise 8 Write a NumPy program to normalize a 3x3 random matrix. (Define norm $|x| = \sqrt{\sum x_i^2}$ and compute $M/|M|$)

```
m = np.random.rand(3,3)
norm = lambda u: np.sqrt(np.sum(m**2))
def normalize(M):
    return M/norm(M)
normalize(m)
```

```
array([[0.25083547, 0.24018983, 0.53775716],
       [0.60598535, 0.0048301 , 0.08123388],
       [0.37595728, 0.26999702, 0.04609624]])
```

Exercise 9 Create 2d array M such of size 3*3 such that $M_{ij} = i \times j$

```
u = np.arange(3)+1 # creates vector with 1,2,3
u[:,None]*u[None,:]
```

```
array([[1, 2, 3],
       [2, 4, 6],
       [3, 6, 9]])
```

Exercise 10 Take a random matrix A of size $N \times 2$ (N=10) where each line represents a different 2d point. Compute the euclidean distance matrix such that E_{ij} is the distance between point i and point j.

```
N = 10
A = np.random.rand( N,2)
B = A[:,None,:]   # B_ijk = A_ik
C = A[None,:,:]   # C_jk = A_jk
np.sqrt(
    np.sum(
        (( B - C)**2),
    axis=2)
)
```

```
array([[0.        , 0.16725802, 0.2789606 , 0.40612636, 0.47504742,
        0.10726855, 0.58071971, 0.17470673, 0.50381449, 0.32924439],
       [0.16725802, 0.        , 0.43701528, 0.24132115, 0.6340258 ,
        0.27065717, 0.68681185, 0.3419181 , 0.41446088, 0.1975022 ],
       [0.2789606 , 0.43701528, 0.        , 0.66004565, 0.19706009,
        0.2148399 , 0.37721702, 0.13451282, 0.64207054, 0.54720166],
       [0.40612636, 0.24132115, 0.66004565, 0.        , 0.85411087,
        0.5114768 , 0.8406523 , 0.57917532, 0.33644344, 0.15013449],
       [0.47504742, 0.6340258 , 0.19706009, 0.85411087, 0.        ,
        0.39997148, 0.33923342, 0.31521581, 0.80074929, 0.73471866],
       [0.10726855, 0.27065717, 0.2148399 , 0.5114768 , 0.39997148,
        0.        , 0.56537958, 0.08478762, 0.59950819, 0.43570146],
       [0.58071971, 0.68681185, 0.37721702, 0.8406523 , 0.33923342,
        0.56537958, 0.        , 0.50096862, 0.65558965, 0.69391475],
       [0.17470673, 0.3419181 , 0.13451282, 0.57917532, 0.31521581,
        0.08478762, 0.50096862, 0.        , 0.62833896, 0.48867058],
       [0.50381449, 0.41446088, 0.64207054, 0.33644344, 0.80074929,
        0.59950819, 0.65558965, 0.62833896, 0.        , 0.23088681],
       [0.32924439, 0.1975022 , 0.54720166, 0.15013449, 0.73471866,
        0.43570146, 0.69391475, 0.48867058, 0.23088681, 0.        ]])
```

Exercise 11 Create A of size 10×3. Create matrix B with the same column as A reordered by sum of absolute values.

```
A = np.random.rand(10,3)
v = np.sum(np.abs(A), axis=1) # values for each row
new_order = np.argsort(v)
A[new_order,:]
```

```
array([[0.37043156, 0.18647922, 0.28380079],
       [0.38707922, 0.38995773, 0.26295399],
       [0.65377111, 0.07227131, 0.38283178],
       [0.22454386, 0.82039156, 0.16386348],
       [0.92318985, 0.07880316, 0.24136368],
       [0.68624188, 0.11444125, 0.49974818],
       [0.49323743, 0.62723799, 0.43132796],
       [0.2432563 , 0.77297125, 0.68497518],
       [0.61888255, 0.57791432, 0.6178024 ],
       [0.4792605 , 0.76334795, 0.77361025]])
```

Simulating an AR1

Take an AR1 process $x_t = Ax_{t-1} + \epsilon_t$ with $\epsilon_t \sim \Sigma$ where Σ is a positive definite matrix.

