

Exercise 2: Intertemporal Consumption

We consider one representative agent. He receives income Y_1 in period 1 and Y_2 in period 2. In each period agent can consume C_1 and C_2 . There is no inflation but he can save from period 1 to period 2 at (real) interest rate r . In choosing consumption levels the agent seeks to maximize $U(C_1, C_2) = \log(C_1) + \beta \log(C_2)$ where β is the time preference discount such that $0 < \beta < 1$.

1. Ricardian Agents

1. Explain why intertemporal budget constraint can be written as $(1 + r)C_1 + C_2 \leq (1 + r)Y_1 + Y_2$
2. In the plane (C_1, C_2) represent indifference curves (aka isoutility curves). Graphically, what is marginal rate of substitution between goods 1 and 2? Compute it, using the functional form for U .
3. In the same plane, represent the budget line. How would you characterize, graphically, the optimal consumption choice? Compute the derivative of the budget line and show that optimal consumption choice is characterized by: $\frac{1/C_1}{\beta/C_2} = \frac{1+r}{1}$. That equation is called Euler equation.
4. Using Euler equation, comment on the effect of parameters r and β
5. Assuming the budget constraint is binding (both sides are equal) and combining it with the Euler equation, compute optimal consumption in period 1, and in period 2.
6. What is the marginal propensity to consume out of temporary income shock (increase in Y_1 only).
7. How does consumption react to interest rate?

2. Non Ricardian Agents (consumption fueled with borrowing)

1. We now assume the agent faces a borrowing constraint in period 1 in the form $C_1 - Y_1 \leq \kappa Y_1$. How would you interpret κ ? Compare with the “keynesian” agents defined in the course.
2. Take the result for the optimal consumption from part 1. At which condition does it satisfy the borrowing constraint?
3. What is the marginal propensity to consume when the borrowing constraint is binding? How does it depend on κ ?
4. Comment.

3. Non Ricardian Agents (consumption fueled with house prices)

Let's denote by $H = (1 + \frac{1}{r})H_0$ the value of a house owned by an agent where H_0 is a fixed constant. We consider the borrowing constraint in period 1 in the form $C_1 - Y_1 \leq \nu H$ where ν is a collateral coverage ratio. We assume that constraint is binding.

1. What is the marginal propensity to consume?
2. How does consumption react to a change in interest rates Δr ?
3. Comment.