Exercise 12 Define 2x2 matrices A and Σ, the latter being symmetric positive definite

```
A = np.array([
    [0.9, 0.4],
    [-0.1, 0.8]
])
Σ = np.array([
    [0.8, 0.1],
    [0.1, 0.6]
])
```

Exercise 13 Compute asymptotic variance using matrix algebra (there is a recursive formula)

Define $S_0 = I$ and define $S_{n+1} = AS_nA' + \Sigma$

```
def asymptotic_variance(A,Σ, K=1000):
    n = Σ.shape[0]
    S = np.eye(n)
    for k in range(K):
        S = A@S@A.T + Σ
    return S
```

```
asymptotic_variance(A,Σ)
```

```
array([[6.53179191, 0.22398844],
       [0.22398844, 1.74855491]])
```

Exercise 14 Simulate N draws for T periods and store the result in `sim`.

```
N = 50 # let's simulate N draws at the same time
def simulate( A, Σ, T, N):
    x0= np.array([0,0]) # initial point
    x = x0[None,:].repeat(N, axis=0)
    mu = Σ[0,:]*0
    sim = [x]
    for t in range(T):
        x = sim[-1]
```

```
        epsilon = np.random.multivariate_normal(mu, Σ, size=N)
        x = x@A.T + epsilon
        sim.append(x)
    # return sim
    # print(sim[0])
    return np.concatenate(
        [u[:,None,:] for u in sim],
        axis=1
    )
```

```
sims = simulate(A, Σ, 100, 10)
sims.shape
# first dimension: draw
# second dimension: time
# third dimension: variable
```

```
(10, 101, 2)
```

Exercise 15 Compute ergodic variance (bonus compute std of the variance estimate)

```
sims = simulate(A, Σ, 100, 1000)
```

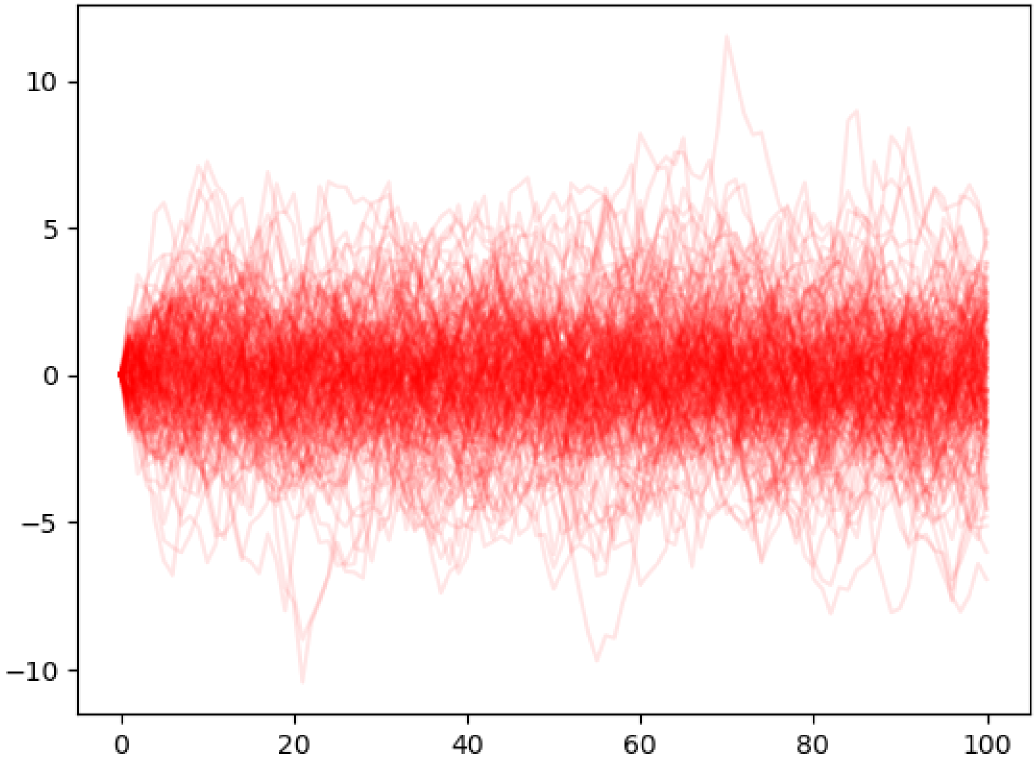
```
np.cov(
    sims[:, -1, :].T # keep only last period
)
#orders of magnitude are similar to theoretical variance computed above
```

```
array([[7.20014718, 0.30583115],
       [0.30583115, 1.83896185]])
```

Exercise 16 Plot a few simulations on the same graph

```
sims = simulate(A, Σ, 100, 100)
```

```
from matplotlib import pyplot as plt
for i in range(100):
    plt.plot(sims[i,:,:], color='red' ,alpha=0.1)
```



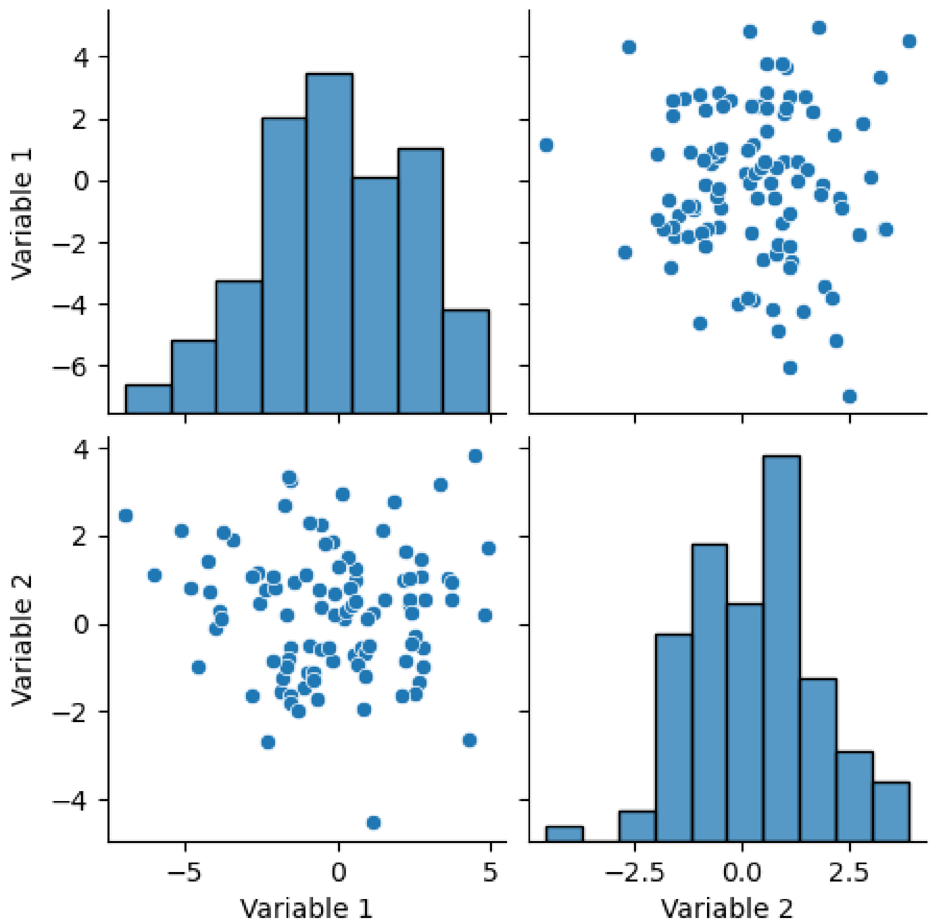
Exercise 17 Plot asymptotic distribution (seaborn)

```
import seaborn as sns
sims = simulate(A, Σ, 100, 1000)
```

```
import pandas as pd
```

```
# seaborn operates naturally on pandas dataframes
df = pd.DataFrame(sims[:,-1,:], columns=["Variable 1", "Variable 2"])
```

```
sns.pairplot(df)
```


Variable 1
Variable 2
Variable 1
Variable 2
4
2
0
−2
−4
−6
4
2
0
−2
−4
−5
0
5
−2.5
0.0
2.5